

4. Binary Search

Solved

Easy

Topics

Companies

Given an array of integers `nums` which is sorted in ascending order, and an integer `target`, write a function to search `target` in `nums`. If `target` exists, then return its index. Otherwise, return -1.

You must write an algorithm with $O(\log n)$ runtime complexity.

Example 1:

Input: `nums = [-1,0,3,5,9,12]`, `target = 9`

Output: 4

Explanation: 9 exists in `nums` and its index is 4

Example 2:

Input: `nums = [-1,0,3,5,9,12]`, `target = 2`

Output: -1

Explanation: 2 does not exist in `nums` so return -1

Constraints:

- $1 \leq \text{nums.length} \leq 10^4$
- $-10^4 < \text{nums}[i], \text{target} < 10^4$
- All the integers in `nums` are **unique**.
- `nums` is sorted in ascending order.

🔍 Problem Summary

You are given:

- A **sorted array** `nums` (ascending order)
- An integer `target`

Your task:

- Return the **index of target** if it exists
- Otherwise, return -1
- **Time complexity must be $O(\log n)$**

💡 Key Idea (Binary Search)

Because the array is **sorted**, we can:

- Repeatedly **divide the search space into half**
- Compare target with the **middle element**

🔗 Binary Search Logic (Step-by-Step)

1. Set two pointers:
 - $l = 0$ (start)
 - $r = n - 1$ (end)
2. While $l \leq r$:
 - Find middle index:
 - $mid = l + (r - l) / 2$
 - If $\text{nums}[mid] == \text{target} \rightarrow$ return mid
 - If $\text{nums}[mid] < \text{target} \rightarrow$ search right half ($l = mid + 1$)
 - Else \rightarrow search left half ($r = mid - 1$)

3. If loop ends → target not found → return -1

```
class Solution {  
    public int search(int[] nums, int target) {  
        int l= 0, r = nums.length - 1;  
        while (l <= r) {  
            int mid = l + (r - l) / 2;  
            if (nums[mid] == target) {  
                return mid;  
            } else if (nums[mid] < target) {  
                l= mid + 1;  
            } else {  
                r = mid - 1;  
            }  
        }  
        return -1;  
    }  
}
```

Example Dry Run

Example 1

nums = [-1,0,3,5,9,12], target = 9

l=0, r=5 → mid=2 → nums[2]=3 < 9 → l=3

l=3, r=5 → mid=4 → nums[4]=9 == target

Output = 4

Example 2

nums = [-1,0,3,5,9,12], target = 2

Target not found

Output = -1

Complexity Analysis

- Time Complexity: O(log n)
- Space Complexity: O(1)

[2529. Maximum Count of Positive Integer and Negative Integer](#)

Solved

Easy

Topics

Companies

Hint

Given an array nums sorted in **non-decreasing** order, return *the maximum between the number of positive integers and the number of negative integers.*

- In other words, if the number of positive integers in nums is pos and the number of negative integers is neg, then return the maximum of pos and neg.

Note that 0 is neither positive nor negative.

Example 1:

Input: nums = [-2,-1,-1,1,2,3]

Output: 3

Explanation: There are 3 positive integers and 3 negative integers. The maximum count among them is 3.

Example 2:

Input: nums = [-3,-2,-1,0,0,1,2]

Output: 3

Explanation: There are 2 positive integers and 3 negative integers. The maximum count among them is 3.

Example 3:

Input: nums = [5,20,66,1314]

Output: 4

Explanation: There are 4 positive integers and 0 negative integers. The maximum count among them is 4.

Constraints:

- $1 \leq \text{nums.length} \leq 2000$
- $-2000 \leq \text{nums}[i] \leq 2000$
- nums is sorted in a **non-decreasing order**.

```
class Solution {  
  
    public int maximumCount(int[] nums) {  
  
        int pos = 0, neg = 0;  
  
        for (int x : nums) {  
  
            if (x > 0) pos++;  
  
            else if (x < 0) neg++;  
  
        }  
  
        return Math.max(pos, neg);  
  
    }  
}
```

Complexity

- **Time:** $O(n)$
- **Space:** $O(1)$

4.4 [Search Insert Position](#)

Solved

Easy

Topics

Companies

Given a sorted array of distinct integers and a target value, return the index if the target is found. If not, return the index where it would be if it were inserted in order.

