Spatial Filtering of Images

$$v(m,n) = T\left[u(m,n)\right]$$

$T \Rightarrow$ 3×3 neighborhood operation

$$U(m,n) = \left\{ \begin{array}{l} u(m-1,n),\ u(m,n),\ u(m+1,n) \\ u(m-1,n-1),\ u(m,n-1),\ u(m+1,n+1) \\ u(m+1,n+1),\ u(m,n+1),\ u(m+1,n+1) \end{array} \right.$$

Spatial Averaging

$$= \sum_{(k,l)\in w}\sum a(k,l)\, y(m-k, n-l)$$

$y(m,n)$ — I/p image

$v(m,n)$ — o/p image

w = window

$a(k,l)$ = Filter

$a(k,l) = 1$ over the window

N_w = # pixels in a window

$$v(m,n) = \frac{1}{N_w} \sum_{(k,l)\in w}\sum y(m-k, n-l)$$

Weighted Average

1	1	1
1	4	1
1	1	1

window size ↑ ⇒ Blurring & sharpness will be lost

<u>Adv</u> Spatial Avg ⇒ LPF, Smoothing effect

Noise removal

$$y(m,n) = u(m,n) + \eta(m,n)$$

↑ white noise zero mean σ_η^2

$$v(m,n) = \frac{1}{N_w} \sum_{(k,l)\in W} u(m-k, n-l) + \bar{\eta}(m,n)$$

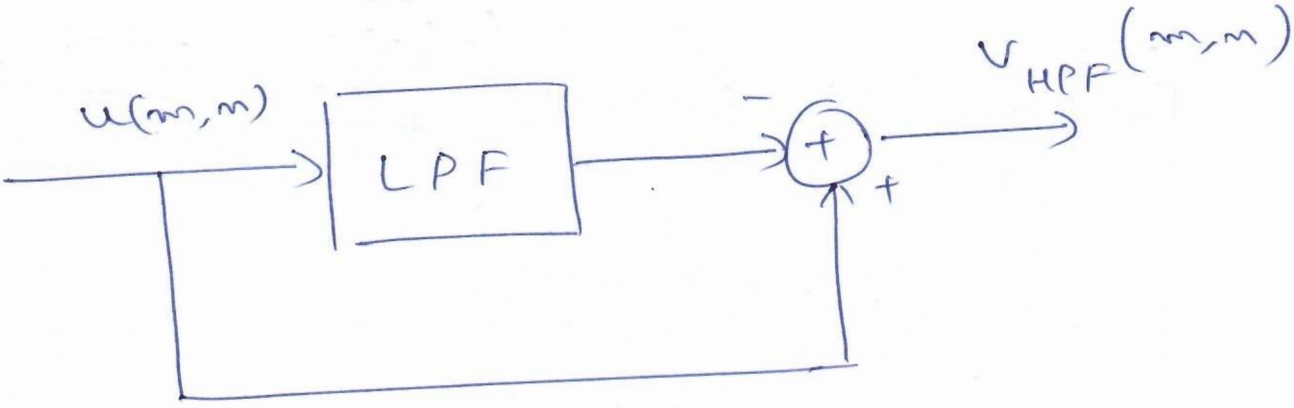
$$\sigma_{\bar{\eta}}^2 = \frac{\sigma_\eta^2}{N_w}$$

