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Image Processing

Q1.(a) A pixel p at coordinates (x, y) has 4 horizontal and vertical neighbours whose coordinates are given by.

$(x+1, y)$, $(x-1, y)$, $(x, y+1)$, $(x, y-1)$

This set of pixels, called the 4-neighbours of p, is denoted by $N_4(p)$.

The 4 diagonal neighbours of p have coordinates.

$(x+1, y+1)$, $(x+1, y-1)$, $(x-1, y+1)$, $(x-1, y-1)$.

and are denoted by $N_D(p)$. These points together with the 4-neighbours are called the 8-neighbours of p denoted by $N_8(p)$.

- 4-adjacency: Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$.
- 8-adjacency: Two pixels p and q with values from V are 8-adjacent if q is in $N_8(p)$.
- m-adjacency (mixed adjacency): Two pixels p and q with values from V are m-adjacent if
  (i) q is in $N_4(p)$, or
  (ii) q is in $N_D(p)$, and $N_4(p) \cap N_4(q)$ has no pixels whose values are from V.

where V is the set of values used to define adjacency.

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Q1 (b) In intensity slicing we first consider an images as a 3D function mapping spatial coordinates to intensities (height). Now, we consider placing planes at certain levels parallel to the coordinate plane. If a value is ono one side of a plane it is rendered in one colour, and if on the other side it is rendered in a different colour.

In general intensity slicing. is

- Let $[0, L-1]$ represent the grey scale.
- Let l_0 represent black $[f(x,y)=0]$ and l_{L-1} represent white $[f(x,y)=L-1]$.
- Suppose P planes perpendicular to intensity axis are defined as levels $l_1, l_2, \ldots, l_p$.
- Assuming that $0 < P < L-1$ then the P planes partition the greyscale into $P+1$ intervals $V_1, V_2, \ldots, V_{P+1}$.
- Grey level color assignments can then be made according to the relation

$$f(x,y) = c_k \quad \text{if } f(x,y) \in V_k.$$

c_k is the colour associated with the kth intensity level. V_k.

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Q1 (c) Adaptive filtering means changing the behaviour according to the values of the grayscales under the mask.

$$m_f^2 + \frac{\sigma_f^2}{\sigma_f^2 + \sigma_g^2}(g - m_f)$$

m_f = mean under the mask.

σ_f^2 = variance under the mask.

σ_g^2 = variance of the image.

g = current grayscale.

If σ_f^2 is high fraction is close to 1 and the output is close to g. This is the case case for significant detail such as edges.

If variance is low then output is close to m_f.

Adaptive median filtering is used to remove salt and paper noise, etc.

The median filter performs relatively well on impulse noise as long as the spatial density of the noise is not large.

The adaptive median filter can perform better. The filter size changes depending on the characteristics of the image.

Let, Z_{min} = minimum grey level in S_{xy}.

Z_{max} = maximum grey level in S_{xy}.

Z_{med} = median of grey levels in S_{xy}.

Z_{xy} = grey level at coordinates (x,y).

S_{max} = max. allowed size of S_{xy}.

S_{xy} is the window size at (x,y)

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Level A: $A1 = Z_{med} - Z_{min}$.
$A2 = Z_{med} - Z_{max}$.

if $A1 > 0$ and $A2 < 0$, Goto level B.
else increase window size. If window size $\leq S_{max}$ repeat level A. else output Z_{med}.

Level B: $B1 = Z_{xy} - Z_{min}$.
$B2 = Z_{xy} - Z_{max}$
if $B1 > 0$ and $B2 < 0$, output Z_{xy} else output Z_{med}.

Q1(d) chain code is a lossless compression algorithm for monochrome images. The basic principle of chain codes is to separately encode each component, or "blob" in the image. For each such region, a point on the boundary is selected and its coordinates are transmitted. The encoder then moves along the boundary of the region and, at each step, transmits a symbol representing the direction of the movement. This continues until the encoder returns to the starting position, at which point the blob has been completely described, and encoding continues with the next blob in the image. This encoding method is particularly effective for images consisting of a reasonably small number of connected components.

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Q1(e) Unsharp masking is used to sharpen images. It consists of subtracting a blurred version of an image from the image itself.. This process called unsharp masking is expressed as.

$$f_s(x,y) = f(x,y) - \bar{f}(x,y).$$

where $f_s(x,y)$ = Sharpened image obtained.