You must write an algorithm with $O(\log n)$ runtime complexity.

Example 1:

Input: nums = [1,3,5,6], target = 5

Output: 2

Example 2:

Input: nums = [1,3,5,6], target = 2

Output: 1

Example 3:

Input: nums = [1,3,5,6], target = 7

Output: 4

Constraints:

- $1 \leq \text{nums.length} \leq 10^4$
- $-10^4 \leq \text{nums}[i] \leq 10^4$
- nums contains **distinct** values sorted in **ascending** order.
- $-10^4 \leq \text{target} \leq 10^4$

class Solution {

```
    public int searchInsert(int[] nums, int target) {
```

```
        int lo = 0;
```

```
        int hi = nums.length - 1;
```

```
        while (lo <= hi) {
```

```
            int mid = lo + (hi - lo) / 2;
```

```
            if (nums[mid] == target) {
```

```
                return mid; // Target found
```

```
            } else if (nums[mid] < target) {
```

```
                lo = mid + 1;
```

```
            } else {
```

```
                hi = mid - 1;
```

```
}
```

```
    }
```

```
    return lo; // Insert position if not found
```

```
}
```

```
}
```

4.4.1 [278. First Bad Version](#)

You are a product manager and currently leading a team to develop a new product. Unfortunately, the latest version of your product fails the quality check. Since each version is developed based on the previous version, all the versions after a bad version are also bad.

Suppose you have n versions [1, 2, ..., n] and you want to find out the first bad one, which causes all the following ones to be bad.

You are given an API bool isBadVersion(version) which returns whether version is bad. Implement a function to find the first bad version. You should minimize the number of calls to the API.

Example 1:

Input: n = 5, bad = 4

Output: 4

Explanation:

call isBadVersion(3) -> false

call isBadVersion(5) -> true

call isBadVersion(4) -> true

Then 4 is the first bad version.

Example 2:

Input: n = 1, bad = 1

Output: 1

Constraints:

- $1 \leq \text{bad} \leq n \leq 2^{31} - 1$

🔍 Problem Summary

You are given versions 1 to n.

- There exists a **first bad version**
- Every version **after it is also bad**
- You can only check a version using:
- `isBadVersion(version)`
- Your goal: **find the first bad version**
- Minimize the number of API calls

💡 Key Insight

This is a **classic Binary Search** problem.

The versions look like:

Good Good Good Bad Bad Bad

Once `isBadVersion(x)` becomes true, it stays true.

⚡ This monotonic behavior allows **binary search**.

㉚ Binary Search Logic (Step-by-Step)

1. Set search range:
2. $l = 1, r = n$
3. While $l < r$:
 - Find middle:
 - $mid = l + (r - l) / 2$
 - If mid is **bad**:
 - The first bad version is **at mid or before**
 - Move left:
 - $r = mid$
 - Else (mid is good):
 - First bad version must be **after mid**
 - Move right:
 - $l = mid + 1$
4. When loop ends:
 - $l == r \rightarrow$ this is the **first bad version**

☑ Java Code (Optimal – O(log n))

```
public class Solution extends VersionControl {
```

```
    public int firstBadVersion(int n) {
```

```
        int l = 1, r = n;
```

```
        while (l < r) {
```

```
            int mid = l + (r - l) / 2;
```

```
            if (isBadVersion(mid)) {
```

```
r = mid; // first bad is at mid or before
} else {
    l = mid + 1; // first bad is after mid
}
}
return l;
}
```

Example Dry Run

Example 1

n = 5, bad = 4

l=1, r=5 → mid=3 → isBadVersion(3)=false → l=4

l=4, r=5 → mid=4 → isBadVersion(4)=true → r=4

Loop ends → l=4

Example 2

n = 1, bad = 1

l=1, r=1 → return 1

Complexity Analysis

- **Time Complexity:** O(log n)
- **Space Complexity:** O(1)
- **API Calls:** Minimal (logarithmic)

4.4.2. Minimum Operations to Exceed Threshold Value I

You are given a **0-indexed** integer array nums, and an integer k.

In one operation, you can remove one occurrence of the smallest element of nums.