Spatial Non-linear filter

$$v(m,n) = \underset{(k,l)\in W}{\text{median}} \left\{ y(m-k, n-l) \right\}$$

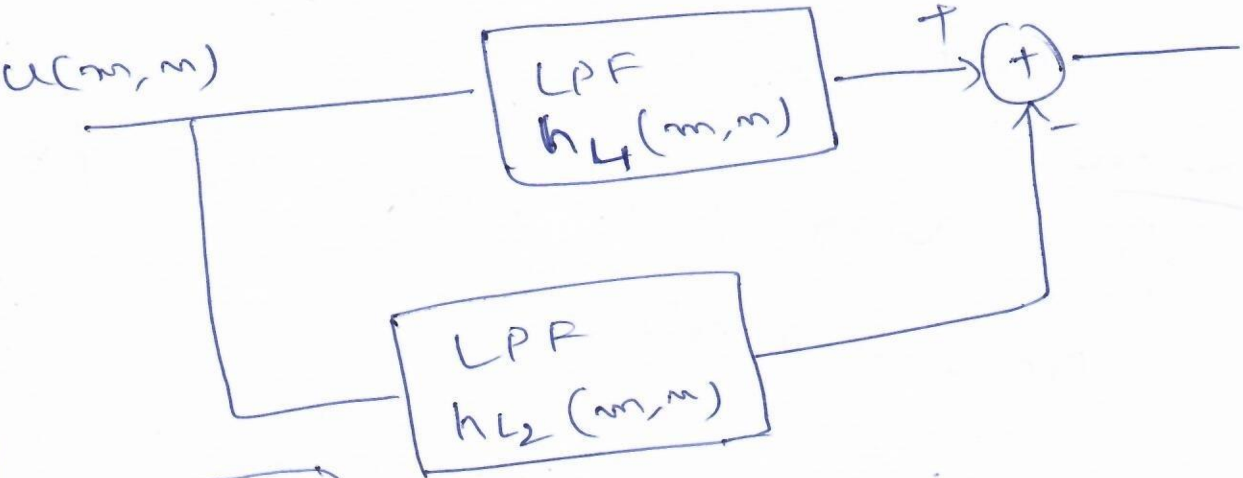
$1\ 2\ 2\ 3\ \boxed{5}\ 5\ 7\ 8\ 18$

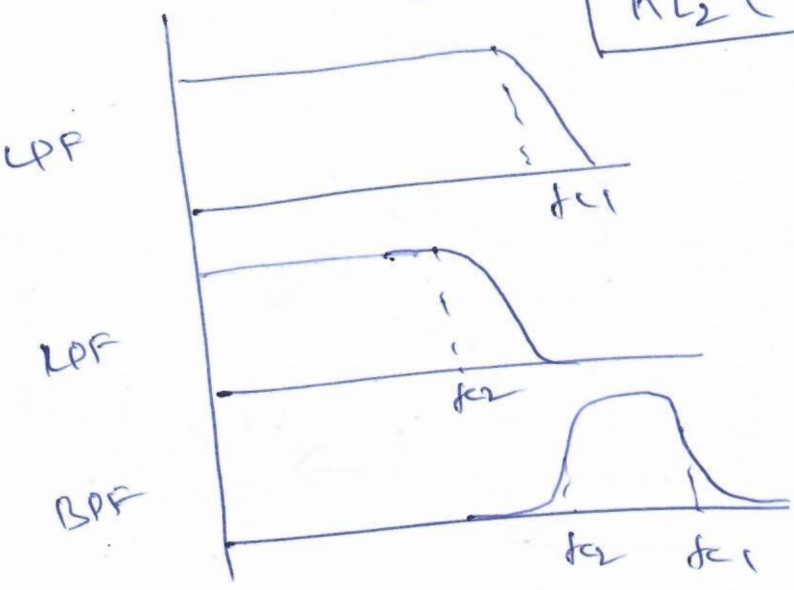
$$5 \leftarrow \begin{bmatrix} 1 & 2 & 5 \\ 8 & 18 & 3 \\ 2 & 7 & 5 \end{bmatrix}$$



HPF of Image — Highlight the details of the image.

BPF of an Image — Removing Noise + sharpening the image





Sharpening Filter (High pass filter)

(1) For uniform intensity regions, the filter o/p should be zero

(2) Details of changes of intensity should be highlighted

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

$\sum z_i = 0$

-1	-1	-1
-1	8	-1
-1	-1	-1

1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	5	1	1
1	1	1	1	1	1

Intensity values may be -ve (centre zero) and more than 255.

