Answer any 6 questions:

1. Write the differential forms of the four Maxwell's equations. Derive the integral form of the Maxwell's first equation starting from the differential form. [5]

   [Hint: Use Gauss divergence theorem]

2. State Ampere's circuital law and Gauss's law of electrostatics and magnetostatics. Write down the mathematical forms of the two laws. [5]

3. State and write the mathematical forms of Gauss theorem and Stokes theorem. Write Poisson equation, Laplace equation and continuity equation. [5]

4. Find the divergence and curl of the vector $A = 2x\,\boldsymbol{i} + 4y\,\boldsymbol{j} + \boldsymbol{k}$ [5]

5. Find the volume charge density for a given region whose potential is $V = 6x^2 + 2y^2 - z^2$ [5]

   [Hint: Use Poisson's equation to calculate the value of volume charge density, ϱ]

6. Explain absorption and spontaneous emission process. [5]

7. Calculate the volume of the parallelepiped whose sides are given by the vectors $\mathbf{A} = 2\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$, $\mathbf{B} = 5\mathbf{i} + 6\mathbf{k}$, and $\mathbf{C} = \mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$. [5]

8. A scalar field is given by $\varphi(x, y, z) = 4x^3y + xy + z^2$. Calculate the magnitude and direction of the gradient of the scalar field. [5]

1. Maxwells equations (Differential form)

In free space:

1. $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$
2. $\vec{\nabla}\cdot\vec{B} = 0$
3. $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$
4. $\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$

1. $\vec{\nabla}\cdot\vec{D} = \rho$
2. $\vec{\nabla}\cdot\vec{H} = 0$
3. $\vec{\nabla}\times\vec{D} = -\mu_0\epsilon_0\frac{\partial\vec{H}}{\partial t}$
4. $\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$

where $\vec{E} \rightarrow$ Electric field vector

$\vec{B} \rightarrow$ Magnetic field vector

$\rho \rightarrow$ Volume charge density

$\epsilon_0 \rightarrow$ Permittivity of the free space

$\vec{D} \rightarrow$ Displacement current vector $[\vec{D} = \epsilon_0\vec{E}]$

$\vec{H} \rightarrow$ Magnetic field vector $[\vec{B} = \mu_0\vec{H}]$

$\vec{J} \rightarrow$ Current density vector.

Integral form of 1st Maxwell equation from differential form:

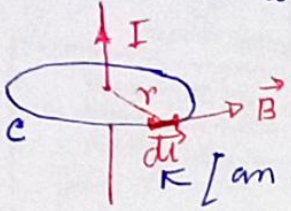
The Maxwell's equation in differential form is

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\Rightarrow \iiint_V (\vec{\nabla}\cdot\vec{E})\,dv = \frac{1}{\epsilon_0}\iiint \rho\,dv = \frac{q}{\epsilon_0} \quad \left[\because q = \iiint \rho\,dv\right]$$

$$\Rightarrow \oiint_S \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0} \quad \left[\text{From Gauss's divergence theorem: } \iiint(\vec{\nabla}\cdot\vec{A})\,dv = \oiint_S \vec{A}\cdot\vec{ds}\right]$$

2. **Ampere's circuital law**: The line integral of the magnetic field vector ($\vec{B}$) around a closed loop is equal to μ_0 times the total current enclosed by the loop. So, mathematically the law is written as



$$\boxed{\oint_c \vec{B}\cdot\vec{dl} = \mu_0 I} \quad \text{------} \quad (2.1)$$

[an example]

where $\vec{B}$ is the magnetic field vector at a point on the loop of radius r and $\vec{dl}$ is a small line element on the loop and I is the current enclosed by the loop [For example].

Important: The law is valid provided the total electric inside the loop remains constant.

- **Gauss's law of electrostatics**: The total electric flux coming out of a closed surface (also called Gaussian surface) is equal to the $(1/\epsilon_0)$ times the total charge enclosed by the surface, i.e.,

$$\boxed{\phi_E = \oint\!\!\oint_S \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0}} \quad \text{-----} \quad (2\text{-}2)$$

where $\vec{E} \rightarrow$ Electric field vector in the vector field produced by the charg $q = \iiint \rho\, dv$.

$\vec{ds} \rightarrow$ small elemental surface in the gaussian surface S.

$\epsilon_0 \rightarrow$ Permittivity of the free space.

