Practicing PL & Resolution & Game tree questions from previous years' question papers

2015

Q4)

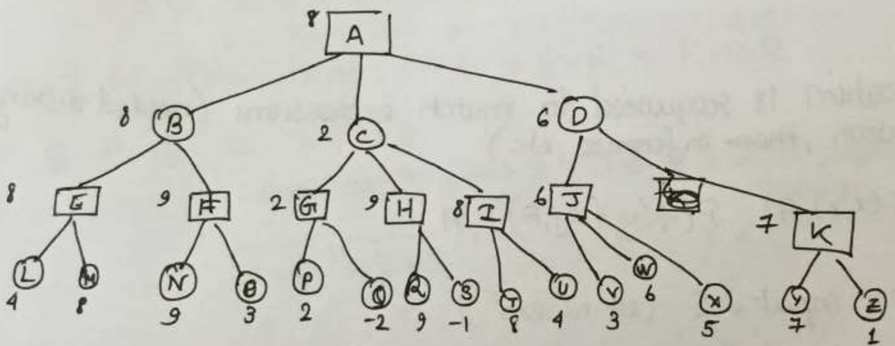
(i) The nodes corresponding the MIN's next move are designated as 'AND' nodes because from MAX's point of view, a win must be obtainable from all possible moves of min

(ii) The ordering of leaf nodes that results in best pruning requires that

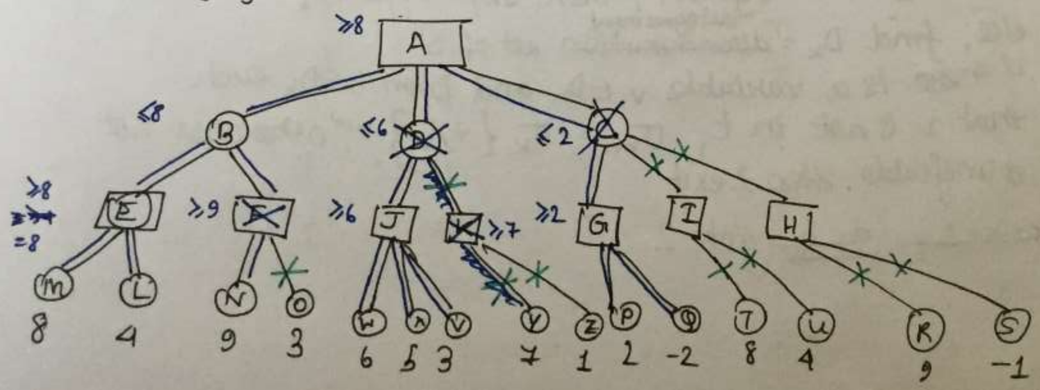
- for every MAX node, successors with largest backed up value be examined first (descending order)
- for every MIN node, successors with smallest backed up value be examined first (ascending order)

~~Performing a mir~~

Backed up values of all nodes



Now, rearranging so that

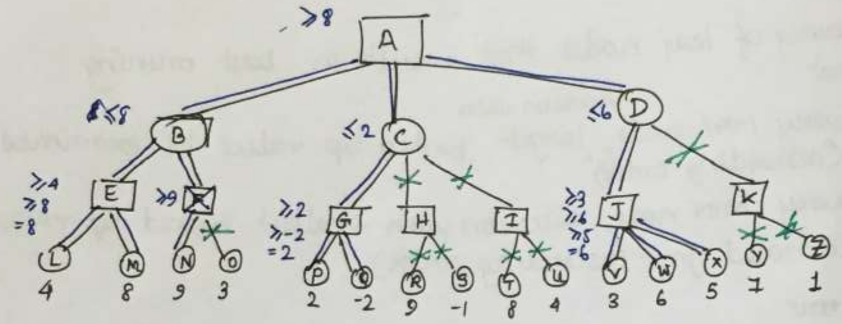


(ii) α-cutoff

→ search can be terminated below MIN node with $\beta \leq \alpha$ of any ancestor

β-cutoff

→ search may be terminated below any MAX node with $\alpha \geq \beta$ (β value of any ancestor)



Q6)

(a) Unification is required to match expressions (needed during resolution, ~~theor~~ inference, etc)

(b) $P(f(x,x),A)$, $P(f(y,f(y,A)),A)$

Unification: input = S (set of exp^r)

