Presentation of Anasua Sarkar - 2

For Soft Computing (CSE/T/425E)

At Computer Science and Engineering Department, Jadavpur University, India

13/01/2020

Topics of Interests

- Hard COMPUTING AND Soft Computing
- Reasoning under Uncertainty
- Fuzzy sets theory
- Type-2 Fuzzy sets
- Applications with Fuzzy Sets

Characteristics of Neuro-Fuzzy & Soft Computing:

1. Human Expertise
2. Biologically inspired computing models
3. New Optimization Techniques
4. Numerical Computation
5. New Application domains
6. Model-free learning
7. Intensive computation
8. Fault tolerance
9. Goal driven characteristics
10. Real world applications

- Intelligent Control Strategies (Components of Soft Computing): The popular soft computing components in designing intelligent control theory are:
- 1. Fuzzy Logic
- 2. Neural Networks
- 3. Evolutionary Algorithms.

http://www.vssut.ac.in/lecture_notes/lecture1423723637.pdf

Neural Networks

- Imitation of the natural intelligence of the brain
- Parallel processing with incomplete information
- Nerve cells function about 10^6 times slower than electronic circuit gates, but human brains process visual and auditory information much faster than modern computers
- The brain is modeled as a continuous-time non linear dynamic system in connectionist architectures
- Distributed representation in the form of weights between a massive set of interconnected neurons

Fuzzy Set Theory

- Human brains interpret imprecise and incomplete sensory information provided by perceptive organs
- - Fuzzy set theory provides a systematic calculus to deal with such information linguistically
- - It performs numerical computation by using linguistic labels stimulated by membership functions
- - It lacks the adaptability to deal with changing external environments ==> incorporate NN learning concepts in fuzzy inference systems: NF modeling
- Human **thinking** and **reasoning** (analysis, logic, interpretation) frequently involved **fuzzy** information.

NF and SC characteristics

- With NF modeling as a backbone, SC can be characterized as:
- - Human expertise (fuzzy if-then rules)
- - Biologically inspired computing models (NN)
- - New optimization techniques (GA, SA, RA)
- - Numerical computation (no symbolic AI, only numerical)

https://notendur.hi.is/benedikt/Courses/ch01_2005.pdf

Fuzzy Logic & NN

- Fuzzy logic is mainly associated to imprecision, approximate reasoning and computing with words, neurocomputing to learning and curve fitting (also to classification), and probabilistic reasoning to uncertainty and belief propagation (belief networks). These methods have in common that they
- 1. are nonlinear,
- 2. have ability to deal with non-linearities,
- 3. follow more human-like reasoning paths than classical methods,
- 4. utilize self-learning,
- 5. utilize yet-to-be-proven theorems,
- 6. are robust in the presence of noise or errors.
- The main dissimilarity between fuzzy logic system (FLS) and neural network is that FLS uses heuristic knowledge to form rules and tunes these rules using sample data, whereas NN forms "rules" based entirely on data.

- Kosko lists the following similarities between fuzzy logic systems and neural networks [Kosko, 1992]:
- estimate functions from sample data
- do not require mathematical model
- are dynamic systems
- can be expressed as a graph which is made up of nodes and edges
- convert numerical inputs to numerical outputs
- process inexact information inexactly
- have the same state space
- produce bounded signals
- a set of n neurons defines n -dimensional fuzzy sets
- learn some unknown probability function p (x)
- can act as associative memories
- can model any system provided the number of nodes is sufficient.

Fuzzy Computing

- In the real world there exists much fuzzy knowledge, that is, knowledge which is vague, imprecise, uncertain, ambiguous, inexact, or probabilistic in nature.
- Human can use such information reasoning frequently involve fuzzy because information, the human thinking and possibly originating from inherently inexact human concepts and matching of similar rather then identical experiences.
- The logic,computing systems, based upon classical set theory and two-valued can not answer to some questions, as human does, because they do not have completely true answers.
- We want, the computing systems should not only give human like answers but also describe their reality levels. These levels need to be calculated using imprecision and the uncertainty of facts and rules that were applied.

This example explains the grade of truth value.

- **tall students** qualify and **not tall students** do not qualify
- if students 1.8 m tall are to be qualified, then should we exclude a student who is $^1/_{10}$" less? or should we exclude a student who is 1" shorter?

▪ Non-Crisp Representation to represent the notion of a tall person.

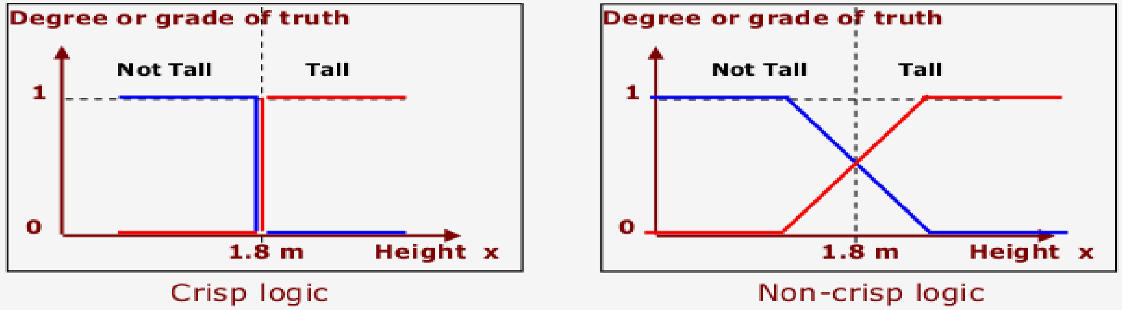


Fig. 1 Set Representation – Degree or grade of truth

A student of height 1.79m would belong to both tall and not tall sets with a particular degree of membership.

As the height increases the membership grade within the tall set would increase whilst the membership grade within the not-tall set would decrease.

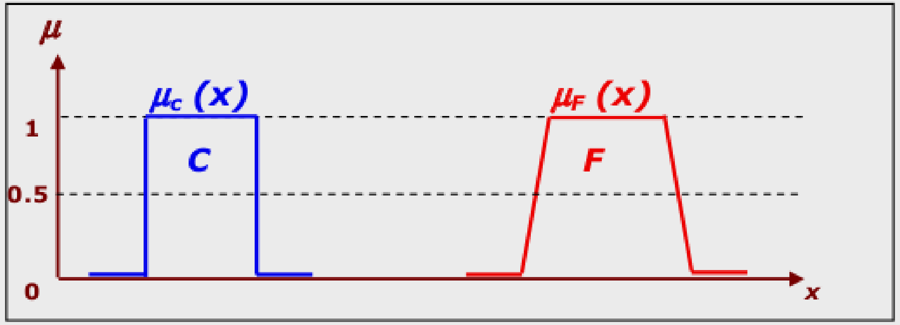


Fig. 2 Membership function of a Crisp set C and Fuzzy set F

▪ In the case of Crisp Sets the members of a set are :

either out of the set, with membership of degree " **0** ",

or in the set, with membership of degree " **1** ",

Therefore, **Crisp Sets ⊆ Fuzzy Sets**

In other words, Crisp Sets are Special cases of Fuzzy Sets.

• Fuzzy Sets

- Extension of Classical Sets
- Fuzzy sets - 1965 Lotfi Zadeh as an extension of classical notation set.
- Not just a membership value of in the set and out the set, 1 and 0
  - ˜ but partial membership value, between 1 and 0

https://sites.google.com/site/savitakumarisheoran79/soft-computing

Fuzzy Sets

- Vagueness in concept formation and representation comes from our inability to describe a precisely defined concept in situations with incomplete information.
- The vagueness is described by differences in representations of the same vague concept in various possible worlds. A fuzzy set is a weighted combination.
- A fuzzy set is defined by a membership function from a universe U to the unit interval [0, 1].
- Typically explicit fuzzy sets are defined in some universe of discourse:
  - Each elopement of the universe is associated with a membership degree.

$$\mu_{\mathcal{A}} : U \longrightarrow [0, 1],$$

$$u_{ik} = \frac{1}{\sum_{j=1}^{c} \left(\frac{d_{ik}}{d_{jk}} \right)^{\frac{2}{m-1}}}$$

$$d_{ik} = \|\mathbf{x}_k - \mathbf{v}_i\|^2$$

- i = assigned Cluster among c clusters, k = pattern to allocate, v=centroid

• The crisp set v.s. the fuzzy set

- The crisp set is defined in such a way as to dichotomize(classification) the individuals in some given universe of discourse into two groups: members and nonmembers.
  - However, many classification concepts do not exhibit this characteristic.
  - For example, the set of tall people, expensive cars, or sunny days.

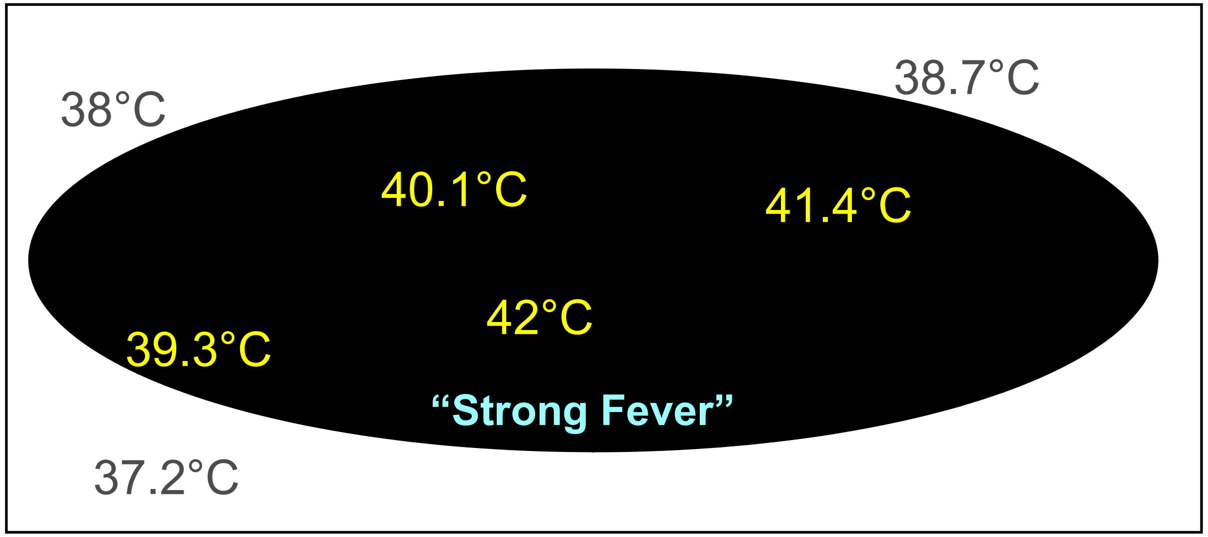
- A fuzzy set can be defined mathematically by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the fuzzy set.
  - For example: a fuzzy set representing our concept of sunny might assign a degree of membership of 1 to a cloud cover of 0%, 0.8 to a cloud cover of 20%, 0.4 to a cloud cover of 30%, and 0 to a cloud cover of 75%.

https://sites.google.com/site/savitakumarisheoran79/soft-computing

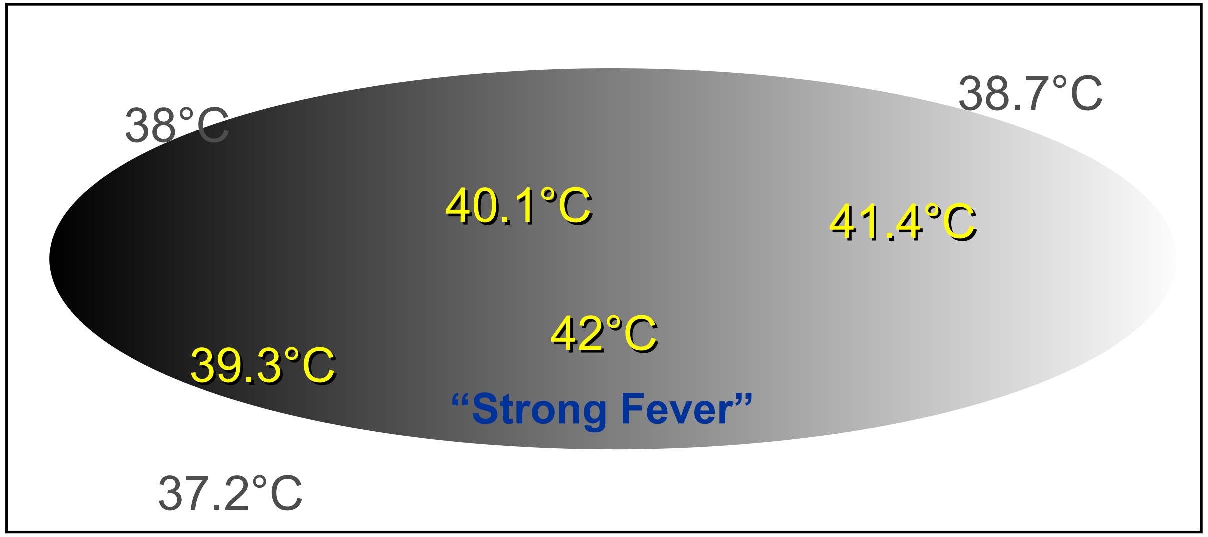
Classical set theory	Fuzzy set theory
• Classes of objects with sharp boundaries.	• Classes of objects with un-sharp boundaries.
• A classical set is defined by crisp(exact) boundaries, i.e., there is no uncertainty about the location of the set boundaries.	• A fuzzy set is defined by its ambiguous boundaries, i.e., there exists uncertainty about the location of the set boundaries.
• Widely used in digital system design	• Used in fuzzy controllers.