∴ Scaling is required

DIS-ADV

Slowly varying intensities are not preserved

DC value is also lost

$$\text{HPF image} = \text{original image} - \text{LPF image}$$

$$= A(\text{original}) - \text{LPF image}$$

$$= (A-1)\,\text{original} + \text{original} - \text{LPF filter}$$

$$= \boxed{(A-1)\,\text{original} + \text{HPF image}} \rightarrow \text{High boost image}$$

(3)

High boost Image => slow intensity variation + details of the image

$$\frac{1}{9} \times \begin{bmatrix} -1 & -1 & -1 \\ -1 & w & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

$w = 9A - 1 \quad A \geq 1$

$A = 1 \Rightarrow$ HPF

$A > 1 \Rightarrow$ High boost filter

LPF => Integration

HPF => Differentiation

1st order Derivative

Gradient of $f(x,y)$ at point (x,y)

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} \; ; \; \underbrace{|\nabla f|}_{M(x,y)} = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

$$\text{mag } \nabla f = \sqrt{\underbrace{(z_5 - z_6)^2}_{\left(\frac{\partial f}{\partial x}\right)} + \underbrace{(z_5 - z_8)^2}_{\left(\frac{\partial f}{\partial y}\right)}}$$

$$\frac{\partial f}{\partial y} \to \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix} \quad \frac{\partial f}{\partial x} \to \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

$$\approx |z_5 - z_6| + |z_5 - z_8|$$

Cross Gradients operators

$$\approx |z_5 - z_9| + |z_6 - z_8|$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Robert's cross gradient operators

Gradients for 3×3 Mask

Prewitt's operator

$\frac{\partial f}{\partial y}$

-1	-1	-1
0	0	0
1	1	1

$\frac{\partial f}{\partial x}$

-1	0	1
-1	0	1
-1	0	1

Sobel's operator

-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1

Edge Detection

* Horizontal & vertical gradients are averaged combined to ... other detection

Prewitt & Sobel operators detect the gradients only in vertical (N & S) & Horizontal (E & W) directions

$$\begin{matrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{matrix} \uparrow N \qquad \begin{matrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & -1 \end{matrix} \nwarrow NW \qquad \begin{matrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{matrix} \nearrow NE \qquad \begin{matrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{matrix} \swarrow SW$$

8-compass gradient operators

Kirsch operator

$$\begin{matrix} 5 & 5 & 5 \\ -3 & 0 & -3 \\ -3 & -3 & -3 \end{matrix} \uparrow N$$

Max [8-compass gradient operators]

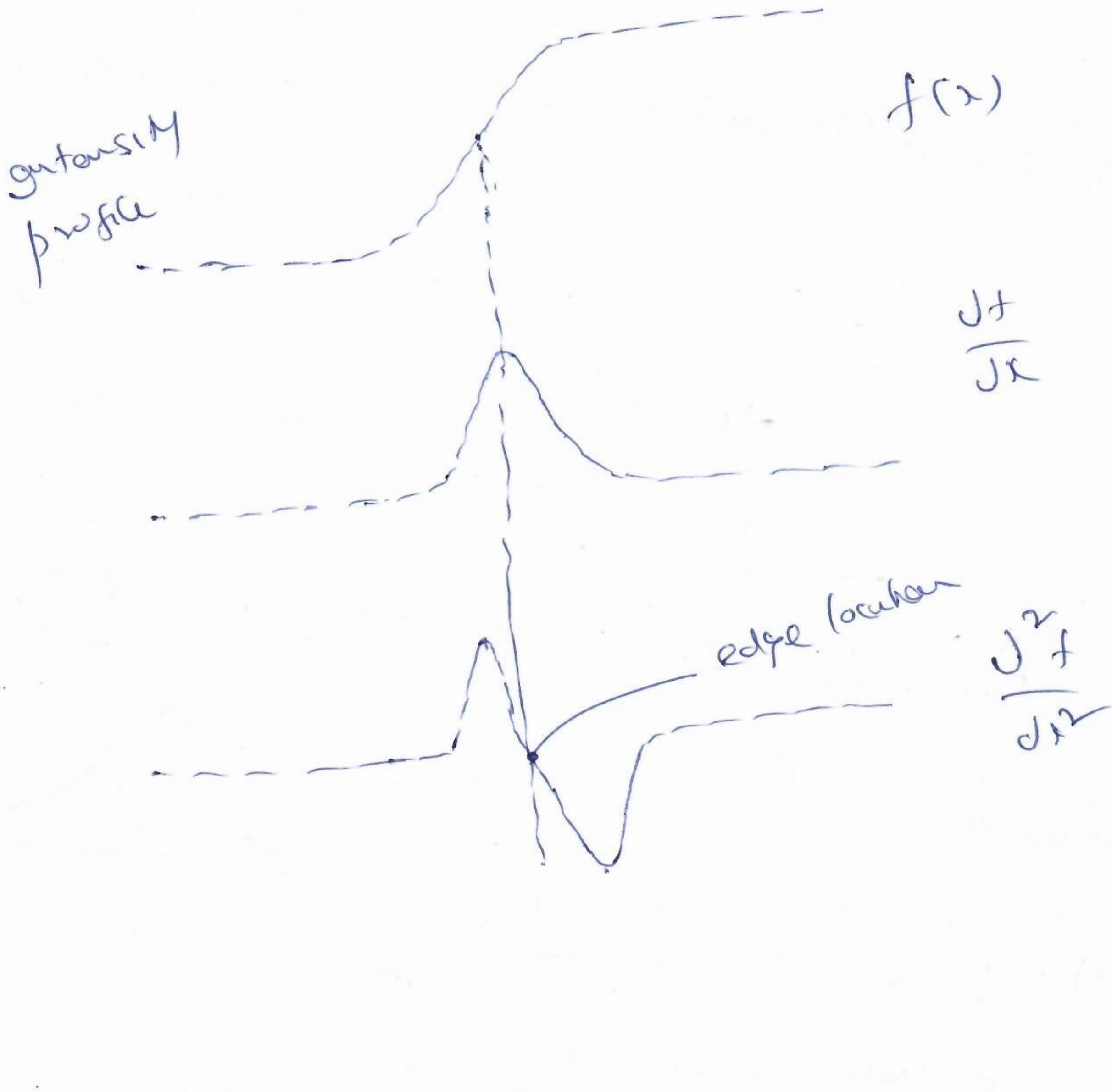
Gradient at location (m,n) is defined as