1. $K=0$ $\quad \sigma_k = \{\}$
2. if $S\sigma_k$ is a singleton, then stop. mgu $= \sigma_k$
   else, find D_k = ~~discrimination~~ disagreement set of S
   if there is a variable $v \in D_k$ and term $t \in D_k$ such that v is not in t, $\sigma_{k+1} = \sigma_k \{t/v\}$, $k=k+1$, otherwise not unifiable. stop & exit
3. ~~$k=k+1$~~, ~~$S=S\sigma$~~ goto 2

k	σ
0	{}
1	{x

Not unifiable

(c)

Resolution is a so

$P \vdash_r Q$

A rule, is

if $P \vdash_r Q$

or, if P deriv

Resolution rule

P ∧

P ∨

∴ we need to

(Also $P \models$

P	Q	
F	F	F
F	F	T
F	T	F
F	T	T
T	F	F
T	F	T
T	T	F
T	T	T

Hence prove

k	σ_k	$S\sigma_k$	D_k	σ_{k+1}
0	$\{\}$	$P(f(x,x),A),\ P(f(y,f(y,A)),A)$	$\{x,y\}$	$\{x/y\}$
1	$\{x/y\}$	$P(f(x,x),A),\ P(f(x,f(x,A)),A)$	$\{x,f(x,A)\}$	Not valid failure

Not unifiable

(c)

Resolution is a sound rule of inference

A rule $P \vdash_r Q$ is a sound rule of inference when

if $P \vdash_r Q$ then $P \models Q$

or, if P derives Q then Q logically follows P

Resolution rule is as follows

$\cancel{P \wedge Q, Q}$

$P \vee Q,\ \sim Q \vee R \vdash P \vee R$

$\therefore$ we need to show that $P \vee Q,\ \sim Q \vee R \models P \vee R$

(Also $P \models Q$ iff $\models P \rightarrow Q$ ~~(read as~~. (that is, $P \rightarrow Q$ is a valid proposition tautology)

P	Q	R	$\underbrace{P \vee Q}_{X}$	$\underbrace{\sim Q \vee R}_{Y}$	$\underbrace{P \vee R}_{Z}$	$X \wedge Y$	$X \wedge Y \rightarrow Z$
F	F	F	F	T	F	F	T
F	F	T	F	T	T	F	T
F	T	F	T	F	F	F	T
F	T	T	T	T	T	T	T
T	F	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	T	F	T	F	T	F	T
T	T	T	T	T	T	T	T

tautology

Hence proved

(d) $(\forall x)\,(\mathrm{CHILD}(x) \rightarrow \mathrm{LOVES}(x, \mathrm{SANTA}))$

$(\forall x)\,(\mathrm{LOVES}(x, \mathrm{SANTA}) \rightarrow \cancel{\mathrm{LOVES}(}\,(\forall y)\,(\cancel{\mathrm{LOVE}}\ \mathrm{REINDEER}(y) \rightarrow \mathrm{LOVES}(x,y)))$

$\mathrm{REINDEER}(\mathrm{RUDOLPH}) \wedge \cancel{\mathrm{REINDEER}(}\ \mathrm{REDNOSE}(\mathrm{RUDOLPH})$

$(\forall x)\,(\mathrm{REDNOSE}(x) \rightarrow \mathrm{WEIRD}(x) \vee \mathrm{CLOWN}(x))$

$(\forall x)\,(\mathrm{REINDEER}(x) \rightarrow \sim\mathrm{CLOWN}(x))$

$\cancel{\sim\mathrm{CHILD}(\mathrm{JOHN})}$

$(\forall x)\,(\mathrm{WEIRD}(x) \rightarrow \sim\mathrm{LOVES}(\mathrm{JOHN}, x))$

Goal: $\sim\mathrm{CHILD}(\mathrm{JOHN})$

Clausal form: $\sim\mathrm{CHILD}(x1) \vee \mathrm{LOVES}(x1, \mathrm{SANTA})$

$\sim\mathrm{LOVES}(x2, \mathrm{SANTA}) \vee \sim\mathrm{REINDEER}(y) \vee \mathrm{LOVES}(x2, y)$

