Fuzzy Logic and the Calculus of Fuzzy If-Then Rules

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Abstract

In contrast to classical logical systems, fuzzy logic is aimed at a formalization of modes of reasoning which are approximate rather than exact. Basically, a fuzzy logical system may be viewed as a result of fuzzifying a standard logical system. Thus, one may speak of fuzzy predicate logic, fuzzy modal logic, fuzzy default logic, fuzzy multivalued logic, fuzzy epistemic logic, etc. In this perspective, fuzzy logic is essentially a union of fuzzified logical systems in which precise reasoning is viewed as a limiting case of approximate reasoning.

During the past few years, fuzzy logic has been finding a rapidly growing number of applications in fields ranging from consumer electronics and photography to medical diagnosis systems and securities management funds. What is exploited in most of these applications is the tolerance for imprecision. In effect, the operative principle of fuzzy logic is: Precision is costly. Minimize the precision needed to perform a task.

Fuzzy logic provides a wide variety of concepts and techniques for representing and inferring from knowledge which is imprecise, uncertain or lacking in reliability. At this juncture, however, what is used in most practical applications is a relatively restricted and yet important part of fuzzy logic centering on the use of fuzzy if-then rules. This part of fuzzy logic is referred to as the calculus of fuzzy if-then rules — or CFR, for short — because it constitutes a fairly self-contained collection of concepts and methods for dealing with varieties of knowledge which can be represented in the form of a system of if-then rules in which the antecedents and/or consequents are fuzzy rather than crisp.

The rules considered in CFR fall into two categories: (a) categorical (unqualified) rules, e.g., Y is B if X is A, where A and B are fuzzy predicates; and (b) qualified rules, e.g., usually (Y is B) if X is A, (Y is B if X is A) is very true; (Y is B) is quite possible if X is A, and Y is B if X is A unless X is E.

The agenda of the calculus of fuzzy if-then rules may be set down as follows:

(1) interpretation of a fuzzy if-then rule; (2) interpretation of a collection of fuzzy if-then rules; (3) representation of propositions in a natural language as collections of fuzzy if-then rules; (4) inference from a collection of fuzzy if-then rules (interpolation); (5) manipulation of blocks of fuzzy if-then rules; (6) algebraic operations on fuzzy if-then rules; and, (7) induction of fuzzy if-then rules from observations.

The importance of the calculus of fuzzy if-then rules stems from the fact that much of human knowledge lends itself to representation in the form of a hierarchy of fuzzy if-then rules. And what is of central importance in practical applications is that the fuzziness of antecedents eliminates the need for a precise match with the input. Thus, in a fuzzy rule-based system each rule is fired to a degree which is a function of the degree of match between its antecedent and the input. The mechanism of imprecise matching provides a basis for interpolation. Interpolation, in turn, serves to minimize the number of fuzzy if-then rules which are needed to describe an input-output relation. As a consequence, fuzzy rule-based systems are simpler, cheaper, and more robust than their conventional counterparts. In our talk, the primary emphasis is on the problem of inference and rule induction in the context of (a) categorical rules; (b) usuality-qualified rules; (c) possibility-qualified rules; and, (d) rules with exceptions.