DEPARTMENT OF INFORMATICS

TECHNISCHE UNIVERSITÄT MÜNCHEN

Bachelor's Thesis in Information Systems

A Comparison of the Impact of Auction Formats Used in the 2015 German Spectrum Auction "Mobiles Breitband - Projekt 2016"

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Ein Vergleich des Einflusses von Auktionsformaten in der Spektrumsauktion "Mobiles Breitband - Projekt 2016" in Deutschland"

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I confirm that this bachelor's thesis in information systems is my own work and I have documented all sources and material used.

Munich, 15.09.2016 Tim Berger

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I dedicate my gratitude to Andor Goetzendorff, without his constant and relentless support this project would have never come to life. I am grateful for the opportunity to have learned so much under his supervision.

Also, I want to thank my parents. Without their unalterable trust in my potential, despite the contradictions raised through out my life, I would never have come so far in achieving my goals and dreams.

This work is dedicated to my mother. May she rest in peace.

Abstract

With specific implementations varying world wide, characteristically spectrum auctions comprise of many licenses being auctioned simultaneously. Allowing bidders to submit bids only on single items such as in the well established *Simultaneous Multi-Round Action (SMRA)*, acquiring certain combinations of complementing objects can arise in strategical problems for bidders. Combinatorial auctions try to mitigate this effect by allowing bids on bundles of items, but computational challenges emerge due to possible allocations being in combinatorial magnitude with growing number of items being offered. To reduce computational effort, *Hierarchical Package Bidding (HPB)* was introduced which arranges items in a hierarchical tree structure. The effort of this thesis is to explore indications of the eligibility of HPB in the context of spectrums auctions by running simulations of the SMRA and HPB auction formats based on the German spectrum auction of 2015. The results indicate, that HPB might be helpful in enabling bidders to find more efficient overall allocations when choosing an appropriate bidding hierarchy. This improvement came with the reduction of overall revenue as bidder signalling improved. SMRA was superior when a less competitive HPB hierarchy was used, indicating that choosing a suitable hierarchy is key to the success of the auction.

Zusammenfassung

Ein Charakteristikum von Spektrumsauktionen ist die simultane Versteigerung von einer Vielzahl von Lizenzen. Wenn es Bietern nur möglich ist, Angebote auf einzelne Objekte abzugeben, wie es beispielsweise in der wohl etablieren *Simultaneous Multi-Round Action (SMRA)* der Fall ist, treffen Bieter auf strategische Herausforderungen, wenn sie Kombinationen von Objekten mit Komplementaritäten erwerben wollen. Kombinatorische Auktionen versuchen diesen Effekt zu lindern, indem sie Gebote auf Objekt-Bündelungen zulassen. Jedoch steigt die Anzahl möglicher Allokationen im kombinatorischen Maße mit der Anzahl an versteigerten Objekten, wodurch es zu Herausforderungen im Rechenaufwand und der Rechenzeit kommen kann. Um den benötigten Rechenaufwand zu reduzieren, wurde das *Hierarchical Package Bidding (HPB)* entwickelt, welches Auktionsobjekte in einer hierarchischen Baumstruktur arrangiert. Das Ziel dieser Arbeit ist es, die Eignung von HPB im Kontext von Spektrumsauktionen zu untersuchen, indem Simulationen der SMRA und HPB Auktionsformate auf Basis der deutschen Spektrumsauktion vom Jahr 2015 durchgeführt wurden. Die Ergebnisse deuten darauf hin, dass HPB es Bietern ermöglicht effizientere Allokationen zu finden, gesetzt dem Fall, es wird eine geeignete Hierarchiestruktur verwenden. Diese Verbesserung kam mit einer Reduktion der Umsätze, da die Kommunikation zwischen den Bietern sich verbesserte. Bei der Verwendung von weniger kompetitiven Hierarchiestrukturen war die SMRA überlegen, was darauf hindeutet, dass die Wahl einer geeigneten Hierarchie eine große Bedeutung für den Erfolg einer Auktion hat.

Contents

1 Introduction

1.1 Motivation

Over the last two decades, auctions have been established as a reliable mechanism to allocate spectrum licenses to businesses that want to provide wireless communication services [Cra02]. Spectrum auctions already have produced billions in revenue for governmental institutions world-wide making it the preferred method of assigning spectrum. Well-designed auction mechanisms showed to be superior to previously used mechanisms like comparative hearings and lotteries, while maintaining the tendency to allocate licenses to parties that have the highest valuation for the bidding items [Cra02].

When auctioning spectrum licenses, often certain license combination create an added value to the bidder. This may be induced by particular geographical arrangements or bundling of frequency blocks enabling new technology and thus produce a competitive advantage effecting the bidder's valuation of this item combination. In this case, the valuation of this bundling of items is greater than the sum of each item's valuation, which is called super-additive valuation.

To achieve those desired combinations, in certain cases bidders might have to strategically bid over an item's valuation if it is part of such an item composition. Being uncertain if they can realise the desired bundling, the bidders *expose* themselves, because they are still in danger of failing to achieve the combination and end up with single item prices that exceed their valuation. To overcome this issue, in combinatorial auctions bidders can place bids on item combinations. Nevertheless the helpful enhancement, the number of possible arrangements growths in combinatorial magnitude with the amount of items auctioned. Hence, finding the optimal allocation in an acceptable time frame becomes challenging as the calculations are computationally expensive and can lead to incomprehensible non-linear price progressions that are non-transparent for bidders.

With the aim of constantly improving the performance of auctions, a wide array of different auction mechanisms were already devised over the last decades. Even though some formats prevail in their use in national spectrum auctions, academia and business partners are always in search for new solutions for questions arising in adopted auction formats, i.e. bidder exposure, non-linear and incomprehensible price calculations or computational complexity in combinatorial auctions.

The idea of hierarchical bidding was introduced by Rothkopf, Pekeč, and Harstad in 1998 and aims to reduce computational complexity in combinatorial auctions by structuring the bidding items into a hierarchy of pre-defined packages [RPH98]. This gives bidders the opportunity to bid on a certain item combinations until their valuation. Goeree and Holt extended the idea of hierarchical package bidding by devising computation algorithms for the assignment and pricing of items with much less computational effort needed [GH10]. The goal of this thesis is to explore indicators of the eligibility of HPB in spectrum auctions by running simulations based on the German spectrum auction from 2015, which will be described in more detail in the following section.

1.2 The German Spectrum Auction in 2015

The German auction was conducted under the project name *"Mobiles Breitband Projekt 2016"* by the German federal network agency *("Bundesnetzagentur")* in 2015 over the course of 4 weeks or 181 bidding rounds and resulted in EUR 5.081bn of revenue. Germany, as the first European country, auctioned off the 700 MHz band to be used for mobile telecommunication purposes. The auction was important, as it offered valuable frequency bands ready to be used for expanding LTE coverage as well as prospective technologies and covered the spectra *700, 900, 1800* as well as *1500 MHz*, totalling in 270 MHz offered. In each band, a certain number of frequency blocks was sold in a 2 x 5 MHz fashion, except the 1500 MHz band, which was offered in 1 x 5 MHz blocks. During the auction, the blocks were treated as *abstract* blocks, meaning that the bidder did not know the specific frequency allocation inside the band yet. After the auction, the blocks were then allocated to specific frequency values. With the exception of 1500 Mhz, the other bands each included one *debased* block which was already tied to a certain frequency allocation due to technological circumstances (e.g. as buffer to other bands). Figure 1.1 shows the amount of frequency offered in each spectrum.

Since 2015, the German mobile network operator (MNO) market is split between three incumbent operators: "Deutsche Telekom AG" (DT), "Vodafone GmbH" (VOD) and "Telefónica Germany GmbH & Co. OHG" (TEF). Just shortly before the auction, TEF merged with the German MNO "Eplus" reducing the number of active operators in the market.

The auction format used was the Simultaneous Multi-Round Auction (SMRA). Including a set of different auction rules, the SMRA can be described as an extension of the English auction, where multiple items are auctioned *simultaneously*: Each round, bidders place increasing bids and the auction stops when the bidders stop expressing new bids. After the end of the auction, every item is allocated to the bidder with the highest bid in the last round. The specific implementation of SMRA differs from

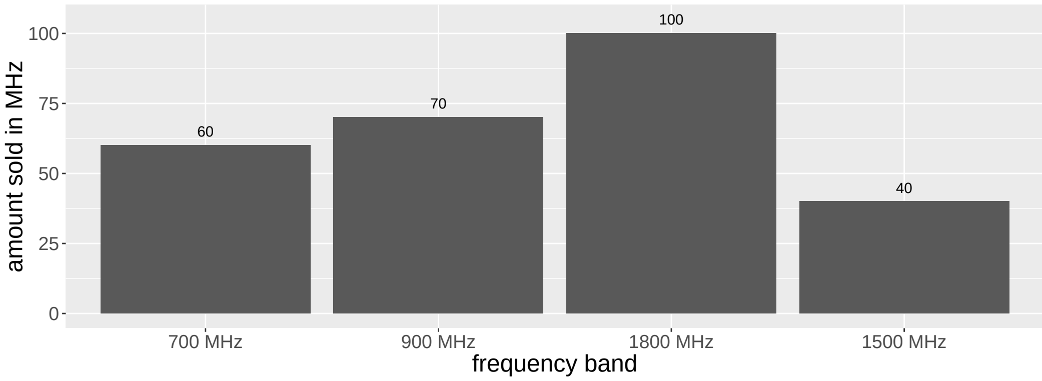

Figure 1.1: Amount of MHz sold in the German auction

country to country. The German auction was standing out due to its high level of transparency. After every round, each bidder had access to a wide array of information, e.g. including the bids placed by every bidders. Thus, bidders were able to reconstruct the whole bidding history of every participating bidder and draw conclusions from the bidding behaviour. The data made available by the German federal network agency included only the highest bids placed and the current winner of each item, but was still valuable for deducting bidding behaviour. Due to the available data and the low number of bidders, the German auction seemed like a suitable basis for the simulation done in this thesis.

