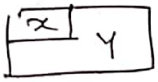
Roll : 001610501020

Q4.

Factory.	W1	W2	W3	W4	Factory cap.
F1	20	56.	34	19	90
F2	1	19	57	20	1500.
F3	33	21	56.	1	1368.
Warehouse Requirement	20	1120	33	2850	

Find initial basic feasible solution using Least Cost Method. For a cell the convention used is

y = cost
x = allocation.



The problem is an unbalanced transportation problem where demand > supply.

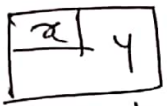
Here ~~demand~~ total demand = 4023.

total supply = 2958

Excess demand = 1065.

∴ we add a dummy row F_d with transportation cost 0.

cell is



x is allocation
y is cost

	W1	W2	W3	W4	supply
F1	20	56	34	19	90
F2	1	19.	57	20	1500.
F3	33	21	56	1	1368
~~F~~ F_d	0	1065 0	0	0	1065
Demand	20	55 1120	33	2850	4023.

Smallest transportation cost is 0. We can select any one cell say $F_d W_2$. We allocate. min (1065, 1120) and F_d row is removed.

Anwiran Chakraborty

Step-2

Roll: 001610501020

	W1	W2	W3	W4	Supply
F1	20	56	34	19	90
F2	1	19	57	20	1500
F3	33	21	56	1	1368
Fd	0	1065 0	0	0	0
Demand	20	55	33	2850	

1420

min cost is 1. select F3W4. Allocate min (1500, 20) = 20. min (1368, 2850) = 1368

Step-3

	W1	W2	W3	W4	Supply
F1	20	56	34	19	90
F2	20 1	19	57	20	1500
F3	33	21	56	1368 1	0
Fd	0	1065 0	0	0	0
Demand	20	55	33	1482	

Allocate min. cost = 1. Select F2W1.
Allocate min (20, 1500) = 20.

Step-4

	W1	W2	W3	W4	Supply
F1	20	56	34	90 19	90
F2	20 1	19	57	20	1480
F3	33	21	56	1368 1	0
Fd	0	1065 0	0	0	0
Demand	0	55	33	1482.	

Amiran Chakraborty

Roll: 001610501020

step-3

min cost = 19. Select F1W4.

Allocate min (90, 1482) = 90.

step-5

	W1	W2	W3	W4	Supply
F1	20	56	34	90 19	0
F2	20	55 19	57	20	1480
F3	33	21	56	1368 1	0.
Fd	0	1065 0	0	0	0
Demand	0	55	33	1392	

min. cost = 19. Select F2W2. Allocate min (55, 1480) = 55.

step-6

	W1	W2	W3	W4	Supply
F1	20	56	34	90 19	0
F2	20 1	55 19	57	1392 20	1425
F3	33	21	56	1368 1	0
Fd	0	1065 0	0	0	0
Demand	0.	0	33	1392	

min. cost = 20. Select F2W4.

Allocate min. (1425, 1392) = 1392.

Anwran Chakraborty

Roll : 001610501020

Step-7

	W1	W2	W3	W4	Supply
F1	20	56	34	90 19	0
F2	20 1	55 19	57	1120 20	33
F3	33	21	56	1368 1	0
Fd	0	1065 0	0	0	0
Demand	0	0	33	0	

min. cost = 57. select F2W2. Allocate 33.

Step-8

	W1	W2	W3	W4	Supply
F1	20	56	34	90 19	0
F2	20 1	55 19	33 57	1120 20	0
F3	33	21	56	1368 1	0
F4	0	1065 0	0	0	0
Demand	0	0	0	0	0

∴ (8) Initial feasible solution

	W1	W2	W3	W4	Supply
F1	20	56	34	90 19	90
F2	20 1	55 19	33 57	1120 20	1500
F3	33	21	56	1368 1	1368
Fd	0	1065 0	0	0	1065
Demand	20	1120	33	2858	

Anwesa Chakraborty

Roll: 001610501020

Initial ~~feasible~~ cost = $90\times19 + 20\times1 + 55\times19 + 33\times57 + 1392\times20 + 1368\times1 + 1065\times0$

$= 33864.$

number of allocated cells = 7.

Number of rows = 4 = n

Number of columns = 4 = m.

$\therefore$ No. of allocated cells = $n+m-1$

$\therefore$ Solution is non-degenerate.

(a) Solve using MODI method.