$\bar{f}(x,y)$ = blurred version of $f(x,y)$.

The origin of unsharp masking is in dark room photography. However, the resulting image although clearer may be a less accurate representation of the image's subject.

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Q2(a) Algorithm for histogram equalization:

Histogram equalisation is spreading out the frequencies in an image in order to improve dark or washed out images.

In order to arrive at a suitable transformation we have to first apply it to continuous function. Let r be the image whose values are normalized between 0 and 1., $r=0$ is black and $r=1$ is white. Later we consider a discrete formula and allow pixel values to be in the interval $[0, L-1]$. We have to find a transformation

$$s = T(r) \quad 0 \leq r \leq 1.$$

that produce a level s for every pixel value r in the original image. Assume,

(a) $T(r)$ is single valued and monotonically increasing in $0 \leq r \leq 1$.

(b) $0 \leq T(r) \leq 1$ for $0 \leq r \leq 1$.

$T(r)$ must be single valued for inverse to be possible; and monotonicity condition preserves the increasing order from black to white in the output image.

$$r = T^{-1}(s) \quad 1 \leq s \leq 0.$$

The gray levels in an image are random variables $p_r(r)$ and $p_s(s)$ denote probability density functions on r and s. We know, from probability.

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$

In IP

$$s = T(r) = \int_0^r p_r(w)\, dw$$

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$$\frac{ds}{dr} = \frac{dT(r)}{dr} = \frac{d}{dr}\left[\int_0^r p_r(w)\,dw\right] = p_r(r).$$

$$\therefore\ p_s(s) = p_r(r)\left|\frac{dr}{ds}\right| = p_r(r).\left|\frac{1}{p_r(r)}\right| = 1 \quad 0 \le s \le 1.$$

$p_s(s)$ is a uniform pdf.

For discrete values, r_k

~~$p_r(r_k) = n_k$~~

$$p_r(r_k) = \frac{n_k}{n} \qquad k = 0, 1, \ldots, L-1$$

n_k = no. of pixels having gray level r_k

r_k = gray level.

n = total ~~no. of pixels in the image.~~ sum of frequency of all levels.

$$\therefore\ S_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j)$$

$$= \sum_{j=0}^{k} \frac{n_j}{n}$$

Thus to perform histogram equalisation apply the transformation.

$$S_k = T(r_k) = \sum_{j=0}^{k} \frac{n_j}{n}$$

to every pixel in the image.

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Algorithm:

1. Convert input image into grayscale image
2. Find frequency of ~~occurrence~~ occurrence for each pixel value i.e. histogram.
3. Calculate cumulative frequency of all pixel values.
4. Divide the cumulative frequencies by total number of pixels and multiply them by maximum graycount in the image.

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Q2(b) Mexican Hat filter for edge detection :

Mexican hat filter for edge detection is also known as Laplacian of Gaussian or the LoG filter. It is a high pass filter.

We know the Laplacian filter of a 2D function $f(x,y)$ is a second order derivative.

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

we know $\frac{\partial f}{\partial x} = f(x+1) - f(x)$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x,y)]$$

This can be represented by a filter matrix.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

However, the derivative operator such as Laplacian is prone to noise so before applying the Laplacian filter a Gaussian filter is applied on the image to remove high frequency noise. Also applying only the Laplacian can result in double edges. Also the Laplacian is unable to detect edge direction. For these reasons we apply LoG.

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~~The Laplacian~~

The Laplacian is combined with smoothing as a precursor to finding edges via zero crossings.

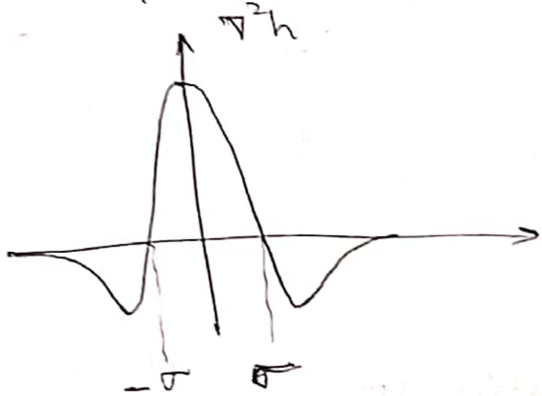
$$h(r) = -e^{-\frac{r^2}{2\sigma^2}}$$

where $r^2 = x^2+y^2$ $\sigma =$ std. deviation.

