Module-2 Propositions

A proposition is a ~~declaration~~ 'declarative' statement which is either True or false but not both.

Ex: Delhi is the capital of India. True
Khushal is a boy. True

⇒ **Primitive Proposition:** Which can be broken down further.

⇒ **Compound Proposition:** If we combine two or more proposition with the help of logical connectives (OR, AND, etc.).

Ex: Umvee is earning & vaibai is counting.

⇒ Logical Connectives:

i) AND —— $\wedge$ —— Conjunction
ii) OR —— $\vee$ —— Disjunction
iii) NOT —— $\neg$ —— Negation
iv) Conditional/Implication —— ($\longrightarrow$) If p then q
v) Biconditional/ Equivalence —— ($\longleftrightarrow$) p if & only if q.

⇒ **Conjunction ($\wedge$)** similar to and.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

★ Disjunction (∨) Similar to OR.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

★ Negation: $\neg$ similar to NOT

p	$\neg p$
T	F
F	T

→ Conditional ($\longrightarrow$): If p then q.

p	q	$p \longrightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

Ex: p: Maria learns Discrete Mathematics.
q: Maria will get a job.
$p \rightarrow q$: If Maria learns Discrete Mathematics then she will get a job.

10-3-21

Biconditional or Equivalence

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

P iff Q {P if and only if Q}

Q. Let P the statement, You can take the flight, Q be the statement, You buy the ticket. Represent the statement as biconditional.

You can take the flight, only if you buy a ticket.
if &

Precedence Of Connectives

Precedence	Connective
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

XOR $P \oplus Q$

P	Q	$P \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

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Q $\neg(p \leftrightarrow q) = p \leftrightarrow \neg q$

p	q	$p \leftrightarrow q$	$\neg q$	$p \leftrightarrow \neg q$
T	T	T	~~F~~	~~F~~
T	F	F	~~T~~	~~T~~
F	T	F	~~F~~	~~T~~
F	F	T	~~T~~	~~T~~

p	$\neg q$	$p \leftrightarrow \neg q$
T	T	T
T	F	F
F	T	F
F	F	T

Tautology

A compound preposition that is always true $\forall$ possible truth values of its variable.
In other words, contain only true (T) in the last column of the truth table is called tautology

Q P.T all are Tautology
a) $p \vee \neg p$
b) $\neg(p \wedge q) \vee q$
c) $p \rightarrow (p \vee q)$

(a)

P	$\neg P$	$P \lor \neg P$
T	F	T
F	T	T

(b) $\neg(P \land Q) \land Q$

P	Q	$P \land Q$	$\neg(P \land Q)$	$(P \land Q) \land Q$	$\neg(P \land Q) \land Q$
T	T	T	F	T	F
T	F	F	T	F	T
F	T	F	T	F	T
F	F	F	T	F	~~F~~ T

* Contradiction

A compound preposition ie always false for possible values of the variables. In other words, contain only F (False) in last column is called contradiction.

Ex $P \land (Q \land \neg P)$

P	Q	$\neg P$	$(Q \land \neg P)$	$P \land (Q \land \neg P)$
T	T	F	F	F
T	F	F	F	F
F	T	T	T	F
F	F	T	F	F

* Contingency

A proposition i.e. neither tautology nor a contradiction is called contingency.

Ques

Paris is in France & England is in London.

Paris is in France or England is in London.

*

* Negation of Conditional *

$$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \sim q \equiv \neg p \leftrightarrow q$$

* Tautologically Implication *

A compound proposition $A(p, q \ldots)$ is said to be tautologically imply or simply imply the compound proposition $B(p, q, \ldots)$ if & only if $A \to B$ is a tautology.

This is denoted by $A \Rightarrow B$ and read as 'A implies B'.

Ex $p \wedge q \Rightarrow q$ since $p \wedge q \to q$ is a tautology.