1.3 Contribution

To contribute to the vector of improvement, this thesis is set-up to research the potential eligibility of the Hierarchical Package Bidding format (HPB) in the realm of national spectrum auctions by running simulations based on data from the German auction in 2015. The key questions that guided the process of this thesis can be summarised as follows:

- By introducing pre-defined packages for bidders, can HPB provide significant progression in terms of efficiency and revenue of the auction?
- How does the set-up of the HPB bidding hierarchy influence the results?
- How *"stable"* is the format in comparison to the well established Simultaneous Multi-Round auction format?

To find answers to those questions, I extended an existing project from the university chair and introduced the specific characteristics of the German auction into the implementation, as well as improvements on existing program code. Accordingly, I derived a value model to describe the valuations of the agents in the simulations. Also, I needed to devise a selector model that let the bidders emulate an optimal bidding behaviour under the constraints of the German auction. To set-up the HPB simulations I developed two different hierarchy structures to explore their effect on the performance of the auction. On the basis of this work, I ran several simulations with different parameter configurations to research the performance and stability of HPB in comparison to SMRA.

1.4 Thesis Outline

This thesis is organized as follows: chapter 2 introduces important concepts and terminology of auction theory as well as establishes an understanding of the auction formats used in the simulation. Subsequently, chapter 3 first gives an overview of the data from the German auction. Based on the insights gained from the analysis and external sources I than devise a value model to describe the bidding behaviour of the agents in the simulation. The following chapter 4 is the heart of this thesis, comprising of the explanation of the simulation process and a description of the results. Lastly, chapter 5 draws final conclusions from the simulation data and gives an outlook for future work.

2 Auction Theory

In this chapter I will establish a basis of auction theory developed by academia. At first I will describe what an auction is and introduce basic auction formats like the First Price and Second Price Auction. Afterwards I will describe the exposure problem. To get an understanding of the auction formats used in the simulation (see chapter 4) I will introduce the Simultaneous Multi-Round Auction (SMRA) and Hierarchical Package Bidding (HPB).

2.1 What is an Auction?

Auctions have become a well-established tool for answering a fundamental question in markets: "Who gets what to which price?". Having themselves proven as an effective tool to sell goods and implement public policies, auctions are now being used in a wide variety of settings, from effectively allocating radio spectrum to mobile network operators, trading electricity and pollution permits, governmental procurement and many more. Auction theory has been studied by academia for decades mostly by economists. With the rise of new auction types like combinatorial formats, where bidders can place bids on packages of items, auction theory became increasingly an interdisciplinary field of economics, operation research and computer science.

Auctions are concerned with an allocation problem, meaning which bidder gets a good at what price. They are micro-foundations of markets, that try to answer this question [CSS06].

The Independent Private Value (IPV) model. Auctions can be defined using different perspectives, e. g. a game theoretic, a contract and mechanism design theory or market microstructure approach. For the following formulations, I will use the game theoretic perspective. A basic auction environment comprises of the following characteristics (based on [Lev04, p. 1]):

- A number of bidders $i = 1, ..., n$
- the object to be auctioned, called *item*
- the signal S_i observed by bidder i with a realization $s_i \in [\bar{s}, \underline{s}]$

- independence of bidder's signals $S_1, ..., S_n$
- Bidder i has a valuation function $v_i(s_i) = s_i$

The IPV[1] can be extended to fit the needs for combinatorial auctions, where not only one item, but m indivisible items are simultaneously being auctioned among the n bidders [Nis+07, p. 267]. Also, the valuation function now maps from a subset S of the m items to their valuation $v(S) \rightarrow \mathbb{R}$, which the bidder i obtains upon winning this specific bundle of items and where the valuation function is monotone and "normalized", meaning $v(\emptyset) = 0$ [Nis+07, p. 268]. This formulation is necessary as the valuation of these subsets does not necessarily equal the sum of its containing items. Furthermore, two types of subsets can be defined, where subsets S and T with $S \cap T = \emptyset$ are called *complements* if $v(S \cup T) > v(S) + v(T)$ (also called *super-additive*) and *substitutes* if $v(S \cup T) < v(S) + v(T)$ [Nis+07, p. 268]. Assuming the non-existence of *externalities*, meaning the bidder's valuation is not dependent on the allocation of items to other bidders, the *utility* or *payoff* π_i of bidder i can be described as $\pi_i = v_i(S) - p$, where p is the price for the current subset S [Nis+07, p. 268].

The mapping of items to bidders is called *allocation* and is written $S_1, ..., S_n$ with $S_i \cap S_j = \emptyset$ for every $i \neq j$. Summing all valuations over the allocation $\sum_i v_i(S_i)$ is called *social welfare*. One goal of an auction can be to maximize this metric in its equilibrium. [Nis+07, p. 268].

Common Value Auctions. Item valuations do not have to be bound by its private valuation, but can also be influenced by the overall allocation of the items. In those cases, bidder i can learn about bidder j's information and might be forced to re-evaluate its valuation for the object [Lev04, p. 8]. Therefore, the signals and information from bidder i and j are dependent. Examples for auctions incorporating this behaviour can be initial public offerings or spectrum auctions. In *common value auctions*, the signals from other bidders influence a bidder's valuation by $v_i(s_i, s_{-i})$. Note, that the IPV is a special case of this formulation with $v_i(s_i, s_{-i}) = s_i$, where the signals $S_1, ..., S_n$ are independent [Lev04, p. 8].

Bayesian Nash Equilibrium. Often, auctions are described by the equilibria they create, meaning the "convergence" of strategies used by the bidders that is induced by the auction format. If a bidder would know every strategy its competitors were following, he could easily deduce a payoff-maximizing action, which is called *best response*. Let $s = (s_i, s_{-i})$ be the *strategy profile* of the game, where s_i is bidder's i strategy and s_{-i} the strategies of its competitors. Bidder's i best response s_i^* to the

[1]The simulation in this thesis will be based on the IPV.

strategy profile of its competitors s_{-i} is a mixed strategy $s_i^* \in S_i$ where $\pi_i(s_i^*, s_{-i}) \geq \pi_i(s_i, s_{-i}) \quad \forall s_i \in S_i$ [SL10, p. 62]. When considering all the bidder's strategies, the strategy profile $s = (s_1, ..., s_n)$ is a *Nash equilibrium* if for all agents i, s_i is the best response to their competitors' strategies s_{-i} [SL10, p. 62]. Therefore, the Nash equilibrium describes the *stable* balance between the different bidder's strategies, where no bidder wants to change his strategy if he knew the strategies its competitors. The *Bayesian Nash equilibrium* takes into account the assumption a bidder makes about the strategies of its competitors. With $I = \{1, 2, ..., n\}$ being the set of players, X_i the set of possible types of agent $i \in I$ and $F(\cdot)$ the probability distribution over the set $X = X_1 \times X_2 \times ... \times X_n$ each player can choose a strategy $s_i \in S_i$ where $s_i : X_i \rightarrow S_i$ [MM05, p. 6 f.]. The *Bayesian Nash equilibrium* concerning the best responses $s_1^*, ..., s_n^*$ and $\forall i \in I, \forall x_i \in X$ and $\forall s_i \in S_i$ can then be described as

$$\int_{x_{-i} \in X_i} \pi_i(s_i^*, s_{-i}^*, x_i, x_{-i}) d\hat{F}_i(x_{-i}|x_i) \geq \int_{x_{-i} \in X_i} \pi_i(s_i, s_{-i}^*, x_i, x_{-i}) d\hat{F}_i(x_{-i}|x_i)$$

where $\hat{F}_i(x_{-i}|x_i)$ describes probability distribution over agent i's competitors' types, given that agent i knows his own type x_i. The agent continuously updates his prior information on the distribution using Bayes rule, when he learned that his type is x_i [MM05, p. 6 f.].

