### **Understanding GMM, HMM, DNN and LSTM**

Pradeep R 12<sup>th</sup> April 2019

## Outline

- Introduction
- K-Means Clustering
- Gaussian Model
- Need for GMM
- Need for HMM
  - Implementation
- Conclusion

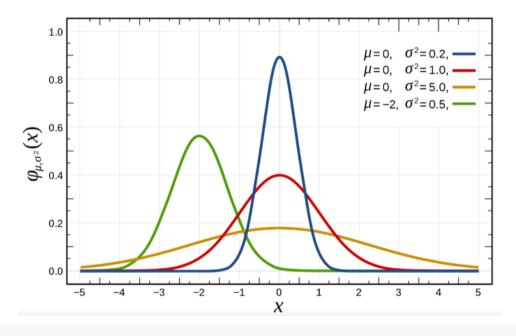
## Introduction

- Applications:
  - Hand Character Recognition
  - Spoken Digit Recognition
  - Weather Prediction
  - Human activity detection
- Central limit theorem states that "when we add large number of independent random variables, irrespective of the original distribution of these variables, their normalized sum tends towards a Gaussian distribution."
- For example, the distribution of total distance covered in an random walk tends towards a Gaussian probability distribution.

## Why Gaussian?

- Gaussian remain as a Gaussian even after transformation
  - Product of two Gaussian is a Gaussian
  - Sum of two independent Gaussian random variables is a Gaussian
  - Convolution of Gaussian with another Gaussian is a Gaussian
  - Fourier transform of Gaussian is a Gaussian
- Simpler
  - The entire distribution can be specified using just two parameters- mean and variance

## Variants of Gaussian



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(\mu-x)^2}{2\sigma^2}}$$

**UniVariate Gaussian** 

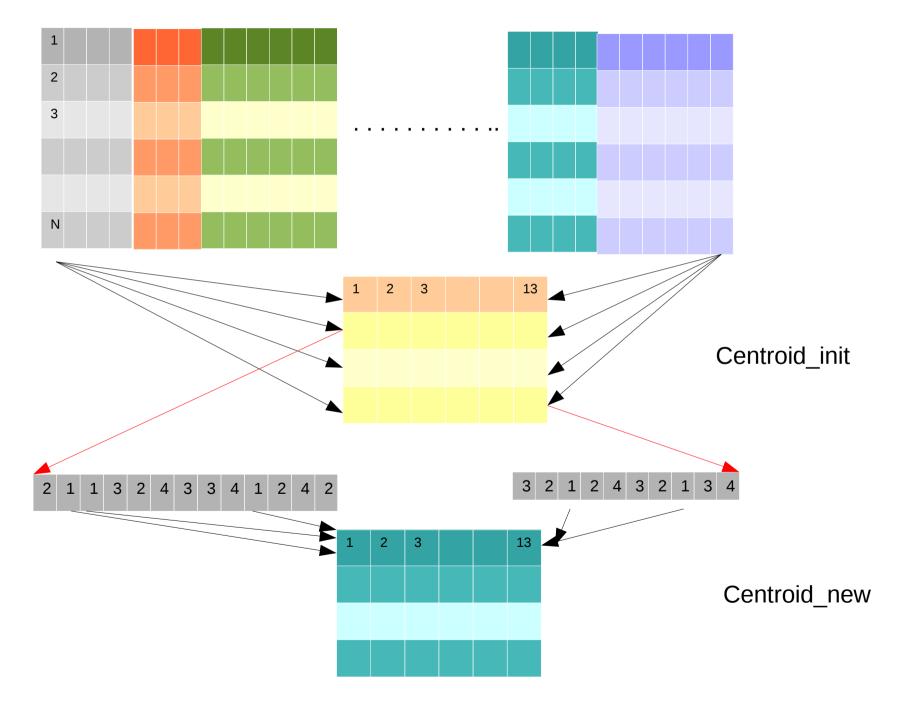
$$p(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{1}{(2\pi)^{\frac{n}{2}} |\boldsymbol{\Sigma}|^{\frac{1}{2}}} \exp\biggl(-\frac{1}{2} \, (\mathbf{x} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{x} - \boldsymbol{\mu})\biggr)$$

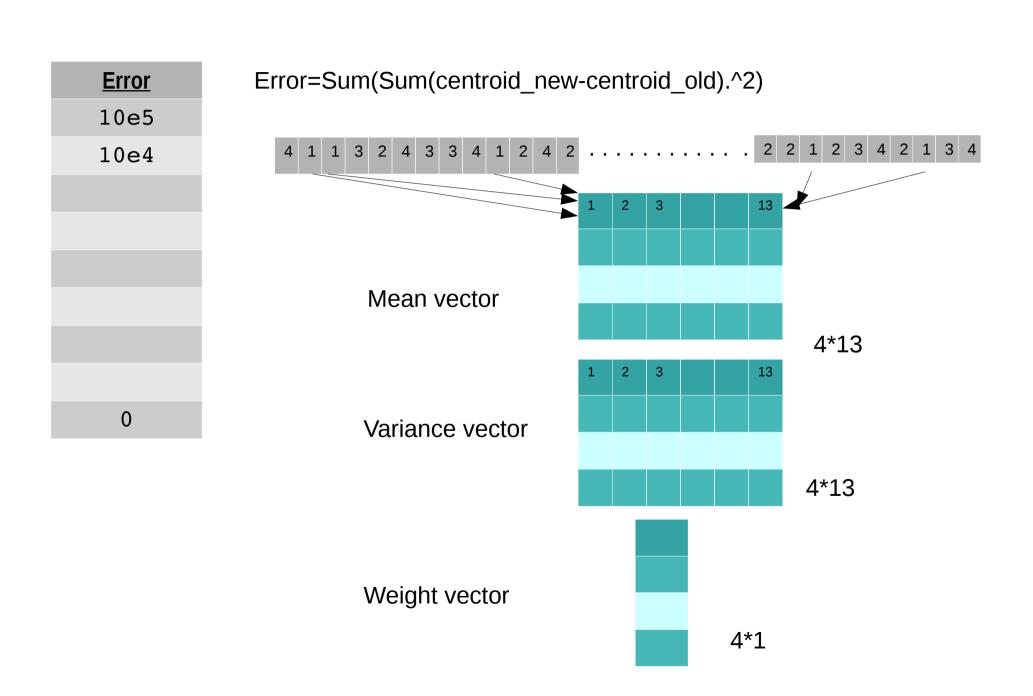
**MultiVariate Gaussian** 

$$f(\mathbf{x}) = \sum_{i=1}^N p_i \cdot rac{1}{(2\pi)^{n/2} \sqrt{\det(\Sigma_i)}} \mathrm{exp}igg(-rac{1}{2} (\mathbf{x} - \mathbf{m}_i) \, \Sigma_i^{-1} (\mathbf{x} - \mathbf{m}_i)^Tigg)$$

**Gaussian Mixture Model** 

## K-Means Clustering - Idea





## GMM Training: Expectation Maximization Algorithm

Initialization: k-means clustering

$$\gamma(z_{nk}) = \frac{\pi_k \mathcal{N}(\mathbf{x}_n | \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)}{\sum_{j=1}^K \pi_j \mathcal{N}(\mathbf{x}_n | \boldsymbol{\mu}_j, \boldsymbol{\Sigma}_j)}$$

M-step

$$N_k = \sum_{n=1}^{N} \gamma(z_{nk})$$

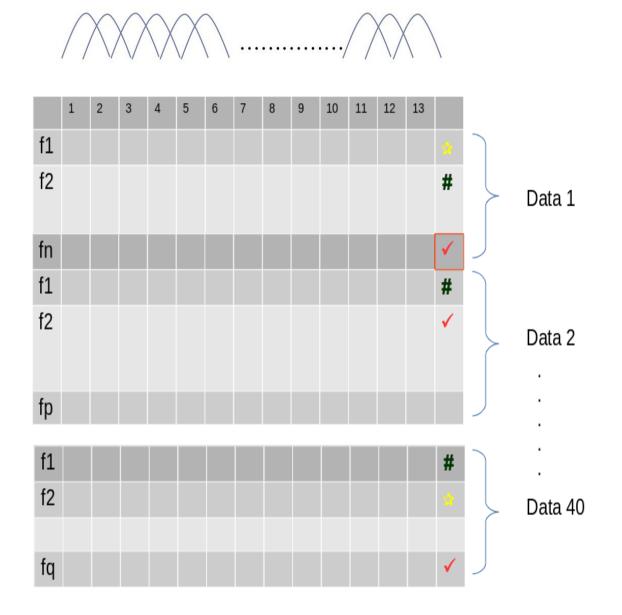
$$\mu_k^{\text{new}} = \frac{1}{N_k} \sum_{n=1}^N \gamma(z_{nk}) \mathbf{x}_n \qquad \pi_k^{\text{new}} = \frac{N_k}{N}$$

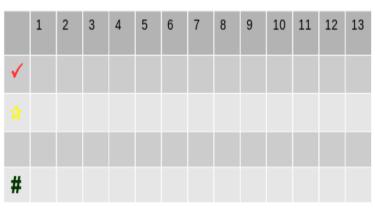
$$\Sigma_k^{\text{new}} = \frac{1}{N_k} \sum_{n=1}^N \gamma(z_{nk}) \left( \mathbf{x}_n - \boldsymbol{\mu}_k^{\text{new}} \right) \left( \mathbf{x}_n - \boldsymbol{\mu}_k^{\text{new}} \right)^{\text{T}}$$

 Evaluate log likelihood and check for convergence

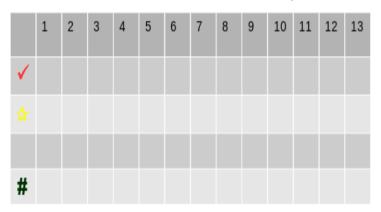
$$\ln p(\mathbf{X}|\boldsymbol{\mu}, \boldsymbol{\Sigma}, \boldsymbol{\pi}) = \sum_{n=1}^{N} \ln \left\{ \sum_{k=1}^{K} \pi_k \mathcal{N}(\mathbf{x}_n | \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k) \right\}$$

## Implementation-Initial Setup



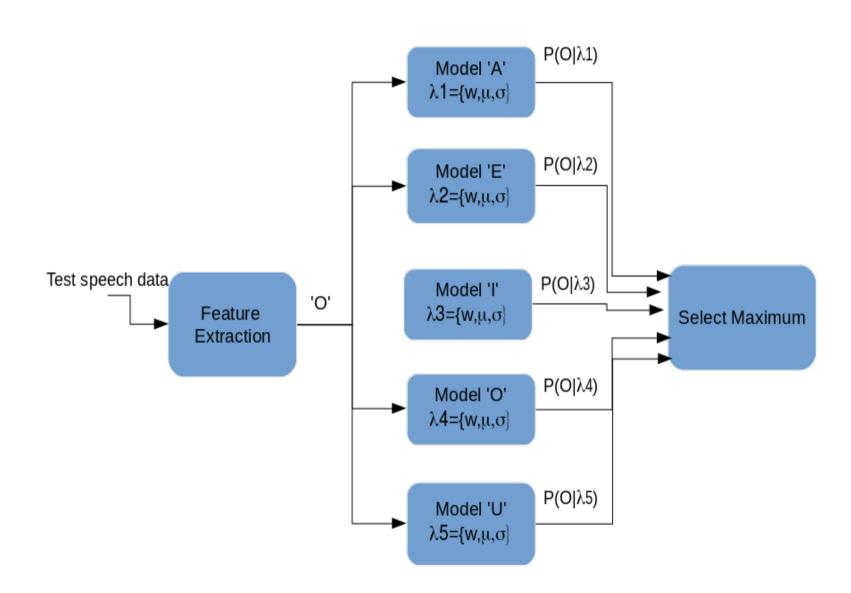


Number of mixtures \* coeff per frame





## **Testing**



## Drawbacks of GMM

- Cannot alone perform time series prediction
- It is necessary to evaluate probabilities along with time in many practical applications.
  - Ex 01: GMM build for word 'Kamal' and 'Kalam' gathers no sequential information
  - GMM 'b' and GMM 'd' may have false substitutions
- Hence it is necessary to capture temporal/sequential information along with spatial information.