Fuzzy Set Theory

Conventional (Boolean) Set Theory:



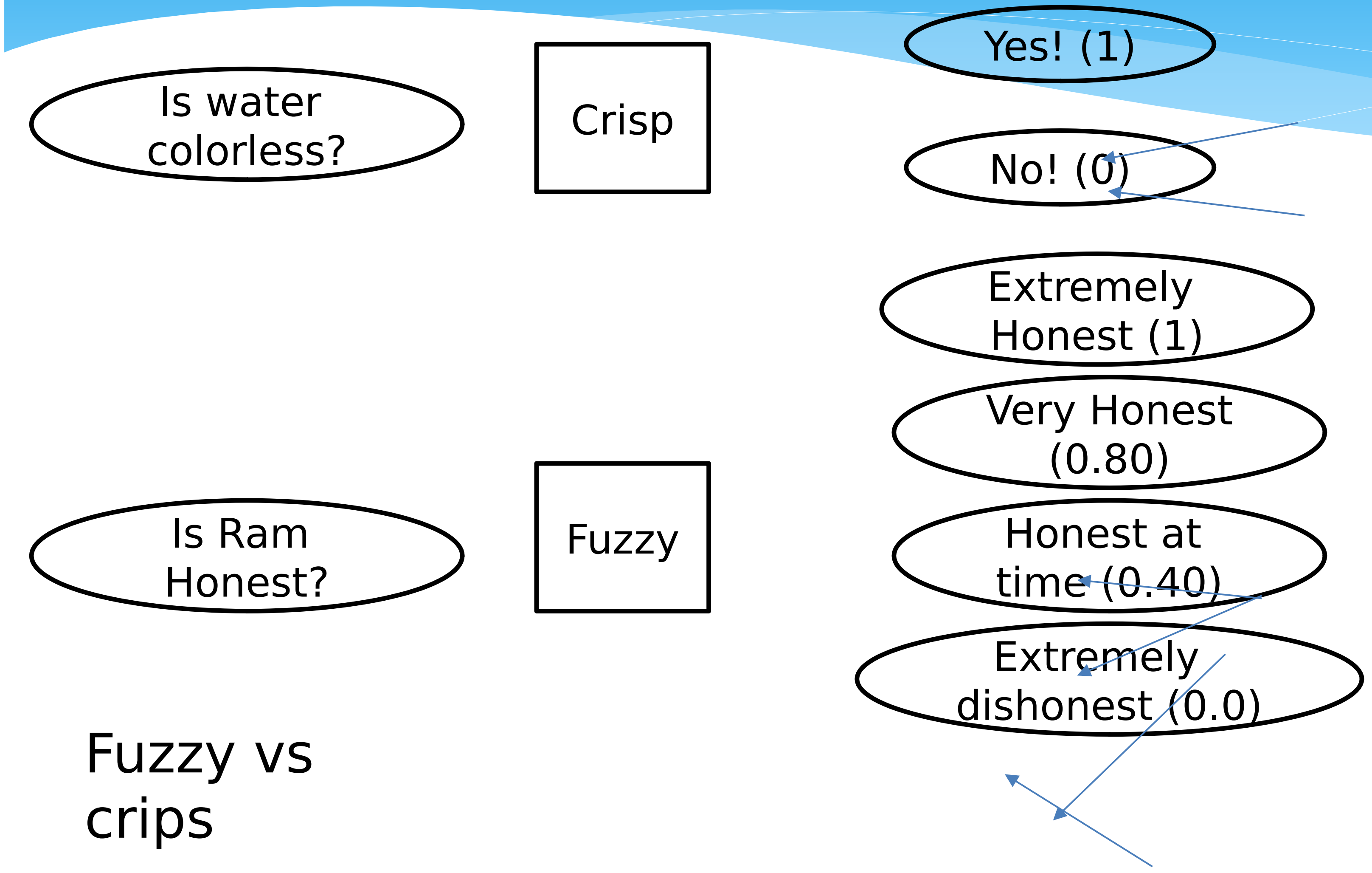
Fuzzy Set Theory:



"More-or-Less" Rather Than "Either-Or" !

Fuzzy Set example

Example



Fuzzy vs crips

• Applications

- Application areas
  - ˜ Fuzzy Control
    - Subway trains
    - Cement kilns
    - Washing Machines
    - Fridges

- Two frameworks for Fuzzy Systems
- 1) Development based on Crisp mathematical model and fuzzifying some quantities :
- Model 1 : Fuzzy Mathematical Model
- Example : Fuzzy – K means clustering
- 2) Development based on Fuzzy Inference rules:
- Model 2 : Fuzzy Logical Model
- Example : Fuzzy decision Support System

https://sites.google.com/site/savitakumarisheoran79/soft-computing ,https://www.iitk.ac.in/eeold/archive/courses/2013/intel-info/d1pdf3.pdf

Example: Height

- Tall people: say taller than or equal to 1.8m
  - 1.8m , 2m, 3m etc member of this set
  - 1.0 m, 1.5m or even 1.79999m not a member

- Real systems have measurement uncertainty
  - so near the border lines, many misclassifications

https://sites.google.com/site/savitakumarisheoran79/soft-computing

Classical set theory

$A = \{x \mid x > 6\},$

- A Set is any well defined collection of objects.
- An object in a set is called an element or member of that set.
- Sets are defined by a simple statement,
- Describing whether a particular element having a certain property belongs to that particular set.
  - $A = \{a_1,a_2,a_3,......,a_n\}$
- If the elements a_i $(i = 1,2,3,....,n)$ of a set A are subset of universal set X, then set A can be represented for all elements $x \in X$ by its characteristics function
- $\mu_A(x) = 1$ if $x \in X$ otherwise 0

Operations on classical set theory

- ***Union:*** the union of two sets A and B is given as
  - A U B = { x | x ϵ A or x ϵ B }
  -
- ***Intersection:*** the intersection of two sets A and B is given as
  - A ∩ B = { x | x ϵ A and x ϵ B }
  -
- ***Complement:*** It is denoted by Ã and is defined as
- Ã = { x | x does not belongs A and x ϵ X }

Law of contradiction	$A \cap \overline{A} = \emptyset$
Law of the excluded middle	$A \cup \overline{A} = X$
Idempotency	$A \cap A = A,\ A \cup A = A$
Involution	$\overline{\overline{A}} = A$
Commutativity	$A \cap B = B \cap A,\ A \cup B = B \cup A$
Associativity	$(A \cup B) \cup C = A \cup (B \cup C)$ $(A \cap B) \cap C = A \cap (B \cap C)$
Distributivity	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Absorption	$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$
Absorption of complement	$A \cup (\overline{A} \cap B) = A \cup B$ $A \cap (\overline{A} \cup B) = A \cap B$
DeMorgan's laws	$\overline{A \cup B} = \overline{A} \cap \overline{B}$ $\overline{A \cap B} = \overline{A} \cup \overline{B}$

•Fuzzy sets: basic types

A set defined by membership functions is a fuzzy set.

- A set without a crisp boundary.
- The transition from “belong to set” to “not belong to set” is gradual.
- This transition is characterized by membership funtions that give fuzzy set flexibility in modeling commonly used linguistic expressions like “water is hot” or “temp is high”.
- A membership function:
  - A characteristic function: the values assigned to the elements of the universal set fall within a specified range and indicate the membership grade of these elements in the set.
  - Larger values denote higher degrees of set membership.

- The most commonly used range of values of membership functions is the unit interval [0,1].
- We think the universal set *X* is always a crisp set.
- Notation:
  - The membership function of a fuzzy set *A* is denoted by : μ_A

    $$\mu_A : X \rightarrow [0,1]$$
  - In the other one, the function is denoted by *A* and has the same form

    $$A : X \rightarrow [0,1]$$
  - In this text, we use the second notation.

• Member Functions

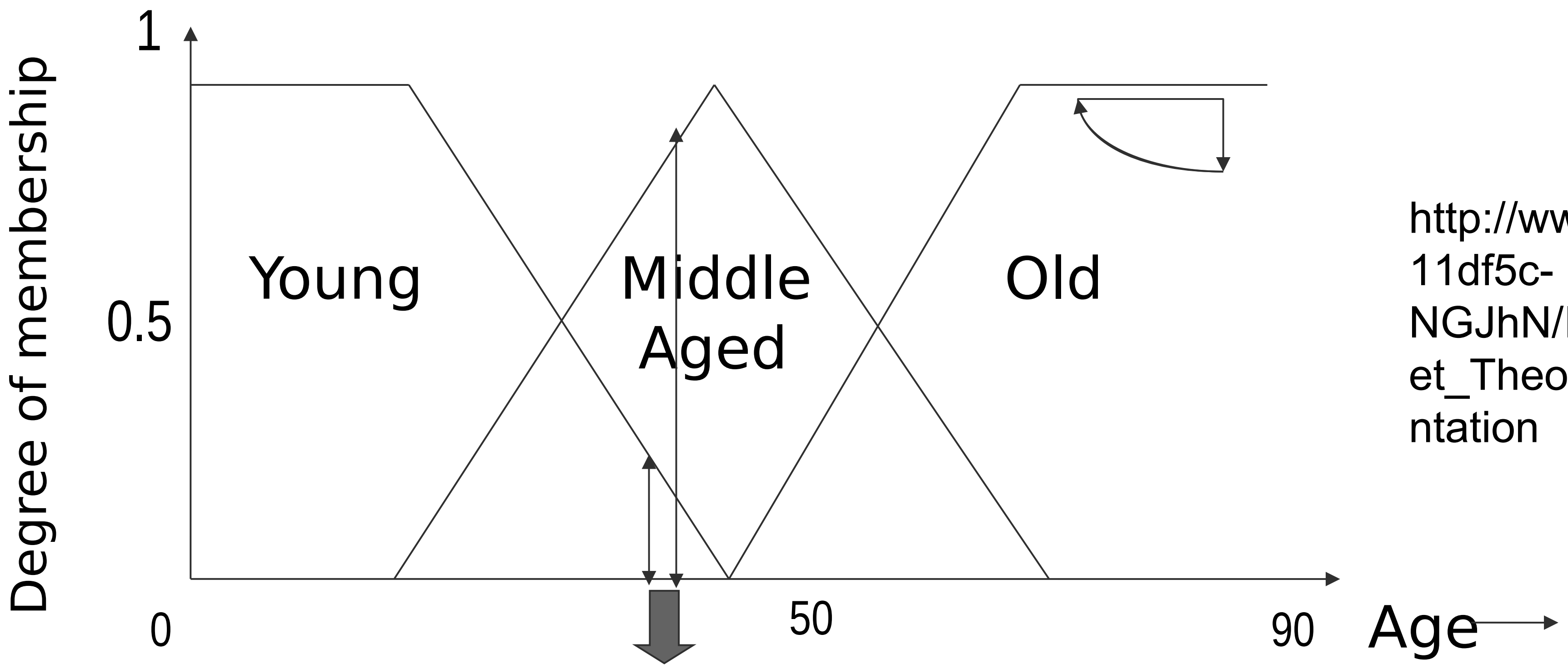
- Membership function
  - better than listing membership values
- e.g. Tall(x) = {
- 1 if x >= 1.9m ,
  - 0 if x <= 1.7m
  - else (x - 1.7) / 0.2 }

[redacted]

- Short(x) = {
- 0 if x >= 1.9m ,
  - 1 if x <= 1.7m
  - else (1.9 - x) /

[redacted]

- A degree of membership between 0 and 1 in several sets (to a combined total of 1).
- Then label these sets using human terms.
- It encapsulates terms with no consensus definition,
- but we might use surveys to define them.