Return the **minimum number of operations needed so that all elements of the array are greater than or equal to k**.

Example 1:

Input: nums = [2,11,10,1,3], k = 10

Output: 3

Explanation: After one operation, nums becomes equal to [2, 11, 10, 3].

After two operations, nums becomes equal to [11, 10, 3].

After three operations, nums becomes equal to [11, 10].

At this stage, all the elements of nums are greater than or equal to 10 so we can stop.

It can be shown that 3 is the minimum number of operations needed so that all elements of the array are greater than or equal to 10.

Example 2:

Input: nums = [1,1,2,4,9], k = 1

Output: 0

Explanation: All elements of the array are greater than or equal to 1 so we do not need to apply any operations on nums.

Example 3:

Input: nums = [1,1,2,4,9], k = 9

Output: 4

Explanation: only a single element of nums is greater than or equal to 9 so we need to apply the operations 4 times on nums.

Constraints:

- $1 \leq \text{nums.length} \leq 50$
- $1 \leq \text{nums}[i] \leq 10^9$
- $1 \leq k \leq 10^9$
- The input is generated such that there is at least one index i such that $\text{nums}[i] \geq k$.

```
class Solution {  
  
    public int minOperations(int[] nums, int k) {  
  
        int cnt = 0;  
  
        for (int x : nums) {  
  
            if (x < k) cnt++;  
  
        }  
  
        return cnt;  
  
    }  
  
}
```

Complexity Analysis

- **Time:** $O(n)$
- **Space:** $O(1)$

2.3 🧩 Problem Understanding

- Koko has **n piles of bananas**
- $\text{piles}[i]$ = number of bananas in the i th pile
- Guards will return in **h hours**
- Koko chooses an **eating speed = k bananas/hour**
- In **one hour**, she eats from **only one pile**
 - If pile has $\geq k$ bananas \rightarrow eats k
 - If pile has $< k$ bananas \rightarrow eats all and stops for that hour
- She wants to **finish all bananas within h hours**
- Goal: **find the minimum possible integer k**

⌚ How to Calculate Time for a Given Speed k

For one pile with x bananas:

$$\text{hours} = \text{ceil}(x / k)$$

In Java, we avoid floating point by using:

$$(x + k - 1) / k$$

Total hours = sum of hours for all piles

🔍 Key Observation

- Minimum possible speed = **1**
- Maximum possible speed = **max(piles)**
(Eating faster than the largest pile gives no extra benefit)

⌚ The answer lies between **1 and max(piles)**

⌚ This range is **sorted**, so we apply **Binary Search on the answer**

⚡ Binary Search Strategy

1. Set:
 - $\text{low} = 1$
 - $\text{high} = \text{max}(\text{piles})$
2. While $\text{low} \leq \text{high}$:

- mid = possible eating speed
- Calculate total hours needed at speed mid

3. Decision:

- If hours $\leq h \rightarrow$ speed is valid \rightarrow try **smaller speed**
- Else \rightarrow speed too slow \rightarrow **increase speed**

Simple Logic to Remember (Interview Trick)

Higher speed \rightarrow fewer hours

Lower speed \rightarrow more hours

```
class Solution {
```

```
    public int minEatingSpeed(int[] piles, int h) {
```

```
        int low = 1;
```

```
        int high = 0;
```

```
        // Find maximum pile
```

```
        for (int p : piles) {
```

```
            high = Math.max(high, p);
```

```
}
```

```
        int answer = high;
```

```
        while (low  $\leq$  high) {
```

```
            int mid = low + (high - low) / 2; // candidate speed
```

```
            long totalHours = 0;
```

```
            // Calculate hours needed at speed = mid
```

```
            for (int p : piles) {
```

```
                totalHours += (p + mid - 1) / mid; // ceil division
```

```
}
```

```
            if (totalHours  $\leq h$ ) {
```

```
                answer = mid; // valid speed
```

```
                high = mid - 1; // try slower
```

```
} else {
```

```
                low = mid + 1; // speed too slow
```

```
}
```

```
}
```

```
return answer;
```

```
}
```

```
}
```

Complexity Analysis

- **Time Complexity:**
 $O(n \log \text{maxPile})$
- **Space Complexity:**
 $O(1)$

