Q2. A company knows that the demand of one of its most important products 1,2,3,4 over the next 4 months. The company must plan the production of 10 units. If any production appears in a period, there is a setup cost of ₹ 5700k. In addition there is a production cost of ₹ 5700k for each produced unit. If a unit is put in the inventory there is an inventory cost of ₹1650k per unit. 5 units at most can be produced in a month and at most 4 units can be put in the inventory. How should the company plan their production to satisfy the demand and minimize production & inventory costs. There are no units in inventory at the beginning of month 1. Solve using dp after formulation.

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setup cost = ₹ 5700 k

Production cost per unit = ₹ 5700k

Inventory cost per unit (I_c) = ₹ 1650k

Cost of making 'p' products

~~$c(p) = 5700 +$~~ All units are in multiple of 10^3.

$c(p) = 5700 + $ ~~5700 p~~ $5700 p$.

$c(0) = 0$ ∴ no setup cost is needed.

no. of items in inventory at the beginning of month m, is i_m.

$$i_m = i_{m-1} + P_{m-1} - d_{m-1}$$

where P_{m-1} is the production in month m-1 and d_{m-1} is " demand in month m-1.

To formulate. let

$f_m(x)$ ~~$f_{(m)}$~~ denote the ~~cost~~ minimum cost at month m, when x items are available in the inventory

~~$f_m(p) = \min_p P_c(p) + I_c(p)$~~

$$f_m(x) = \min_p \left[c(p) + I_c (x+p-d_m) + f_{m+1}(x+p-D_m) \right]$$

$$p \le 5, \quad x \le 4, \quad x+p-d_m \le 4.,$$

$$x, p, d_m \ge 0.$$

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Month 4 $d_4 = 4$.

	$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	f_4
					28500	28500
$x=0$						24450
$x=1$				24450		20400.
$x=2$			20400			
$x=3$		16350				16350.
$x=4$	660					

$\therefore$ month 4 is last $\Rightarrow$ $x+p = d_4 = 4$.

$c(4) = 5700 + 5700 \times 4 = ₹28500.$

$f_4(0) = \min\limits_{p} \left[c(p) + I_c(0+p-4) + 0 \right]$

$\quad = \min\limits_{p} \left[c(p) \right]$

$\quad = ₹28500$

$f_4(1) = c(3) = 22800.$

$f_4(2) = c(2) = 20400$

$f_4(3) = c(1) = 16350$

$f_4(4) = c(0) = 660.$

month 4 $x+p=4$ ∴ last month.

	p=0	p=1	p=2	p=3	P=4	f_4
x=0					28500	28500
x=1				22800		22800
x=2			17100			17100.
x=3		11400				11400
x=4	0					⓪ ← minimum.

$$f_4(0) = c(4) = 28500.$$
$$f_4(1) = c(3) = 22800$$
$$f_4(2) = c(2) = 17100$$
$$f_4(3) = c(1) = 11400$$
$$f_4(4) = c(0) = 0$$

$$c(5) = 6 \times 5700$$
$$= 34200$$

month 3

	p=0	p=1	p=2	p=3	p=4	p=5	f_3
x=0				51300	52950	54600	51300
x=1			45600	47250	48900	50550	45600.
x=2		39900	41550	43200	44850	40800	39900,
x=3	28500	35850	37500	39150	35100		28500.
x=4	24450	31800	33450	35100.			24450.

$$d_3 = 3$$
$$x+p-d \leq 4. \text{ for next stage}$$
$$\text{inventory.}$$
$$x+p \geq d.$$