38yr old = 10% Young + 90% Middle Aged

http://www.powershow.com/view/11df5c-NGJhN/Introduction_to_Fuzzy_Set_Theory_powerpoint_ppt_presentation

Expansion of fuzzy set

- Type-n Fuzzy Set
- The value of membership degree might include uncertainty.
- If the value of membership function is given by a fuzzy set, it is a type-2 fuzzy set.
- This concept can be extended up to Type-n fuzzy set.

Example (Type-n Fuzzy Set)

- Fuzzy sets of type 2: $A : X \to \mathcal{F}([0,1])$,
- $\mathcal{F}([0,1])$: the set of all ordinary fuzzy sets that can be defined with the universal set [0,1].
- $\mathcal{F}([0,1])$ is also called a fuzzy power set of [0,1].

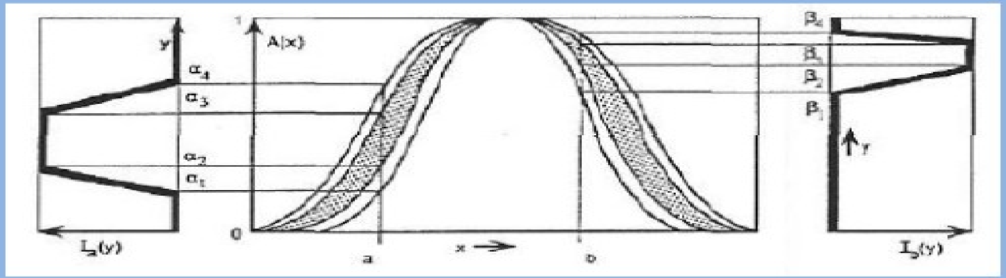


Fig : Fuzzy Set of Type-2

Type-2 fuzzy sets

- Membership degree treated as a single number in [0,1]
- Type-2 fuzzy set: admit membership modeled as fuzzy sets defined in [0,1]
- For type-II fuzzy set $\hat{B}$, a membership function for type-II is defined as: $\mu_{\hat{B}}(y,u)$,

  where $y \in Y$ and $u \in J_y \subseteq [0,1]$ where, $0 \le \mu_{\hat{B}}(y,u) \le 1$

$$\hat{B} = \{(y, \mu_U(y), \mu_L(y)) \mid y \in Y,$$

$$\mu_L(y) \le \mu(y) \le \mu_U(y), \mu \in [0,1]\}$$

- μ_U and μ_L are the upper and lower membership degrees of the skeleton (main) membership function μ

•Fuzzy Set Operators

- Fuzzy Set:
  - Union
  - Intersection
  - Complement
  - Containment or subset
- Many possible definitions
  - we introduce one possibility

https://sites.google.com/site/savitakumarisheoran79/soft-computing

• Fuzzy Set Union

- Union ($f_A(x)$ and $f_B(x)$) = max ($f_A(x)$, $f_B(x)$)
- Union (Tall(x) and Short(x))

$C = A \cup B$ $\qquad\qquad$ $\mu_C(x) = \max(\mu_A(x), \mu_B(x)) = \mu_A(x) \vee \mu_B(x).$

https://sites.google.com/site/savitakumarisheoran79/soft-computing

• Fuzzy Set Intersection

- Intersection ($f_A(x)$ and $f_B(x)$) = min ($f_A(x)$, $f_B(x)$)
- Intersection (Tall(x) and Short(x))

$$C = A \cap B \qquad \mu_C(x) = \min(\mu_A(x), \mu_B(x)) = \mu_A(x) \wedge \mu_B(x).$$

https://sites.google.com/site/savitakumarisheoran79/soft-computing

•Fuzzy Set Complement

- Complement($f_A(x)$) = 1 - $f_A(x)$
- Not (Tall(x))

$\overline{A}$ ($\neg A$, NOT A)

$$\mu_{\overline{A}}(x) = 1 - \mu_A(x).$$

https://sites.google.com/site/savitakumarisheoran79/soft-computing

Fuzzy Set Subset

Fuzzy set A is **contained** in fuzzy set B (or, equivalently, A is a **subset** of B, or A is smaller than or equal to B) if and only if $\mu_A(x) \le \mu_B(x)$ for all x. In symbols,

$$A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x). \tag{2.12}$$

□

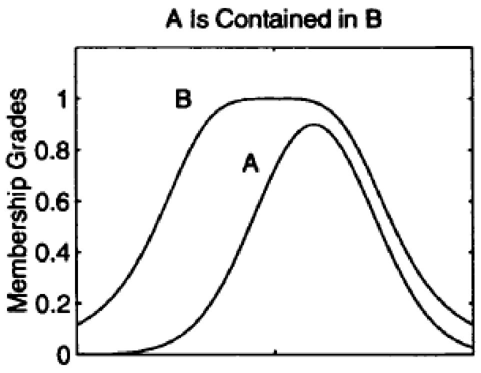


Figure 2.5. *The concept of $A \subseteq B$.* (MATLAB file: **subset.m**)

Definition 2.17 *Cartesian product and co-product*

Let A and B be fuzzy sets in X and Y, respectively. The **Cartesian product** of A and B, denoted by $A \times B$, is a fuzzy set in the product space $X \times Y$ with the membership function

$$\mu_{A\times B}(x,y) = \min(\mu_A(x), \mu_B(y)). \tag{2.16}$$

Similarly, the **Cartesian co-product** $A + B$ is a fuzzy set with the membership function

$$\mu_{A+B}(x,y) = \max(\mu_A(x), \mu_B(y)). \tag{2.17}$$

Both $A\times B$ and $A+B$ are characterized by two-dimensional MFs, which are explored

Operations on Fuzzy Sets

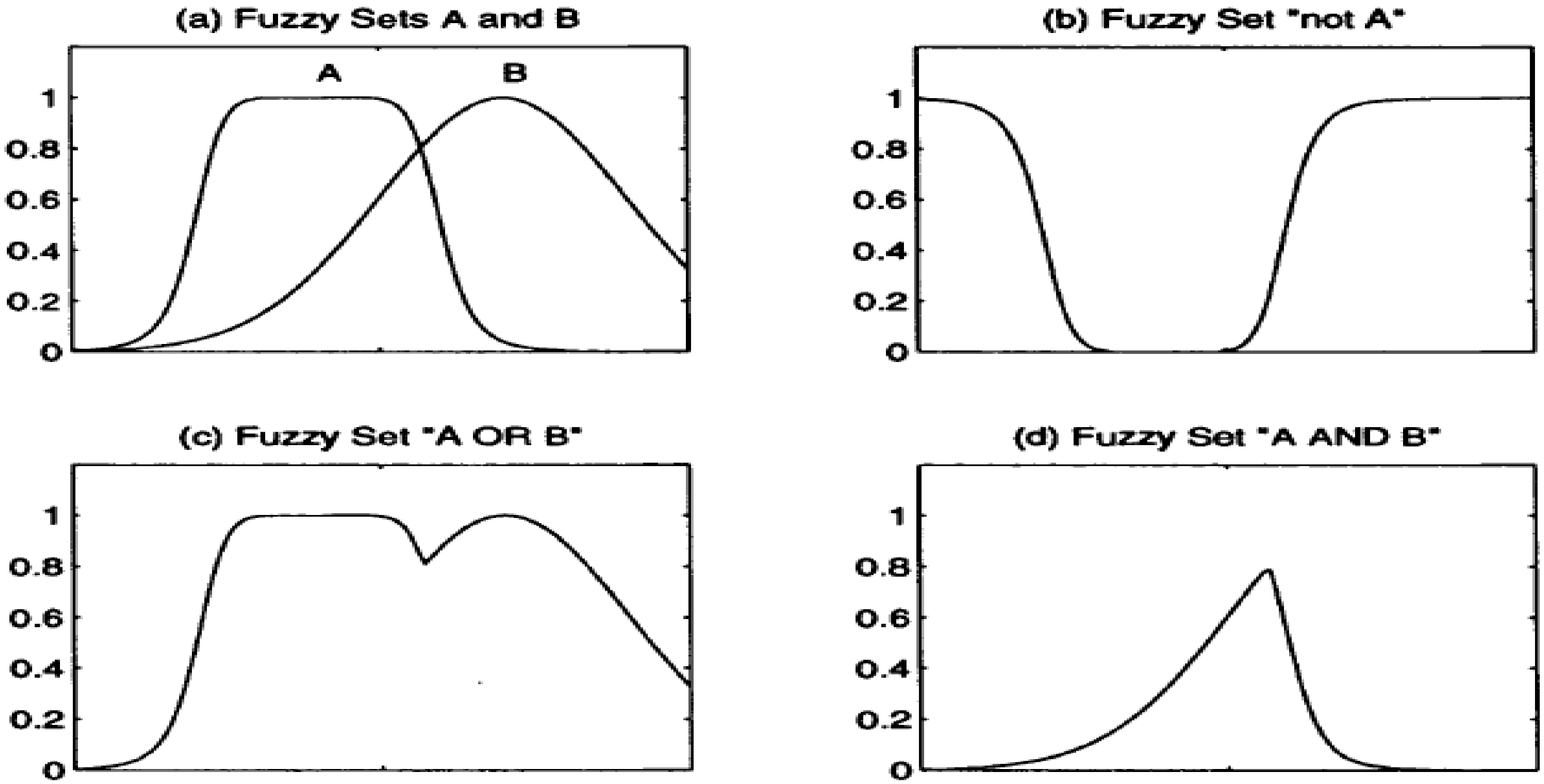


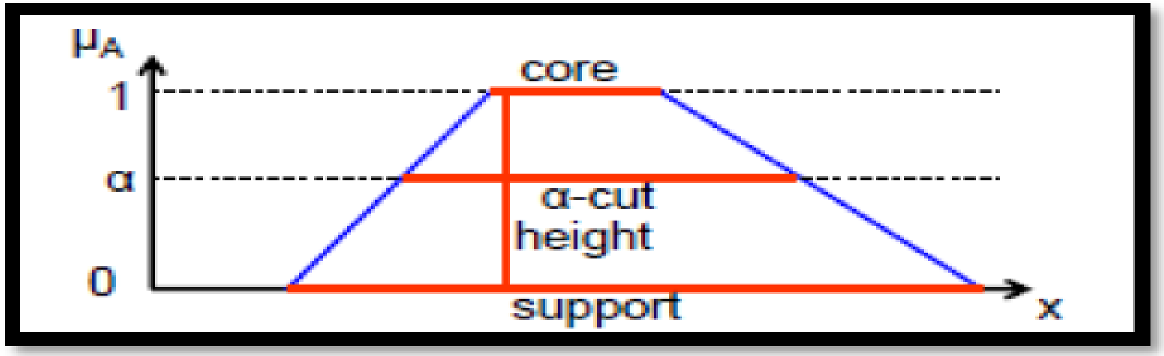
Figure 2.6. *Operations on fuzzy sets: (a) two fuzzy sets A and B; (b)* $\overline{A}$*; (c)* $A \cup B$*; (d)* $A \cap B$*.* (MATLAB file: `fuzsetop.m`)

MF: basic concepts

- **Fuzzy singleton**- A fuzzy set with a membership function that is unity at a one particular point and zero everywhere else.
- **Singleton output function**- An output function that is given by a spike at a single number rather than a continuous curve. In the Fuzzy Logic Toolbox it is only supported as part of a zero-order Sugeno model.

Fuzzy Membership Function: Basic Concepts

- **Support**: elements having non-zero degree of membership.
- **Core**: set with elements having degree of 1.
- ***α*-Cut**: set of elements with degree >= α.
- **Height**: maximum degree of membership.



Membership Functions in the Fuzzy Logic Toolbox

@Dr. Quadri
hamarsheh

Membership Functions

The Fuzzy Logic Toolbox includes 11 built-in membership function types. These 11 functions are, in turn, built from several basic functions:

o Piecewise linear functions.
o Gaussian distribution function.
o Sigmoid curve.
o Quadratic polynomial curves.
o Cubic polynomial curves.

The simplest membership functions are formed using straight lines. These straight line membership functions have the advantage of simplicity.

o Triangular membership function: trimf.
o Trapezoidal membership function: trapmf.