Convolving this function with an image blurs the image, degree of blurring being determined by σ.

$$\nabla^2 h = -\left[\frac{r^2-\sigma^2}{\sigma^4}\right] e^{-\frac{r^2}{2\sigma^2}}$$

This function is known as the Laplacian of Gaussian (LOG) function.



An approximate 5x5 filter is shown below which captures the shape of the graph

$$\begin{bmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & -2 & -1 & 0 \\ -1 & -2 & 16 & -2 & -1 \\ 0 & -1 & -2 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix}$$

Sum of the values in the mask must be equal to 0 so as not to change in areas of constant gray level.

Thus the purpose of the Gaussian function is to smooth the image, and the Laplacian is to provide an image with 0 crossings used to establish the location of the edges.

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Q2(c) In order to define opening and closing morphological operations we first have to define Erosion and Dilation.

Dilation:

With A and B sets in Z^2, the dilation of A by B, denoted by $A \oplus B$ is defined as

~~$A \oplus B = \{ z \mid (B)$~~

$$A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \phi \}.$$

This is based on obtaining the reflection of B about its origin and shifting this reflection by z. The dilation of A by B then is the set of all displacements, z, such that $\hat{B}$ and A overlap by at least one element.

Erosion:

For set A and B in Z^2 the erosion of A by B denoted by $A \ominus B$ is defined as.

$$A \ominus B = \{ z \mid (B)_z \subseteq A \}.$$

That is the set of all points z such that B, translated by z is contained in A.

Opening: Opening generally smoothes the contour of an object, ~~to~~ breaks narrow isthmuses, and eliminates thin protrusions. The opening set of A by structural element B may be defined as.

$$A \circ B = (A \ominus B) \oplus B$$

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Closing : Closing also tends to smooth sections of contours but, as opposed to opening, it generally fuses narrow breaks and long thin gulfs, eliminates small holes and fills gaps in the contour. The closing set A by structuring element B, denoted by $A \bullet B$ is

$$A \bullet B = (A \oplus B) \ominus B$$

Boundary Extraction :

The boundary of a set A, denoted by $\beta(A)$ can be obtained by first eroding A by B and then performing the set difference between A and its erosion.

$$\beta(A) = A - (A \ominus B)$$

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Q3 (a) Weiner filter or minimum mean squared error filter is used for image restoration.

~~Let $\hat{f}(u,v)$ denote the undegraded image and~~

In this method we aim to find an ~~els~~ estimate $\hat{f}$ of the uncorruped ~~image~~ image f such that the mean squared error between them is minimized. This error measure is given by

$$e^2 = E\{(f-\hat{f})^2\}$$

where $E\{\}$ is the expected value of the argument. Generally, we take sum of squared errors.

It is assumed that the noise and the image are uncorrelated; that one or the other has zero mean; and that the gray levels in the estimate are a linear function of the levels in the degraded image. Based on these conditions the minimum of error ~~of the~~ function is given in the frequency domain by the expression.

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)\,S_f(u,v)}{S_f(u,v)|H(u,v)|^2 + S_\eta(u,v)}\right]G(u,v)$$

$$= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)$$

$$= \left[\frac{1}{H(u,v)}\,\frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)$$

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where, $H(u,v)$ = degradation function.

$H^*(u,v)$ = complex conjugate of $H(u,v)$

$|H(u,v)|^2 = H^*(u,v)\,H(u,v)$

$S_\eta(u,v) = |N(u,v)|^2$ = power spectrum of the noise

$N(u,v)$ = noise function.

$S_f(u,v) = |F(u,v)|^2$ = power spectrum of undegraded image.

$G(u,v)$ = transform of the degraded image.

$\hat{F}(u,v)$ = fourier transform of the undegraded image.

When dealing with spectrally white noise, the spectrum $|N(u,v)|^2$ is constant.

If $|F(u,v)|^2$ is not known

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + K}\right] G(u,v)$$

K is a constant.

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