Ex 1. $p \Rightarrow p \vee q$ 2. $\sim p \Rightarrow p \to q$ 3. $q \Rightarrow p \to q$

4. $\sim(p \to q) \Rightarrow p$ 5. $\sim(p \to q) \Rightarrow \sim q$ 6. $p \wedge (p \to q) \Rightarrow q$

7. $\sim q \wedge (p \to q) \Rightarrow \sim p$ 8. $\sim p \wedge (p \vee q) \Rightarrow q$

9. $(p \to q) \wedge (q \to r) \Rightarrow p \to r$

10. $(p \vee q) \wedge (p \to r) \wedge (q \to r) \Rightarrow r$

24-03-22

Arguement.

An arguement is a sequence of statements.
All statements except the final one are called premises or hypothesis or assumptions.

* The final statement is called conclusion.

Ex:

Ex: If he works hard, he will pass. : Premise 1
If he pass, he will be happy. : Premise 2
Hence, Hardwork makes happy.

Valid Arguements

An Arguement is said to be valid if the conclusion is true whenever all premises are true.

An arguement is .

Me

1. Identify all the premises & conclusion of the arguement.
2. Construct the Truth Table, showing the truth values for all the premises and conclusion.
3. Find the rows called critical rows in which all the premises are true.
4. In each critical rows check if the conclusion is true, ~~the arguement is valid if there is one critical rows in which conclusion if is~~ false Then arguement is fallacy.

(a) If in each critical row the conclusion is also true, then then arguement form is valid.
(b) If there is ~~atleast~~ at least one critical row in which the conclusion is false, the arguement form is invalid.

Cont

A pre

Ques

Par

Ne

Ques Show the arguement valid/fallacy.

$p \to q$, $\sim q$ → Premises

$\sim p$ → conclusion.

p	q	$p \to q$	$\sim q$	$\sim p$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Valid

If it rains today then we will not have a party today.
If we do not have party today, then we will have a party tomorrow.
Therefore if it rains today then we will have party tomorrow.

p : It rains today

q : We will have a party today.

r : We will have a party tomorrow.

Pre 1 : $p \to \sim q$

Pre 2 : $\sim q \to r$

~~Pre 3 :~~

Critical Statement : $\boxed{p \to \sim q \to r}$

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p	q	r	$\sim q$	$p \to \sim q$	$\sim q \to r$	$(p \to \sim q) \to r$
T	T	T	F	F	T	T
T	T	F	F	F	T	T
T	F	T	T	T	T	T
T	F	F	T	T	F	T
F	T	T	F	T	T	T
F	T	F	F	T	T	T
F	F	T	T	T	T	T
F	F	F	T	F	T	T

Statement 1. ~~Statement 2~~.

Q If a no. % 6 == 0 Then no. % 3 == 0.

This no. % 3 != 0 state

This no. % 6 != 0

Q Either Ram is not Guilty or Shyam is ~~not~~ telling the truth. Shyam is not telling the truth. Hence Ram is not Guilty.

Logical Equivalences

* Compound propositions that have same truth values in all possible cases are logically equivalent.
* The compound propositions p and q are called logically equiva if $p \leftrightarrow q$ is a tautology
* The notation $p \equiv q$ denotes that p and q are logically equival

Equivalence	Name
1. $p \wedge T \equiv p$ $p \vee F \equiv p$	Identity Laws
2 $p \vee T \equiv T$ $p \wedge F \equiv F$	Domination Laws.
3 $p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent Laws
4 $\neg(\neg p) \equiv p$	Double Negation Law
5 $p \vee q \equiv q \vee p$, $p \wedge q \equiv q \wedge p$	Commutative Laws
6 $(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative Laws
7 $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$.	Distributive Laws
8 $\neg(p \wedge q) \equiv \neg p \vee \neg q$, $\neg(p \vee q) \equiv \neg p \wedge \neg q$	Demorgan's Laws
9 $p \vee (p \wedge q) \equiv p$, $p \wedge (p \vee q) \equiv p$	Absorption Laws
10 $p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$	Negation Laws

Logical Equivalences Involving Conditional Statements.