Example 1

piles = [3, 6, 7, 11]

h = 8

Step 1: Define Search Space

- Minimum speed low = 1
- Maximum speed high = max(piles) = 11
- Answer will be between 1 and 11

Step 2: Start Binary Search

❖ Iteration 1

low = 1, high = 11

mid = $(1 + 11) / 2 = 6$

Calculate hours needed if k = 6

Pile Bananas Hours = ceil(pile / 6)

3	3	1
6	6	1
7	7	2
11	11	2

Total hours = $1 + 1 + 2 + 2 = 6$

✓ $6 \leq 8 \rightarrow$ Speed 6 is valid

➡ Try slower speed

answer = 6

high = 5

❖ Iteration 2

low = 1, high = 5

mid = $(1 + 5) / 2 = 3$

Calculate hours if k = 3

Pile Bananas Hours

3	3	1
6	6	2
7	7	3
11	11	4

Total hours = $1 + 2 + 3 + 4 = 10$

✗ $10 > 8 \rightarrow$ Speed too slow

➡ Increase speed

low = 4

❖ Iteration 3

low = 4, high = 5

mid = $(4 + 5) / 2 = 4$

Calculate hours if k = 4

Pile Bananas Hours

3	3	1
6	6	2
7	7	2
11	11	3

Total hours = $1 + 2 + 2 + 3 = 8$

✓ $8 \leq 8 \rightarrow$ Speed 4 is valid

➡ Try slower

answer = 4

high = 3

Step 3: Stop Condition

low = 4, high = 3 → loop ends

Final Answer

Minimum eating speed = 4

[69. Sqrt\(x\)](#)

This is **LeetCode 69 – Sqrt(x)**.

Best and expected approach is **Binary Search**

Key Idea (Why Binary Search?)

- Square root is **monotonic**
If $\text{mid}^2 \leq x$, then all values $< \text{mid}$ are also valid.
- We want the **largest integer k such that $k^2 \leq x$** .
- Binary search gives **$O(\log x)$** time → very efficient.

Important Edge Cases

- $x = 0 \rightarrow \text{answer} = 0$
- $x = 1 \rightarrow \text{answer} = 1$
- Avoid overflow → use long for $\text{mid} * \text{mid}$

Binary Search Logic

Search range:

low = 0

high = x

At every step:

1. $\text{mid} = (\text{low} + \text{high}) / 2$
2. If $\text{mid} * \text{mid} == x \rightarrow \text{perfect square} \rightarrow \text{return mid}$
3. If $\text{mid} * \text{mid} < x \rightarrow \text{possible answer} \rightarrow \text{move right}$
4. If $\text{mid} * \text{mid} > x \rightarrow \text{move left}$

Final answer will be stored in ans.

Java Code (Recommended for Exams & Interviews)

```
class Solution {  
  
    public int mySqrt(int x) {  
  
        if(x == 0 || x == 1)  
            return x;  
  
        int low = 0, high = x;  
  
        int ans = 0;  
  
        while (low <= high) {  
  
            int mid = low + (high - low) / 2;  
  
            long sq = (long) mid * mid; // avoid overflow  
  
            if(sq == x) {  
                return mid;  
            } else if(sq < x) {  
                ans = mid; // store possible answer  
                low = mid + 1;  
            } else {  
                high = mid - 1;  
            }  
        }  
        return ans;  
    }  
}
```

```
} else {  
    high = mid - 1;  
}  
}  
}  
}  
return ans;  
}  
}
```

✍ Dry Run (x = 8)

```
low = 0, high = 8, ans = 0  
mid = 4 → 4*4 = 16 > 8 → high = 3  
mid = 1 → 1*1 = 1 < 8 → ans = 1, low = 2  
mid = 2 → 2*2 = 4 < 8 → ans = 2, low = 3  
mid = 3 → 3*3 = 9 > 8 → high = 2  
loop ends → return ans = 2
```

✓ Output = 2

⌚ Time & Space Complexity

- **Time:** O(log x)
- **Space:** O(1)

📝 Exam / Interview One-Line Mnemonic

“Find the largest number whose square is \leq x using binary search.”