2.1.1 Sealed Bid Auction - First Price Auction

The sealed bid or first price auction is easy to imagine. Each bidder places sealed bids $b_1, ..., b_n$ and the bidder with the highest bid wins, *paying the amount he bid*. Because of the nature of this rule, bidders have an incentive to not bid their true valuation of the object, because this would result in $\pi_i = v(s_i) - b_i = 0$ [Lev04, p. 3]. To circumvent that, bidders might bid below their actual valuation to potentially increase their profit. Interestingly, it can be shown that the equilibrium for the sealed-bid first price auction with n bidders can be described as

$$b(v(s_i)) = \left(\frac{n-1}{n}\right) v(s_i)$$

e. g. in a sealed bid auction with two bidders, each bidder would bid half its valuation of the object [Vic61].

2.1.2 Vickrey Auction - Second Price Auction

The Vickrey auction is a special case of the sealed bid auction, where bidders submit sealed bids $b_1, ...b_n$ and the bidder with the highest bids wins, but only *pays the amount of the second highest bid*. It can be shown, that in equilibrium, each bidder will bid its valuation of the object $b_i(s_i) = s_i$ [Lev04, p. 2]. The Nash equilibria of the Vickrey auction and the open English auction are the same. When auctioning items in an open and ascending (English) manner, the equilibrium will settle at the second highest valuation [Lev04, p. 2]. With the item prices rising from zero upwards, bidders can drop out of the auction when they reached their valuation of the object. The winner will have to pay the amount of the second highest valuation as its bidder will have dropped out as the price reached its valuation, making it the last bidder in the auction.

2.2 SAA - Simultaneous Ascending Auctions

In contrast to the normal English auction, were one item is sold at the same time using an ascending price system, spectrum auctions sell many goods. When selling items sequentially, bidders' available information and responding possibilities are limited and thus bidders might risk missing the opportunity to buy items at low prices early or even being forced to buy them at a much higher prices later on. Also they might fail to bundle desired packages of items together, which would be more valuable to them. Bidders are forced to make good predictions about the outcome of the auction and bid accordingly. Most times this leads to less efficient auctions, meaning less frequently bidders achieve to acquire the items or item combinations they value the most [CSS06, p. 185].

Characteristics. To reduce the amount of exposure bidders will be subjected to, simultaneous ascending auctions were introduced. In this format, many items are sold, were each item can be bid on simultaneously. Bidders can not submit bids on packages of items, but have to place bids on single objects. Like in an English auction, prices rise with each valid bid placed until the auction is finished, e.g. when each bidder stopped raising bids or when times runs out. If certain item combination are complementarities, meaning their combination has super-additive valuation, bidders in SAA tend to suffer from bidder exposure [CSS06, p. 209]. To mitigate this effect, withdrawal of standing bids can be implemented, that enables bidders to back out of failed item aggregations.

Conclusion and discussion. SAA are a well established format used in high-stake auctions like spectrum, energy or pollution permit auctions. With mild complementarities, SAA yield in competitive equilibria, that also hold stand in practical usage [CSS06, p. 209]. While SAA manage to successfully combine auctioning of multiple items with simple mechanism design, SAA can incentivise bidders to expose themselves in order to achieve certain item combinations when complementarities exist. In those environments, package bids should be used to increase efficiency of the auction[CSS06,

p. 185]. Also, revenue-reducing strategies and bidder collusion can occur in markets with weak competition, as well as bid-signalling to cooperatively split items between competing bidders [CSS06, p. 187., p. 209].

2.3 The Exposure Problem

During an auction with many offered items, bidders often try to acquire certain item combinations, because those packages of items might have super-additive valuation, meaning their package valuation is higher than the sum of the valuation of each item. In the field of spectrum auctions these might be a specific number of blocks obtained in a certain frequency band that enables better technology, e.g. LTE instead of GSM, or the clustering of adjacent geographical regions that allows a bidder to provide service for a bigger market while reducing costs by optimally placing transmission towers in the regions. When auction formats only allow bidders to place bids on single items, not packages of items, they run into the risk of failing to achieve the desired item combination. In order to acquire those bundles, bidders might have to bid above the valuation of a particular item but might not be able to acquire the remaining items in the bundle. This is called the exposure problem.

A simple example. A bidder tries to acquire the synergistic combination of items A and B. The bidder's valuation for each single item is $v(A) = v(B) = 10$, but when combining both, let's say because it enables the bidder to use more cost-effective technology, the valuation of both items together is $v(AB) = 40$, so twice the summed valuation of the single items.

Now, let's consider the following situation: The current prices are $p(A) = 9$ and $p(B) = 14$. Normally, the bidder would set its bid for item A with $bid(A, B) = (10, 0)$, thus excluding item B from its bid, because the current price already exceeds the bidder's internal valuation of the item. However, because the sum of both prices are still less than the combined valuation of the items, hence, the bidder has an incentive to also bid on B. For the sake of the argument, our bidder now placed its bid with $bid(A, B) = (10, 15)$, the auction finished, but a competitor achieved to acquire item A with a higher price. Now, our bidder bought item B with $p(B) = 15$ and let's remember that the single item internal valuation of item B was $v(B) = 10$. Instead of gaining a surplus through obtaining one of the auctioned items, our bidder now made a loss (example based on [Lev09]).

In auctions with many items and thus a high number of item combinations it is hard to keep track of the dependencies. Academia tackles the exposure problem by researching new auction formats or mechanisms that try to mitigate the exposure subjected to the bidders. One mechanism is the possibility to retract bids like they are offered in implementations of the SMRA (see subsection 2.5.1), where "failed bids", e.g. bids that failed to aggregate a desired item combination, can be withdrawn to free up budget, which the bidder can then focus on other valuation maximizing bids. This is a well establish mechanism, even though according to Cramton, this can lead to "undesirable gaming behavior" within the auction and has to be constrained to effectively control such behaviour [CSS06]. Other efforts strive to reduce exposure of bidders by enabling them to bid on (pre-)defined packages, like the HPB format (more on that in subsection 2.5.2).

2.4 What is the Advantage of Round Based Auctions?

In standard auction models, like in sealed bid auctions, it is assumed that each bidder knows its valuation for each possible package a priori [CSS06]. In fact, determining valuation is a costly process, because it can depend on information of other bidders and especially when a high number of items are involved, create a solution space of combinatorial magnitude.

Round based auctions try to mitigate this problem as each round, bidding information is released and bidders can identify target licenses more easily [Lev09]. The extent of the information range depends on the model and its implementation, but exemplary might contain price information like the highest winning bid and all bids placed by competitors. With the newly revealed information at hand, bidder's uncertainty is reduced so that they can set their future bids more aggressively and can focus on the valuation maximazing parts of the item space, which improves efficieny according to Cramton [CSS06]. Over time, bidders increasingly get an understanding of what the overall allocation and prices at the end of the auction might look like and can bid according to this judgement.

Using this advantage, round based formats were used in national spectrum auctions in the U.S. (and later on in Europe) since 1994 when it was proposed to the U.S. Federal Communication Commission by Milgrom, Wilsen and McAfee.

2.5 Auction Formats used in Spectrum Auctions

Even though the success of simultaneous ascending auction formats in national spectrum auctions led to a widespread use of formats like the SMRA, a number of other formats were used in recent times or new formats were proposed by academia. In the following section I will describe the auction processes and rules for the Simultaneous

Multi-Round Auction (SMRA). and the Hierarchical Package Bidding Auction (HBP). Both formats will be compared in the simulation (see chapter 4).

2.5.1 SMRA - Simultaneous Multi-Round Auction

The Simultaneous Multi-Round Auction (SMRA) is most commonly used auction format used for selling spectrum world wide. Since its development in the early 90's for the US Federal Communications Commission its usage has become wide spread, making it the de facto standard. Its success stems from the fact, that it often leads to good allocations [Bic16], but it suffers from strategic challenges for bidder when used in an environment where complementarities exist.

Characteristics. The SMRA extends the SAA, by adding a round based system. Multiple items are auctioned at the same time - in contrast to the English auction - in a round based fashion, where each round has a time limit in which bidders can place bids. When a round is finished, bidding data is published that contains the *current* winner of each item, which corresponds with the highest bid placed on each item [Bic16]. The degree of transparency provided by the auctioneer depends on the implementation, e.g. the German auction in 2015 provided very high transparency by showing all bids placed by each participant [Bun15]. Often, only highest bid submitted is published. In each round, bidders have to exceed the provisional prices of the previous round by a pre-defined *increment* if they wish to claim the item. In the German auction, bidders could submit bids using a *clickbox* system, which offered pre-defined multiples of the current increment. The increment can change over time, depending on the current *phase* of the auction. The auction ends, when no new bids are being submitted. Each bidders than acquire the items where he possesses the highest active bid and pays the prices accordingly.