## Hidden Markov Models

- Terminologies:
  - State
  - State Transition
  - Emission Probability

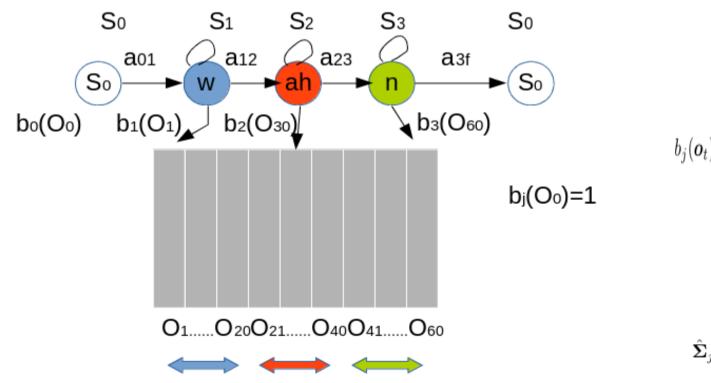
$$b_j(\boldsymbol{o}_t) = \frac{1}{\sqrt{(2\pi)^n |\boldsymbol{\Sigma}_j|}} e^{-\frac{1}{2}(\boldsymbol{o}_t - \boldsymbol{\mu}_j)' \boldsymbol{\Sigma}_j^{-1}(\boldsymbol{o}_t - \boldsymbol{\mu}_j)}$$

$$\hat{\boldsymbol{\mu}}_j = \frac{1}{T} \sum_{t=1}^T \boldsymbol{o}_t$$

$$\hat{\boldsymbol{\Sigma}}_j = \frac{1}{T} \sum_{t=1}^T (\boldsymbol{o}_t - \boldsymbol{\mu}_j) (\boldsymbol{o}_t - \boldsymbol{\mu}_j)'$$

## Illustration of phone level modeling

- Train utterance.way
  - Trancription: One
  - Phonetic equivalence: One -- w ah n

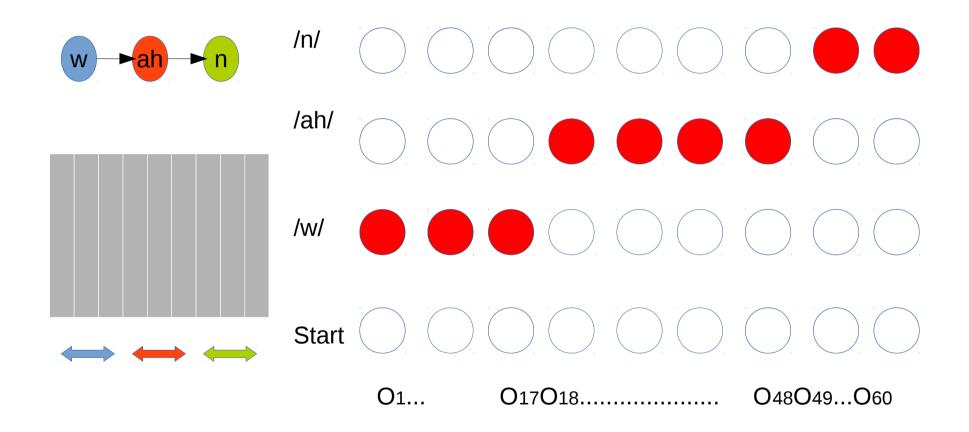


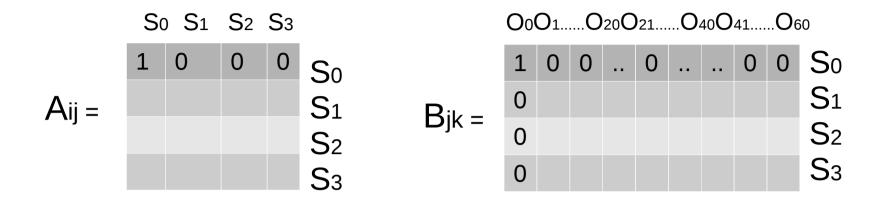
$$b_j(\boldsymbol{o}_t) = \frac{1}{\sqrt{(2\pi)^n |\boldsymbol{\Sigma}_i|}} e^{-\frac{1}{2}(\boldsymbol{o}_t - \boldsymbol{\mu}_j)' \boldsymbol{\Sigma}_j^{-1}(\boldsymbol{o}_t - \boldsymbol{\mu}_j)}$$

$$\hat{\boldsymbol{\mu}}_j = \frac{1}{T} \sum_{t=1}^{T} \boldsymbol{o}_t$$

$$\hat{\boldsymbol{\Sigma}}_j = \frac{1}{T} \sum_{t=1}^{T} (\boldsymbol{o}_t - \boldsymbol{\mu}_j) (\boldsymbol{o}_t - \boldsymbol{\mu}_j)'$$

## **Expected Outcome**





#### **Compute Forward probabilities: Forward Recursion**

• The goal here is to find the probablity that the HMM is in state 'i' at time 't' -----  $\alpha_i(t)$ 

0; t=0 & j!=initial state 
$$\alpha_j(t)=1$$
; t=0 & j=initial state  $\alpha_j(t)=1$ ; t=0 & j=initial state  $\alpha_j(t)=1$ ; t=0 & j=initial state

0; Si(t)!=0 & t=final state 
$$\beta i(t)=1$$
; Si(t)=0 & t=final state sum( $\alpha i(t+1)*aii$ )\*bi(Ot+1)

0; t=0 & j!=initial state  $\alpha_j(t)=1$ ; t=0 & j=initial state  $\alpha_j(t)=1$ ; t=0 & j=initial state  $\alpha_j(t)=1$ ; t=0 & j=initial state

S <sub>0</sub>	S1	S2	S <sub>3</sub>
1	0	0	0
0.2	0.3	0.1	0.4
0.2	0.5	0.2	0.1
0.7	0.1	0.1	0.1

<b>O</b> 0	O1	O25	O46	O0
1	0	0	0	0
0	0.3	0.1	0.4	0.2
0	0.1	0.7	0.1	0.1
0	0.5	0.1	0.2	0.2

S3 0 0
--------

[A] [B]







 $\alpha_j(0)$   $\alpha_j(1)$   $\alpha_j(2)$   $\alpha_j(3)$   $\alpha_j(T=4)$ 

#### **Assumption:**

- Let the HMM be at state S1 at t=0
- $\alpha_1(0)=1$
- Find P(O<sup>4</sup>|θ)



Use **Viterbi decoding** to find the hidden state sequence that generated the particular observation  $P(O^4|\theta)=\alpha O(T)=0.0018$   $P(O^4)=\beta i(0)$ 

$$\begin{array}{l} t{=}1\\ \alpha 0(1){=}\{\alpha 0(0)a00{+}\alpha 1(0)a10{+}\alpha 2(0)a20{+}\alpha 3(0)a30\}*b0(O1){=}0\\ \alpha 1(1){=}\{\alpha 0(0)a01{+}\alpha 1(0)a11{+}\alpha 2(0)a21{+}\alpha 3(0)a31\}*b1(O25){=}0.09\\ \alpha 2(1){=}\{\alpha 0(0)a02{+}\alpha 1(0)a12{+}\alpha 2(0)a22{+}\alpha 3(0)a32\}*b2(O46){=}0.02\\ \alpha 3(1){=}\{\alpha 0(0)a03{+}\alpha 1(0)a13{+}\alpha 2(0)a23{+}\alpha 3(0)a33\}*b3(O0){=}0.05 \end{array}$$

In matrix form  $[0\ 1\ 0\ 0]^*[A]^*[B(:,2)] = [0.2\ 0.01\ 0.09\ 0]$ 

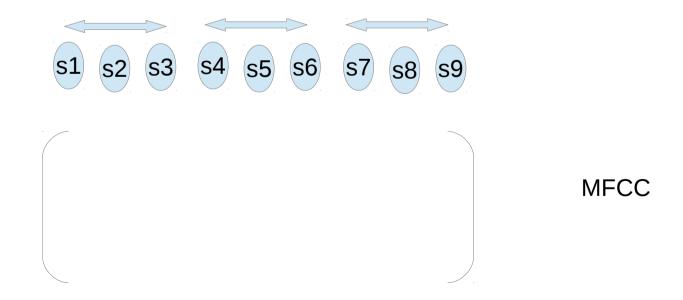
$$\hat{a}_{ij} = \frac{\sum_{t=1}^{T-1} \gamma_t(i,j)}{\sum_{t=1}^{T-1} \gamma_t(i)}$$

