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RL = 20
MN = ~~8902689001~~ 01
FN = (65 + 78 + 85 + 82 + 65 + 78) % 70 = 453 % 70 = 33.
LN = (67 + 72 + 65 + 75 + 82 + 65 + 66 + 79 + 82 + 84 + 89) % 70
= 826 % 70 = 56.

FLN = 1279 % 70 = 19

RLM = 20 + 01 = 21
MFN = 01 + 33 = 34
MLN = 01 + 56 = 57

Q1. max. $z = 34x_1 + 57x_2$

Subject to
$$x_1 + 20x_2 \le 1000$$
$$x_1 + x_2 \le 800.$$
$$x_1 + x_2 \le 400.$$
$$x_1, x_2 \ge 0.$$

Convert to canonical form by adding ~~slack~~ variables.

$$z - 34x_1 - 57x_2 + 0s_1 + 0s_2 + 0s_3 = 0.$$
$$x_1 + 20x_2 + s_1 = 1000.$$
$$x_1 + x_2 + s_2 = 800$$
$$x_1 + x_2 + s_3 = 400.$$
$$x_1, x_2 \ge 0.$$

~~max. $z = 34x_1 + 57x_2$~~

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For any iteration. Roll : 001610501020

$$\max.\ Z = C_B X_B + C_N X_N$$

s.t.

$$[B, N] \cdot \begin{bmatrix} X_B \\ X_N \end{bmatrix} = b \quad \text{and} \quad \begin{bmatrix} x_B \\ x_N \end{bmatrix} \geq 0.$$

$\because C_N = 0.$

$$\therefore Z = C_B X_B + C_N X_N = C_B X_B = C_B B^{-1} b.$$

$$B x_B + N x_N = b$$

$$B x_B = b - N x_N$$

$$x_B = B^{-1} b - B^{-1} N x_N$$

$$x_B = B^{-1} b.$$

$$\boxed{\begin{aligned} x_B &= B^{-1} b \\ Z &= C_B B^{-1} b \end{aligned}}$$

Iteration 0

$$x_B = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = B^{-1}$$

$$\therefore \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1000 \\ 800 \\ 400 \end{bmatrix} = \begin{bmatrix} 1000 \\ 800 \\ 400 \end{bmatrix}$$

$$C_B = [0 \quad 0 \quad 0] \qquad \therefore Z = [0 \quad 0 \quad 0] \begin{bmatrix} 1000 \\ 800 \\ 400 \end{bmatrix} = 0$$

~~Iteration 1~~

~~$x_B = b$~~

$$c_1 - z_1 = c_1 - y P_1$$

$$= 34 - y \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

y is the dual.

$$y = C_B B^{-1} = [0\ 0\ 0] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$c_1 - z_1 = 34 - [0\ 0\ 0] \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 34$$

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Initialization Iteration 0.

$$c = [34 \quad 57] \qquad [A, I] = \left[\begin{array}{cc|ccc} 1 & 20 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$b = \begin{bmatrix} 1000 \\ 800 \\ 400 \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad x_s = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$$x_B = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix}, \quad B = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = B^{-1}$$

$$x_B = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix} = B^{-1} b = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1000 \\ 800 \\ 400 \end{bmatrix} = \begin{bmatrix} 1000 \\ 800 \\ 400 \end{bmatrix}$$

$$C_B = [0 \quad 0 \quad 0] \qquad \text{So } z = [0 \quad 0 \quad 0] \begin{bmatrix} 1000 \\ 800 \\ 400 \end{bmatrix} = 0.$$

Optimality test : $$C_B B^{-1} A - c = [0 \; 0 \; 0] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 20 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} - [34 \; 57]$$

$$= [-34 \quad -57]$$

Now ~~all are~~ there is atleast one -ve value.

$-c_2 = -57 < -c_1 = -34 \quad \therefore \; x_2$ is the entering variable.

Iteration 1

x_2 is entering variable.

To determine leaving variable.

$$B^{-1} A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 20 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 20 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$$

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We check only the column for x_2.

$a_{12} = 20, \quad a_{22} = 1, \quad a_{32} = 1.$

$\frac{b_1}{a_{12}} = \frac{1000}{20} = 50, \quad \frac{b_2}{a_{22}} = \frac{800}{1} = 800$

$\frac{b_3}{a_{32}} = \frac{400}{1} = 400.$

$\frac{b_1}{a_{12}}$ is min so row 1, x_3 is the leaving variable

Entering : x_2 [2nd col]
Leaving : x_3 [1st row]
(1,2) is the pivot)

$$x_B = \begin{bmatrix} x_2 \\ x_4 \\ x_5 \end{bmatrix}$$

Obtain new B^{-1}.