- Two membership functions are built on the Gaussian distribution curve: a simple Gaussian curve and a two-sided composite of two different Gaussian curves. The two functions are gaussmf and gauss2mf.
- The generalized bell membership function is specified by three parameters and has the function name gbellmf.
- Sigmoidal membership function: sigmf.
- Polynomial based curves: Three related membership functions are the Z, S, and Pi curves, all named because of their shape (The functions zmf, smf and pimf).

Fuzzy Logic Toolbox also allows you to create your own membership functions.

Example:

```
x = (0:0.1:10)';
y1 = trapmf (x, [2 3 7 9]);
y2 = trapmf (x, [3 4 6 8]);
y3 = trapmf (x, [4 5 5 7]);
y4 = trapmf (x, [5 6 4 6]);
plot (x, [y1 y2 y3 y4]);
```

•Fuzzy Logic Operators

- Fuzzy Logic:
  - NOT (A) = 1 - A
  - A AND B = min(A, B)
  - A OR B = max(A, B)

https://sites.google.com/site/savitakumarisheoran79/soft-computing

- Fuzzy Logic NOT

A	NOT A
0	1
0.25	0.75
0.5	0.5
0.75	0.25
1	0

https://sites.google.com/site/savitakumarisheoran79/soft-computing

Fuzzy Logic AND

A AND B / B → A ↓	0	0.25	0.5	0.75	1.0
0	0	0	0	0	0
0.25	0	0.25	0.25	0.25	0.25
0.5	0	0.25	0.5	0.5	0.5
0.75	0	0.25	0.5	0.75	0.75
1	0	0.25	0.5	0.75	1

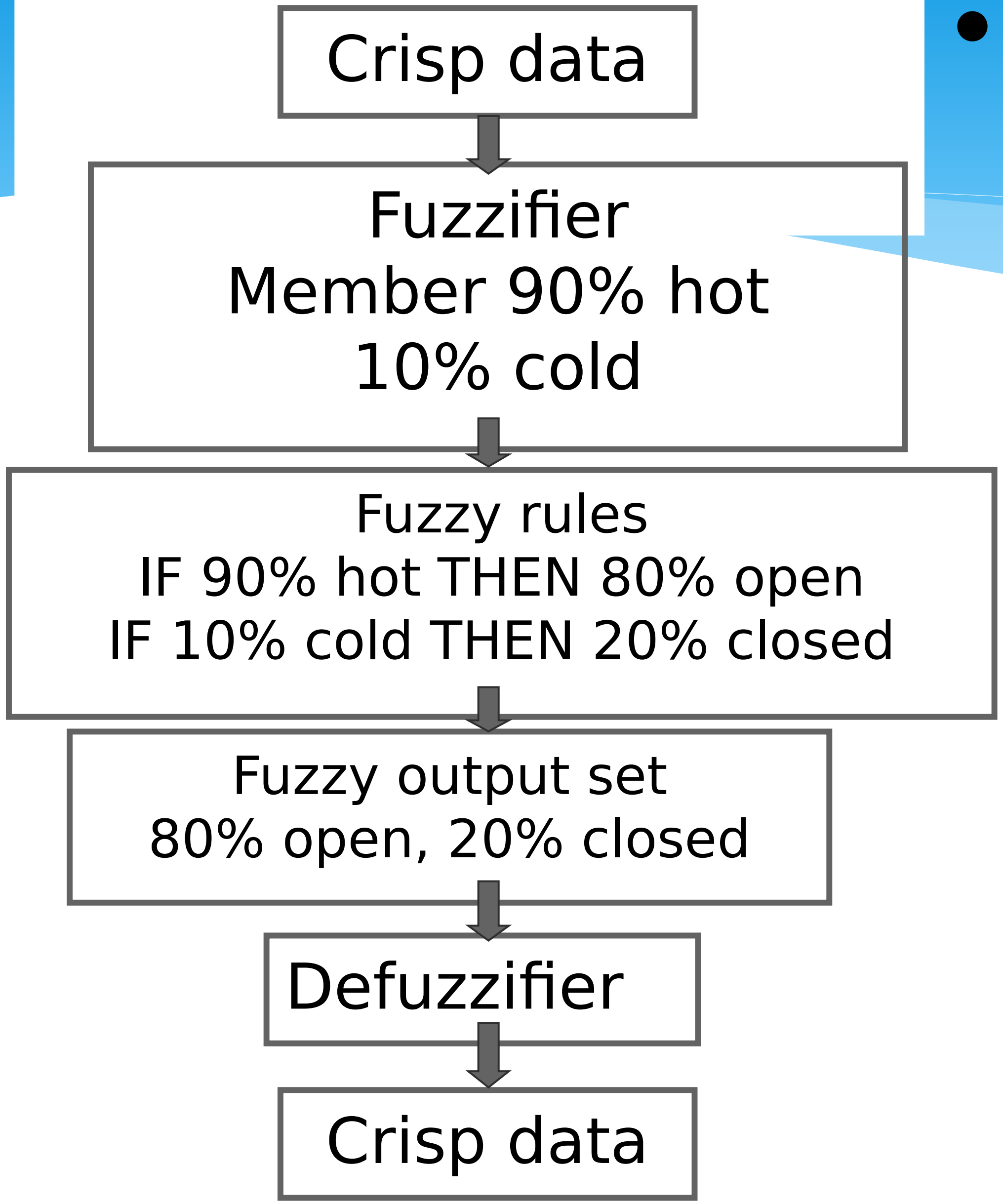
https://sites.google.com/site/savitakumarisheoran79/soft-computing

• Fuzzy Logic OR

A OR B / B / A	0	0.25	0.5	0.75	1.0
0	0	0.25	0.5	0.75	1.0
0.25	0.25	0.25	0.5	0.75	1.0
0.5	0.5	0.5	0.5	0.75	1.0
0.75	0.75	0.75	0.75	0.75	1.0
1	1.0	1.0	1.0	1.0	1.0

https://sites.google.com/site/savitakumarisheoran79/soft-computing

How the models work



Inputs converted to degrees of membership of fuzzy sets.

Fuzzy rules applied to get new sets of members.

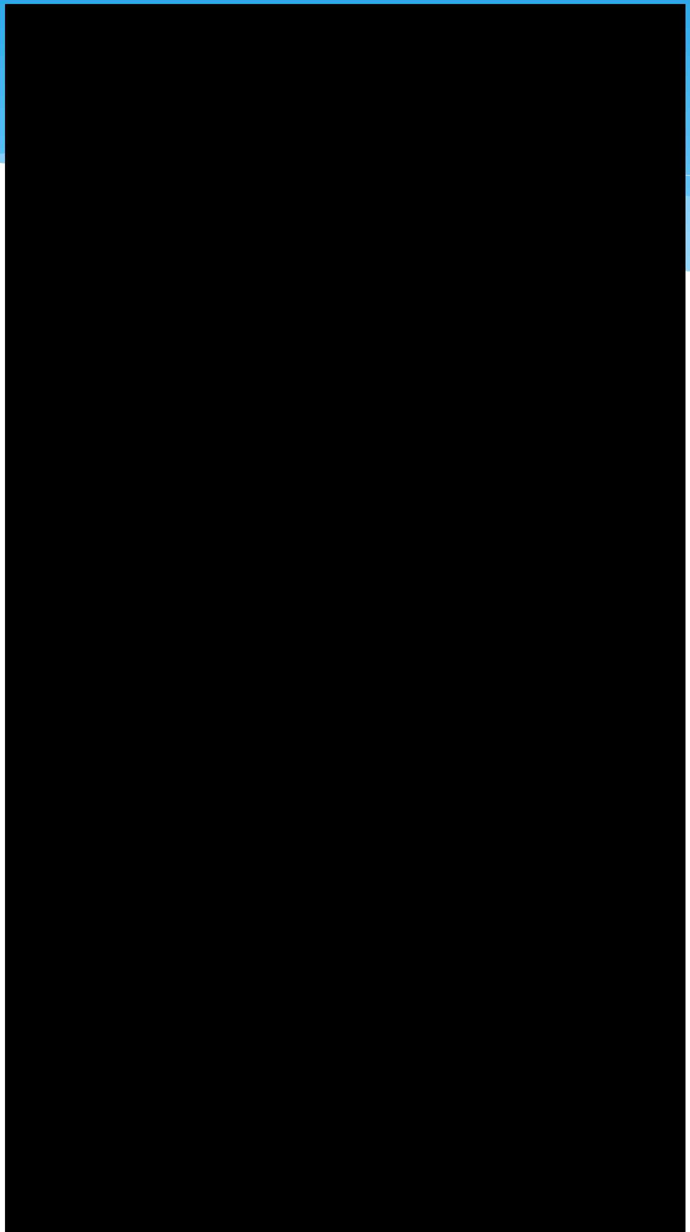
These sets are then converted back to real numbers.

http://www.powershow.com/view/11df5c-NGJhN/Introduction_to_Fuzzy_Set_Theory_powerpoint_ppt_presentation

•Fuzzy Controllers

- Used to control a physical system

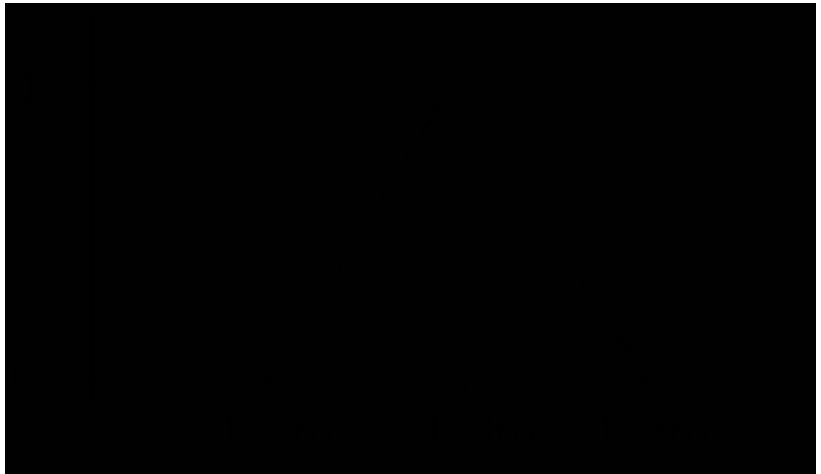
- Structure of a Fuzzy Controller



https://sites.google.com/site/savitakumarisheoran79/soft-computing

• Fuzzification

- Conversion of real input to fuzzy set values
- e.g. Medium (x) = {
  - ˜ 0 if x >= 1.90 or x < 1.70,
  - ˜ (1.90 - x)/0.1 if x >= 1.80 and x < 1.90,
  - ˜ (x- 1.70)/0.1 if x >= 1.70 and x < 1.80 }



https://sites.google.com/site/savitakumarisheoran79/soft-computing

Inference Engine

- Fuzzy rules
  - based on fuzzy premises and fuzzy consequences

- e.g.
  - If height is Short and weight is Light then feet are Small
  - Short(height) AND Light(weight) => Small(feet)

https://sites.google.com/site/savitakumarisheoran79/soft-computing

• Fuzzification & Inference Example

- If height is 1.7m and weight is 55kg
  - what is the value of Size(feet)

https://sites.google.com/site/savitakumarisheoran79/soft-computing

• Defuzzification

- Rule base has many rules
  - so some of the output fuzzy sets will have membership value > 0