$\cancel{\sim\mathrm{LOVES}(x3, \mathrm{SANTA}) \vee \mathrm{LOVES}(x3, y2)}$

$\mathrm{REINDEER}(\mathrm{RUDOLPH})$

$\mathrm{REDNOSE}(\mathrm{RUDOLPH})$

$\sim\mathrm{REDNOSE}(x3) \vee \mathrm{WEIRD}(x3) \vee \mathrm{CLOWN}(x3)$

$\sim\mathrm{REINDEER}(x4) \vee \sim\mathrm{CLOWN}(x4)$

$\sim\mathrm{WEIRD}(x5) \vee \sim\mathrm{LOVES}(\mathrm{JOHN}, x)$

Rewrite as: $\sim C(x1) \vee L(x1, \mathrm{SANTA})$

$\sim L(x2, \mathrm{SANTA}) \vee \sim R(y) \vee L(x2, y)$

$R(\mathrm{RUDOLPH})$

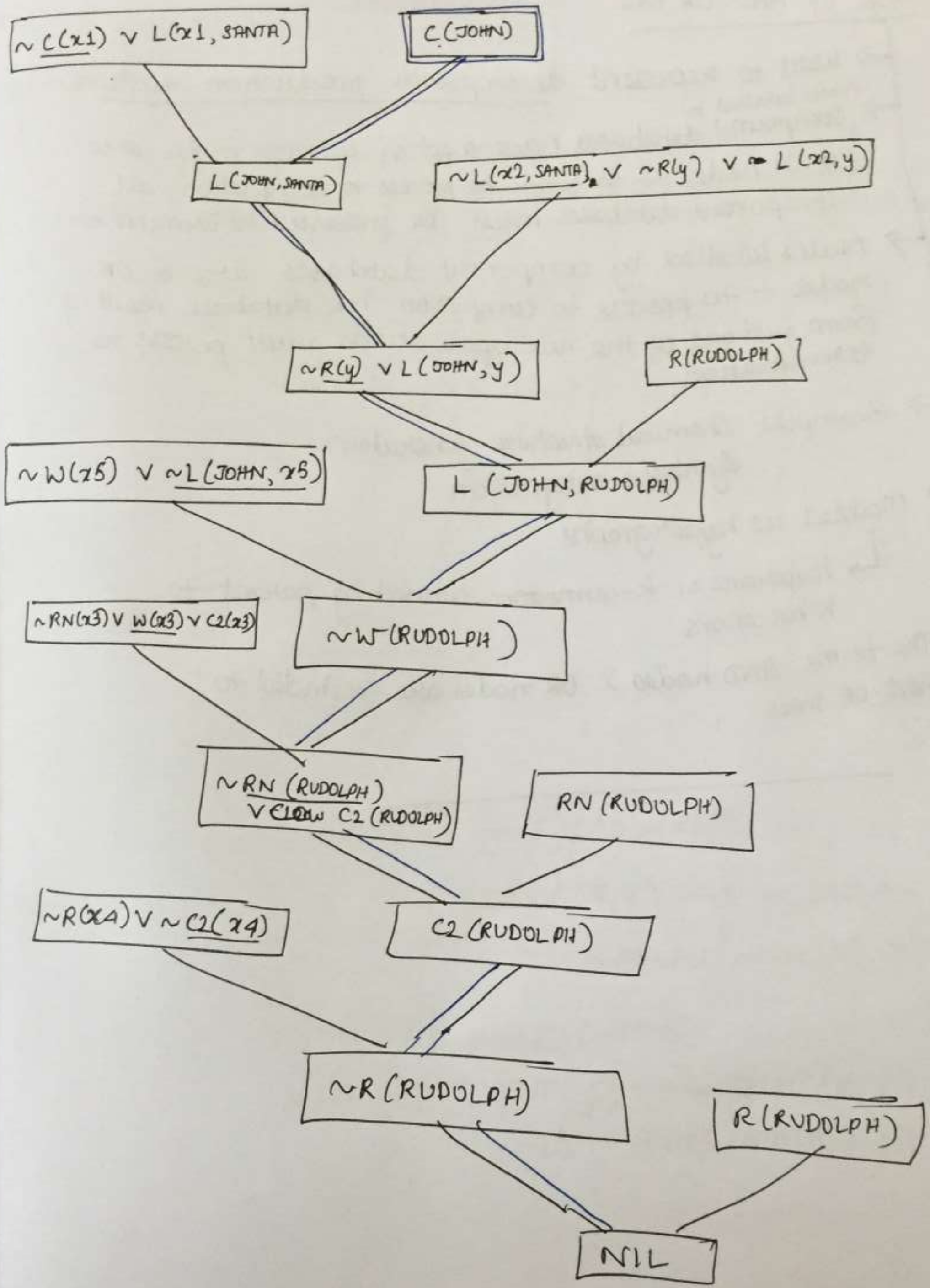
$RN(\mathrm{RUDOLPH})$

$\sim RN(x3) \vee W(x3) \vee C(x3)$

$\sim R(x4) \vee \sim C(x4)$

$\sim W(x5) \vee \sim L(\mathrm{JOHN}, x5)$

Inverted goal: $C(\mathrm{JOHN})$



~C(x1) V L(x1, SANTA)
C(JOHN)
L(JOHN, SANTA)
~L(x2, SANTA) V ~R(y) V L(x2, y)
~R(y) V L(JOHN, y)
R(RUDOLPH)
~W(x5) V ~L(JOHN, x5)
L(JOHN, RUDOLPH)
~RN(x3) V W(x3) V C2(x3)
~W(RUDOLPH)
~RN(RUDOLPH) V C2(RUDOLPH)
RN(RUDOLPH)
~R(x4) V ~C2(x4)
C2(RUDOLPH)
~R(RUDOLPH)
R(RUDOLPH)
NIL

Usage of AND-OR tree

- used to represent decomposable production systems
- Nodes labelled by compound databases have a set of successor nodes are AND nodes — in order to process to completion, all component databases must be processed to termination
- Nodes labelled by component databases are ~~eo~~ OR nodes — to process to completion, the database resulting from just one of the rule applications must process to termination
- Examples: Chemical structure generation
  Symbolic integration
- Modeled as hypergraphs
  - hyperarcs or k-connectors connecting parent to k successors

