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Q3(b). The ~~de~~ derivatives of digital functions are defined in terms of differences. There are many ways to define these differences. We require that any definition we use for a first derivate·

(1) must be zero in flat segments (areas of constant gray-level values)

(2) must be non-zero at the onset of a gray level step or ramp.

(3) must be non-zero along ramps.

The basic definition of the first-order derivative of a one-dimensional function $f(x)$ is the difference

$$\cancel{\frac{\partial f}{\partial x} = f(x+1) - f(x)}$$

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

$$\cancel{\frac{\partial f}{\partial y} = f(y+1) - f(y)}$$

$$\cancel{\nabla f(x,y) = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}}$$

$$\cancel{= f(x+1,y) - f(x,y) + f(x,y+1) - f(x,y)}$$

$$\cancel{= f(x+1,y) + f(x,y+1) - 2f(x,y)}$$

$$\cancel{\therefore \text{filter} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 1 & 0 \end{bmatrix}}$$

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First derivatives in image processing are implemented using the magnitude of the gradient. For a function $f(x,y)$, the gradient of f at coordinates (x,y) is defined as the two dimensional column vector.

$$\nabla f = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

The magnitude of this vector

$$\nabla f = \text{mag}(\nabla f)$$

$$= \left[G_x^2 + G_y^2 \right]^{1/2}$$

$$= \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 \right]^{1/2} \quad — (1)$$

Since the above equation is computationally expensive and not trivial we approximate the magnitude by using absolute values.

$$\nabla f \approx |G_x| + |G_y| \quad — (2)$$

Now let a window by represented by

$$\begin{bmatrix} z_1 & z_2 & z_3 \\ z_4 & z_5 & z_6 \\ z_7 & z_8 & z_9 \end{bmatrix}$$

z_5 denotes $f(x,y)$.

z_1 denotes $f(x-1, y-1)$

Simplest approximation to a first order derivate is $G_x = (z_5 - z_8)$ $G_y = (z_6 - z_5)$

if ~~we~~ use the exact calculation of ~~eqn (1)~~

$$\cancel{\nabla f = \left[(z_9 - z_5)^2 + (z_8 - z_6)^2 \right]^{1/2}}$$

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~~If we use eqn (2)~~

$$\cancel{\nabla f \approx |z_9 - z_5| + |z_8 - z_6|}$$

Also if we use Robert's cross differences

$$G_x = (z_9 - z_5) \quad G_y = (z_8 - z_6)$$

then using eqn (1)

$$\nabla f = \left[(z_9 - z_5)^2 + (z_8 - z_6)^2 \right]^{1/2}$$

using eqn (2)

$$\nabla f \approx |z_9 - z_5| + |z_8 - z_6|$$

Another approximation using 3x3 filter mask is

$$\nabla f \approx \left| (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3) \right| + \left| (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7) \right|$$

This may be represented by the masks.

$$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

(a) (b)

The above masks (a) approximates the derivative in x-direction. (b) approximates in y-direction. The above masks are called Sobel operators. The idea behind using a weight of 2 is to achieve smoothing by giving more importance to the center point. The mask coefficients sum to 0 thus gives 0 in area of constant gray level as expected.

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