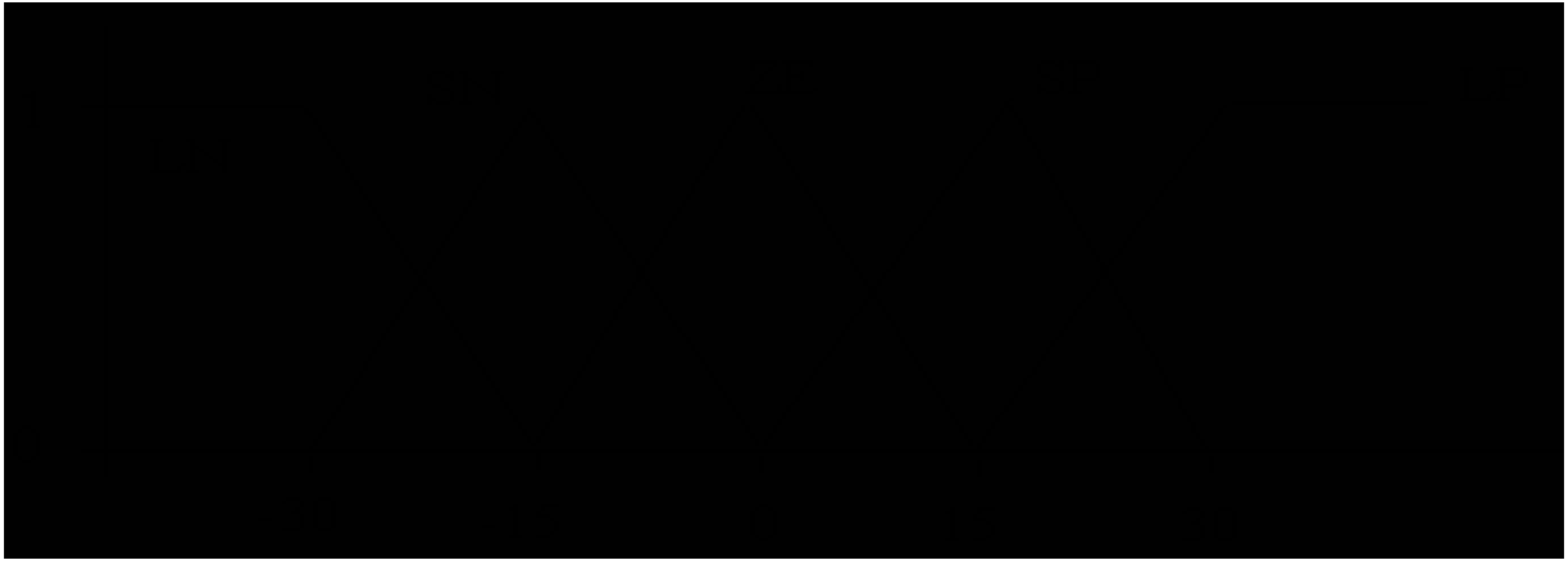
  - Defuzzify to get a real value from the fuzzy outputs
    - One approach is to use a centre of gravity method

https://sites.google.com/site/savitakumarisheoran79/soft-computing

• Fuzzy Control Example

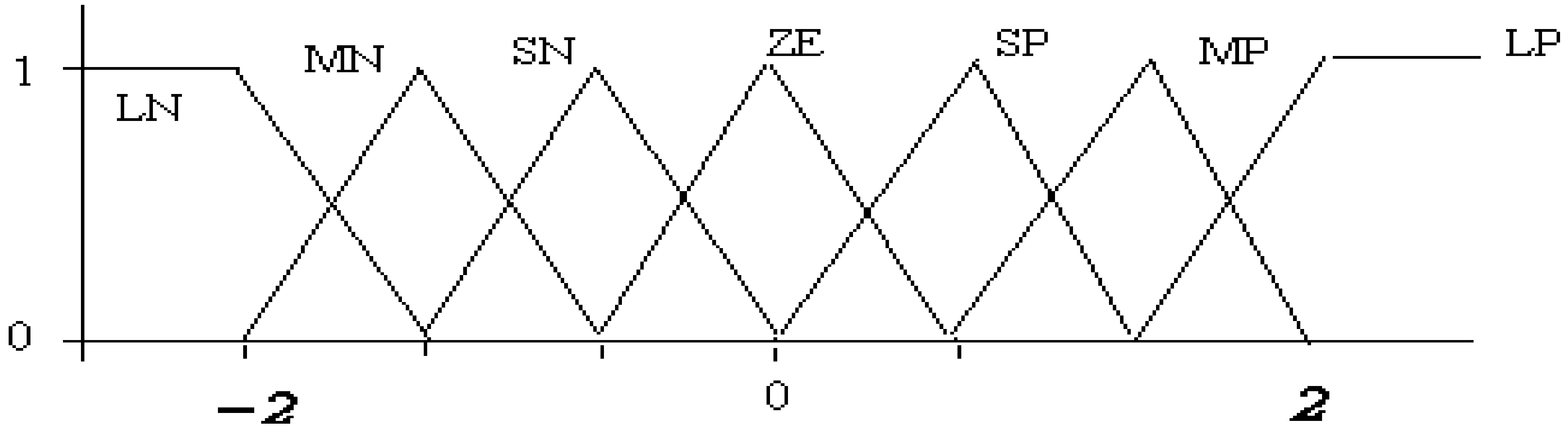
• Input Fuzzy Sets

- Angle:- -30 to 30 degrees



• Output Fuzzy Sets

- Car velocity:- -2.0 to 2.0 meters per second



https://sites.google.com/site/savitakumarisheoran79/soft-computing

• Fuzzy Rules

- If Angle is Zero then output ?
- If Angle is SP then output ?
- If Angle is SN then output ?
- If Angle is LP then output ?
- If Angle is LN then output ?

• Fuzzy Rule Table

Angle	Output Velocity
LN	MP
SN	SP
ZE	ZE
SP	SN
LP	MN

• Extended System

- Make use of additional information
  - angular velocity:- -5.0 to 5.0 degrees/ second
- Gives better control

https://sites.google.com/site/savitakumarisheoran79/soft-computing

•New Fuzzy Rules

- Make use of old Fuzzy rules for angular velocity Zero
- If Angle is Zero and **Angular vel is Zero**
  - then output Zero velocity
- If Angle is SP and **Angular vel is Zero**
  - then output SN velocity
- If Angle is SN and **Angular vel is Zero**
  - then output SP velocity

• Table format

AngleVel Angle	LN	SN	ZE	SP	LP
LN			MP		
SN			SP		
ZE			ZE		
SP			SN		
LP			MN		

https://sites.google.com/site/savitakumarisheoran79/soft-computing

•Complete Table

- When angular velocity is opposite to the angle do nothing
  - System can correct itself
- If Angle is SP and Angular velocity is SN
  - then output ZE velocity
- etc

https://sites.google.com/site/savitakumarisheoran79/soft-computing

• Example

- Inputs:10 degrees, -3.5 degrees/sec
- Fuzzified Values

- Inference Rules

- Output Fuzzy Sets

- Defuzzified Values

https://sites.google.com/site/savitakumarisheoran79/soft-computing

•Fuzzy Reasoning

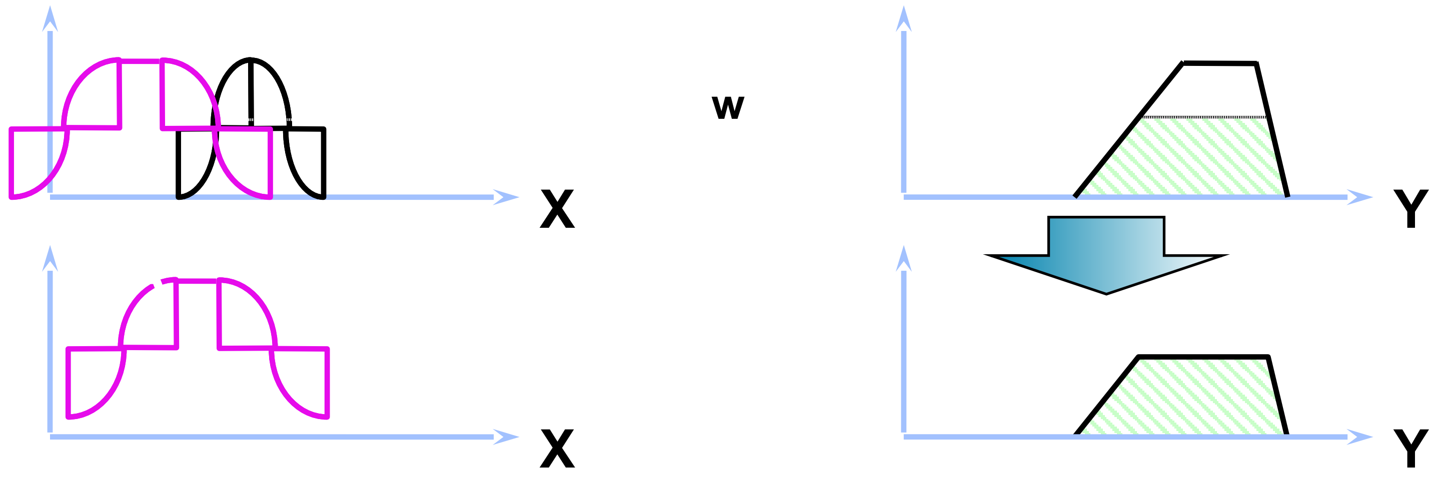
- Single rule with single antecedent

  Rule: if x is A then y is B

  Fact: x is A’

  Conclusion: y is B’

- Graphic Representation:



https://sites.google.com/site/savitakumarisheoran79/soft-computing

•Fuzzy Reasoning

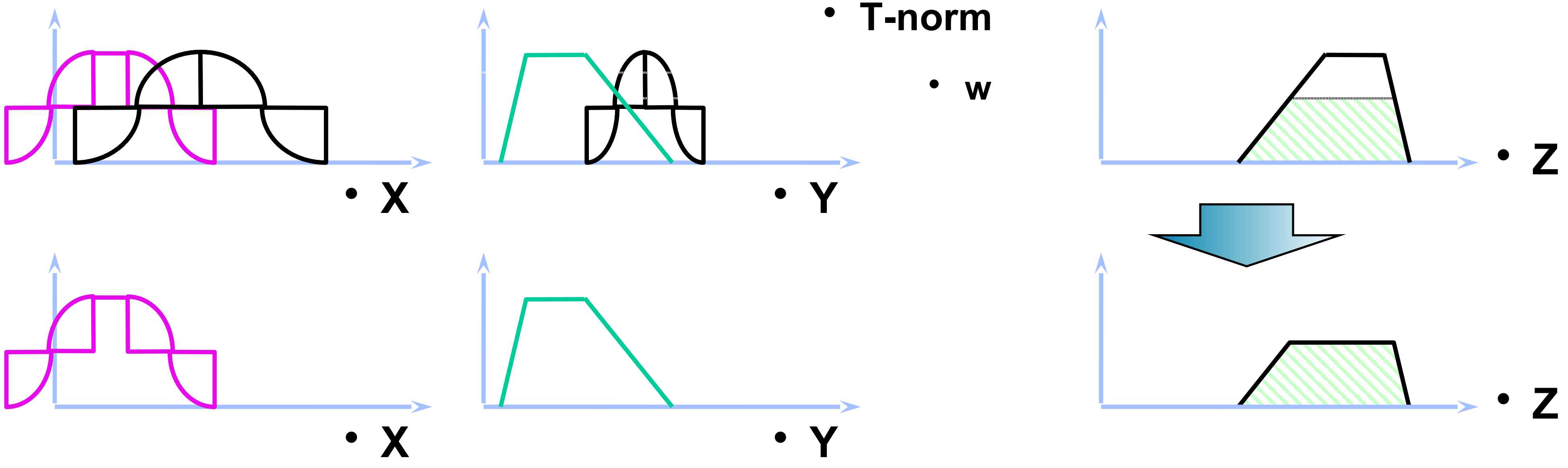
- Single rule with multiple antecedent

  Rule: if x is A and y is B then z is C

  Fact: x is A' and y is B'

  Conclusion: z is C'