To incentivise bidding right from the start, SMRA makes use of *activity rules*. This is often realised by *eligibility points (EP)* that corresponds to a bidder's bidding extent. Each bidder starts with a pre-defined number of EP - often comprising of the maximum number of licenses allowed to bid on - and each item can be matched to a certain number of EP necessary to bid on them. If the EP from the items won in the previous round and the bids submitted in the new round are less than the current *activity level*, a bidder's EP gets deducted by the amount it "under-bid". An example for an activity level can be 65% of the starting EP, as used in the German auction in the first bidding phase. From then on, the activity level subsequently rose to 80% and 100% [Bun15, p. 15]. Additionally, auctioneers using SMRA can use a wide range of tools to influence the outcome of the auction. These can include:

- *minimum prices* for the items

- *bid increments* and how their value changes over the course of the auction
- *bid withdrawals* and *waivers*, which allow the bidder to either withdraw a bid submitted in a previous round or to suspend himself from bidding for a round (without consequences)
- *bidding floors* and *caps* which regulate how many items a bidders is allowed to obtain in a specific frequency band to prevent monopolies or businesses to drop out of the market due to insufficient supply.

Conclusion and discussion. The SMRA is an easy to implement auction format which clear and simple rules which often realises efficient allocations. In environments where complementarities exist (such as synergetic values between items), the SMRA fails to resolve at a Walrasian equilibrium [Bic16]. Due to its lack of package bidding, bidders might be incentivised to expose themselves in order to acquire certain item combinations, making the exposure problem a central challenge in SMRA strategies. The activity rules have shown to be fairly similar world wide, but the degree of transparency differs widely, as well as the rules in procurement, which have impact on the performance of the auction. Nevertheless, the SMRA is a well established and frequently used format for spectrum auctions and other common value auctions.

2.5.2 HPB - Hierarchical Package Bidding

While the Hierarchical Package Bidding Auction (HPB) has never been used in a national spectrum auction before, this thesis will run simulations using this format to explore indicators of its eligibility as basis for further research. The implementation used in this thesis is based on the propositions of Goeree and Holt from 2007.

The paper hypothesised that HPB might help mitigating the exposure problem (see section 2.3) by pre-packaging bidding items into bundles inside a tree like hierarchy structure. This allows bidders to bid on packages, signalling a desired allocation. By that, the bidder can bid on a certain package until its valuation, without the need to possibly strategically bid over the valuation of single items in that package. In the end, the bidder either manages to acquire every item in the package and combining them to realise super-additive valuation or does not win any of the items. Of course, bidders are still able to bid on single items.

Combinatorial auctions try to solve the problem by allowing bidders to bid on any desired item combinations. Determining the provisional allocation after each round shows to be an intricate and computationally expensive problem in the magnitude of *NP-hard* [GH10], meaning non-deterministic polynomial-time hard, because the number of possible arrangements growth exponentially with the number of objects. Thus, it is not guaranteed to find the optimal solution within a reasonable amount of time and mostly the best current solution yielded within a pre-defined time frame will be chosen. This approach is problematic as price calculations seem non-transparent and even experts claim the ability to rig auctions based on this computational downside [GH10].

The HPB idea. Rothkopf, Pekeč, and Harstad proposed a hierarchical pre-packaging of items to avoid the issues of computational expensiveness of combinational auctions in 1998 [RPH98]. The work of Goeree and Holt was based on this proposition and extended it by developing a recursive pricing formula for combinatorial auctions using the pre-defined hierarchies. The basic idea of the pricing formula is that prices for single items would be increased by *"lump-sum taxes"* handed down from packages in the higher hierarchy levels. In the case a package bid is winning, the excess prices will be proportionally propagated down towards the single item level of the hierarchy. The goal is to create a transparent and computationally inexpensive way to calculate the prices.

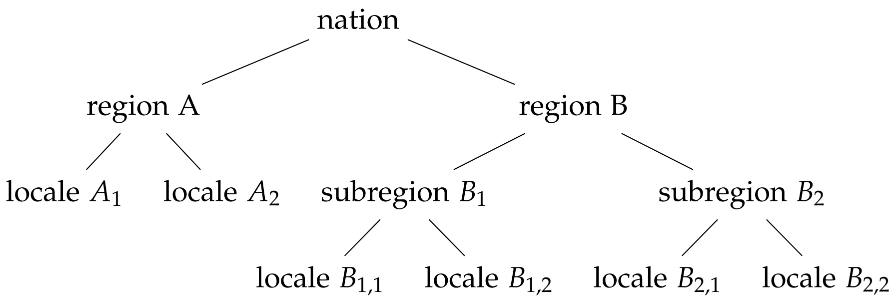

Figure 2.1: Example of an HPB Hierarchy for spectrum auctions

The mathematical formulation. An example for a HPB hierarchy can be seen in Figure 2.1, which subsequently unites items to regional and finally a nation wide package. Formulated in [GH10], a hierarchy has $H \geq 1$ hierarchy levels which are subsequently labeled $h = 1, ..., H$ and each level h contains I_h packages. The single items can be found as leaves in the tree in $h = 1$, from there the package grow bottom up. Packages in level h are written as $P^h_{i_h}$ for $i_h = 1, ..., I_h$. Each of the packages consists of $\alpha^h_{i_h}$ bidding items and the number of packages in a level falls while propagating upward in the hierarchy. As an important note, all packages in a hierarchy are non-overlapping, meaning $\forall$ level-h packages $P^h_{i_h} \neq P^h_{i_h} \implies P^h_{i_h} \cap P^h_{i_h} = \emptyset$. Therefore, each package is contained in only one package in the following upper level $h' > h$ and there is only one unique level-h' package $P^{h'}_{j_{h'}}$ with $P_{j^{h'}_{h'}} \supset P^h_{i_h}$. This results in the number of items being contained in a package is the sum of items it covers in level-1.

$$\sum_{P^1_{i_1} \subset P^h_{i_h}} \alpha^1_{i_1} = \alpha^h_{i_h} \tag{2.1}$$

Every hierarchy level contains all items from level-1: $\sum_{i_h=1}^{I_h} \alpha^h_{i_h} = \alpha \; \forall$ h in h $= 1, ..., H$ with α being the total number of *(level-1)* items in the auction. After defining the hierarchy and its properties, Goeree and Holt devised two recursive algorithms for calculating the *revenues* $R(P^h_{i_h})$ and *prices* $p(P^1_{i_1})$.

Revenues and the assignment problem: To assign items to bidders, first the highest bid $b^{max}(P^h_{i_h})$ on the packages have to be found. Finding the optimal assignment and revenue can be describes as follows:

1. Set $h = 1$, for this level set revenues to $R(P^1_{i_1}) = b^{max}(P^1_{i_1})$ for $i_1 = 1, ..., I_1$, those bids are marked as *"provisionally winning"*
2. if $h < H \implies h = h + 1$ and continue with step 3, otherwise *stop*
3. if $b^{max}(P^h_{i_h}) > \sum_{P^{h-1}_{i_{h-1}} \subset P^h_{i_h}} R(P^{h-1}_{i_{h-1}}) \implies R(P^h_{i_h}) = b^{max}(P^h_{i_h})$ and label $b^{max}(P^h_{i_h})$ as *provisionally winning*, bids from all lower levels on packages that overlap with $P^h_{i_h}$ are unmarked. Otherwise, $R(P^h_{i_h}) = \sum_{P^{h-1}_{i_{h-1}} \subset P^h_{i_h}} R(P^{h-1}_{i_{h-1}})$ and *return to step 2*.

Prices: Now, with the assignment set, the prices have to be updated. They are assigned to the level-1 items in the hierarchy by adding *"lump sum tax"* to the items if the revenue of package is less than the one the next level upwards. With $p(P^1_{i_1})$ being the price of level-1 package $P^1_{i_1}$:

1. Set $h = 1$ and define prices for all packages in this level as $p(P^1_{i_1}) = b^{max}(P^1_{i_1})$ $\forall i_1 = 1, ..., I_1$
2. if $h < H \implies h = h + 1$ and continue with *step 3*, otherwise *stop*
3. For each package $P^h_{i_h}$ in level-h calculate

$$\tau^h(P^1_{i_1}) = \frac{\alpha^1_{i_1}}{\alpha^h_{i_h}} \Big(R(P^h_{i_h}) \quad - \sum_{P^{h-1}_{i_{h-1}} \subset P^h_{i_h}} R(P^{h-1}_{i_{h-1}}) \Big) \geq 0$$

   and add $\tau^h(P^1_{i_1})$ to the price $p(P^1_{i_1})$ of each level-1 package $P^1_{i_1}$ contained in $P^h_{i_h}$. Return to *step 2*

In comparison to the calculations needed in other combinatorial auction formats, the computations done in the two formulations above are computationally inexpensive. This is one of the advantages of HPB. Goeree and Holt derived a calculation of the level-1 prices as follows:

$$p(P^1_{i_1}) = b^{max}(P^1_{i_1}) + \sum_{P^h_{i_h} \supset P^1_{i_1}} \frac{\alpha^1_{i_1}}{\alpha^h_{i_h}} \Big(R(P^h_{i_h}) \quad - \sum_{P^{h-1}_{i_{h-1}} \subset P^h_{i_h}} R(P_{i^{h-1}_{h-1}}) \Big) \tag{2.2}$$

Summary and discussion. HPB shows to be less computationally expensive as other combinatorial auction formats but maintains the possibility for bidders to bid on *pre-defined* packages within a hierarchy that is built bottom up from the single items upwards. The assignment problem as well as the price calculations can be done using a recursive function, where the number of comparisons needed is only linear in relation to the pre-defined packages [GH10].