1. $p \rightarrow q \equiv \neg p \vee q$
2. $p \rightarrow q \equiv \neg q \rightarrow \neg p$
3. $p \vee q \equiv \neg p \rightarrow q$
4. $p \wedge q \equiv \neg(p \rightarrow \neg q)$
5. $\neg(p \rightarrow q) \equiv p \wedge \neg q$
6. $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
7. $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$
8. $(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$
9. $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

Logical Equivalences Involving Biconditionals.

1. $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
2. $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
3. $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
4. $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Logical Equivalences.

Two propositions can be proved logically equivalent in two ways:

* Using Truth tables.
* Using Logical Identities.

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Arguement:

- An arguement in propositional logical is a sequence of propositions.
- All but the final proposition in the arguement are called premises or hypothesis and the final proposition is called the conclusion.

Ex: • If you have a current password then you can log on to the n/w.
• You have a current password.
• Therefore
• "You can log onto the network.
• This is an arguement with two premises and a conclusion.

Validity of Arguements

- An arguement is valid if the truth of all the premises implies that the conclusion is true.

Arguement Form

- An arguement form in propositional logic is a sequence of compound propositions involving propositional variables.
- An arguement form with premises $p_1, p_2, p_3, \ldots, p_n$ and conclusion q is valid, when $(p_1 \wedge p_2 \wedge p_3 \wedge \ldots \wedge p_n) \rightarrow q$ is a tautology.
- The key to showing that an arguement in propositional logic is valid is to show that its arguement form is valid.

Ex: Consider an arguement: If you have a current password, then you can log onto the network. You have a current password. Therefore, You can log onto the network.

- Arguement Form
- Let p represents "You have a current password."
- and q represents "You can log onto the network".
- Then, the arguement has the form.

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

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Rule Of Inference	Tautology	Name.
$\begin{array}{l} p \\ p \to q \\ \hline \therefore q \end{array}$	$p \wedge (p \to q) \longrightarrow q$	Modus Ponens
$\begin{array}{l} \neg q \\ p \to q \\ \hline \therefore \neg p \end{array}$	$\neg q \wedge (p \to q) \longrightarrow \neg p$	Modus tollens.
$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$	$(p \to q) \wedge (q \to r) \longrightarrow p \to r$	Hypothetical Syllogism
$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$	$(p \vee q) \wedge \neg p \longrightarrow q$	Disjunctive Syllogism
$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}$	$p \to (p \vee q)$	Addition
$\begin{array}{l} p \wedge q \\ \hline \therefore p \end{array}$	$(p \wedge q) \longrightarrow p$	Simplification
$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$	$p \wedge q \to (p \wedge q)$	Conjunction
$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array}$	$(p \vee q) \wedge (\sim p \vee r) \to q \vee r$	Resolution

Resolution Principle

* **Literal** : A variable or negation
* **Clause**: Disjunction of literals.
* Resolution rule of inference is based on the tautology

$$((p \vee q) \wedge (\neg p \vee r)) \longrightarrow (q \vee r)$$

* The final disjunction in the resolution rule, $q \vee r$, is called the resolvent.

Argument validity by Resolution Principle

* Consider an argument where $P_1, P_2, P_3 \ldots P_n$ are premises and C is the conclusion.
* To prove the validity of the argument by resolution principle, Put $P_1, P_2, \ldots P_n$ in clause form and add to it $\sim C$ in clause form.

From this sequence if $\square$ (Empty clause) can be derived, the argument is valid.

Ques If today is tuesday, I have a test in Mathematics or Economics. If my Economics Professor is sick, I will not have a test in Economics. Today is tuesday and my Economics Professor is sick. Therefore, I have a test in Mathematics.

Converting to logical notation.

Let T denote 'Today is tuesday'.

M : I have a test in Mathematics.

E : I have a test in Economics.

S : My professor is sick.