- Graphic Representation:



https://sites.google.com/site/savitakumarisheoran79/soft-computing

•Fuzzy Reasoning

- Multiple rules with multiple antecedent

  Rule 1: if x is A_1 and y is B_1 then z is C_1

  Rule 2: if x is A_2 and y is B_2 then z is C_2

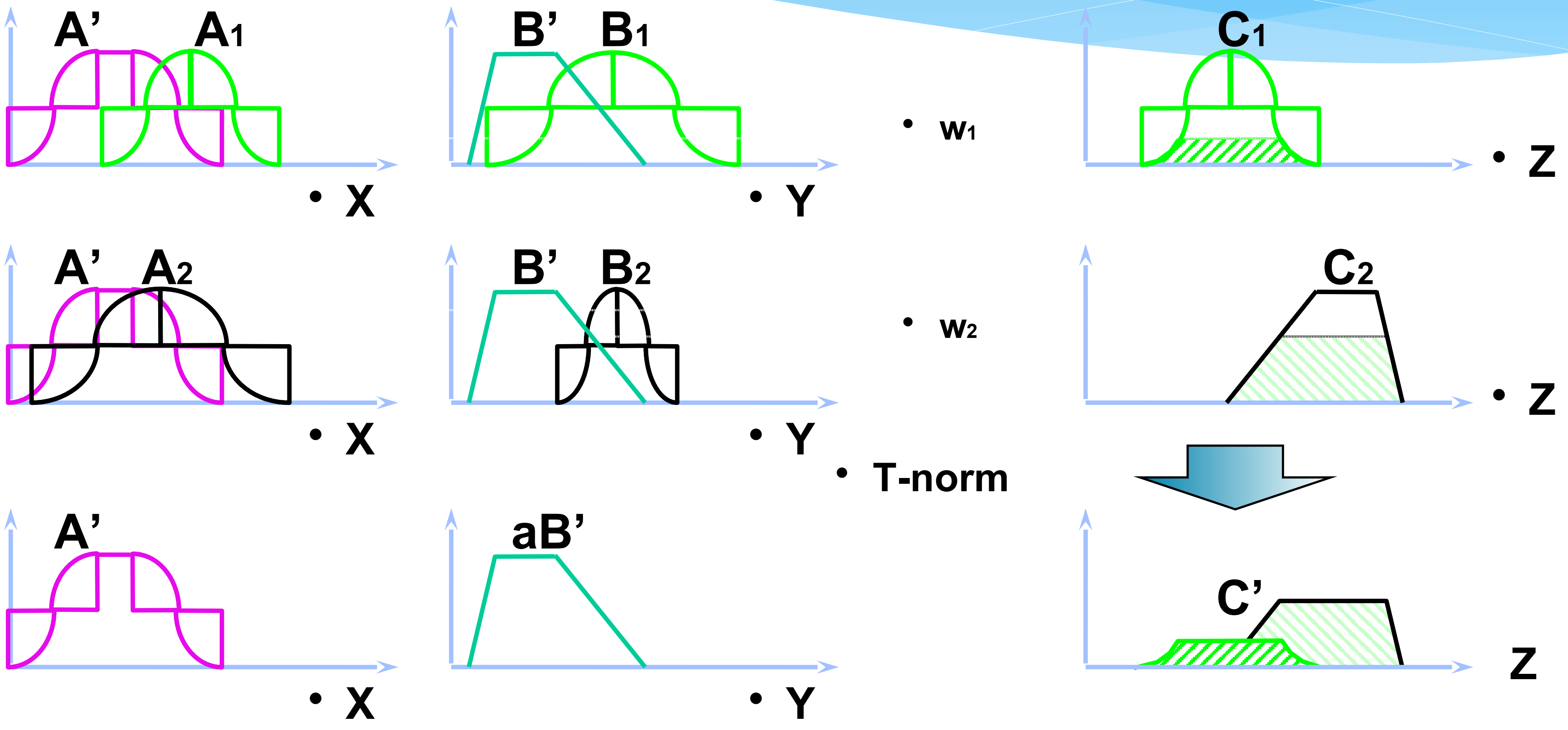
  Fact: x is A' and y is B'

  Conclusion: z is C'

- Graphic Representation: (next slide)

• Fuzzy Reasoning

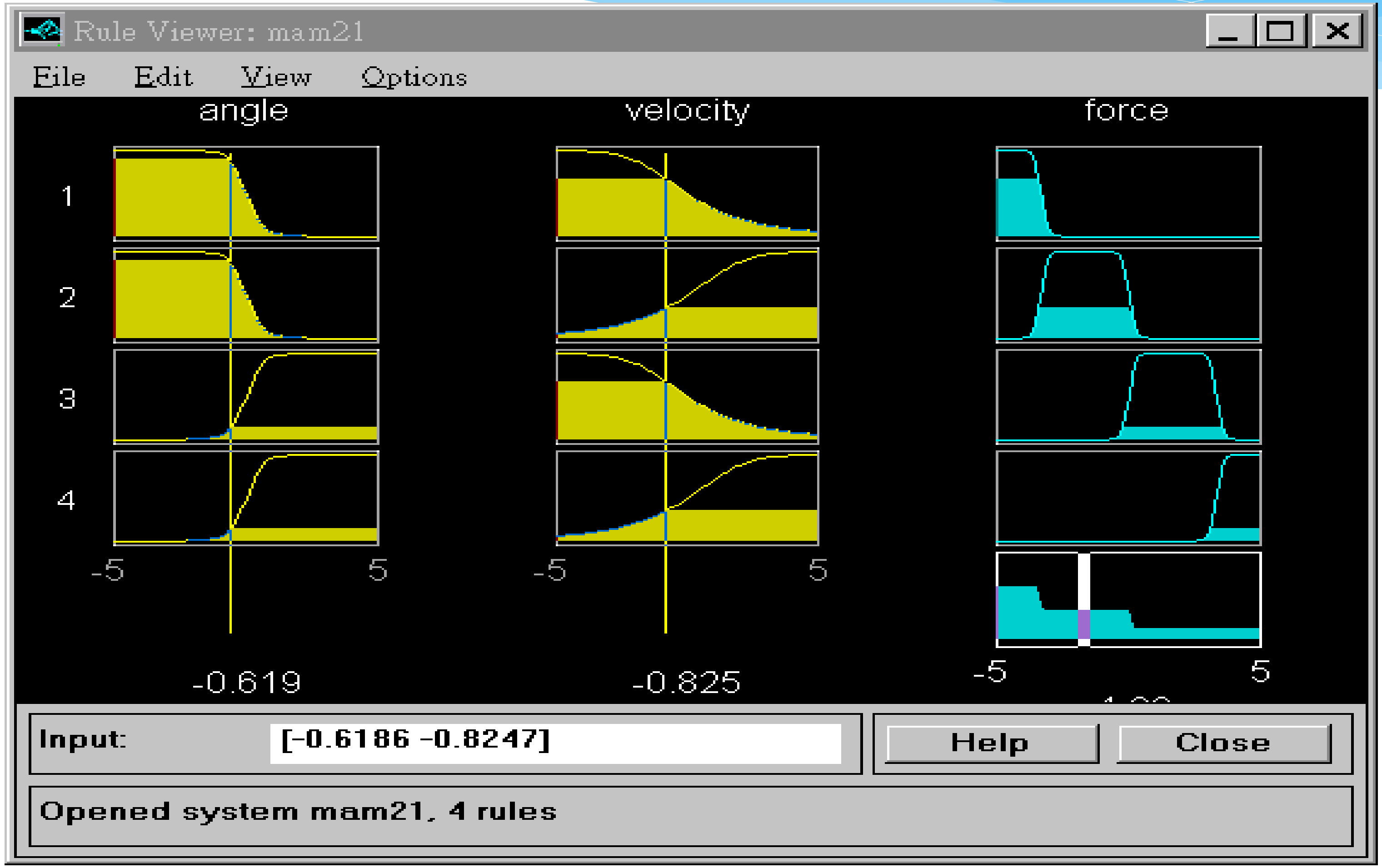
- Graphics representation:



https://sites.google.com/site/savitakumarisheoran79/soft-computing

Fuzzy Reasoning: MATLAB Demo

- >> rule view mam21



https://sites.google.com/site/savitakumarisheoran79/soft-computing

• Other Variants

- Some terminology:
  - Degrees of compatibility (match)
  - Firing strength
  - Qualified (induced) MFs
  - Overall output MF

Fuzzy Inference

Contents

- Mamdani Fuzzy Inference
  - Fuzzification of the input variables
  - Rule evaluation
  - Aggregation of the rule outputs
  - Defuzzification
- Sugeno Fuzzy Inference
- Mamdani or Sugeno?

• Mamdani Fuzzy Inference

- The most commonly used fuzzy inference technique is the so-called **Mamdani** method.
- In 1975, Professor Ebrahim Mamdani of London University built one of the first fuzzy systems to control a steam engine and boiler combination. He applied a set of fuzzy rules supplied by experienced human operators.
- The Mamdani-style fuzzy inference process is performed in four steps:

1. Fuzzification of the input variables
2. Rule evaluation (inference)
3. Aggregation of the rule outputs (composition)
4. Defuzzification.

https://sites.google.com/site/savitakumarisheoran79/soft-computing

• Mamdani Fuzzy Inference

We examine a simple two-input one-output problem that includes three rules:

Rule: 1	IF x is A3	OR	y is B1	THEN	z is C1
Rule: 2	IF x is A2	AND	y is B2	THEN	z is C2
Rule: 3	IF x is A1			THEN	z is C3

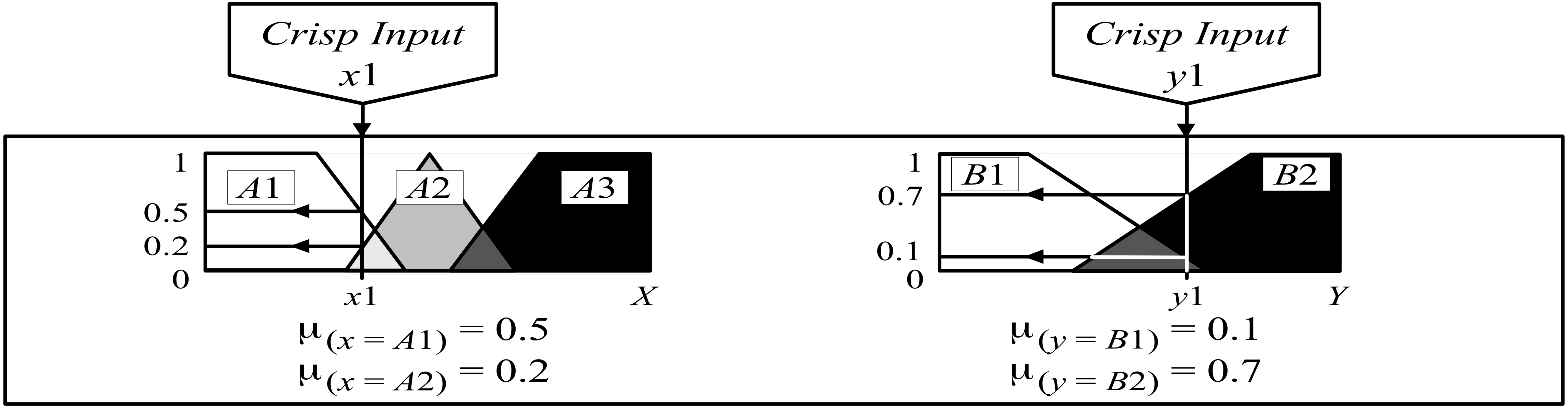
Real-life example for these kinds of rules:

Rule: 1	IF project_funding is adequate	OR	project_staffing is small	THEN	risk is low
Rule: 2	IF project_funding is marginal	AND	project_staffing is large	THEN	risk is normal
Rule: 3	IF project_funding is inadequate			THEN	risk is high

https://sites.google.com/site/savitakumarisheoran79/soft-computing

•Step 1: Fuzzification

- The first step is to take the crisp inputs, x1 and y1 (*project funding* and *project staffing*), and determine the degree to which these inputs belong to each of the appropriate fuzzy sets.



https://sites.google.com/site/savitakumarisheoran79/soft-computing

• Step 2: Rule Evaluation

- The second step is to take the fuzzified inputs, $\mu_{(x=A1)} = 0.5$, $\mu_{(x=A2)} = 0.2$, $\mu_{(y=B1)} = 0.1$ and $\mu_{(y=B2)} = 0.7$, and apply them to the antecedents of the fuzzy rules.
- If a given fuzzy rule has multiple antecedents, the fuzzy operator (AND or OR) is used to obtain a single number that represents the result of the antecedent evaluation.