Goeree and Holt also mention in their paper that the pre-packaging will disadvantage small bidders who also contribute to finding optimal allocations. The impact of this needs to be researched in further studies. Also, Goeree and Holt argue that non-overlapping package structures may not always be able to reflect the interests of the participating bidders. It still needs to be verified to what extent the hierarchy influences auction outcomes. Also, for each auction there might be an array of alternative hierarchies. To choose the appropriate hierarchy seems to be key in using HPB efficiently.

2.5.3 Other Auction Formats

In spectrum auctions world wide a variety of different auctions formats are used. The reason for that are different performance properties of efficiently allocating items to bidders as well as maximizing revenue or social welfare. For example, Canada used a the combinatorial clock auction (CCA) format to sell the 2500 MHz spectrum in the 2014 . It used an OR-bidding language, where bidders were able to express that they want to acquire a license for a certain price *and / or* a different license for a different price [IC14]. The auction involved a price discovery stage that was similar to the SMRA auction format, but the CCA allowed bidders to submit bids on packages rather than only on single items. This was necessary due to the regional nature of the licenses being auctioned and the complementarities that existed between them [IC14].

Despite the fact that combinatorial auction allows bidding on packages, it still inherits a lot of issues like non-linear price progressions that makes it hard for bidders to compare prices, as well as computational issues [Nis+07, p. 290], which results in combinatorial auction not being frequently used in spectrum auctions.

In 2015, the French 700 MHz spectrum auction was even conducted in a sealed bid manner [Fie15], raising EUR 2.8bn in the process. The sealed bid part in the auction was used to decide where in the spectrum band auctioned blocks should be placed.

3 Experimental Design

In this chapter I first describe the data of the German spectrum auction from 2015. Afterwards, a value model is introduced that was built from the insights of the data analysis and external resources to describe the valuation of each bidder in the auction. The value model serves as a cornerstone of the simulation in chapter 4. In the last section I describe on which basis the performance of different auction formats can be compared.

3.1 Data from the German Spectrum Auction 2015

To simulate the German spectrum auction "Mobiles Breitband - Projekt 2016" (MBP16, see section 1.2) a value model needs to be derived for each bidder in the auction. To achieve that, the respective data was analysed to give insights into bidding behaviour and valuation.

3.1.1 Data Aquisition

The German Federal Network Agency (BNetzA) made the MBP16 data available to the public via their website[1]. The data contains the highest bidder and the winning bid for each block in each round, as well as the the summed value of all retracted bids (see an example in Figure 3.1). During the auction every bidder could see the bids placed by their competitors after each round ended [Bun15]. This data is not contained in the public data set.

As the round data was published as image files, there was a need to systematically extract the information from the images and save it to a suitable data format. To achieve this, a Python script loaded all the images and used Optical Character Recognition (OCR), a computer vision algorithm for pattern recognition, to scan them. Afterwards, all the data was written into a CSV file.

[1] `http://www.bundesnetzagentur.de/DE/Sachgebiete/Telekommunikation/Unternehmen_Institutionen/Frequenzen/OeffentlicheNetze/Mobilfunknetze/Projekt2016/projekt2016-node.html` (accessed on 11/07/2016)

gepart - paired
ungepart - unpaired

Rundenergebnis der Runde 181

Frequenzbereich	Block	Ausstattung	Höchstbieter highest bidder	Höchstgebot (€ in Tsd.)
700 MHz (gepaart)	700 A	2x5 MHz konkret	TEF DE	166.397
	700 B	2x5 MHz abstrakt	Vodafone	165.509
	700 C	2x5 MHz abstrakt	TEF DE	166.847
	700 D	2x5 MHz abstrakt	Telekom	166.567
	700 E	2x5 MHz abstrakt	Telekom	171.649
	700 F	2x5 MHz abstrakt	Vodafone	163.476
900 MHz (gepaart)	900 A	2x5 MHz konkret	TEF DE	195.520
	900 B	2x5 MHz abstrakt	Vodafone	211.807
	900 C	2x5 MHz abstrakt	Vodafone	203.298
	900 D	2x5 MHz abstrakt	Telekom	183.671
	900 E	2x5 MHz abstrakt	Telekom	180.968
	900 F	2x5 MHz abstrakt	Telekom	180.465
	900 G	2x5 MHz abstrakt	TEF DE	189.958
1,8 GHz (gepaart)	1800 A	2x5 MHz abstrakt	Vodafone	237.494
	1800 B	2x5 MHz abstrakt	Telekom	248.054
	1800 C	2x5 MHz abstrakt	Vodafone	258.247
	1800 D	2x5 MHz abstrakt	Vodafone	249.133
	1800 E	2x5 MHz abstrakt	Telekom	248.101
	1800 F	2x5 MHz abstrakt	Vodafone	255.967
	1800 G	2x5 MHz abstrakt	TEF DE	239.228
	1800 H	2x5 MHz abstrakt	Telekom	248.784
	1800 I	2x5 MHz abstrakt	TEF DE	240.288
	1800 J	2x5 MHz konkret	Vodafone	180.153
1,5 GHz (ungepaart)	1500 A	1x5 MHz abstrakt	Vodafone	40.939
	1500 B	1x5 MHz abstrakt	Vodafone	40.939
	1500 C	1x5 MHz abstrakt	Vodafone	40.919
	1500 D	1x5 MHz abstrakt	Telekom	42.964
	1500 E	1x5 MHz abstrakt	Vodafone	42.961
	1500 F	1x5 MHz abstrakt	Telekom	39.011
	1500 G	1x5 MHz abstrakt	Telekom	40.961
	1500 H	1x5 MHz abstrakt	Telekom	40.961
Summe aller gehaltenen Höchstgebote (€ in Tsd.)				5.081.236
Zahlungsverpflichtung aufgrund zurückgenommener Höchstgebote (€ in Tsd.)				0
Summe (€ in Tsd.)				5.081.236

Figure 3.1: Example of the available bidding data

3.1.2 Data Analysis

Now, using the processed data I could have a look the bidding behaviour and derive insights for the value model. The paper of Bichler, Gretschko, and Janssen states that three different bidding phases can be found during the German Auction: the coordination phase, the competition phase and the end phase. Analysing the bids submitted during the auction, these phases could be shown using the data.

Supporting in the process will be Figure 3.3, which visualises the different phases by showing which bidder placed the highest bids in each round, thus temporarily winning the corresponding frequency block. Additionally, Figure 3.4 shows the development of the mean bids by spectrum submitted by each bidder over the course of the auction to give an overview of the price progression.

In this following segment I will give a quick overview of the course of the bidding behaviour according to Bichler, Gretschko, and Janssen. The **coordination phase** reached from the beginning of the auction until round 30. It was marked by a careful search

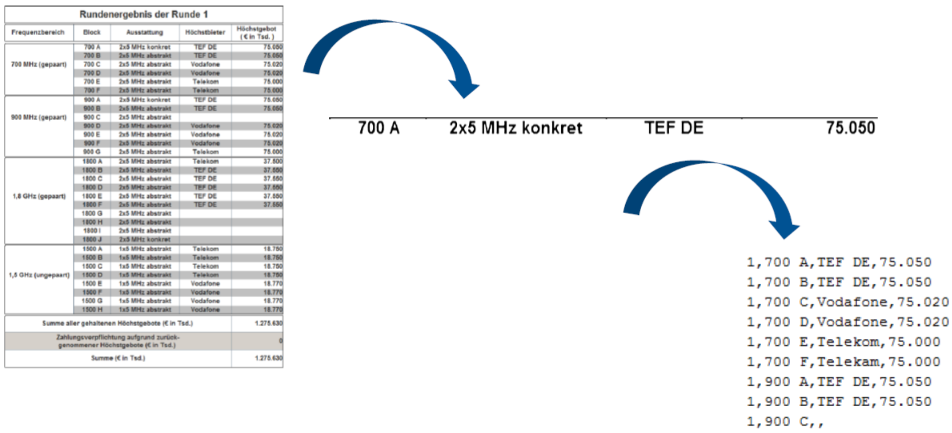

Figure 3.2: The OCR process

for possible allocations that would satisfy all the bidders which resulted in cooperative bidding behaviour, hence the naming. A distinctive feature of this phase was that prices only rose slowly, trying to signal possible mutually compatible allocations between the bidders. An interesting behaviour can be seen in the 700 MHz band. All three bidders placed bids on two blocks in the segment. As can be seen in Figure 3.3, over the course of this phase (and as we will see later, much longer than that) no new bids were placed after the initial bids in the first round, which resulted in stable prices and indicating a lack of excess demand and a demand equilibrium. In the other bands, the bidders tried to communicate their demand and signal a potential overall allocation. At the end of this phase, the total revenue over all items was at EUR 2 billion.