Assign u_i for rows and v_j for columns.

$C_{ij} = u_i + v_j$

	V_1	V_2	V_3	V_4		
	W1	W2	W3	W4	supply	
F1	20	56	34.	90 19	90	u_4
F2	20 1	55 19	33 57	1392 20	1500	u_2
F3	33	21	56	1368 1	1368	u_3
Fd.	0	1065 0	0	0	1065	u_4
	~~v_1~~	~~v_2~~	~~v_3~~	~~v_4~~		
Demand	20	1120	33	2850		

$C_{14} = u_4 + v_4 = 19$ $\quad C_{34} = u_3 + v_4 = 1$

$C_{21} = u_2 + v_1 = 1$ $\quad C_{42} = u_4 + v_2 = 0.$

$C_{22} = u_2 + v_2 = 19$

$C_{23} = u_2 + v_3 = 57$

$C_{24} = u_2 + v_4 = 20.$

Anuran Chakraborty.

Roll : 30161050101028

putting $u_2 = 0$. We get

$v_1 = 1$
$v_2 = 19$
$v_3 = 57$
$v_4 = 20$

$u_1 = c_{14} - v_4 = 19 - 20 = -1$
$u_2 = 0$
$u_3 = c_{34} - v_4 = 1 - 20 = -19$
$u_4 = c_{42} - v_2 = 0 - 19 = -19$

$v_1 = 1$	$u_1 = -1$
$v_2 = 19$	$u_2 = 0$
$v_3 = 57$	$u_3 = -19$
$v_4 = 20$	$u_4 = -19$

Find d_{ij}

$d_{11} = c_{11} - (u_1 + v_1) = 20$
$d_{12} = c_{12} - (u_1 + v_2) = 56 - (-1+19) = 38$
$d_{13} = c_{13} - (u_1 + v_3) = 34 - (-1+57) = -22$
~~$d_{21} = c_{21} - (u_2 + v_1) = 33 - (-1+1) = 33$~~
$d_{31} = c_{31} - (u_3 + v_1) = 33 - (-19+1) = 51$
$d_{32} = c_{32} - (u_3 + v_2) = 21 - (-19+19) = 21$
$d_{33} = c_{33} - (u_3 + v_3) = 56 - (-19+57) = 18$
$d_{41} = c_{41} - (u_4 + v_1) = 0 - (-19+1) = 18$
$d_{43} = c_{43} - (u_4 + v_3) = 0 - (-19+57) = -38$
$d_{44} = c_{44} - (u_4 + v_4) = 0 - (-19+20) = -1$

As all dij are not +ve ~~not~~ not optimal

Choose d_{43} as it is most -ve.

Anuran Chakraborty

u_i		W_1	W_2	W_3	W_4	Supply
$u_1=-1$	F1	20	56	34	19	90
$u_2=0$	F2	1	19	57	20	1500
$u_3=-19$	F3	33	21	56	1	1368
$u_4=-19$	Fd	0	0	0	0	1065
v_i		$v_1=1$	$v_2=19$	$v_3=57$	$v_4=20$	
Demand		20	1120	33	2850	

minimum allocated value = 33. Subtract 33 if (−) else add 33.

	W_1	W_2	W_3	W_4	Supply
F1	20	56	34	90 19	90
F2	20 1	88 19	57.	1392 20.	1500
F3	33	21	56	1368 1	1368
Fd	0.	1032 0	33 0	0	1065
Demand	20	1120	33	2850	

Again calculate u_i, v_j..

$c_{14} = u_4 + v_4 = 19$

$c_{21} = u_2 + v_1 = 1$

$c_{22} = u_2 + v_2 = 19$

$c_{24} = u_2 + v_4 = 20$

$c_{34} = u_3 + v_4 = 1$

$c_{42} = u_4 + v_2 = 0.$

$c_{43} = u_4 + v_3 = 0.$

Anwesan Chakraborty

Roll: 00161050102O

put $u_2 = 0$.

$v_1 = 1$ $\quad$ $u_4 = -v_2 = -19$

$v_2 = 19$ $\quad$ $u_3 = 1 - v_4 = -19$

$v_4 = 20$ $\quad$ $u_4 = 19 - v_4 = -1$.

$v_3 = -u_4 = 19$ $\quad$ $u_2 = 1 - v_1 = 0$.