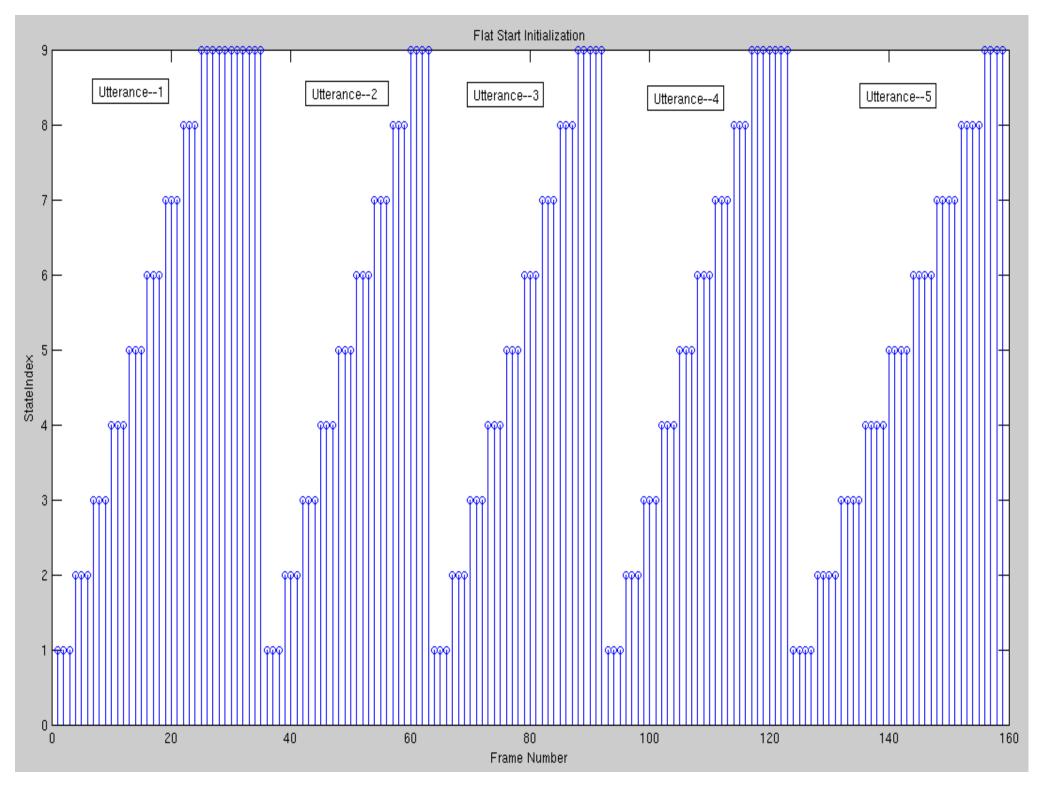
$$\gamma_i(t) = P(X_t = i | Y, \theta) = \frac{\alpha_i(t)\beta_i(t)}{\sum_{j=1}^N \alpha_j(t)\beta_j(t)} \qquad \hat{b}_j(o_k) = \frac{\sum_{t:O_t = o_k} \gamma_t(j)}{\sum_{t=1}^T \gamma_t(j)}$$

$$\xi t(i,j) = \frac{\alpha_i(t) a_{ij} \beta_j(t+1) b_j(y_{t+1})}{\sum_{k=1}^{N} \sum_{l=1}^{N} \alpha_k(t) a_{kl} \beta_l(t+1) b_l(y_{t+1})} = \frac{\alpha_i(t) a_{ij} \beta_j(t+1) b_j(y_{t+1})}{\sum_{k=1}^{N} \alpha_k(t) \beta_k(t)}$$

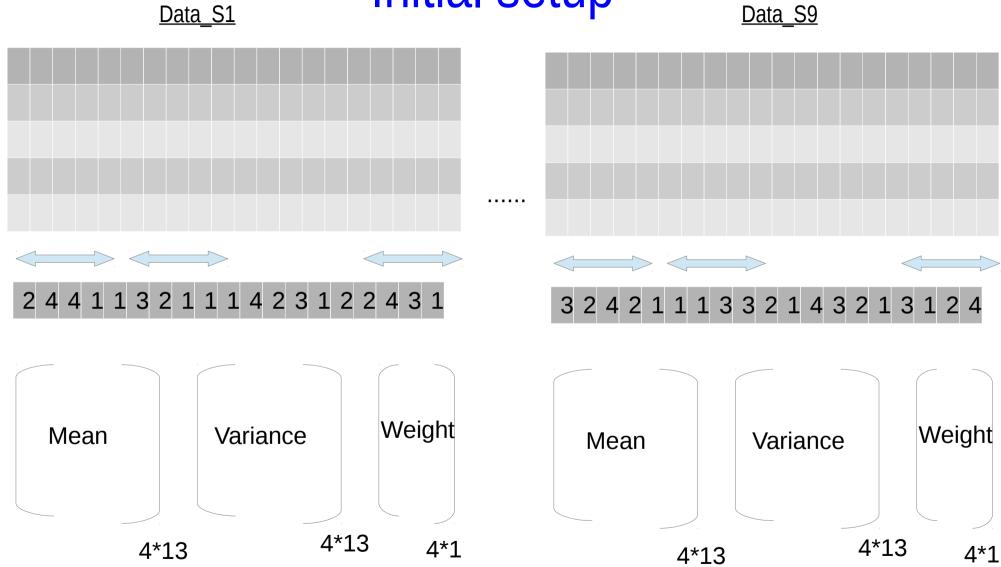
## Illustration

- Data:
  - Ten words of the utterance one
  - One: wah n
- Build 3 state HMM per phone





## Initial setup



#### **Forward Algorithm**

#### 1) Initialization:

$$\alpha_1(i) = \pi_i b_i(O_1), \qquad 1 \le i \le N.$$

2) Induction:

$$\alpha_{t+1}(j) = \left[\sum_{i=1}^{N} \alpha_t(i) a_{ij}\right] b_j(O_{t+1}), \qquad 1 \le t \le T-1$$

$$1 \le j \le N.$$

#### 3) Termination:

$$P(O|\lambda) = \sum_{i=1}^{N} \alpha_{T}(i).$$

#### **Backward Probabilities**

1) Initialization:

$$\beta_T(i) = 1, \quad 1 \leq i \leq N.$$

2) Induction:

$$\beta_{t}(i) = \sum_{j=1}^{N} a_{ij}b_{j}(O_{t+1}) \beta_{t+1}(j),$$

$$t = T - 1, T - 2, \dots, 1, 1 \le i \le N.$$

#### **Viterbi Algorithm**

1) Initialization:

$$\delta_1(i) = \pi_i b_i(O_1), \qquad 1 \le i \le N$$

$$\psi_1(i) = 0.$$

2) Recursion:

$$\delta_t(j) = \max_{1 \le i \le N} [\delta_{t-1}(i)a_{ij}]b_j(O_t), \qquad 2 \le t \le T$$

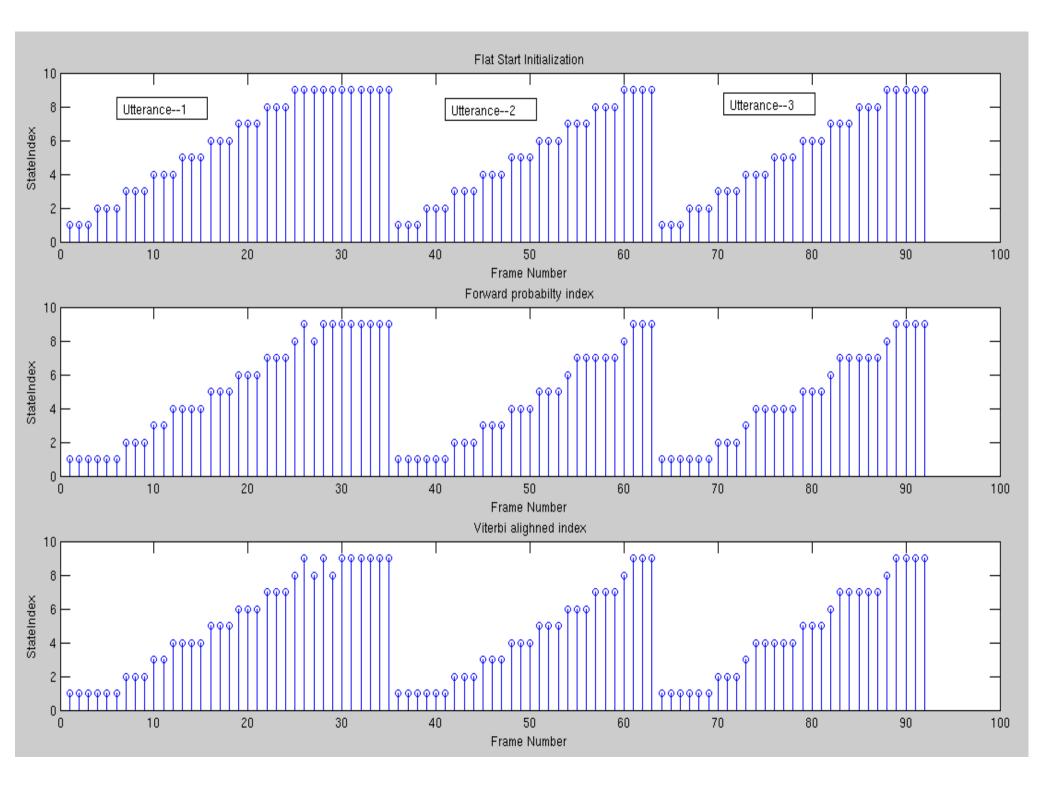
$$1 \le j \le N$$

$$\psi_t(j) = \underset{1 \le i \le N}{\operatorname{argmax}} [\delta_{t-1}(i) a_{ij}], \qquad 2 \le t \le T$$

 $1 \le j \le N$ .

3) Termination:

$$P^* = \max_{1 \le i \le N} [\delta_T(i)]$$
$$q_T^* = \operatorname{argmax} [\delta_T(i)].$$



$$\gamma_t(i) = \frac{\alpha_t(i) \ \beta_t(i)}{P(O|\lambda)} = \frac{\alpha_t(i) \ \beta_t(i)}{\sum_{i=1}^{N} \alpha_t(i) \ \beta_t(i)}$$

 $\overline{\pi}_i$  = expected frequency (number of times) in state  $S_i$  at time (t = 1)

$$\overline{a}_{ij} = \frac{\text{expected number of transitions from state } S_i \text{ to state } S_j}{\text{expected number of transitions from state } S_i}$$

$$=\frac{\sum_{t=1}^{T-1} \xi_t(i,j)}{\sum_{t=1}^{T-1} \gamma_t(i)}$$

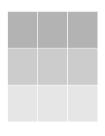
F1

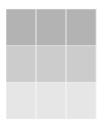
F2

.....