The **competition phase**, reaching from round 31 to 154, started with an excess demand of 2 blocks in the 1800 MHz band [BGJ16]. From signalling potential overall allocations the behaviour shifted to an aggressive outbidding of competitors, especially in the 1800 MHz spectrum. As can be seen in Figure 3.4, prices in this band increased rapidly and a lengthy phase of "war of attrition" began to solve the excess demand problem by showing competitors that they can withstand the price increases and a capitulation would be more advisable. During the last part of this phase, the price war slowly shifted towards the 900 MHz band, which hitherto showed to be more stable in comparison, as an attempt to solve the conflict in the 1800 MHz band.

During the **end phase** of the auction, the price war in the 1800 MHz band was slowing down due to a sudden shift to the 700 MHz spectrum. The band which stayed untouched till the beginning of the auction now experienced a strong increase in prices to solve to excess demand problems in the 900 and 1800 MHz bands and prices in the 700 and 900 MHz band approached each other. The strategy showed its impact by TEF

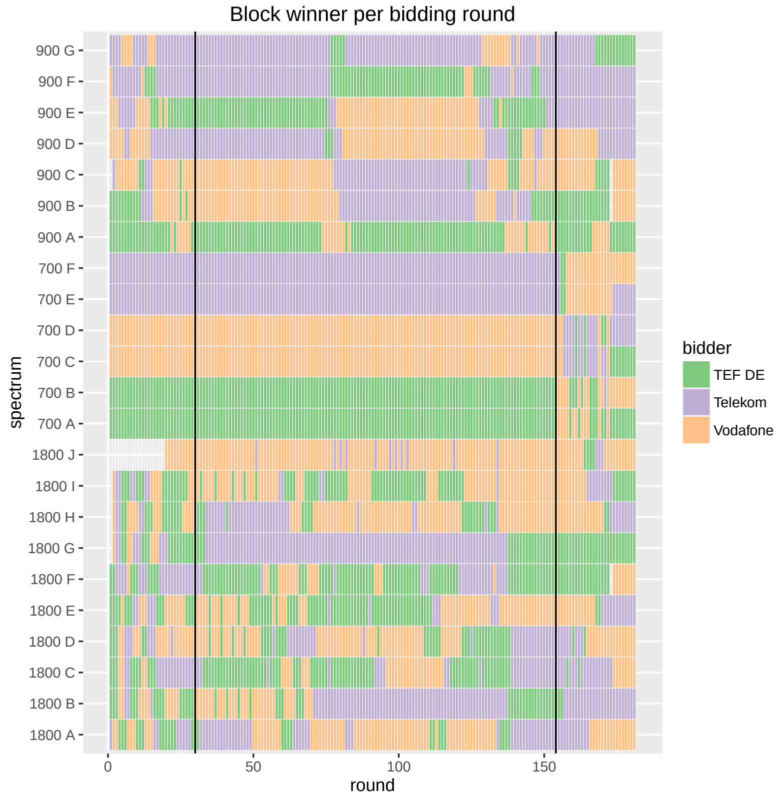

Figure 3.3: Block winner per round

withdrawing bids from the 1800 MHz, dissolving the excess demand in the band. I conjecture that TEF was nearing its valuation for the 1800 MHz blocks and reduced its demand to avoid further price increases. This is an interesting point that will be utilized in the formulation of the value model. After the rough patch in the 700 MHz band it returned to an equilibrium that had the same shape as in the beginning of the auction and each bidder won two blocks in the spectrum.

Table 3.1 shows the final allocation at the end of the auction by the number of blocks obtained by each bidder in the spectrum. In conclusion, the data analysis indicated that the bidding behaviour did not clearly follow a pure straightforward pattern, but the prices were a function of the different bidding strategies. Knowing this, an useful approach to build a value model is needed.

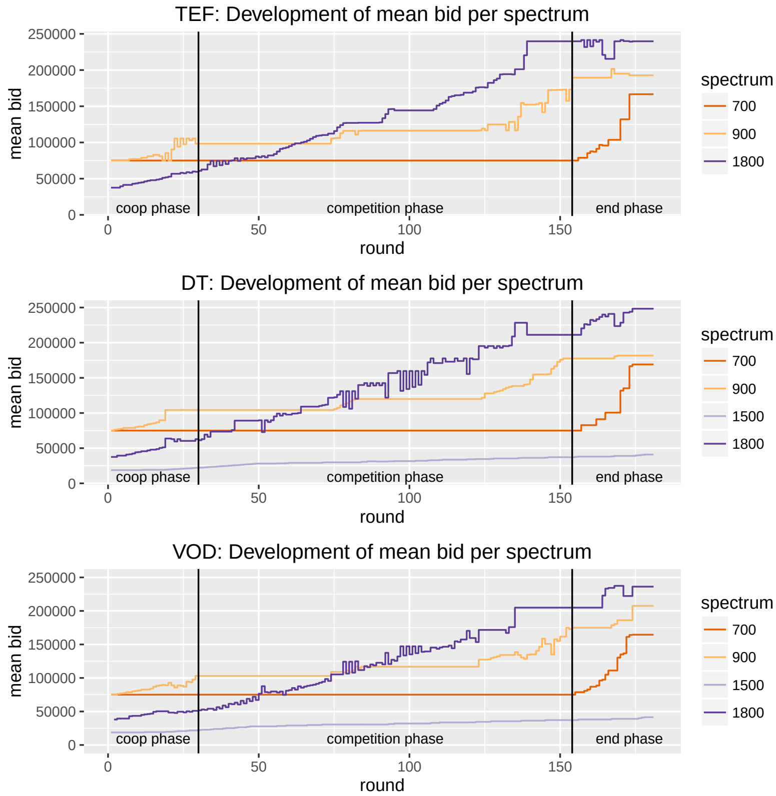

Figure 3.4: Development of mean bids per spectrum over time, bids in thousand EUR

3.1.3 Spectrum specific Information

Before building the model for the German spectrum auction in 2015, it is of help to look at the probable usage of the different frequency bands as they give clues to the bidders' valuation. Below I compiled a listing on how the different bands might be used after the auction, based on the paper of Bichler, Gretschko, and Janssen. Note, that due to its relatively small importance in the auction, the 1500 MHz band will be excluded from the simulation and thus the value model.

700 MHz: This spectrum was of importance as this band was not held by any of the MNOs before the auction. Its usage after the auction is most likely enabling LTE for their customers. Due to better propagation characteristics of lower frequency bands the 700 MHz is of great value to the participating bidders.

900 MHz: Similar to the 700 MHz band, the 900 MHz spectrum is a low frequency band that is of great value to the MNOs. Following Bichler, Gretschko, and Janssen the value of acquiring three blocks in this spectrum is much higher than that of two

MNO	700 MHz	900 MHz	1800 MHz	1500 MHz
TEF	2	2	2	0
DT	2	3	3	4
VOD	2	2	5	4

Table 3.1: Final allocation of the German Auction

blocks. As with three blocks a bidder can use two of them for LTE or use all of them for GSM. This flexibility adds a big surplus on this bundling. Important to note is the spectrum cap of 3 blocks per bidder in the auction. The cap was incorporated to ensure that every bidder at least acquires one block in this spectrum.

1800 MHz: Nine out of the ten blocks offered in this spectrum were already in use by the participating bidders, but were re-auctioned during the MBP16 project. As with the preceding frequency bands, the goal to use this band for LTE is most likely. If utilized in this way, assembling 4 or 6 blocks in this spectrum would enable this technology, thus making it the most valuable combination. Apart from prohibiting a competing MNO to acquire the desired bundle, winning a fifth block would only have a marginal effect on the intrinsic valuation. If a bidder would not achieve the wanted bundling, any blocks from this band would most likely be used for GSM and would have equal value.

1500 MHz: The 1500 MHz band experienced only small competition (e. g. TEF did not even bid in this band), prices only rose slowly over the course of the auction, indicating an inferiority of this non-mainstream band to the other bands. Additionally, with the technological status quo, no mobile devices were able to utilize this frequency band at the time of auction. Thus, this spectrum will be excluded from the simulation to focus on the more important frequency bands.

From the listing it is evident that in particular the bundling of three blocks in 900 MHz as well as bundling 4 or 6 blocks together in 1800 MHz is of great value for the bidders due to their potential usage for LTE. This has to be included into the value model.