☒ Why long fixes it

- long range = $\pm 9 \times 10^{18}$
- Safely stores mid * mid even for large values

long sq = (long) mid * mid;

Now:

- No overflow
- Correct comparison
- Correct answer ☒

⌚ Easy Exam Mnemonic

“ mid^2 can overflow int, so use long.”

367. Valid Perfect Square

Given a positive integer num, return true if num is a perfect square or false otherwise.

A **perfect square** is an integer that is the square of an integer. In other words, it is the product of some integer with itself.

You must not use any built-in library function, such as sqrt.

Example 1:

Input: num = 16

Output: true

Explanation: We return true because $4 * 4 = 16$ and 4 is an integer.

Example 2:

Input: num = 14

Output: false

Explanation: We return false because $3.742 * 3.742 = 14$ and 3.742 is not an integer.

Constraints:

- $1 \leq num \leq 2^{31} - 1$

Key Idea

A number num is a **perfect square** if there exists an integer k such that:

$$k * k == num$$

Since squares grow **monotonically**, we can apply **binary search**.

⌚ Binary Search Strategy

- Search space: $1 \rightarrow num$
- For each mid:
 - if $mid * mid == num \rightarrow \checkmark$ perfect square
 - if $mid * mid < num \rightarrow$ search right
 - if $mid * mid > num \rightarrow$ search left
- Use **long** to avoid overflow (same reason as in \sqrt{x})

☑ Java Code (Recommended)

```
class Solution {  
  
    public boolean isPerfectSquare(int num) {  
  
        int low = 1, high = num;  
  
        while (low <= high) {  
  
            int mid = low + (high - low) / 2;  
  
            long sq = (long) mid * mid; // avoid overflow  
  
            if (sq == num) {  
                return true;  
            } else if (sq < num) {  
                low = mid + 1;  
            } else {  
                high = mid - 1;  
            }  
        }  
        return false;  
    }  
}
```

❖ Dry Run 1 (num = 16)

low = 1, high = 16

mid = 8 $\rightarrow 8*8 = 64 > 16 \rightarrow$ high = 7

mid = 4 $\rightarrow 4*4 = 16 == 16 \rightarrow$ return true

✓ Output: true

❖ Dry Run 2 (num = 14)

low = 1, high = 14

mid = 7 → 49 > 14 → high = 6

mid = 3 → 9 < 14 → low = 4

mid = 5 → 25 > 14 → high = 4

mid = 4 → 16 > 14 → high = 3

loop ends → return false

✓ Output: false

Time & Space Complexity

- Time: $O(\log \text{num})$
- Space: $O(1)$

3. Sum of Square Numbers

Given a non-negative integer c , decide whether there're two integers a and b such that $a^2 + b^2 = c$.

Example 1:

Input: $c = 5$

Output: true

Explanation: $1 * 1 + 2 * 2 = 5$

Example 2:

Input: $c = 3$

Output: false

Constraints:

- $0 \leq c \leq 2^{31} - 1$

🔗 Best Approach: Two Pointers (\sqrt{c} range)

Why this works

- a^2 and b^2 are **non-negative**
- Maximum value of a or b can be \sqrt{c}
- Squares are **monotonic**, so we can move pointers smartly

㉚ Algorithm (Two Pointer Logic)

1. Set
2. $\text{left} = 0$
3. $\text{right} = \text{floor}(\sqrt{c})$
4. While $\text{left} \leq \text{right}$:
 - Compute sum = $\text{left}^2 + \text{right}^2$
 - If sum == c → return true
 - If sum < c → increase left
 - If sum > c → decrease right
5. If loop ends → return false

⚠ Use **long** to avoid overflow.

Java Code (Interview Ready)

```
class Solution {  
  
    public boolean judgeSquareSum(int c) {  
  
        long left = 0;  
  
        long right = (long) Math.sqrt(c);  
  
        while (left <= right) {  
            long sum = left * left + right * right;  
  
            if (sum == c) {  
                return true;  
            } else if (sum < c) {  
                left++;  
            } else {  
                right--;  
            }  
        }  
        return false;  
    }  
}
```

```

while (left <= right) {

    long sum = left * left + right * right;

    if (sum == c) {
        return true;
    } else if (sum < c) {
        left++;
    } else {
        right--;
    }
}

return false;
}
}

```

Dry Run 1 ($c = 5$)

left = 0, right = 2

$$0^2 + 2^2 = 4 < 5 \rightarrow \text{left} = 1$$

$$1^2 + 2^2 = 5 == 5 \rightarrow \text{return true}$$

✓ Output: **true**

Easy Exam Mnemonic

“Fix two pointers at 0 and \sqrt{c} , move based on sum of squares.”