So, premises are

$$T \longrightarrow (M \vee E)$$
$$S \longrightarrow \sim E$$
$$T \wedge S$$
$$\overline{\therefore M}$$

Putting into clause form: $C_1 : \sim T \vee M \vee E$

$C_2 : \sim S \vee \sim E$

$C_3 : T$

$C_4 : S$

$C_5 : \sim M$ (negation of conclusion)

From C_1 and C_2 : $C_6 : \sim T \vee M \vee \sim S$

C_6 and C_3 $C_7 : M \vee \sim S$

C_7 and C_4 $C_8 : M$

C_8 and C_5 $C_9 : \square$

Hence argument is valid.

NORMAL FORMS

$\vee$ denoted by $+$ $\wedge$

$\wedge$ denoted by $\cdot$

ELEMENTARY PRODUCT

A product of variables and their negations is called an Elementary product

$p, q, p\wedge q, p\wedge \sim q, \sim p\wedge q, \sim p\wedge \sim q$

Elementary Sum $\vee$

A sum of variables and their negations is called an elementary sum. $(\wedge \cdot)\ \vee\ (\wedge)$

$p, q, p\vee q, p\vee \sim q, \sim p\vee q, \sim p\vee \sim q$

DNF : Disjunctive Normal Forms.

Statement which consists of a *sum of elementary products of* ~~prod~~ propositional variables

This form is not unique for the given variables.

$(p\wedge q) \vee (p\wedge \sim q)$

Method To Obtain DNF

1. Replace $\rightarrow$ and $\leftrightarrow$ by $\wedge$ and, $\vee$ and $\sim$ in the statement.
2. Manipulate to get an equivalent form which is the sum of Elementary product.

Q Obtain DNF of $p\rightarrow q \wedge \sim q$

$(\sim p \vee q) \wedge (\sim q)$

$(\sim p\wedge \sim q) \vee (q\wedge \sim q)$ * {Distributive Law}

~~$(\sim p \vee \sim q) \vee F$~~

Ques Obtain DNF of (i) $p\wedge (p\rightarrow q)$ (b)

Sol $p\wedge (\sim p \vee q)$

$(p\wedge \sim p) \vee (p\wedge q)$

b) $p \vee (\sim p \to q \vee ((q \to \sim r)))$

$p \vee (\sim p \to q \vee (\sim q \vee r)))$

$\cancel{p \vee (\sim p \to ((q \vee \sim q) \vee (q \vee r))))}$

$\cancel{p \vee (p \vee ((q \vee \sim q) \vee (q \vee r)))}$

$\cancel{p \vee ((p \vee q \vee \sim q) \vee (p \vee q \vee r))}$

$\cancel{p \vee ((p \vee T) \vee (p \vee q \vee r))}$

$p \vee (\sim p \to (q \vee (q \to \sim r)))$

$p \vee (\sim p (q \vee (\sim q \vee \sim r)))$

$p \vee (\sim p \to (q \vee \sim q \vee \sim r))$

$\cancel{p \vee (\sim p\, q}$

$p \vee (p \vee (q \vee \sim q \vee \sim r))$

$p \vee p \vee q \vee \sim q \vee \sim r$

$p + p + q + q' + r'$

$p + q + q' + r'$

c) $p \to ((p \to q) \wedge \sim(\sim q \vee \sim p)))$

$p \to ((\sim p \vee q) \wedge (q \wedge p))$

$p \to ((p \to q) \cdot (q \cdot p))$

$p \to (p' + q) \cdot (q\, p)$

$p \to (p'qp + qpq)$

$\sim p + pq(p' + qp)$

$p' + pq$

Q $p \to ((p \to q) \wedge \sim(\sim q \vee \sim p))$

$p \to ((p \to q) \wedge (q \wedge p)$

CNF : Constructive Normal form

Product of Elementary Sum.