$$f_3(0) = \min \{h(0,p)\}$$

$$h(a,p) = c(p) + I_c(a+p-d) \cdot 0 + \cancel{f_4(a+p-d)} f_4(a+p-d)$$

$$h(0,3) = c(3) + 0 + f_4(0)$$
$$= 22800 + 0 + 28500$$
$$= 51300.$$

$$h(0,4) = c(4) + I_c + f_4(1)$$
$$= 52950.$$

$$h(0,5) = c(5) + 2 \times I_c + f_4(2)$$
$$= 54600$$

$$h(1,2) = c(2) + 0 \times I + f_4(0)$$
$$= 17100 + 0 + 28500 = 45600.$$

$$h(1,3) = c(3) + I_c + f_4(1)$$
$$= 47250.$$

$$h(1,4) = c(4) + 2I_c + f_4(2)$$
$$= 48900.$$

$$h(1,5) = c(5) + 3I_c + f_4(3) = 50550$$

$$h(2,1) = c(1) + 0 I_c + f_4(0) = 39900.$$

$$h(2,2) = c(2) + I_c + f_4(1) = 41550.$$

$$h(2,3) = c(3) + 2I_c + f_4(2) = \cancel{31800} 43200$$

$$h(2,4) = c(4) + 3I_c + f_4(3) = 44850$$

$$h(2,5) = c(5) + 4I_c + f_4(4) = 40800.$$

$$h(3,0) = c(0) + 0 I_c + f_4(0) = 28500.$$

$$h(3,1) = c(1) + I_c + f_4(1) = 35850.$$

$$h(3,2) = c(2) + 2I_c + f_4(2) = 37500.$$

$$h(3,3) = c(3) + 3I_c + f_4(3) = \cancel{38} 39150$$

$$h(3,4) = c(4) + 4I_c + f_4(4) = 35100.$$

$$h(4,0) = c(0) + \otimes I_c + f_4(1) = 24450.$$

$$h(4,1) = c(1) + 2I_c + f_4(2) = 31800.$$

$$h(4,2) = c(2) + 3I_c + f_4(3) = 33450$$

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$h(4,3) = c(4) + 4I_c + f_4(4) = 35100.$

month 2

	p=0	p=1	p=2	p=3	p=4	p=5	f_2
x=0			68400.	70050	71700	67650	67650.
x=1		62700	64350	66000	61950	65250	61950.
x=2	51300	60300	56250	56950	59550		51300
x=3	47250	54600	50550	53850			47250.
x=4	43200	44850	48150				43200.

$d_2 = 2$ $x + p \geqslant 2$
$x + p - 2 \leq 4$.

$h(0,2) = c(2) + 0 \cdot I_c + f_3(0) = 68400.$
$h(0,3) = c(3) + I_c + f_3(1) = 70050$
$h(0,4) = c(4) + 2I_c + f_3(2) = 71700$
$h(0,5) = c(5) + 3I_c + f_3(3) = 67650.$
$h(1,1) = c(1) + 0 \cdot I_c + f_3(0) = 62700.$
$h(1,2) = c(2) + I_c + f_3(1) = 64350$
$h(1,3) = c(3) + 2I_c + f_3(2) = 66000$
$h(1,4) = c(4) + 3I_c + f_3(3) = 61950$
$h(1,5) = c(5) + 4I_c + f_3(4) = 65250$
$h(2,0) = c(0) + 0 \cdot I_c + f_3(0) = 51300$
$h(2,1) = c(1) + I_c + f_3(1) = 58650$
$h(2,2) = c(2) + 2I_c + f_3(2) = 60300$
$h(2,3) = c(3) + 3I_c + f_3(3) = 56250$

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$h(2,4) = c(4) + 4 \times I_c + f_3(4)$
$$= 59550$$

$h(3,0) = e(0) + I_c + f_3(1) = 47250$

$h(3,1) = c(1) + 2I_c + f_3(2) = 54600$

$h(3,2) = c(2) + 3I_c + f_3(3) = 50550$

$h(3,3) = c(3) + 4I_c + f_3(4) = 53850$

$h(4,0) = c(0) + 2I_c + f_3(2) = 43200$

$h(4,1) = c(1) + 3I_c + f_3(3) = 44850.$

$h(4,2) = c(2) + 4I_c + f_3(4) = 48150.$

month 1

$d_1 = 1.$ $\qquad x = 0, \qquad x + p - 1 \leq 4$

$\qquad\qquad\qquad\qquad\qquad x + p \geq 1.$

	p=0	p=1	p=2	p=3	p=4	p=5	f_1
x=0	—	79050.	80700	77400	80700	84000	86200 77400.

$h(0,1) = c(1) + 0 \cdot I_c + f_2(0) = 79050$

$h(0,2) = c(2) + I_c + f_2(1) = 80700$

$h(0,3) = c(3) + 2I_c + f_2(2) = 77400$

$h(0,4) = c(4) + 3I_c + f_2(3) = 80700$

$h(0,5) = e(5) + 4I_c + f_2(4) = 84000$

∴ min cost = 780700 K

Path followed : month 1 : p = 3
month 2 : p = 0
month 3 : p = 3
month 4 : p = 4

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