~~Q3. Minimize $f(x_1, x_2) =$~~

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Q3 min. $f(x_1, x_2) = (x_1 - 1)^2 + (x_2 + 56)^2 + x_1 x_2$

s.t. $x_1^2 + x_2^2 \leq 9$

$56x_1 + x_2 \geq 6.$

$x_1, x_2 \geq 0$

1. Determine a point x that satifies KKT conditions.

For KKT conditions.

min. $f(x)$

s.t. $g_i(x) \leq 0$

$x_1^2 + x_2^2 - 9 \leq 0$

$-56x_1 - x_2 + 6 \leq 0$

$$\nabla f(\bar{x}) + \sum_{i=1}^{m} \lambda_i \nabla g_i(\bar{x}) = 0.$$

$\bar{\lambda}_i \, g_i(\bar{x}) = 0 \quad \forall i$

$g_i(\bar{x}) \leq 0 \quad \forall i$

$\bar{\lambda}_i \geq 0$ λ_i are KKT multipliers.

if $\bar{x}$ is a point of minimum.

However in order to apply KKT we have make sure the function is convex

$x_1^2 + x_2^2 - 9 \leq 0$ is convex as it is a circle

$-56x_1 - x_2 + 6 \leq 0$ is linear so convex.

$$\nabla f = \left[2(x_1 - 1) + x_2 \quad 2(x_2 + 56) + x_1\right]$$

$$H = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

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Find eigen values of H.

$$H = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = 0 \qquad (2-\lambda)^2 - 1 = 0$$

$$(2-\lambda)^2 = 1$$

$$(2-\lambda) = \pm 1 \qquad \lambda = 2 \pm 1$$

$$\lambda = 3, 1.$$

$\therefore$ +ve definite.

$\therefore$ $f(x)$ is convex and KKT maybe applied. Suppose x'_0 is a point where KKT holds.

~~$\nabla f(x_0) + \lambda_1 g_1(x_0)$~~

$$\nabla f(x') + \lambda_1 \nabla g_1(x'_0) + \lambda_2 \nabla g_2(x').$$

$$= [2(x_1'-1) + x_2' \quad 2(x_2'+56) + x_1'] + \lambda_1 [2x_1' \quad 2x_2'] + \lambda_2 [-56 \quad -1] = 0.$$

$$= [2x_1' - 2 + x_2 + 2\lambda_1 x_1' - 56\lambda_2 \quad 2x_2' + 112 + 2x_1' + \cancel{x_2} + 2\lambda_1 x_2' - \lambda_2] = 0.$$

$$(2+2\lambda_1) x_1' + x_2' - 2 - 56\lambda_2 = 0$$

$$2x_1' + (2+2\lambda_1) x_2' + 112 - \lambda_2 = 0.$$

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$\lambda_1 g_1(x') = 0$ $\qquad$ $\lambda_1 \geq 0, \lambda_2 \geq 0$

$\lambda_1 (x_1'^2 + x_2'^2 - 9) = 0.$ $\qquad$ $x_1'^2 + x_2'^2 - 9 \leq 0$

$\lambda_2 (-56x_1' - x_2' + 6) = 0$ $\qquad$ $-56x_1' - x_2' + 6 \leq 0$

Case I: $\lambda_1 = 0, \lambda_2 = 0$

$$\boxed{\begin{aligned} &(2+2\lambda_1)x_1' + x_2' - 2 - 56\lambda_2 = 0 \\ &2x_1' + (2+2\lambda_1)x_2' + 112 - \lambda_2 = 0. \end{aligned}}$$

put $\lambda_1 = \lambda_2 = 0.$

$$2x_1' + x_2' = 2.$$
$$2x_1' + 2x_2' = -112$$
$$-x_2' = 114$$

$$\boxed{\begin{aligned} x_2' &= -114. \\ x_1' &= 58 \end{aligned}}$$

This does not satisfy $x_1'^2 + x_2'^2 - 9 \leq 0$

$\therefore$ Discarded.

Case II: $\lambda_1 = 0.$, $56x_1' + x_2' - 6 = 0.$

$$2x_1' + x_2' - 2 - 56\lambda_2 = 0.$$
$$2x_1' + x_2' + 112 - \lambda_2 = 0$$

Solving we get

$x_1' = \frac{2972}{1595}$ & $x_2' = \frac{-176002}{1595}$ $\qquad \lambda_2 = -2.07$

Now, $x_1'^2 + x_2'^2 > 0$

$\therefore$ discarded.

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case III: $\lambda_2 = 0$, $x_1'^2 + x_2'^2 - 9 = 0$. 001610501020

$$(2+2\lambda_1)x_1' + x_2' - 2 = 0.$$

$$2x_1' + (2+2\lambda_1)x_2' + 112 = 0.$$

$$2x_1'\lambda_1 + 2x_1' + x_2' - 2 = 0.$$

$$2x_1' + 2\lambda_1 x_2' + 2x_2' + 112 = 0.$$

$$x_1'^2 + x_2'^2 - 9 = 0.$$

On solving this we get complex roots.

case IV: $x_1'^2 + x_2'^2 - 9 = 0.$

$$-56x_1' - x_2' + 6 = 0.$$

$$2x_1' + 2\lambda_1 x_1' + x_2' - 2 - 56\lambda_2 = 0$$

$$2x_1' + 2x_2' + 2\lambda_1 x_2' + 112 - \lambda_2 = 0$$

Solving we get

$x_1' = 0.16.$	$x_1' = 0.05$
$x_2' = -2.99.$	$x_2' = 2.99$
$\lambda_1 = -30.1$	$\lambda_1 = -100.34.$
$\lambda_2 = -0.25$	$\lambda_2 = -0.17$

$\because \lambda_i < 0$ $\therefore$ These are also reject.

Thus we get no such points.

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