1. $(p \to q) \wedge \sim q$

$(\sim p \vee q) \wedge (\sim q)$

2. $q \vee (p \wedge r) \wedge \sim [(p \vee r) \wedge q]$

$q \vee (p \wedge r) \wedge [(\sim p \wedge \sim r) \vee \sim q]$

$\cancel{(q \vee p \wedge r) \wedge [}$

$\cancel{(q + pr) \cdot [(pr)' + q']}$

$\cancel{q + pr [p' + r'}$

$\cancel{q + (pr) \cdot \sim[(p+r) \cdot q]}$

$\cancel{(q + pr) [(p+r)' + q']}$

$\cancel{(q + pr) [p'r' + q']}$

$\cancel{p'qr' + qq' + prp'r' + pq'r}$

$\cancel{q \vee ((p \wedge r) \wedge \sim(p \vee r))}$

$q \vee (p \wedge r) \wedge [(\sim p \wedge \sim r) \vee \sim q]$

$(q \vee p) \wedge (q \vee r) \wedge [(\sim p \wedge \sim r) \vee \sim q]$

$(q \vee p) \wedge (q \vee r) \wedge [(\sim p \wedge \sim r) \vee \sim q]$

8-4-22

Minterms

consists of product ~~of sum~~. of p or its negation and product of q or its negation are called maxterm of p and q.

$p \wedge q$, $\sim p \wedge q$, $p \wedge \sim q$, $\sim p \wedge \sim q$

Maxterms

Let p and q be two propositional variables.

All possible formulas which consist sum of p or its negation and sum of q or its negation are called maxterm of p and q.

Ex: $p \vee q$, $\sim p \vee q$, $p \vee \sim q$, $\sim p \vee \sim q$

Principle Disjunctive Normal Form (PDNF)

i) For a given formula, an equivalent formula consisting of disjunction of minterms, is known as PDNF.

ii) Each minterm has truth value T (true) for exactly one combina[tion] of truth value.

iii) A formula which is tautology which has all minterms.

iv) A formula which is contradiction will have no minterms.

Method to find PDNF

i) Construct truth table for the given formula.

ii) Identify the row in which the formula has true as truth value.

iii) Construct minterm from each row by ~~making~~ taking

  i) The variable with true as variable itself.

  ii) The variable with false as negated variable.

iv) Sum of these minterms will be PDNF.

1

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Q Obtain $p \to q$, $p \leftrightarrow q$ PDNF

p	q	$p \to q$	
T	T	T	✓
T	F	F	
F	T	T	✓
F	F	T	✓

$(p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

$p \cdot q + p' \cdot q + p' \cdot q'$

~~BCNF~~

PCNF (Principal Constructive Normal form)

In PCNF we take conjunction of maxterms.

i) Construct the truth table.
ii) Find the false valued row.
iii) The

iv) Product of these maxterms will be PCNF.

Q Obtain $p \to q$, $p \leftrightarrow q$

p	q	$p \to q$
T	T	
T	F	
F	T	
F	F	

Q. Obtain PDNF & PCNF of $[(p \wedge q) \vee (\sim p \wedge r) \vee (q \wedge r)]$

p	q	r	$p \wedge q$	$\sim p$	$\sim p \wedge r$	$q \wedge r$	$(p \wedge q) \vee (\sim p \wedge r) \vee (q \wedge r)$
T	T	T	T	F	F	T	T
T	T	F	T	F	F	F	T
T	F	T	F	F	F	F	F
T	F	F	F	F	F	F	F
F	T	T	F	T	T	T	T
F	T	F	F	T	F	F	F
F	F	T	F	T	T	F	T
F	F	F	F	T	F	F	F

PCNF ~~PDNF~~ $= (p \vee \sim q \vee r) \wedge (p \vee \sim q \vee \sim r) \wedge (\sim p \vee q \vee \sim r) \wedge (\sim p \vee \sim q \vee r)$

PDNF $= (p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (\sim p \wedge q \wedge r) \vee (\sim p \wedge \sim q \wedge r)$

~~Predicates & Quantifiers:~~