$u_4 = -1$	$v_1 = 1$
$u_2 = 0$	$v_2 = 19$
$u_3 = -19$	$v_3 = 19$
$u_4 = -19$	$v_4 = 20$.

$d_{11} = c_{11} - (u_1 + v_1) =$ 20 $- (-1+1) = 20$.

$d_{12} = c_{12} - (u_1 + v_2) = 56 - (-1+19) = 38$.

$d_{13} = c_{13} - (u_1 + v_3) = 34 - (-1+19) = 16$.

$d_{23} = c_{23} - (u_2 + v_3) = 57 - (0+19) = 38$

$d_{31} = c_{31} - (u_3 + v_1) = 33 - (-19+1) = 51$

$d_{32} = c_{32} - (u_3 + v_2) = 21 - (-19+19) = 21$.

$d_{33} = c_{33} - (u_3 + v_3) = 56 - (-19+19) = 56$.

$d_{41} = c_{41} - (u_4 + v_1) = 0 - (-19+1) = 18$

$d_{44} = c_{44} - (u_4 + v_4) = 0 - (-19+20) = -1$.

$d_{44} < 0$. So solution is not optimal.

Anuran Chakraborty

Roll : 001610501020

	W1	W2	W3	W4	Supply
F1	20	56	34	90 / 19	90
F2	20 / 1	88 / (+) 19	57	1392 / (-) 20	1500
F3	33	21	56	1368 / 1	1368
Fd	0	1032 / (-) 0.	0	(+) 0	1065
Demand	20	1120	33	2850	

min allocated value among all -ve position
= 1032.

	W1	W2	W3	W4	Supply
F1	20	56	34	90 / 19	90
F2	20 / 1	1120 / 19	57	360 / 20	1500
F3	33	21	56	1368 / 1	1368
Fd.	0.	0	33 / 0.	1032 / 0	1065
Demand	20	1120	33	2850.	

$c_{14} = u_1 + v_4 = 19$

$c_{21} = u_2 + v_1 = 1.$

$c_{22} = u_2 + v_2 = 19.$

$c_{23} = u_2 + v_3 = 20$

$c_{34} = u_3 + v_4 = 1.$

$c_{43} = u_4 + v_3 = 0$

$c_{44} = u_4 + v_4 = 0.$

Anuran Chakraborty.

Roll : 001610501020

Put $v_4 = 0$.

$u_1 = 19$ $\quad v_1 = 1 - u_2 = -19$

$u_2 = 20$ $\quad v_2 = 19 - u_2 = -1$

$u_3 = 1$ $\quad v_3 = -u_1 = 0$

$u_4 = 0$ $\quad v_4 = 1 - u_3 = 0$.

Find dij.

$d_{11} = c_{11} - (u_1 + v_1) = 20 - (19 - 19) = 20.$

$d_{12} = c_{12} - (u_1 + v_2) = 56 - (19 - 1) = 38$

$d_{13} = c_{13} - (u_1 + v_3) = 34 - (19 + 0) = 15$

$d_{23} = c_{23} - (u_2 + v_3) = 57 - (20 + 0) = 37$

$d_{31} = c_{31} - (u_3 + v_1) = 33 - (1 - 19) = 51$

$d_{32} = c_{32} - (u_3 + v_2) = 21 - (1 - 1) = 21$

$d_{33} = c_{33} - (u_3 + v_3) = 56 - (1 + 0) = 55$

$d_{41} = c_{41} - (u_4 + v_1) = 0 - (0 - 19) = 19$

$d_{42} = c_{42} - (u_4 + v_2) = 0 - (0 - 1) = 1.$

$\because$ all $d_{ij} \geq 0$ $\quad \therefore$ The solution obtained is optimal.

min total cost = $90 \times 19 + 20 \times 1 + 55 \times 19 + 33 \times 57 + 1392 \times 20 + 1368 \times 1 + 1065 \times 0$

$= 31578.$

Anuran Chakraborty

Roll: 00161050102O

b) Solved using stepping stone method.

Initial feasible solution:
Only lowest -ve cost path is shown in diagram.

	W1	W2	W3	W4	Supply
F1	20	56.	34	90 19	90
F2	20 1	55 19 (+)	33 (-) 57	1392 20.	1500
F3	33	21	56	1368 1	1368
Fd	0	1065 0 (-)	(+) 0	0	1065
Demand	20	1120	33	2850.	