• Gauss's law of magnetostatics: The total magnetic flux coming out of a closed surface is equal to zero, ie,

$$\boxed{\oiint_S \vec{B} \cdot \vec{ds} = 0} \quad \text{- - - - - (2.3)}$$

• Mathematical forms:

Amperes law: $\oint_C \vec{B} \cdot \vec{dl} = \mu_0 I$

Gauss's law : $\oiint_S \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}$ → Electrostatics

$\oiint_S \vec{B} \cdot \vec{ds} = 0$ → Magnetostatics

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3. Gauss's theorem: The volume integral of the divergence of a vector field $\vec{A}$ over a closed volume (V) is equal to the surface integral of the vector $\vec{A}$ around the closed surface (S). i.e.,

$$\boxed{\iiint_V (\vec{\nabla} \cdot \vec{A})\, dv = \oiint_S \vec{A} \cdot \vec{ds}} \quad \text{- - - - (3.1)}$$

Stokes theorem: The surface integral of curl of a vector quantity $(\vec{A})$ is equal to the line integral of the vector around the closed loop (c)

, ie, the boundary line of the open surface (S). ie,

$$\boxed{\oiint (\vec{\nabla} \times \vec{A}) \cdot \vec{ds} = \oint_C \vec{A} \cdot \vec{dl}} \quad \text{------ } (3.2)$$

(ii) Poisson equation : $\boxed{\nabla^2 V = -\frac{\rho}{\epsilon_0}}$

Laplace equation : $\boxed{\nabla^2 V = 0}$

Continuity equation : $\boxed{\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0}$

---- (3.3)

where $V \rightarrow$ Electric potential

$\rho \rightarrow$ Volume charge density

$\epsilon_0 \rightarrow$ Free space permittivity.

$\vec{J} \rightarrow$ Current density.

$t \rightarrow$ time.

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4. The given vector is

$$\vec{A} = 2x\,\hat{i} + 4y\,\hat{j} + \hat{k}$$

(iii) Divergence of $\vec{A} = \vec{\nabla} \cdot \vec{A}$

$$= \frac{\partial}{\partial x}(2x) + \frac{\partial}{\partial y}(4y) + \frac{\partial}{\partial z}(1)$$

$$= 2 + 4 + 0$$

$$= 6$$

(1) Curl $\vec{A} = \vec{\nabla} \times \vec{A}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x & 4y & 1 \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial}{\partial y}(1) - \frac{\partial}{\partial z}(4y)\right] + \hat{j}\left[\frac{\partial}{\partial z}(2x) - \frac{\partial}{\partial x}(1)\right] + \hat{k}\left[\frac{\partial}{\partial x}(4y) - \frac{\partial}{\partial y}(2x)\right]$$

$$= \hat{i}[0-0] + \hat{j}[0-0] + \hat{k}[0-0]$$

$$= \vec{0}$$

$$= 0$$

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5. The potential in the region is given by

$$V = 6x^2 + 2y^2 - z^2$$

From Poisson's eq^n we know

$$\nabla^2 V = -\frac{\rho}{\epsilon_0}$$

$\Delta = \bar{\nabla}\cdot\bar{\nabla} = \nabla^2$

Laplacian operator

$$\Rightarrow \rho = -\epsilon_0 \nabla^2 V.$$

Now $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

$$= 12 + 4 - 2$$

$$= 14.$$

$\therefore \rho = -\epsilon_0 \nabla^2 V = -14\epsilon_0$

$\Rightarrow \boxed{\rho = -14\epsilon_0 \text{ unit}}$

∴ The volume charge density in the given region
is $-14\epsilon_0$ unit.