s3





s1 s2 s3

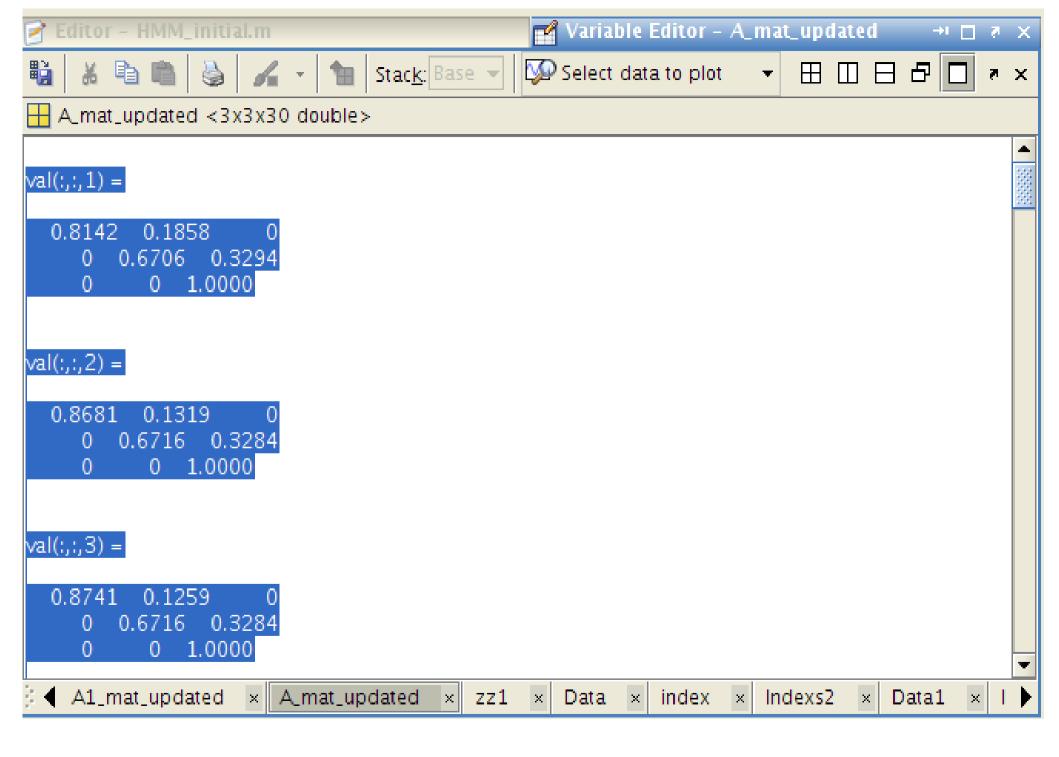
Γ

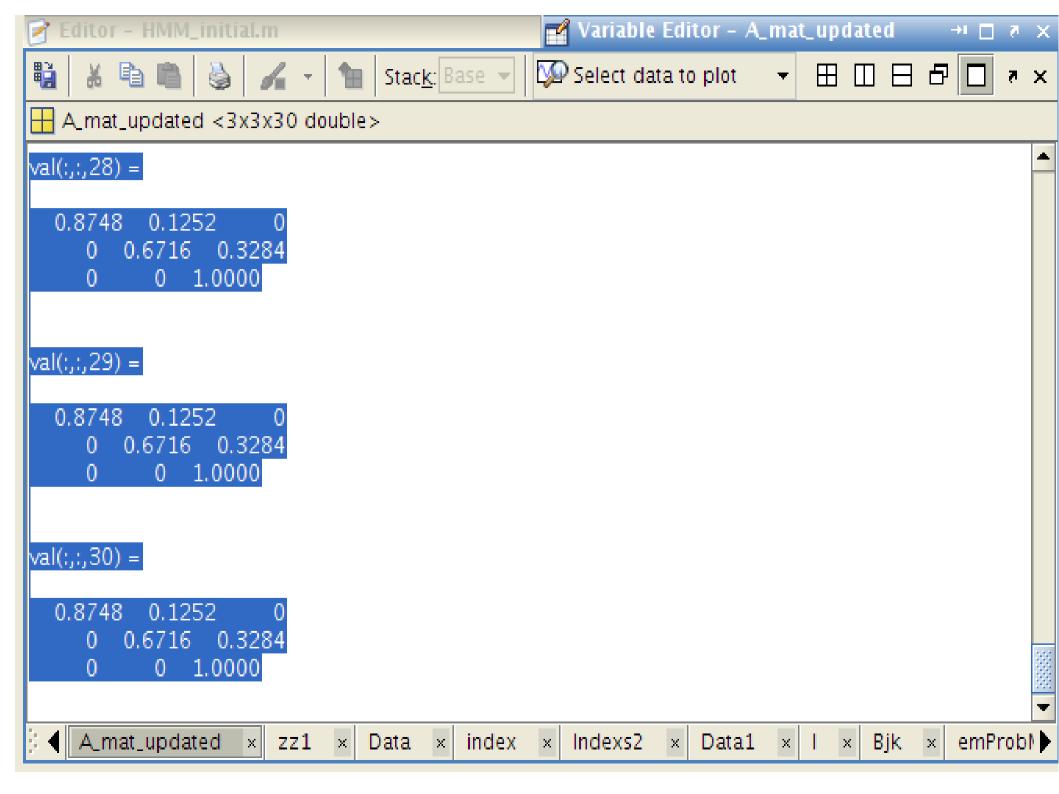


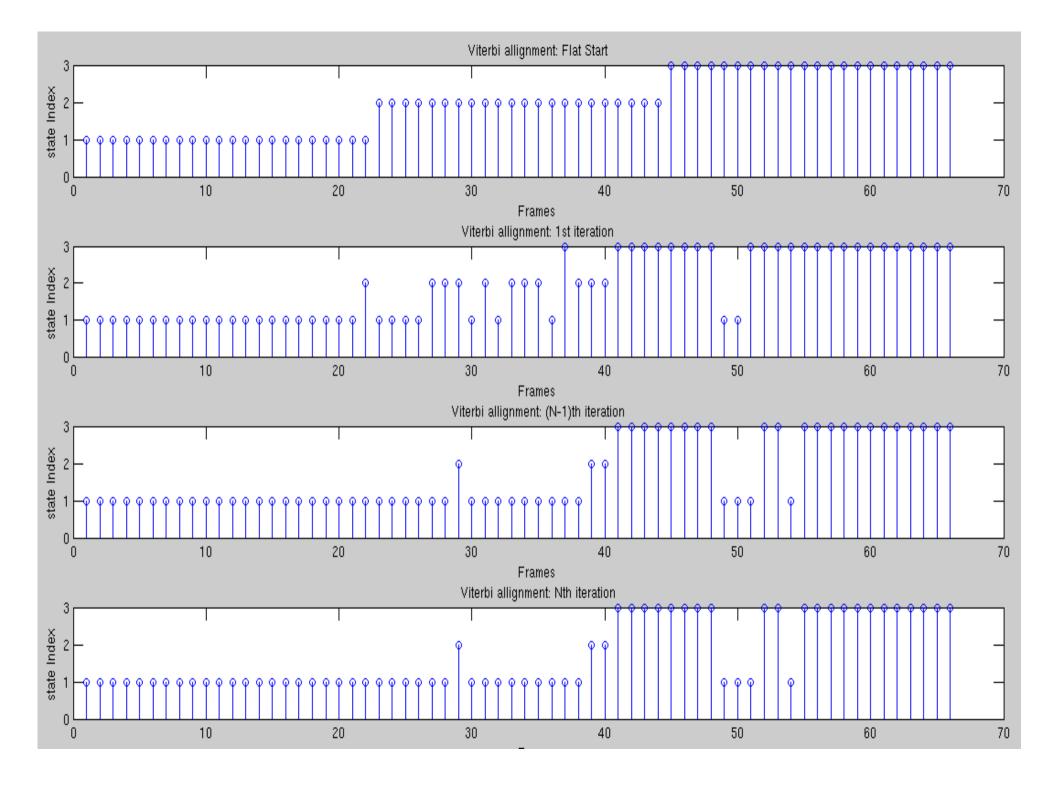
A(S1,:) = zeeta(S1,:)./[S1 S1 S1]

$$\xi_{t}(i,j) = \frac{\alpha_{t}(i) \ a_{ij}b_{j}(O_{t+1}) \ \beta_{t+1}(j)}{P(O|\lambda)}$$

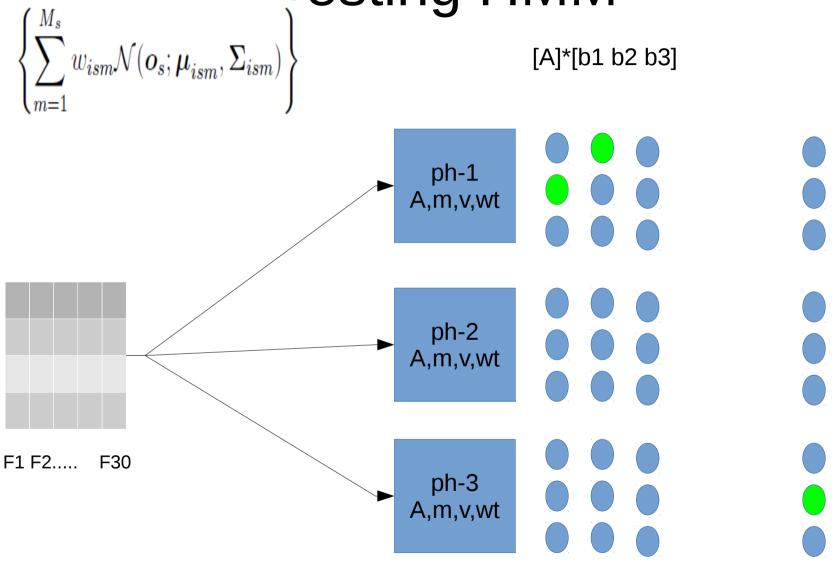
$$= \frac{\alpha_{t}(i) \ a_{ij}b_{j}(O_{t+1}) \ \beta_{t+1}(j)}{\sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_{t}(i) \ a_{ij}b_{j}(O_{t+1}) \ \beta_{t+1}(j)}$$