RECALL: To evaluate the disjunction of the rule antecedents, we use the **OR** fuzzy operation. Typically, fuzzy expert systems make use of the classical fuzzy operation union:

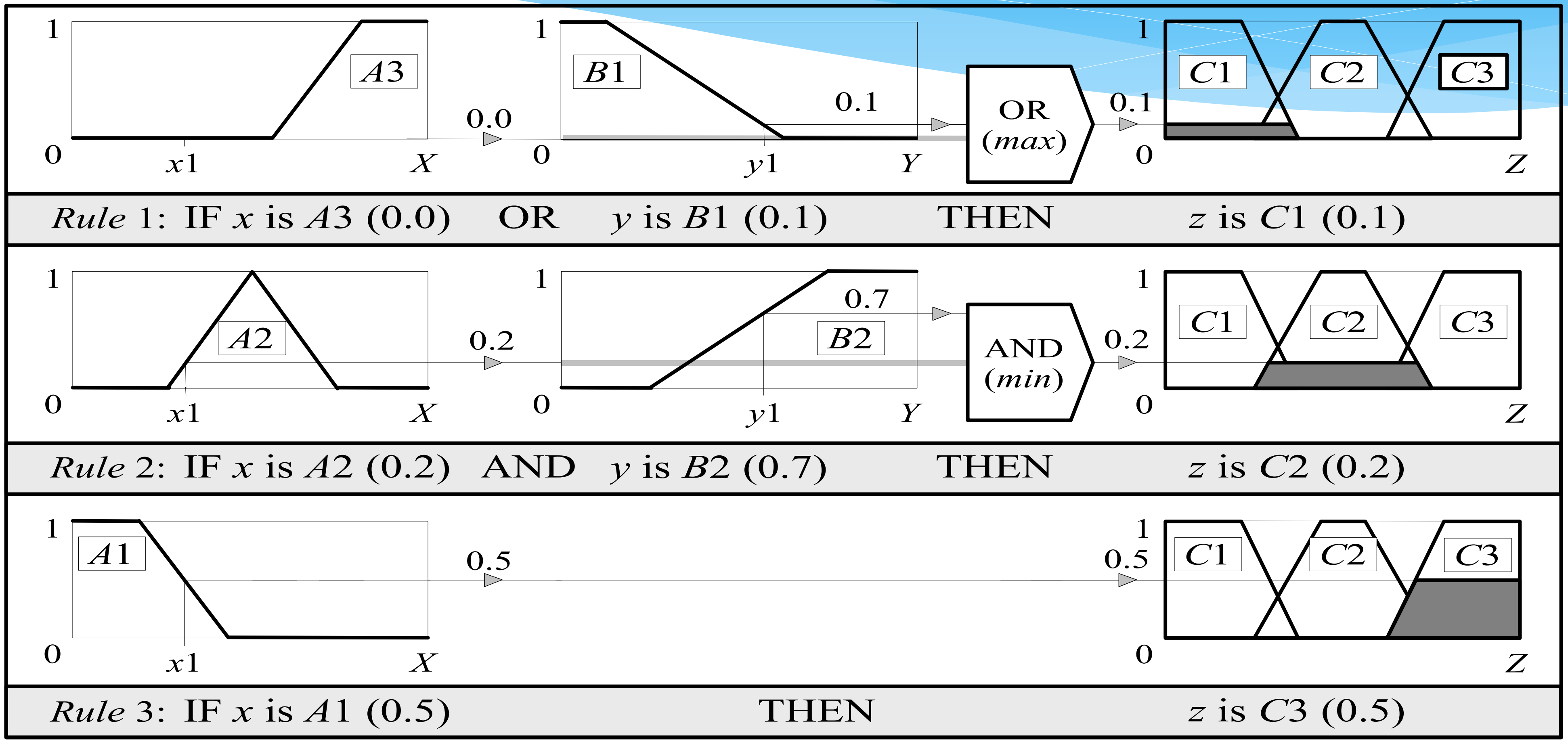
$$\mu_{A \cup B}(x) = \max\,[\mu_A(x),\ \mu_B(x)]$$

Similarly, in order to evaluate the conjunction of the rule antecedents, we apply the **AND** fuzzy operation intersection:

$$\mu_{A \cap B}(x) = \min\,[\mu_A(x),\ \mu_B(x)]$$

https://sites.google.com/site/savitakumarisheoran79/soft-computing

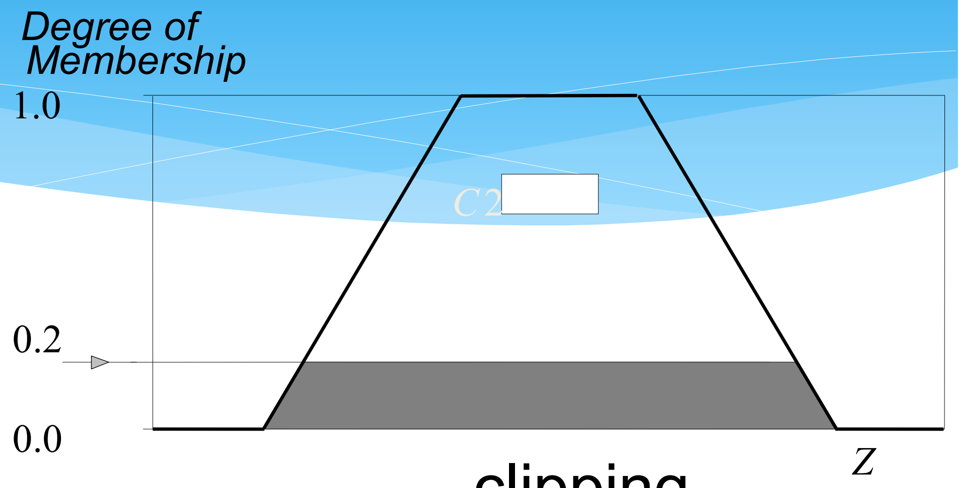
• Step 2: Rule Evaluation



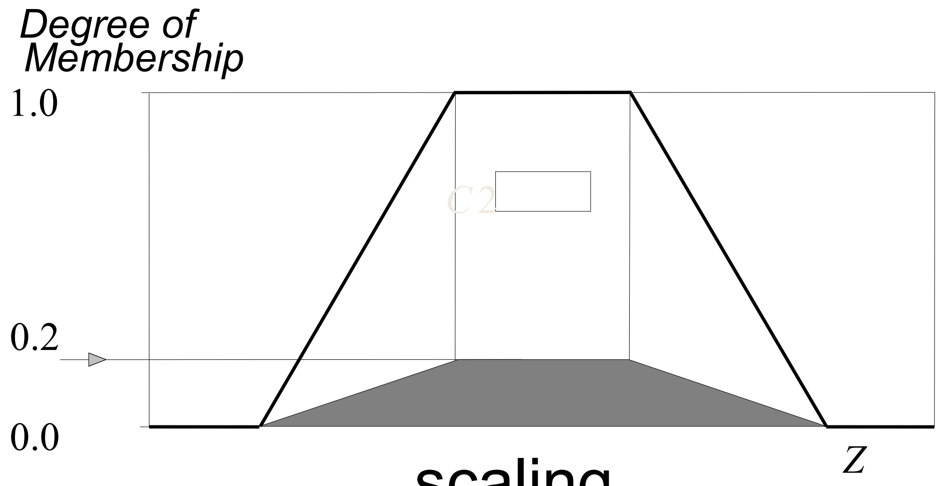
https://sites.google.com/site/savitakumarisheoran79/soft-computing

• Step 2: Rule Evaluation

- Now the result of the antecedent evaluation can be applied to the membership function of the consequent.
- The most common method is to cut the consequent membership function at the level of the antecedent truth. This method is called **clipping** (alpha-cut).
  - Since the top of the membership function is sliced, the clipped fuzzy set loses some information.
  - However, clipping is still often preferred because it involves less complex and faster mathematics, and generates an aggregated output surface that is easier to defuzzify.
- While clipping is a frequently used method, **scaling** offers a better approach for preserving the original shape of the fuzzy set.
  - The original membership function of the rule consequent is adjusted by multiplying all its membership degrees by the truth value of the rule antecedent.
  - This method, which generally loses less information, can be very useful in fuzzy expert systems.



clipping

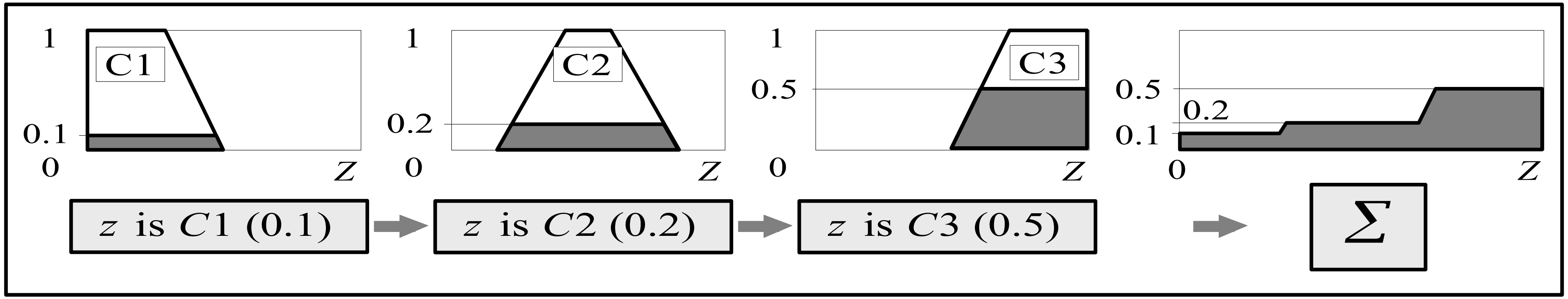


scaling

https://sites.google.com/site/savitakumarisheoran79/soft-computing

• Step 3: Aggregation of the Rule Outputs

- Aggregation is the process of unification of the outputs of all rules.
- We take the membership functions of all rule consequents previously clipped or scaled and combine them into a single fuzzy set.
- The input of the aggregation process is the list of clipped or scaled consequent membership functions, and the output is one fuzzy set for each output variable.



https://sites.google.com/site/savitakumarisheoran79/soft-computing

Step 4: Defuzzification

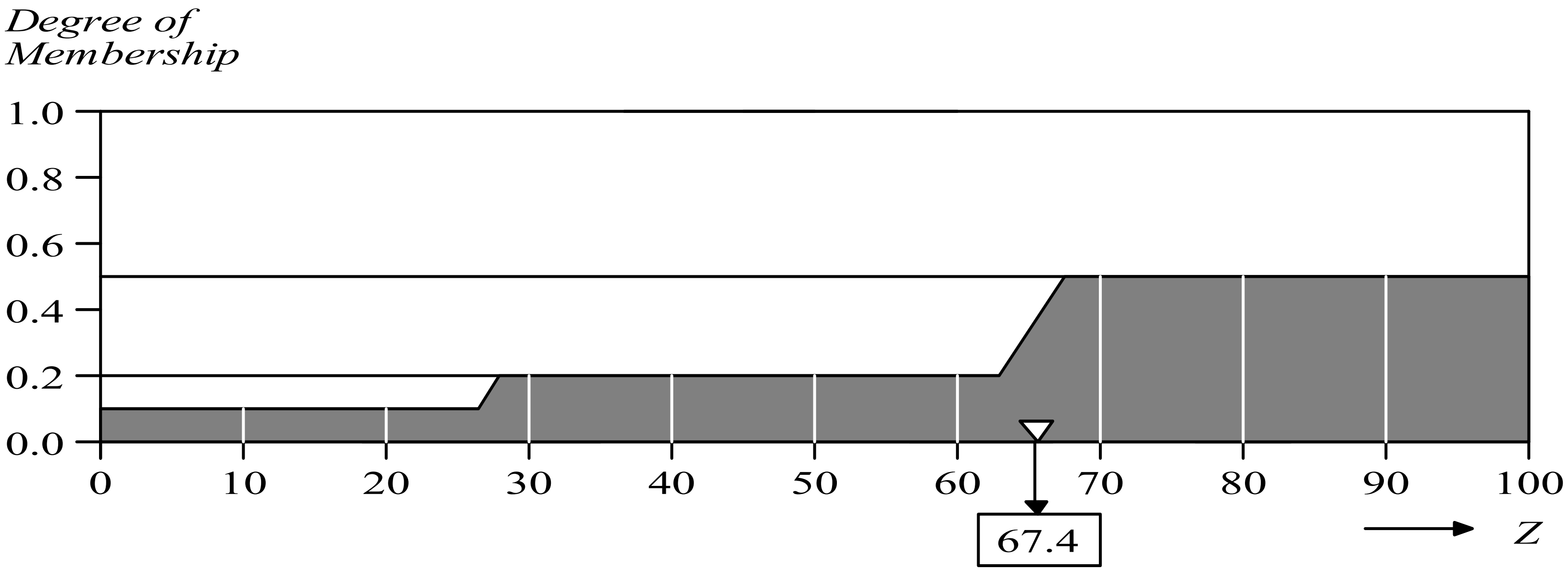
- The last step in the fuzzy inference process is defuzzification.
- Fuzziness helps us to evaluate the rules, but the final output of a fuzzy system has to be a crisp number.
- The input for the defuzzification process is the aggregate output fuzzy set and the output is a single number.
- There are several defuzzification methods, but probably the most popular one is the **centroid technique**. It finds the point where a vertical line would slice the aggregate set into two equal masses. Mathematically this **centre of gravity** (**COG**) can be expressed as:

$$COG = \frac{\int_a^b \mu_A(x)\, x\, dx}{\int_a^b \mu_A(x)\, dx}$$

https://sites.google.com/site/savitakumarisheoran79/soft-computing

• Step 4: Defuzzification

- Centroid defuzzification method finds a point representing the centre of gravity of the aggregated fuzzy set *A*, on the interval [*a, b*].
- A reasonable estimate can be obtained by calculating it over a sample of points.



$$COG = \frac{(0+10+20)\times 0.1 + (30+40+50+60)\times 0.2 + (70+80+90+100)\times 0.5}{0.1+0.1+0.1+0.2+0.2+0.2+0.2+0.5+0.5+0.5+0.5} = 67.4$$

https://sites.google.com/site/savitakumarisheoran79/soft-computing

•Sugeno Fuzzy Inference

- Mamdani-style inference, as we have just seen, requires us to find the centroid of a two-dimensional shape by integrating across a continuously varying function. In general, this process is not computationally efficient.