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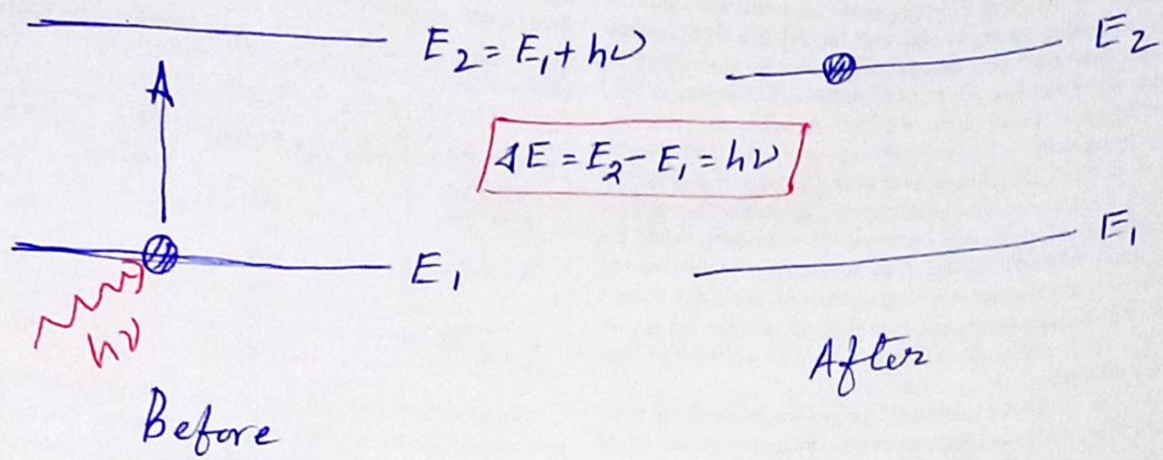
6. <u>Absorption</u> : When light (EM waves / photons) falls
on a material some portion of the
light may get absorbed by the
atoms of the material depending on the characteris
ties (absorption coefficients) of the atoms, i.e, their
energy levels. For a typical absorption ~~pro~~ of a
photon, ~~when the~~ by a single atom, when a
photon is incident on the atom, if the photon
energy (hν) is equal to the difference between the
ground state and excited state, it gets
absorbed by the atom and the atom goes to the
first excited state. This process is called absor-
ption of 1 photon by 1 atom. Naturally a material
l has many atoms and hence a good absorber
usually absorbs considerable amount of light.
This process is called absorption of light. The
process is shown below with the help of energy
level diagram. Due to this process dark lines
at appropriate wavelengths appear in the absorp-
tion spectra of a material.



7. Volume of the parallelopiped $= \vec{A}\cdot(\vec{B}\times\vec{C})$

$$= \begin{vmatrix} 2 & 5 & 2 \\ 5 & 0 & 6 \\ 1 & 6 & 2 \end{vmatrix}$$

$$= 2(0-36) + 5(6-10) + 2(30-0)$$

$$= -72 - 20 + 60$$

$$= -32$$

$\therefore$ Volume of the parallelopiped $= \underline{-32 \text{ unit}}$!

−ve volume! Wait a min! Strange! Does not make any sense!!

Comment: The given vectors ~~are~~ were not chosen appropria-tely, i.e, the can't be three sides of a parallelopiped. !

Lesson: We should not stop thinking after we get a result. We always have think if it makes sense,

8. The given scalar field is

$$\phi(x, y, z) = 4x^3y + xy + z^2$$

$$\therefore \text{grad}\,\phi = \bar{\nabla}\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

$$= (12x^2y + y)\hat{i} + (4x^3 + x)\hat{j} + 2z\hat{k}$$

$$\therefore \text{Magnitude of } \bar{\nabla}\phi = |\bar{\nabla}\phi| = \sqrt{(12x^2y+y)^2 + (4x^3+x)^2 + (2z)^2}$$

$$\text{Direction of gradient} = \frac{\bar{\nabla}\phi}{|\nabla\phi|}$$

$$= \frac{(12x^2y+y)\hat{i} + (4x^3+x)\hat{j} + 2z\hat{k}}{\sqrt{(12x^2y+y)^2 + (4x^3+x)^2 + (2z)^2}}$$

Extra: What is the value of $\bar{\nabla}\phi$ @ $(1, 2, 0)$ point?

$$\bar{\nabla}\phi\Big|_{(1,2,0)} = 26\,\hat{i} + (5)\hat{j} + 0\,\hat{k}$$

$$= 26\hat{i} + 5\hat{j}$$

Instructions for review:

1. Review your answers ~~while~~ during the review in class after Diwali break carefully.
2. Check (your + my) mistakes and read comments.
3. Learn from your mistakes.

Cheers,
→ Sourav.

Spontaneous emission: It's a process in which an atom in excited state (E_2) falls back to the ground state (E_1) spontaneously and emitts a photon of energy $h\nu = \Delta E = E_2 - E_1$. This process is called spontaneous emission of one photon. However, considering a material consists of many atoms many such emissions happens spontaneously and bright lines at appropriate wavelengths or frequency appear on emission spectra of a material. The process is shown below with the help of energy level diagrams.