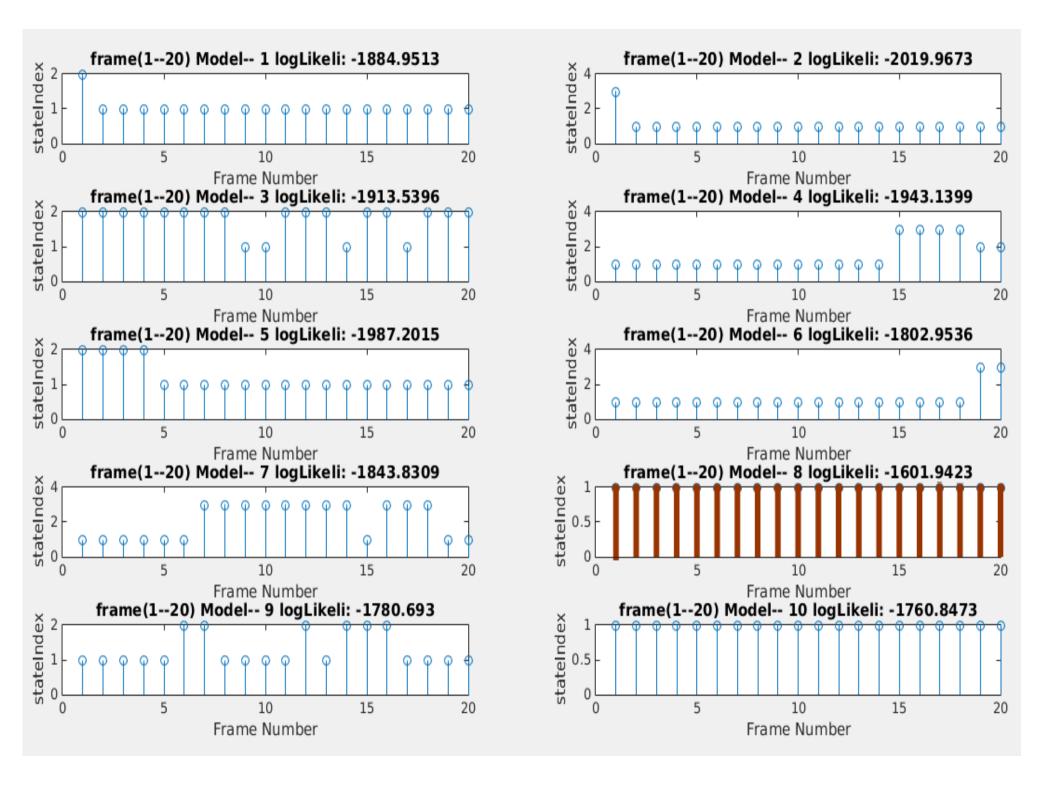
## **Testing HMM**

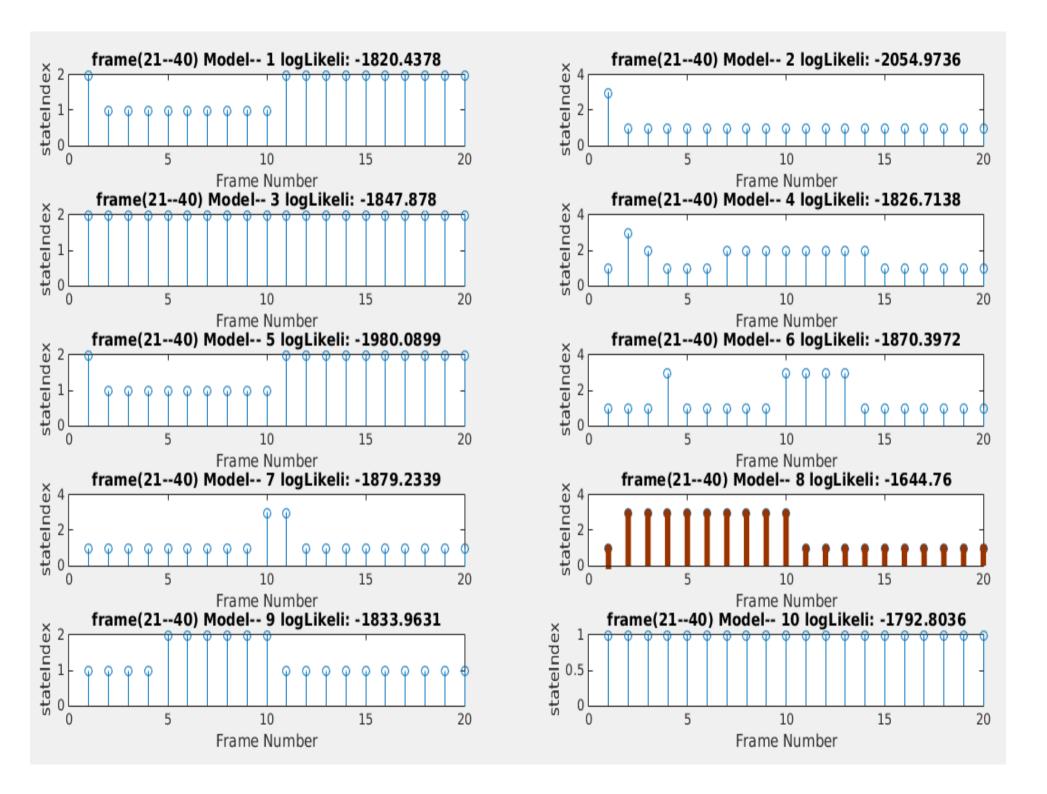


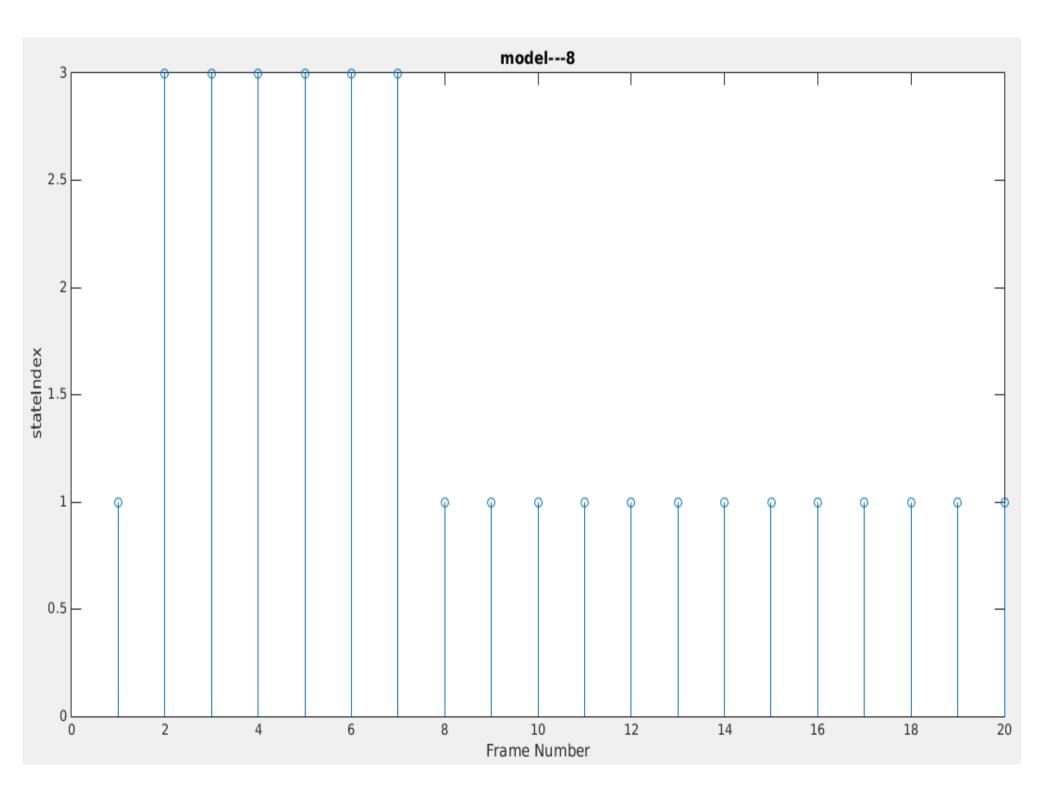
F1 F2 ..... F30

# Calculating log-Likelihood from updated parameters

- Trained parameters for each state of a class:
  - Mean vector
  - Diagonal covariance matrix
  - Weighting factors for each Gaussian in a GMM
  - Updated Transition matrix (N\*N)
  - priors
- Finding Likelihood:
  - $-\alpha(:,t) = B(:,t) .* (A' * \alpha(:,t-1));$
  - logLikelihood = log(sum( $\alpha(:, t)$ ));







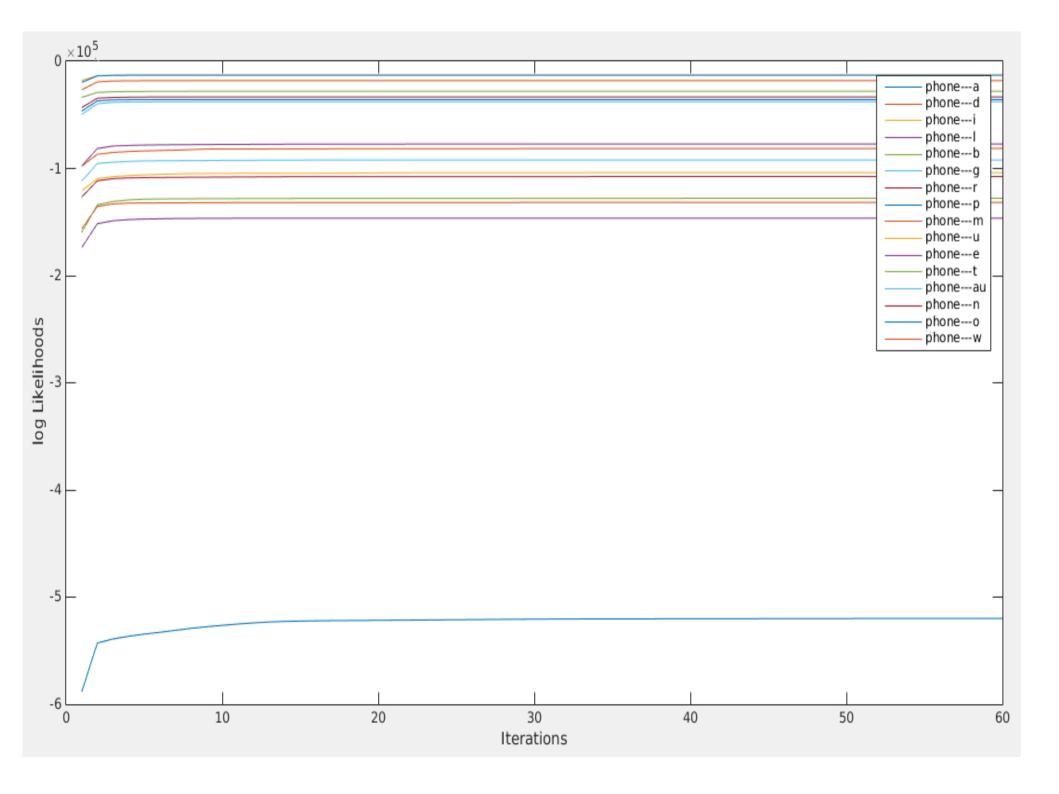
## Building a 3-state Phone HMM

adilabad
agra
alapi
amalapuram
a m b a l a
a m e t i
aurangabad
bangalore
b e l g a u m
bilwara

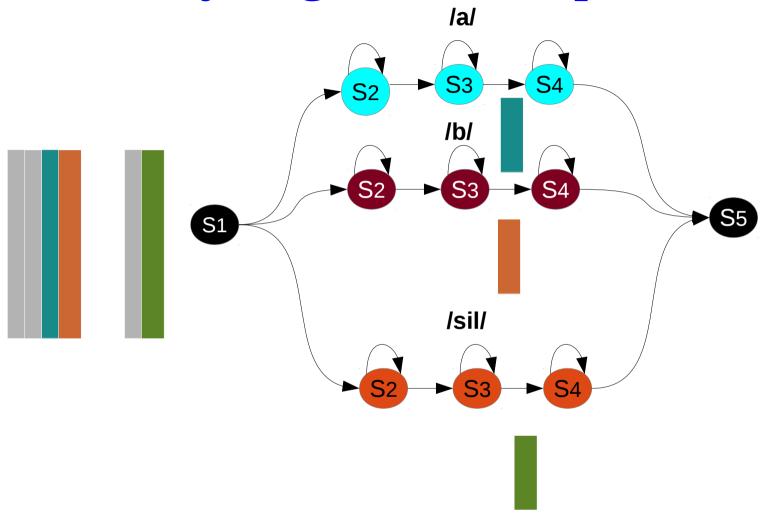
#### **Steps:**

- Flat start initialization----ex: 1 2 3 4 1 5 1 2
- Compute forward and backward probabilities
- Update the HMM parameters
- Stop training if the changes in HMM parameters is negligible