$$\eta = \begin{bmatrix} \frac{1}{a_{12}} \\ -\frac{a_{22}}{a_{12}} \\ -\frac{a_{32}}{a_{12}} \end{bmatrix} = \begin{bmatrix} \frac{1}{20} \\ -\frac{1}{20} \\ -\frac{1}{20} \end{bmatrix}$$

$$E = \begin{bmatrix} \frac{1}{20} & 0 & 0 \\ -\frac{1}{20} & 1 & 0 \\ -\frac{1}{20} & 0 & 1 \end{bmatrix}$$

$$B^{-1}_{new} = E B^{-1}_{old} = \begin{bmatrix} \frac{1}{20} & 0 & 0 \\ -\frac{1}{20} & 1 & 0 \\ -\frac{1}{20} & 0 & 1 \end{bmatrix}$$

$$x_B = \begin{bmatrix} x_2 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} \frac{1}{20} & 0 & 0 \\ -\frac{1}{20} & 1 & 0 \\ -\frac{1}{20} & 0 & 1 \end{bmatrix} \begin{bmatrix} 1000 \\ 800 \\ 400 \end{bmatrix} = \begin{bmatrix} 50 \\ 750 \\ 350 \end{bmatrix}$$

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$$c_B B^{-1} A - c = [57 \quad 0 \quad 0] \begin{bmatrix} \frac{1}{20} & 0 & 0 \\ -\frac{1}{20} & 1 & 0 \\ -\frac{1}{20} & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 20 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} - [34 \quad 57]$$

$$\cancel{= [57/20 \quad 0 \quad 0] \begin{bmatrix} 1 & 20 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} - [34 \quad 57]}$$

$$\cancel{= [57/20 \quad 57] - [34 \quad 57]}$$

$$= \cancel{[-31.15 \quad 0]} = [-31.15 \quad -]$$

$c_1 < 0$ $\therefore$ still not optimal.

Iteration 2

x_4 is the entering variable.

To determine leaving variable.

$$A' = B^{-1} A = \begin{bmatrix} \frac{1}{20} & 0 & 0 \\ -\frac{1}{20} & 1 & 0 \\ -\frac{1}{20} & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 20 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/20 & - \\ 19/20 & - \\ 19/20 & - \end{bmatrix}$$

We check only the column for x_4

$a_{11} = 1/20 \quad a_{12} = 19/20 \quad a_{13} = \cancel{8}.\ 21/20.$

$$\frac{b_1}{a'_{11}} = \frac{1000}{1/20} \qquad \frac{b_2}{a'_{12}} = \frac{800}{19/20} \qquad \frac{b_3}{a'_{13}} = \frac{400}{19/20}$$

row 3 is min.

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$\therefore x_5$ is the leaving varriable.

Entering : x_4 (Col 1)

Leaving : x_5 (row 3)

$\therefore$ Pivot element (3,1)

$$x_B = \begin{bmatrix} x_2 \\ x_4 \\ x_4 \end{bmatrix}$$

$$\eta = \begin{bmatrix} -\frac{a'_{11}}{a'_{31}} \\ -\frac{a'_{21}}{a_{31}} \\ \frac{1}{a'_{31}} \end{bmatrix} = \begin{bmatrix} -\frac{1}{19} \\ -1 \\ \frac{20}{19} \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 0 & -\frac{1}{19} \\ 0 & 1 & -1 \\ 0 & 0 & \frac{20}{19} \end{bmatrix}$$

$$B^{-1}_{New} = E B^{-1}_{old} = \begin{bmatrix} 1 & 0 & -\frac{1}{19} \\ 0 & 1 & -1 \\ 0 & 0 & \frac{20}{19} \end{bmatrix} \begin{bmatrix} \frac{1}{20} & 0 & 0 \\ -\frac{1}{20} & 1 & 0 \\ -\frac{1}{20} & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/19 & 0 & -1/19 \\ 0 & 1 & -1 \\ -1/19 & 0 & 20/19 \end{bmatrix}$$

$$C_B = [57 \quad 0 \quad 34]$$

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$$c_B B^{-1} A = \begin{bmatrix} 57 & 0 & 34 \end{bmatrix} \begin{bmatrix} 1/19 & 0 & -1/19 \\ 0 & 1 & -1 \\ -1/19 & 0 & 20/19 \end{bmatrix} \begin{bmatrix} \cancel{1/20} \\ \cancel{19/20} \\ \cancel{19/20} \end{bmatrix}$$

$$= \cancel{593/19} \begin{bmatrix} 23/19 & 0 & 623/19 \end{bmatrix} > 0.$$

$$\cancel{c_B B^{-1} A - c_1 = \frac{593}{19} - 34 > 0}$$

~~coeffn of x_4~~

non basic variables are x_3, x_5.

$\therefore$ optimality reached.

$$x_B = \begin{bmatrix} x_2 \\ x_4 \\ x_4 \end{bmatrix} = B^{-1} b =$$

$$\begin{bmatrix} 1/19 & 0 & -1/19 \\ 0 & 1 & -1 \\ -1/19 & 0 & 20/19 \end{bmatrix} \begin{bmatrix} 1000 \\ 800 \\ 400 \end{bmatrix}$$

$$= \begin{bmatrix} 600/19 \\ 400 \\ 7000/19 \end{bmatrix}$$

$\therefore x_2 = \frac{600}{19}$ $\quad x_4 = \frac{7000}{19}$

$$\boxed{\begin{aligned} x_4 &= 368.4211 \\ x_2 &= 31.5789 \\ Z &= 34x_4 + 57x_2 = 14326.3158 \end{aligned}}$$

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