The terms AND nodes & OR nodes are restricted to AND-OR trees

2013

Q6)

k	σ_k	$S\sigma_k$	D_k	σ_{k+1}
0	$\{\}$	$P(x, g(y,A,h(y,B)))$ $P(h(A,B), g(A,y,x))$	$\{x, h(A,B)\}$	$\{h(A,B)/x\}$
1	$\{h(A,B)/x\}$	$P(h(A,B), g(y,A,h(y,B)))$ $P(h(A,B), g(A,y,h(A,B)))$	$\{y, A\}$	$\{A/y\}$
2	$\{h(A,B)/x,$ $A/y\}$	$P(h(A,B), g(A,A,h(A,B)))$	—	—

singleton exit

= mgu

$\therefore$ mgu $= \{h(A,B)/x,\ A/y\}$

e) (i) ~~$(\forall x)(\forall y)$~~

$(\forall x)(\text{COUNTRY}(x) \rightarrow (\forall y)(\text{COUNTRY}(y) \rightarrow$
$\text{NEIGHBOR}(x,y) \rightarrow$ ~~$\sim$ COLOUR~~
$\text{NEQUALS}(\text{colour}(x), \text{colour}(y))))$

(ii) $(\exists x)(\text{STUDENT}(x) \wedge \text{FAILED}(\text{MATHS})$
$\wedge (\forall y)(\text{STUDENT}(y) \rightarrow \sim\text{EQUALS}(x,y)$
$\vee \sim\text{FAILED}(\text{MATHS})))$

(d) ~~Zebras~~

$(\forall x)(Z(x) \rightarrow M(x) \wedge S(x) \wedge MedSize(x))$

$(\forall x)(M(x) \rightarrow A(x) \wedge Wb(x))$

$(\forall x)(S(x) \rightarrow NonSolid(x) \wedge NonSpotted(x))$

$(\forall x)(\cancel{M(x) \rightarrow}$
$MedSize(x) \rightarrow \sim Large(x) \wedge \sim Small(x))$

~~Zeke~~
$Z(Zeke)$

Goal: ~~Zeke~~ $\sim Large(Zeke)$

Clausal form

$\sim Z(x1) \vee M(x1)$

$\sim Z(x2) \vee S(x2)$

$\sim Z(x3) \vee MedSize(x3)$

$\sim M(x4) \vee A(x4)$

$\sim M(x5) \vee Wb(x5)$

$\sim S(x6) \vee NonSolid(x6)$

$\sim S(x7) \vee NonSpotted(x7)$

$\sim MedSize(x8) \vee \sim Large(x8)$

~~~Small(x9)~~

$\sim MedSize(x9) \vee \sim Small(x9)$

$Z(zeke)$

$Large(Zeke)$
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