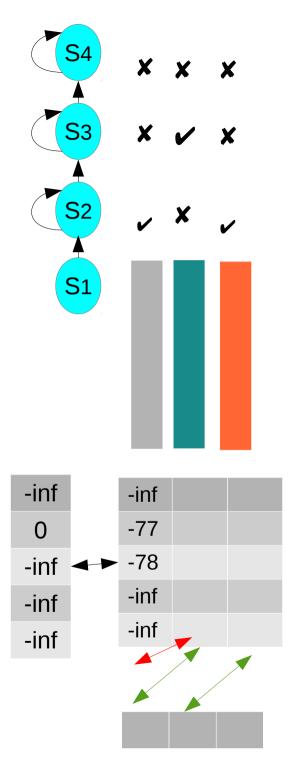
/a/	1
/d/	2
/i/	3
/1/	4
/b/	5
/g/	6
/r/	7
/p/	8
/m/	9
/u/	10
/e/	11
/t/	12
/au/	13
/n/	14
/o/	15
/w/	16

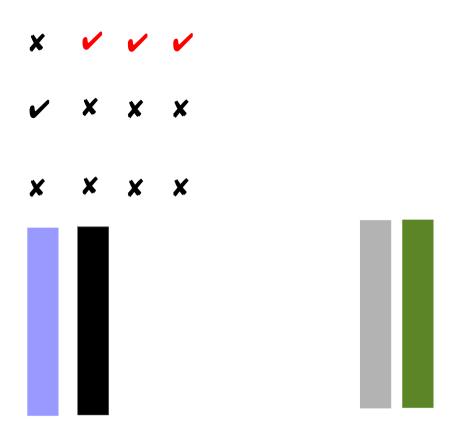


## Analysing the break points



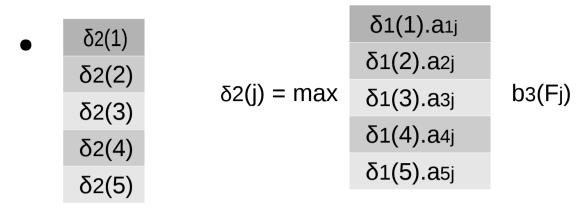
- Key idea:
  - Find the hidden state index where the final emitting state starts emitting the MFCC feature vectors
  - Analyze the corresponding viterbi path and its score w.r.t all trained models





- Identify the path with less significant Viterbi score
- Prune the Viterbi path by considering only the effect of maximum state sequence.

• t=2



At t=1, **p=2** 

δ2(1)		δ1(1).ap <sub>1</sub>	b3(Fp)
δ2(2)		δ1(2).ap2	b3(Fp)
δ2(3)	=	δ1(3).ap3	b3(Fp)
δ2(4)		δ1(4).ap4	b3(Fp)
δ2(5)		δ1(5).aps	b3(Fp)

### Token passing

- Keep the best token arriving at each state
- Propagate these tokens to the next states
- Find the probabilty of the most likely state alignment

## Practical Implementation of ANNs for Speech Applications

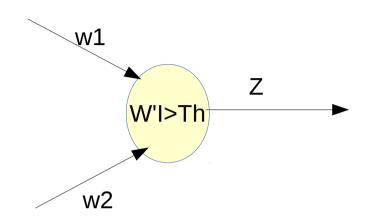
## Introduction

### **OR-Gate**

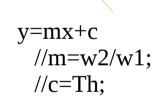
X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	1

### **XOR-Gate**

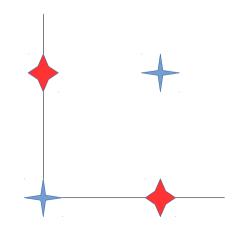
X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	0



Soln: 
$$Th = 0.1; W = [0.15 \ 0.2]$$

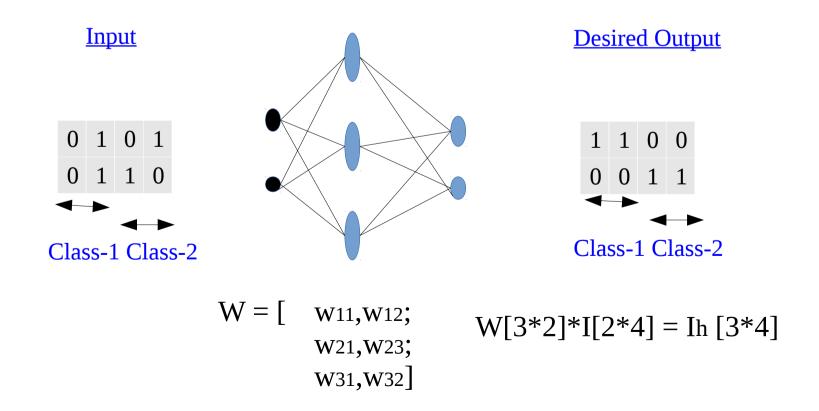


Χ



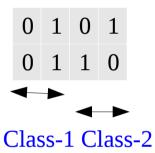
## Need of Hidden Layer

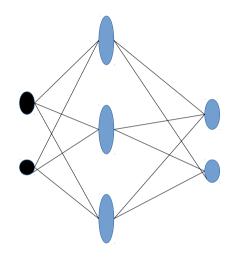
- Project the lower dimensional data into higher one if they are not separable.
- These projections are via weight vectors.



## Trained weights

### **Input**





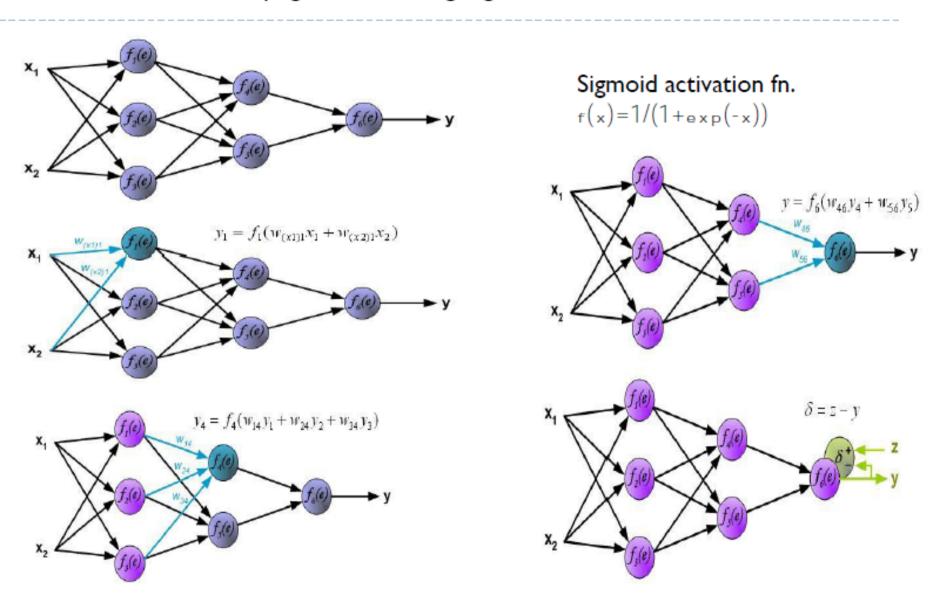
### **Desired Output**

$$W[3*2]*I[2*4] = Ih[3*4]$$

Actual output (after training)

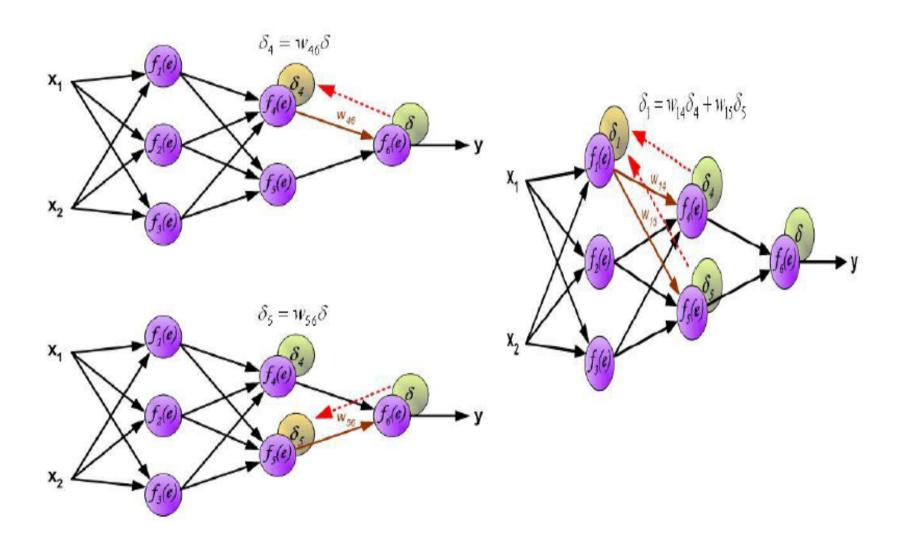
0.93	0.99	0.04	0.04
0.06	0.01	0.96	0.96

### Illustration of Back-Propagation learning algorithm



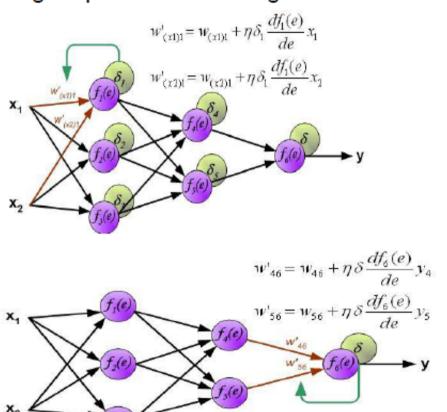
http://home.agh.edu.pl/~vlsi/AI/backp\_t\_en/backprop.html