2778. Sum of Squares of Special Elements

You are given a **1-indexed** integer array `nums` of length `n`.

An element `nums[i]` of `nums` is called **special** if `i` divides `n`, i.e. `n % i == 0`.

Return the *sum of the squares* of all **special** elements of `nums`.

Example 1:

Input: `nums = [1,2,3,4]`

Output: 21

Explanation: There are exactly 3 special elements in `nums`: `nums[1]` since 1 divides 4, `nums[2]` since 2 divides 4, and `nums[4]` since 4 divides 4.

Hence, the sum of the squares of all special elements of `nums` is $\text{nums}[1] * \text{nums}[1] + \text{nums}[2] * \text{nums}[2] + \text{nums}[4] * \text{nums}[4] = 1 * 1 + 2 * 2 + 4 * 4 = 21$.

Problem Understanding

- Array is **1-indexed** (important!)
- Length = `n`
- An index `i` is **special** if:
- `n % i == 0`
- For every such index, add:
- `nums[i]^2`

Core Logic

1. Find `n = nums.length`
2. Loop `i` from 1 to `n`
3. If `n % i == 0`:
 - Take element at index `i-1` (because Java is 0-indexed)
 - Add its square to answer

Java Code (Clean & Exam-Ready)

```
class Solution {  
  
    public int sumOfSquares(int[] nums) {  
  
        int n = nums.length;  
  
        int sum = 0;  
  
        for (int i = 1; i <= n; i++) {  
  
            if (n % i == 0) {  
  
                sum += nums[i - 1] * nums[i - 1];  
  
            }  
  
        }  
  
        return sum;  
  
    }  
  
}
```

Dry Run 1

Input: nums = [1,2,3,4]

n = 4

i n % i special? nums[i] square

1	0	<input checked="" type="checkbox"/>	1	1
2	0	<input checked="" type="checkbox"/>	2	4
3	1	<input checked="" type="checkbox"/>	—	—
4	0	<input checked="" type="checkbox"/>	4	16

Sum = 1 + 4 + 16 = 21 ✓

Dry Run 2

Input: nums = [2,7,1,19,18,3]

n = 6

Special indices: 1, 2, 3, 6

$$2^2 + 7^2 + 1^2 + 3^2$$

$$= 4 + 49 + 1 + 9$$

$$= 63$$

✓ Output = 63

Complexity

- **Time:** O(n)
- **Space:** O(1)

Easy Exam Mnemonic

“If index divides n, square it and add.”

[1588. Sum of All Odd Length Subarrays](#)

Given an array of positive integers arr, return *the sum of all possible odd-length subarrays* of arr.

A **subarray** is a contiguous subsequence of the array.

Example 1:

Input: arr = [1,4,2,5,3]

Output: 58

Explanation: The odd-length subarrays of arr and their sums are:

[1] = 1

[4] = 4

[2] = 2

[5] = 5

[3] = 3

[1,4,2] = 7

[4,2,5] = 11

[2,5,3] = 10

[1,4,2,5,3] = 15

If we add all these together we get $1 + 4 + 2 + 5 + 3 + 7 + 11 + 10 + 15 = 58$

Approach 1: Brute Force (Beginner Friendly)

Idea

- Generate all subarrays
- Check if length is odd
- Add their sum

Java Code

```
class Solution {  
  
    public int sumOddLengthSubarrays(int[] arr) {  
  
        int n = arr.length;  
  
        int ans = 0;  
  
        for (int i = 0; i < n; i++) {  
  
            int sum = 0;  
  
            for (int j = i; j < n; j++) {  
  
                sum += arr[j];  
  
                int len = j - i + 1;  
  
                if (len % 2 == 1) {  
  
                    ans += sum;  
  
                }  
            }  
        }  
        return ans;  
    }  
}
```

Complexity

- **Time:** $O(n^2)$
- **Space:** $O(1)$

Works because $n \leq 100$, but **not optimal**.