3.2 Value Model

To simulate the bidding behaviour a value model is needed. The value model is a function

$$v : (input) \longrightarrow \mathbb{R}$$

that, given a certain set of bidding items, determines the valuation of bidding items of a bidder. During the simulation, each bidding agent can compute its internal valuation for each bidding combination and thus determine its bidding strategy. Hence, The value model is key to the auction simulation (see chapter 4)

Forging all the information from the preceding sections into a model to describe the valuation of the bidders results in the following mathematical formulation. Each mobile network operator $MNO \in \{TEF, DT, VOD\}$ can bid on any number of available blocks $q_{700}, q_{900}, q_{1800} \in \mathbb{N}_0$ in the frequency bands included in the simulation (700 MHz, 900 MHz and 1800 MHz). Additionally, the values for single items and bundles of items have to be incorporated into the model. As stated in subsection 3.1.3, bundles of 3 blocks in 900 MHz as well as 4 and 6 blocks in the 1800 MHz bands have a major additional value in comparison to the normal multiples of the baseline valuations due to their usefulness in enabling LTE. Due to the high competition and prices in the 1800 MHz band it was unlikely that one of the participating MNOs would achieve the bundling of six blocks in this spectrum. Accounting for this fact, the six-block-bundle was not incorporated into the value model. To formulate the bundling options, band specific constants are introduced into the model. Let $items \in \{700, 900, 900_3, 1800, 1800_4\}$ be the set of different item combinations, where $\{700, 900, 1800\}$ stand for the valuation of one single item in the respective band and $\{900_3, 1800_4\}$ for the bundles of respective band items.

Multiplying the quantities with the respective single item valuation V_{MNO}^{items} and adding the bundle effects yields the respective valuation for each bidder and each combination.

$$
\begin{aligned}
v : (q_{700}, q_{900}, q_{1800}) &\longrightarrow \mathbb{R} && (3.1)\\
v_{MNO}(q_{700}, q_{900}, q_{1800}) &= v_{700}(q_{700}) + v_{900}(q_{900}) + v_{1800}(q_{1800}) && (3.2)\\
v_{700}(q_{700}) &= V_{MNO}^{700} * q_{700} && (3.3)\\
v_{900}(q_{900}) &= V_{MNO}^{900} * q_{900} + \mathbb{1}_{\{q_{900} \geq 3\}} V_{MNO}^{900_3} && (3.4)\\
v_{1800}(q_{1800}) &= V_{MNO}^{1800} * q_{1800} + \mathbb{1}_{\{q_{1800} \geq 4\}} V_{MNO}^{1800_4} && (3.5)
\end{aligned}
$$

Note, that the spectrum cap of maximum three blocks per MNO is not depicted in the value model itself, but will be implemented in the auction model (see chapter 4).

As stated in [BGJ16] and seen in the data analysis section before, the prices of the auction items were merely a function of the cooperation and competition strategies of the bidders and resulted in relatively low prices for the lower frequency bands. This is surprising as due to the better propagation properties the prices of lower frequency bands should have been higher. To get a comparison and an understanding of the 2015 prices, it is worthwhile looking at the German spectrum auction of 2010[2] where

[2] `http://www.bundesnetzagentur.de/DE/Sachgebiete/Telekommunikation/Unternehmen_Institutionen/Frequenzen/OeffentlicheNetze/Mobilfunknetze/Z_Auktion2010.html` (accessed 19/07/2016)

similar frequency spectra were auctioned. Table 3.2 shows the yielded mean prices of the offered spectrums. In 2010 the 800 MHz spectrum yielded a mean price of roughly EUR 600m per block, where as the 700 and 900 MHz spectrums in 2010 resulted in 166m and 194m respectively. The 800 MHz band in 2010 was destined to be used for LTE coverage, thus having a high potential to create competitive advantages in the downstream market. The 700 MHz band offered in 2015 will most likely be used for LTE, too. Therefore it is surprising that the auction yielded much lower prices. As noted before, cooperation between the bidders might be accountable for that.

	2010		**2015**	
Spectrum	**Mean price**	**Deviation**	**Mean price**	**Deviation**
700 MHz	-	-	166,741	1,886
800 MHz	596,079	13,569	-	-
900 MHz	-	-	193,998	10,591
1800 MHz	20,983	560	241,423	5,084

Table 3.2: Mean prices yielded for selected spectrum in the 2010 and 2015 auctions (prices in thousand EUR)

To compute the different valuation constants of the *items*, a baseline model approach was chosen, with the baseline being the mean price paid by TEF in the 1800 MHz band at the end of the auction. The decision for this baseline was motivated by the fact that the bids for the 1800 MHz band were probably much closer the bidders' valuation after a long series of bidding rounds that let the prices in this band soar rapidly. The price inclination was so steep that afterwards the bidding war shifted from 1800 MHz to other bands to avoid further price increases. This might give a good indication that the bidders were nearing the point where prices might have exceed the bidders' valuation and thus being much closer to the true valuation than in other bands. From the bids in the 1800 MHz band the TEF mean bid was chosen as I argue that TEF might have bid much closer to its own valuation. TEF merged with the German MNO E-plus only shortly before the beginning of the auction. All the single items valuations were derived by scaling based on assumptions regarding the spectrum specific characteristics $\delta_{spectrum-relation}$ in comparison to the 1800 MHz band and the relative strength s_{MNO} of the participating bidders in comparison to TEF. Also, the prices from the 2010 and 2015 auction were taken into account.

$$V_{MNO}^{700} = V_{TEF}^{1800} * \delta_{1800-700} * s_{MNO} \tag{3.6a}$$
$$V_{MNO}^{900} = V_{TEF}^{1800} * \delta_{1800-900} * s_{MNO} \tag{3.6b}$$
$$V_{MNO}^{1800} = V_{TEF}^{1800} * \delta_{1800-1800} * s_{MNO} \tag{3.6c}$$

Table 3.3 shows the chosen weighting factors $\delta_{spectrum-relation}$ and s_{MNO}. The inter-spectrum scaling was chosen based on the prices for 800 MHz in the 2010 auction. $\delta_{1800-700}$ got a higher weight as to reflect it usefulness to enable LTE. Also, s_{MNO} should reflect the budgets of each MNO, e. g. DT has a much higher budget (revenue of EUR 22.4bn[3] in 2015) than VOD (EUR 10.6bn[4]) and especially TEF (EUR 7.9bn [5]) after the merger. Also, looking at the market shares (in terms of percentage of service revenues in the sector) of 2014, DT realized 36%, VOD 34% and TEF 30% [BGJ16] which roughly translates into the values below, when using TEF as baseline.

$\delta_{spectrum-relation}$	
$\delta_{1800-700}$	3
$\delta_{1800-900}$	2
$\delta_{1800-1800}$	1

s_{MNO}	
s_{TEF}	1
s_{DT}	1,25
s_{VOD}	1,15

Table 3.3: Weighting factors chosen to compute single items valuations

To derive the bundle valuations, an additional scaling factor β_{item} is introduced.

$$V_{MNO}^{900_3} = V_{MNO}^{900} * \beta_{900_3} \tag{3.7a}$$
$$V_{MNO}^{1800_4} = V_{MNO}^{1800} * \beta_{1800_4} \tag{3.7b}$$

For $\beta_{900_3} = \beta_{1800_4} = 1$ were chosen as they both reflect that the value of each bundles equals the value of acquiring one additional block. Following Equation 3.4 and Equation 3.5 in case $q_{900} \geq 3$ or $q_{1800} \geq 4$, $V_{MNO}^{900_3}$ or $V_{MNO}^{1800_4}$ will be added as intra-band synergy surplus to the single item multiples in the respective band valuation.

From the formulations above now the actual values have to be derived. With the mean bid from TEF in the 1800 MHz band being EUR 239m and following the mathematical formulations in Equation 3.6 and Equation 3.7 the following valuations can be computed.

[3] `https://www.telekom.com/konzern/weltweit/profile/23110` accessed on 22/07/2016
[4] `http://www.vodafone.de/unternehmen/umsatz.html` accessed on 22/07/2016
[5] `https://www.telefonica.de/file/public/826/Konzernabschluss_2015_TDH_deutsch_online.pdf` accessed on 22/07/2016

V_{MNO}^{item}	**TEF**	**DT**	**VOD**
V_{MNO}^{700}	719,000	899,000	827,000
V_{MNO}^{900}	480,000	600,000	552,000
$V_{MNO}^{900_3}$	480,000	600,000	552,000
V_{MNO}^{1800}	240,000	300,000	276,000
$V_{MNO}^{1800_4}$	240,000	300,000	276,000

Table 3.4: Overview of the different valuation constants used in the value model, prices in thousand EUR and $\beta_{900_3} = \beta_{1800_4} = 1$

Due to the choice of $\beta_{900_3} = \beta_{1800_4} = 1$ the additional surplus equals the valuation for one single item in the respective spectrum. Taking the formulations above together, a value model for the simulation can now be implemented (see chapter 4).

3.3 Methodology of Evaluation

To obtain an understanding of how well auction formats 'performed' in relation to other formats, I will shortly introduce compare metrics.

As with varying parameters and configurations the outcome of auctions changes, there is a need for a baseline to compare the two auction formats to. To create such baseline, for each parameter configuration a sealed-bid model is run, which determines an optimal allocation of items to bidders. To achieve this optimal allocation I extended an existing linear program for sealed bid auctions from the university chair to incorporate specific characteristics and constraints of the German Auction. See the extended part below.