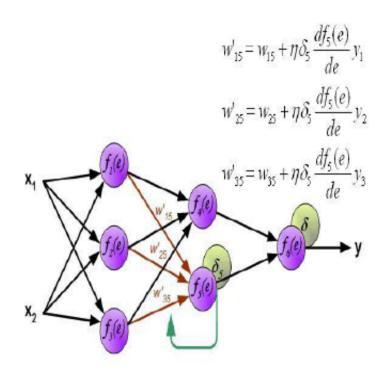
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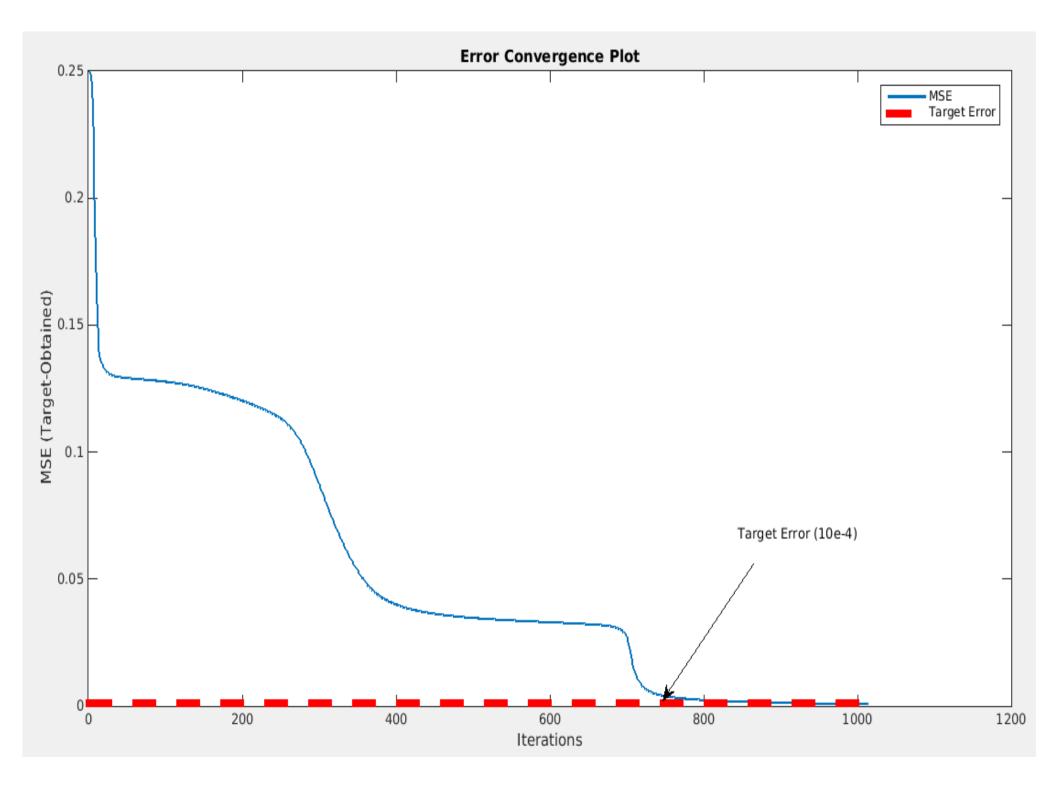
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### ■ Weight Update: Delta learning rule





■Stopping Criteria: Mean square error <= Goal error

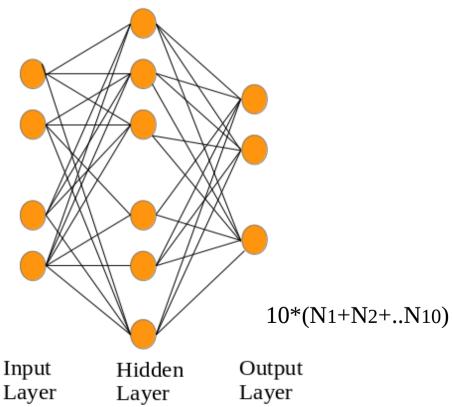


## ANN for Speech Applications



### Applicable for

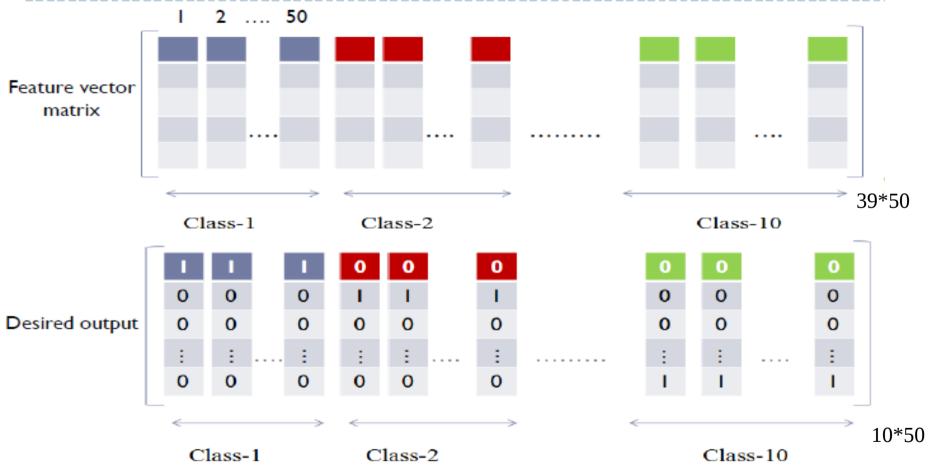
Speech Recognition Language Identification Speech Synthesis....



39\*(N1+N2+..N10)

Input Layer

### Training Feed forward Neural networks

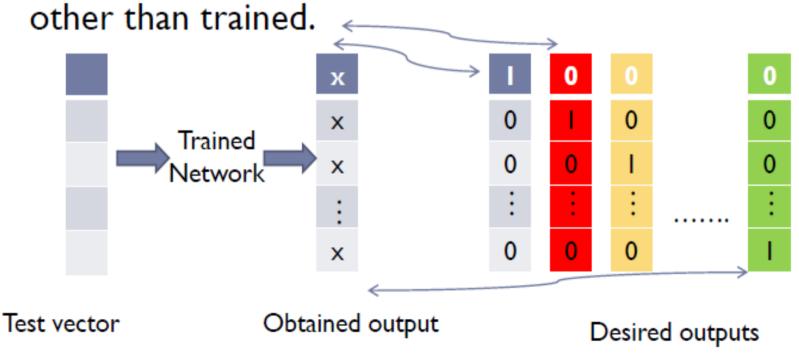


- ■Learning rate=0.001
- ■Min. MSE=0.000 I

## Testing FFNN

Weights are updated implying that the network has learnt.

Check the algorithm efficiency by feeding a new input



## Comparing GMMs and ANNs

GMMs	ANNs	
$\hat{w} = \operatorname*{argmax}_{w} p(w \mid \mathbf{x}) = \operatorname*{argmax}_{w} p(\mathbf{x} \mid w) p(w) / p(\mathbf{x})$	$y_j = logistic(x_j) = \frac{1}{1 + e^{-x_j}}, \qquad x_j = b_j + \sum_i y_i w_{ij},$	
<ul><li>Maximize the Likelihood score</li></ul>	<ul><li>Maximize the posterior probabilities</li></ul>	
<ul><li>Needs appropriate choice of Mixture</li></ul>	<ul><li>Choice of nodes and layers plays a vital</li></ul>	
components	role	

### **Future Work**

- To create HMM-ANN baseline
- HMM State Transitions



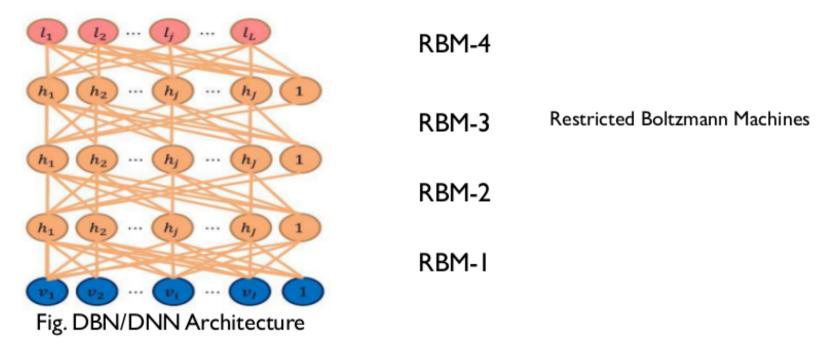
Viterbi Aligned Result

**HMM-ANN** Baseline

## Deep Neural Networks(DNNs)

[Hinton et. al, Neural Computation, 2006]

■DNNs or Deep Belief Networks (DBNs) are Multi layer perceptrons with many hidden layers but the difference exist in the training phase.



- G.Hinton discovered learning strategy for Multilayer Perceptrons
- a) Pre-train each layer from bottom to top
- b) Each pair of layers is an Restricted Boltzmann Machine(RBM)
- c) Jointly fine-tune all layers using back-propagation

### Restricted Boltzmann Machines

[Hinton et. al, Neural Computation, 2006]

■DNNs are formed by stacking several RBMs one upon another.

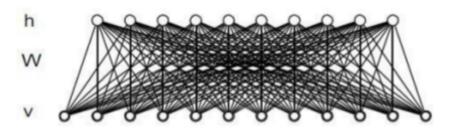


Fig. Restricted Boltzmann Machine (RBM)

#### Bernoulli-Bernoulli RBM

# $p(h_j = 1 | \mathbf{v}; \theta) = \sigma \left( \sum_{i=1}^{I} w_{ij} v_i + a_j \right),$ $p(v_i = 1 | \mathbf{h}; \theta) = \sigma \left( \sum_{j=1}^{J} w_{ij} h_j + b_i \right),$

### Gaussian-Bernoulli RBM

$$p(h_j = 1 | \mathbf{v}; \theta) = \sigma \left( \sum_{i=1}^{I} w_{ij} v_i + a_j \right),$$
$$p(v_i | \mathbf{h}; \theta) = \mathcal{N} \left( \sum_{j=1}^{J} w_{ij} h_j + b_i, 1 \right),$$

$$\sigma(x)=1(1+exp(-x))$$

## Unsupervised Pre-training

- $v=x_I$  is the input distribution.
- Find activations at the hidden layer

$$p(h_j = 1 \mid v; \theta) = \sigma(\sum_{i=1}^{V} w_{ij} v_i + a_j)$$

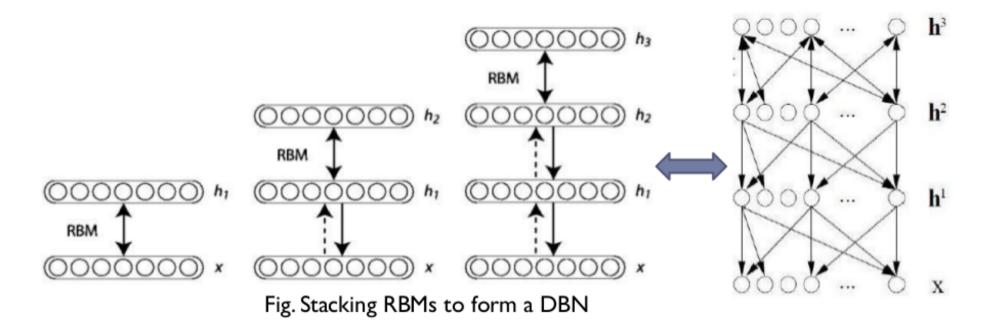
- Convert activation probabilities to binary representation, h2
- Compute the activations at the visible unit given the binary data at the hidden unit.

$$p(v_i = 1 | h; \theta) = N(\sum_{j=1}^{H} w_{ij} h_j + b_i)$$

- \* Again the activation probabilities at the hidden units given the new input distribution are found,  $P(h_2=1|x_2)$ .
- $\bullet$  The weight update rule for  $1^{\rm st}$  iteration as described by the above procedure is given as