- Michio Sugeno suggested to use a single spike, a singleton, as the membership function of the rule consequent.

- A singleton, or more precisely a fuzzy singleton, is a fuzzy set with a membership function that is unity at a single particular point on the universe of discourse and zero everywhere else.

• Sugeno Fuzzy Inference

- Sugeno-style fuzzy inference is very similar to the Mamdani method.
- Sugeno changed only a rule consequent: instead of a fuzzy set, he used a mathematical function of the input variable.
- The format of the **Sugeno-style fuzzy rule** is

IF ***x* is *A*** **AND** ***y* is *B*** **THEN** ***z* is *f*(*x*, *y*)**

where:

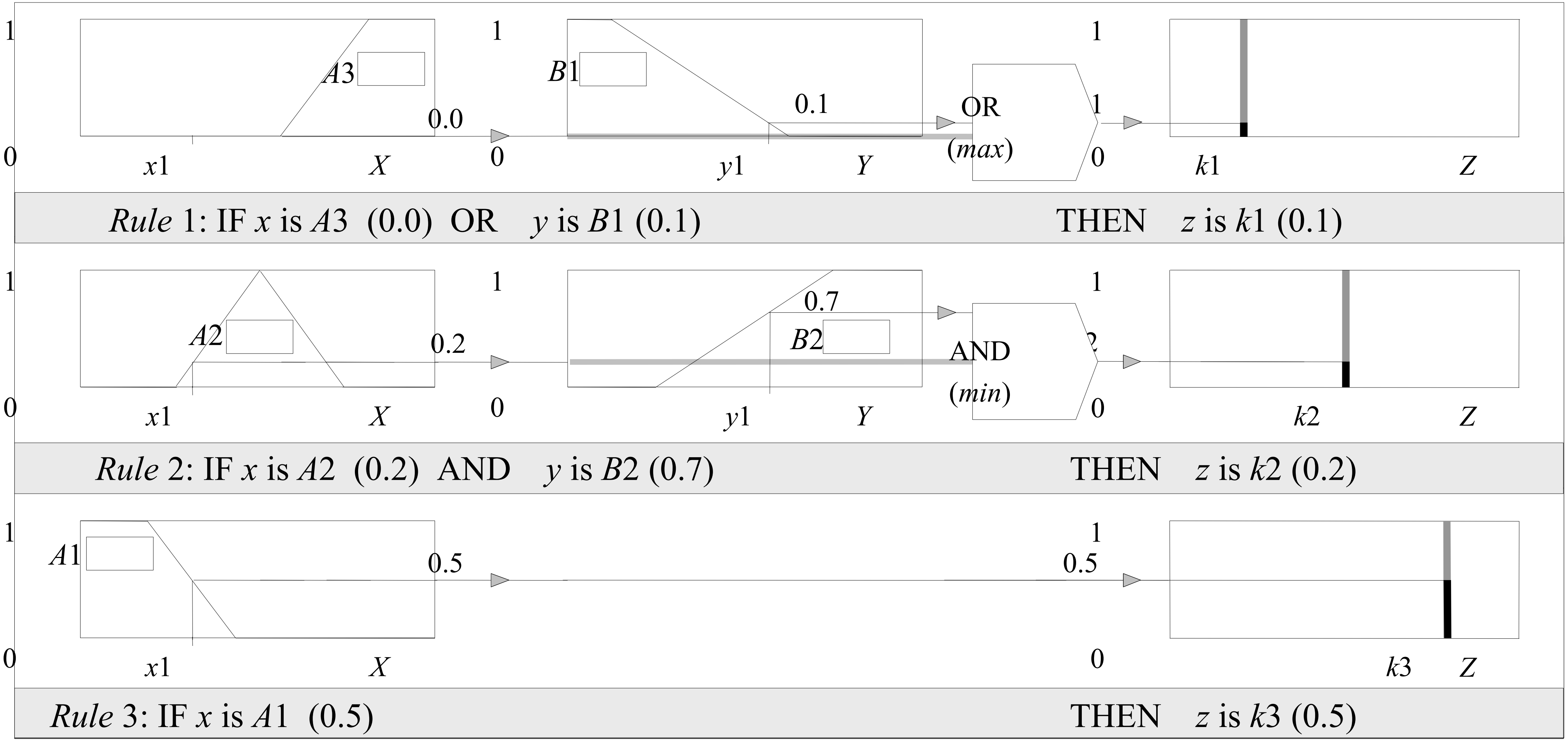
- *x*, *y* and *z* are linguistic variables;
- A and *B* are fuzzy sets on universe of discourses *X* and *Y*, respectively;
- *f* (*x*, *y*) is a mathematical function.

- The most commonly used **zero-order Sugeno fuzzy model** applies fuzzy rules in the following form:

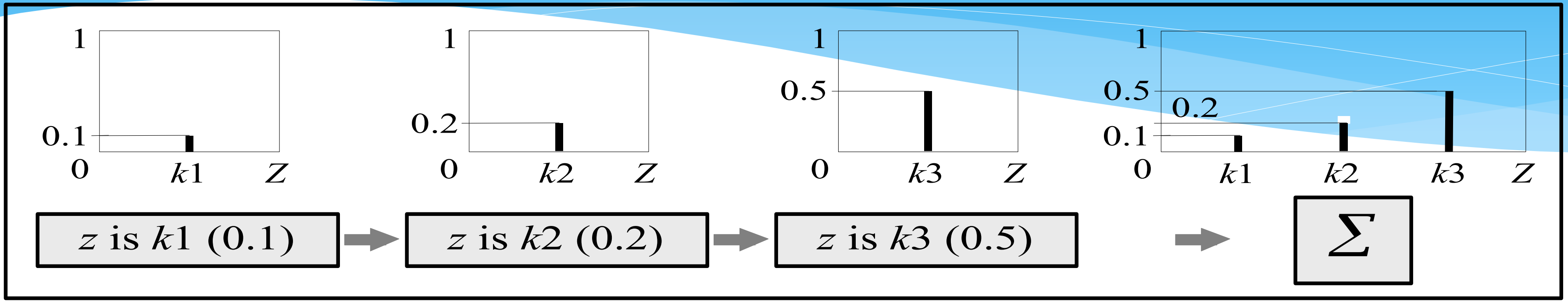
IF ***x* is *A*** **AND** ***y* is *B*** **THEN** ***z* is *k***

- where *k* is a constant.
- In this case, the output of each fuzzy rule is constant and all consequent **membership functions** are represented by **singleton spikes.**

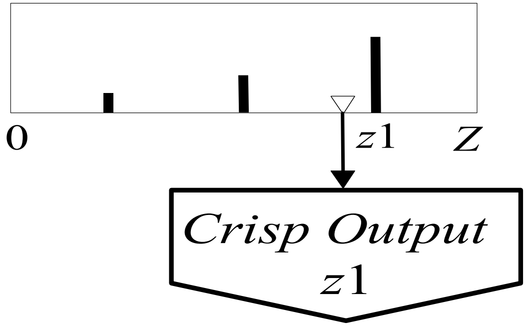
• Sugeno Rule Evaluation



• Sugeno Aggregation and Defuzzification



COG becomes Weighted Average (WA)



$$WA = \frac{\mu(k1) \times k1 + \mu(k2) \times k2 + \mu(k3) \times k3}{\mu(k1) + \mu(k2) + \mu(k3)} = \frac{0.1 \times 20 + 0.2 \times 50 + 0.5 \times 80}{0.1 + 0.2 + 0.5} = 65$$

Mamdani or Sugeno?

- Mamdani method is widely accepted for capturing expert knowledge. It allows us to describe the expertise in more intuitive, more human-like manner. However, Mamdani-type fuzzy inference entails a substantial computational burden.

- On the other hand, Sugeno method is computationally effective and works well with optimization and adaptive techniques, which makes it very attractive in control problems, particularly for dynamic nonlinear systems.

Fuzzy Applications to Deep Neural Networks

Fuzzy Neural Network with Generalized Hamming Network

- Generalized hamming network with induced fuzzy XOR
- A generalized hamming network (GHN) is any networks consisting of neurons, whose outputs

  $\mathbf{h} \in \mathcal{H}^L$ are related to neuron inputs $\mathbf{x} \in \mathcal{H}^L$ and weights $\mathbf{w} \in \mathcal{H}^L$ by element-wise $\boxed{\mathbf{h} = \mathbf{x} \oplus^L \mathbf{w}}$.
- For the generalized case where $U = \mathbb{R}$, the fuzzy membership μ can be defined by a sigmoid function such as logistic, tanh or any function : $U \to I$. In this work authors adopt the logistic function $\mu(a) = \frac{1}{1+\exp(0.5-a)}$ and the resulting fuzzy XOR connective is given by following membership function:

$$\mu_R(i, j) = \frac{1}{1 + \exp\left(0.5 - \mu^{-1}(i) \oplus \mu^{-1}(j)\right)},$$

- Where $\mu^{-1}(a) = -\ln(\frac{1}{a} - 1) + \frac{1}{2}$ is the inverse of $\mu(a)$ $\mu : U \to I : \mu(a) = i, \mu(b) = j$

generalized hamming distance (GHD), denoted by $\oplus$.

Revisit Fuzzy Neural Network: Demystifying Batch Normalization and ReLU with Generalized Hamming Network, Lixin Fan, 31st Conference on Neural Information Processing Systems (NIPS 2017), Long Beach, CA, USA., pp. 1-10.

Fuzzy Logic and Autoencoder Neural Network to improve data privacy

- Their proposed model is divided into two tasks.
  - The first task is based on hiding the sensitive information using fuzzy membership functions – Triangle, Gaussian and S-shaped.
  - The second task is t data to different o autoencoder.

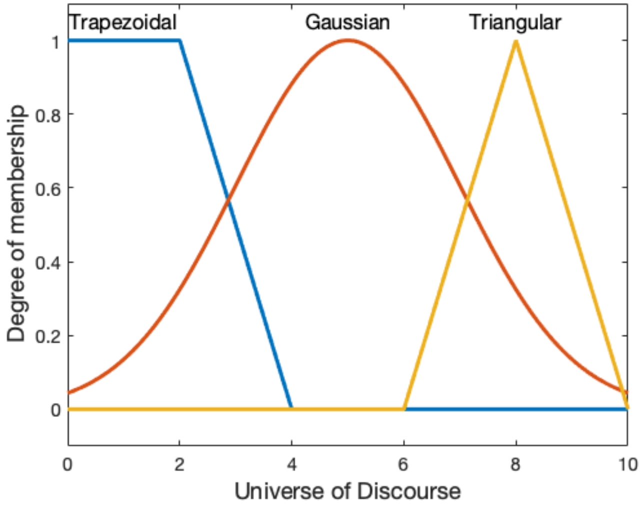


TABLE II
ACCURACY AND LOSS VALUE OF DROP COLUMN DATA SET AND WITH SPARSITY CONSTRAINTS

Membership function	Loss function	Loss value	Accuracy
Gaussian	Mean_Absolute	0.1313	0.7409
Gaussian	MSE	0.1260	**0.8734**
Gaussian	Categorical_crossentropy	0.9451	0.0058
Gaussian	Logcosh	0.1263	0.8122
Gaussian	Hinge	0.7091	0.1135
S-shaped	Mean_Absolute	0.1326	0.8180
S-shaped	MSE	0.1261	0.8457
S-shaped	Categorical_crossentropy	1.0220	0.0291
S-shaped	Logcosh	0.1266	0.8239
S-shaped	Hinge	0.6954	0.1397
Triangular	Mean_Absolute	0.1313	0.8122
Triangular	MSE	0.1278	0.8457
Triangular	Categorical_crossentropy	0.0936	0.0015
Triangular	Logcosh	0.1267	0.8384
Triangular	Hinge	0.7059	0.1266