$$g(m,n) = \max_k \left\{ \left| g_k(m,n) \right| \right\} \quad \text{over } k = 0, 1, \ldots 7$$

$$\text{gradient direction} = \theta_k = \frac{\pi}{2} + k\frac{\pi}{4}$$

$$k = 0, 1, \ldots 7$$

Edge-Map (Binary Image)



Second Derivative in 2D (Laplacian operator)

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$f(x-1, y+1)$	$f(x, y+1)$	$f(x+1, y+1)$
$f(x-1, y)$	$f(x, y)$	$f(x+1, y)$
$f(x-1, y+1)$	$f(x, y+1)$	$f(x+1, y-1)$

$$\Delta_1 = f(x,y) - f(x-1,y)$$

$$\Delta_2 = f(x+1,y) - f(x,y)$$

$$\left.\begin{aligned} \frac{\partial^2 f}{\partial x^2} &= \Delta_2 - \Delta_1 = f(x+1,y) + f(x-1,y) - 2f(x,y) \\ \frac{\partial^2 f}{\partial y^2} &= f(x,y+1) + f(x,y-1) - 2f(x,y) \end{aligned}\right]$$

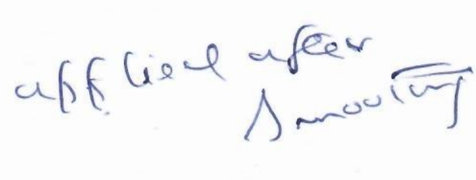
$$\nabla^2 f = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

0	1	0
1	-4	1
0	1	0

Along with edges, Noisy pixels also highlighted

∴ Laplacian operator will be applied after Smoothing

Gaussian Smoothing filter



$$g(x,y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right)$$

σ controls the extent of blurring.
Multi-Scale filtering.

Laplacian of Gaussian operator filtering — L-O-G filtering

$$\nabla^2 G(x,y) = c\left[1 - \frac{(x^2+y^2)}{\sigma^2}\right] \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

Performance Detection of Edge detection operator

n_0 — # Edge pixels ~~detected~~ declared

n_1 — # mixed of New edge pixels after noise addition

Edge detection error rate $P_e = \frac{n_1}{n_0}$

Figure of Merit $P = \frac{1}{\max(N_I, N_D)} \sum_{i=1}^{N_D} \frac{1}{1+\alpha d_i^2}$

N_I = # Ideal edge pixels

N_D = # detected edge pixels.

d_i = distance bet a pixel declared as edge and the nearest ideal edge pixels.