Now for every unoccupied cell we have to construct closed path

Unoccupied cell	Path	cost
F1W1	F1W1 → F1W4 → F2W4 → F2W1	20-19+20-1=20
F1W2	F1W2 → F1W4 → F2W4 → F2W2	56-19+20-19=38
F1W3	F1W3 → F1W4 → F2W4 → F2W3	34-19+20-57=-22
F3W1	F3W1 → F3W4 → F2W4 → F2W1	33-1+20-1=51
F3W2	F3W2 → F3W4 → F2W4 → F2W2	21-1+20-19=21
F3W3	F3W3 → F3W4 → F2W4 → F2W3	56-1+20-57 =18
FdW1	FdW1 → FdW2 → F2W2 → F2W1	0-0+19-1=18
FdW3	FdW3 → FdW2 → F2W2 → F2W3	0-0+19-57=-38
FdW4	FdW4 → FdW2 → F2W2 → F2W4	0-0+19-20=-1

Anwan Chakraborty

Lowest -ve value is -38

minimum allocated value on -ve path is 33.

New allocation.

	W1	W2	W3	W4	Supply
F1	20	56	34	90 19	90.
F2	20 1	88 19	57	1392 20	1500
F3	33	21	56	1368 1	1368
Fd	0	1032 0	33 0	0	1065
Demand	20	1120	33	2850	

Again find closed paths.

Unoccupied cell	Path.	cost.
F1W1	F1W1 → F1W4 → F2W4 → F2W1	20-19+20-1 = 20
F1W2	F1W2 → F1W4 → F2W4 → F2W2	56-19+20-19=38
F1W3	F1W3 → F1W4 → F2W4 → F2W2 → FdW2 → FdW3 → F1W3	34-19+20-19+0-0=16
F2W3	F2W3 → F2W2 → FdW2 → FdW3	57-19+0-0 = 38
F3W1	F3W1 → F3W4 → F2W4 → F2W1	33-1+20-1 = 51
F3W2	F3W2 → F3W4 → F2W4 → F2W2	21-1+20-19=21
F3W3	F3W3 → F3W4 → F2W4 → F2W2 → FdW2 → FdW3 → F3W3	56-1+20-19+0-0 = 56.
FdW1	FdW1 → FdW2 → F2W2 → F2W1	0-0+19-1=18
FdW4	FdW4 → FdW2 → F2W2 → F2W4	0-0+19-20=-

Anwesan Chakraborty

Roll: 001610501020

Lowest -ve value is -1 ∴ Not optimal.

min. allocated value of -ve path is 1032.

New allocation.

	W1	W2	W3	W4	Supply
F1	20	56	34	70 19	90
F2	20 1	1120 19	57	360 20	1500
F3	33	21	56	1068 1	1368
Fd	0	0	33 0	1032 0	1065
Demand	20	1120	33	2850	

Again find closed paths.

Unoccupied cell	Path	Cost
F1W1	F1W1 → F1W4 → F2W4 → F2W1	20-19+20-1 = 20
F1W2	F1W2 → F1W4 → F2W4 → F2W2	56-19+20-19 = 38.
F1W3	F1W3 → F1W4 → FdW4 → FdW3	34-19+0-0 = 15
F2W3	F2W3 → F2W4 → FdW4 → FdW3	57-20+0-0 = 37
F3W1	F3W1 → F3W4 → F2W4 → F2W1	33-1+20-1 = 51.
F3W2	F3W2 → F3W4 → F2W4 → F2W2	21-1+20-19 = 21.
F3W3	F3W3 → F3W4 → FdW4 → FdW3	56-1+0-0 = 55
FdW1	FdW1 → FdW4 → F2W4 → F2W1	0-0+20-1 = 19.
FdW2	FdW2 → FdW4 → F2W4 → F2W2	0-0+20-19 = 1

Anwesa Chakraborty.

Roll : 001610501030

Since there is no gain ∴ the allocation is optimal.

Final allocation

	W1	W2	W3	W4	Supply
F1	20	56	34	90 19	90.
F2	20 1	1120 19	57	360 20	1500.
F3	33	21	56	1368 1	1368
Fd	0	0	33 0	1032 0	1065
Demand	20	1120	33	2850	

Total min. transportation cost =

$90\times19 + 20\times1 + 1120\times19 + 360\times20 + 1368\times1 + 33\times0 + 1032\times0$

$= 31578.$

Anuran Chakraborty.