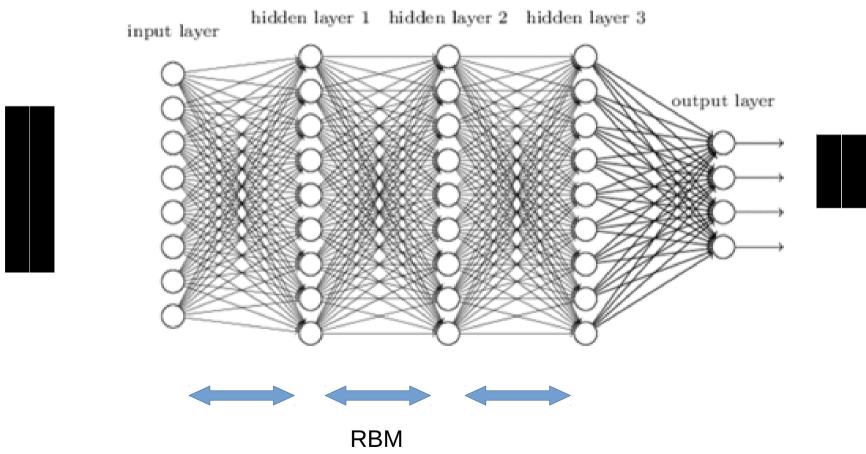
$$W=W+(h_1x_1^T-P(h_2=1|x_2)x_2^T)$$
 Contrastive Divergence

## Stacking RBMs



- •The activation probabilities of hidden units are used as the visible data for the next layer.
- This process is repeated to create other layers.
- Now Back-propagation algorithm is applied to adjust the DBN weights (fine-tuning phase)

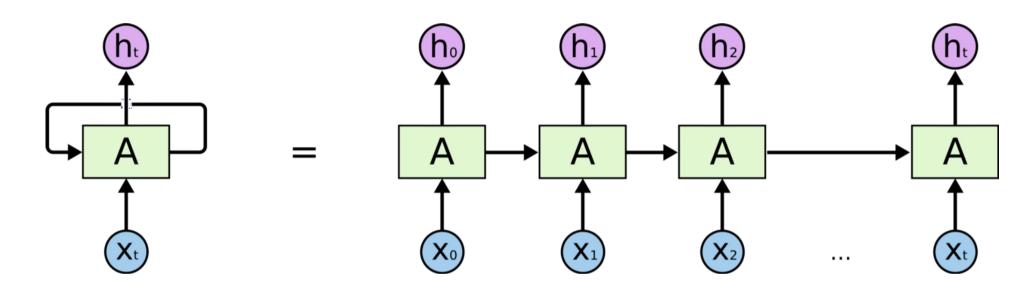
## **DNN**



### Training:

- Unsupervised Pre-training
- Supervised Fine tuning

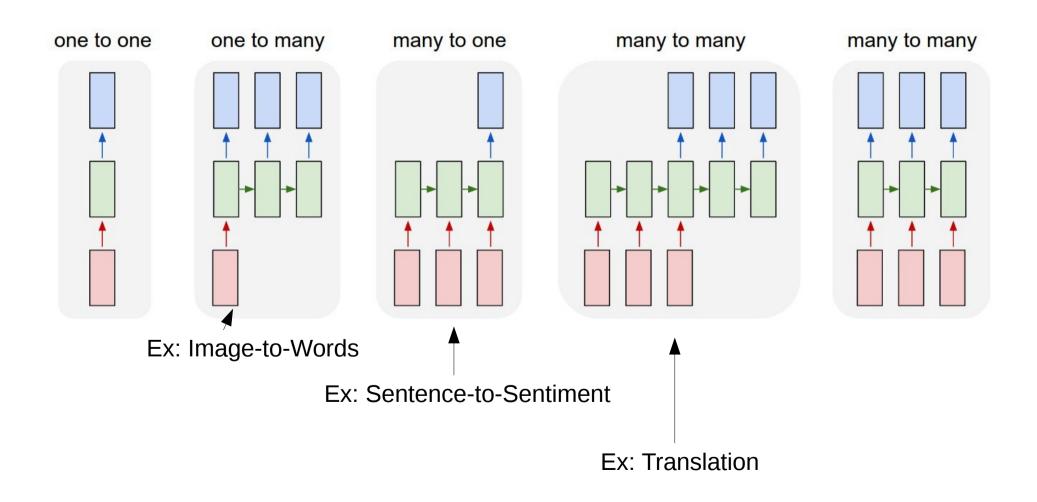
## **RNN**



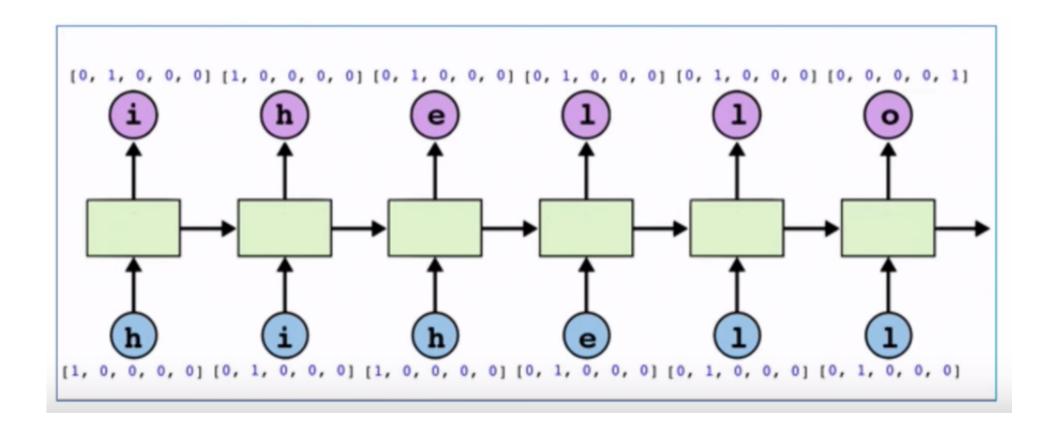
### Applications:

- Time series prediction
- Language modeling
- Text sentiment analysis
- Speech recognition
- Translation

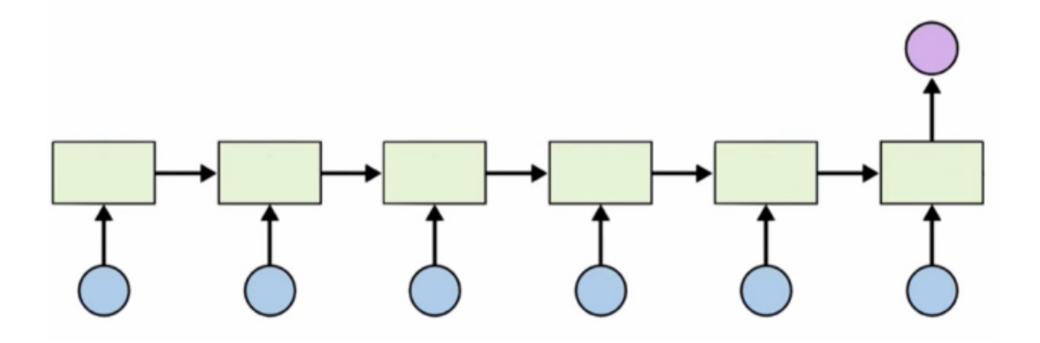
## Variations of RNN



## RNNs- Language modeling



## RNNs -- Classification

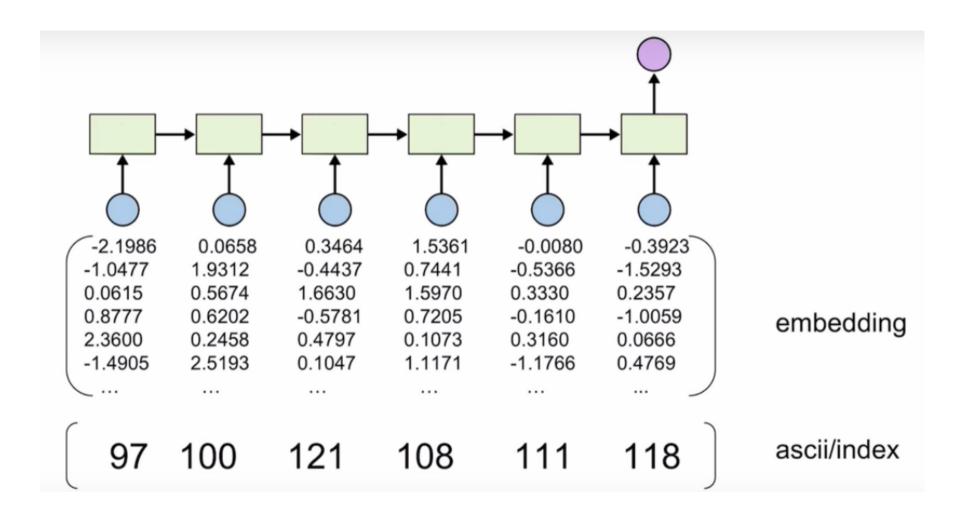


### Names Language

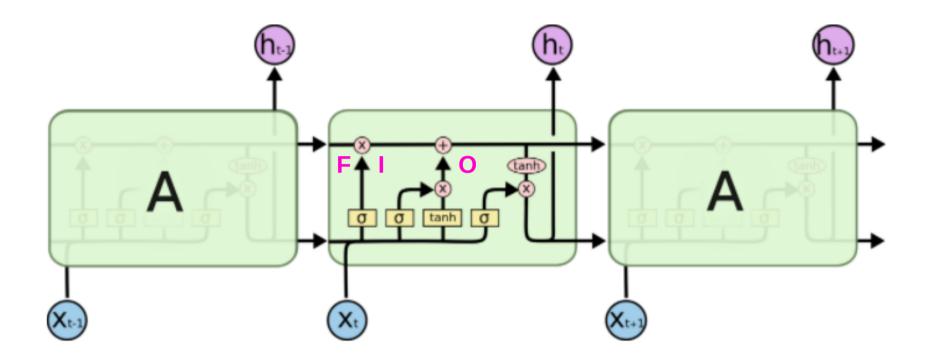
- Adalad German
- Adele German
- Lin Dan Chinese
- Chang Lee Chinese

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### RNNs--Classification



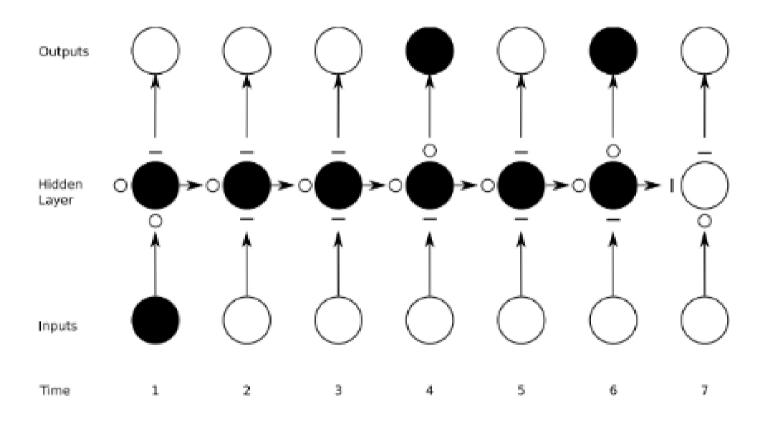
### LSTM



### Gates:

- Forget gate: Determines whether current contents of memory will be forgotten (erased)
- Input gate: Determines whether the input will be stored in the memory cell
- Output Gate: Determines if current memory contents will be output

## LSTM full network



Graves et al 2013

## Thank You