The total of 23 items in each of the three bands 700, 900, 1800 MHz were successively numbered, e.g. for the 900 MHz band: $900_1, 900_2, ..., 900_7$. The collection of all items is called $Items = \{700_1, ..., 700_6, 900_1, ..., 900_7, 1800_1, ..., 1800_{10}\}$ with their respective band subsets $Items_{700}, Items_{900}, Items_{1800}$. Each item has a valuation V_{ij}. To formulate the non-additive bonuses for each bidder $j \in Bidders = \{TEF, DT, VOD\}$ in the frequency bands 900 MHz and 1800 MHz (see subsection 3.1.3), bonus variables $b_{900}^j, b_{1800}^j \in \{0, 1\}$ and their respective valuations $bonusV_{900}^j, bonusV_{1800}^j$ are introduced, that correspond to $V_{MNO}^{900_3}$ and $V_{MNO}^{1800_4}$ respectively, where $j = MNO$.

$$
\begin{aligned}
\text{maximize} \quad & \sum_{i \in Items} \sum_{j \in Bidders} ass_{ij} * V_{ij} + \sum_{j \in Bidders} b^j_{900} * bonusV^j_{900} + b^j_{1800} * bonusV^j_{1800} \\
\text{subject to} \quad & \sum_{i \in Items_{900}} ass_{ij} \geq 3 * b^j_{900}, \qquad \forall j \in Bidders \qquad (Bonus\ 900\ MHz) \\
& \sum_{i \in Items_{1800}} ass_{ij} \geq 4 * b^j_{1800}, \qquad \forall j \in Bidders \qquad (Bonus\ 1800\ MHz) \\
& \sum_{i \in Items_{900}} ass_{ij} \leq 3, \qquad \forall j \in Bidders \qquad (Cap\ 900\ MHz) \\
\text{where} \quad & V_{ij}, bonusV^j_{900}, bonusV^j_{1800} \in \mathbb{N}, \forall i \in Items, \forall j \in Bidders \\
& ass_{ij}, b^j_{900}, b^j_{1800} \in \{0, 1\}
\end{aligned}
$$

With this optimal allocation, the outcomes of the auction formats can now be set into relation. To compare the formats, the terms **efficiency** and **revenue** are mostly used.

To understand both metrics, I quickly introduce the term of **welfare**. Said figure equals the valuation of all won items by the bidder. It therefore is an internal measure of valuation.

Efficiency is defined by the ratio of the sum of each bidder's welfare W_j obtained in the auction and the sum of each bidder's welfare in the optimal allocation $W_{optimal}$. Following this definition, the sealed-bid auction (baseline model from above) has an efficiency of 1.

$$
efficiency = \frac{\sum_{j \in Bidders} W_j}{W_{optimal}} \times 100\% \tag{3.8}
$$

Similar to that, revenue can defined as the normalized sum of all the payments P_i made by each bidder.

$$
revenue = \frac{\sum_{j \in Bidders} P_j}{W_{optimal}} \times 100\% \tag{3.9}
$$

4 Simulation

To find out how the HPB format influences efficiency and revenue in comparison to the SMRA, I ran several simulations based on the findings in the previous chapter. For the bidders to act as agents in the auction simulations, they need a strategy and a suitable selection heuristic to model their behaviour and thus to place bids. In this chapter I will at first give an overview of the structure of the implementation, followed by a further description of the simulation process by defining the setup as well as the strategy and the selector model. Afterwards, I will present the findings of the experiment.

4.1 Structure of the Project

In order to run the simulations I extended an existing project from the university chair. In this project, an abstract structure on how to run auction simulations was given by an already defined software architecture. Figure 4.1 visualises the structure I used for my experiment. The whole project was implemented in Python.

As can be seen from the visualisation, a main **Auction** object exists, that has an own **AuctionFormat** which specifies the auction characteristics and the procedure, as well as a collection of agents participating. Every **Agent** possesses a bidding **Strategy** which in turn holds a **Selector**. The latter computes optimal bidding vectors based on the chosen selector model (e.g. a naive exhaust approach by iterating over each possible bid). Then the strategy translates these into according actions like submitting a bid based on the selector's result or calling a waiver to exclude the agent from bidding in the current round. The strategy class realises this by comparing the payoff $\pi_{combination}$ of different combinations, where payoff is defined as a *quasilinear utility function* by subtracting the prices $p(combination)$ a bidder has to pay from the bidder's valuation $v(combination)$ for a certain bidding combination.

$$\pi_{combination} = v(combination) - p(combination) \tag{4.1}$$

The valuation $v(combination)$ can be obtained by the strategy class using its **ValueModel**. This model has access to the **ItemStructure**, which holds the **Items** of the auction as well as possible packages, mapping (e.g. to frequency bands), or hierarchies. Before submitting a bid, the action gets validated by the **BidderInfo** class (e.g. to check if spectrum cap instructions were followed). After each successful round, the important information like *payments*, *prices*, and the current *allocation* are saved in the **RoundInfo**.

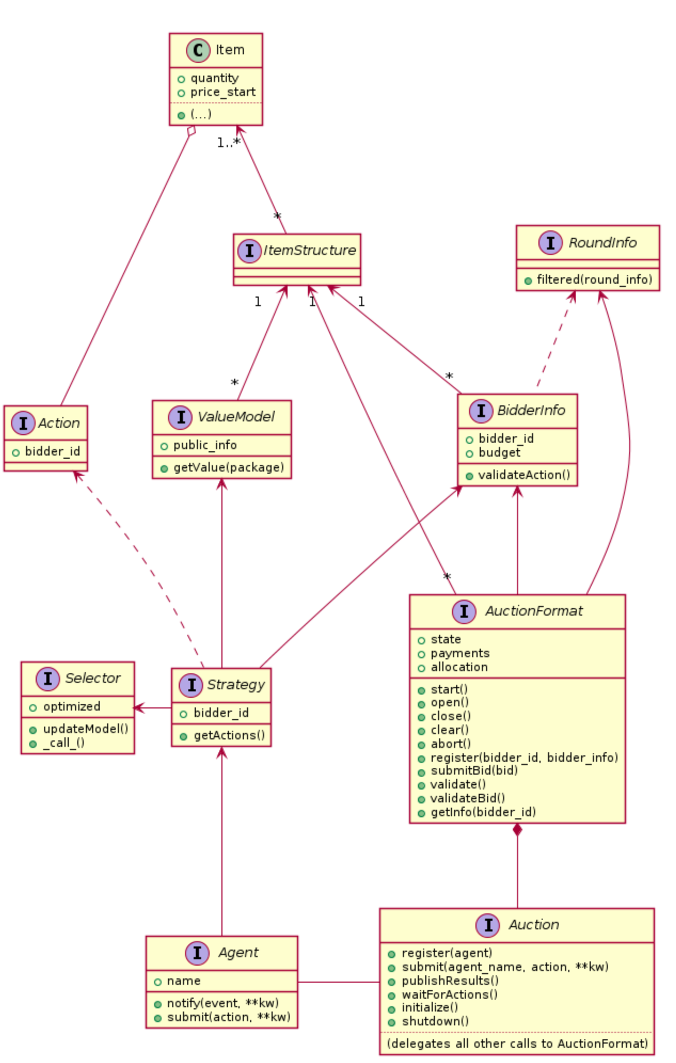

Figure 4.1: Structure of the simulation implementation

To simulate the auctions I had to extend the existing auction formats for SMRA and HPB. Making it possible to fill them with the specific characteristics of the German auction. In particular, this included implementing the spectrum cap of three blocks in the 900 MHz band and extending mechanisms in the HPB format. Also, I needed to implement the mathematical formulation of the value model (see section 3.2) as well as the specific selector model, which is based on the value model. Furthermore, I implemented a sealed bid model to calculate the optimal allocation needed to compare the two auction formats.

4.2 The Simulation Process

As stated in the previous section, in order to run the simulations based on the information and data of the German Auction I needed to extend and further develop the existing project. In this section I will describe the set-up of the auctions (e.g. the items and hierarchies used) as well as the implementation of the value, strategy and selector model.

4.2.1 Auction Set-Up

As discussed in the Value Model section, the simulation concentrated on the 700, 900 and 1800 MHz frequency bands, which include a total of 23 items. Each bidder was able to bid on every item and item combination. For the first round, the starting prices were appointed according to the official definition of the German Auction [Bun15] as provided in Table 4.1.

block	quantity	starting price	eligibility points
700 MHz	6	EUR 75 mn	2
900 MHz	7	EUR 75 mn	2
1800 MHz	10	EUR 37.5 mn	2

Table 4.1: Starting parameters for the items in the auction

The activity level was chosen to be 65% of the eligibility points held by each bidder in every round. This is the activity level chosen in the first phase of the German Auction. As no mathematical progression rule for the phase shifts exists, because they are decided by the auction moderator [Bun15], the activity level stayed the same throughout the auction simulations. Furthermore, the exposure ratio (the degree to which a bidder can overbid the valuation of an item) was set very high, practically eliminating exposure. This decision was made, as deriving a fitting exposure ratio for the simulation was out of the scope of this thesis. I will discuss the implications in chapter 5.

As HPB makes use of pre-defined packaging of items using a tree structure, I needed to develop a useful item hierarchy to be used in the simulation. To show how the

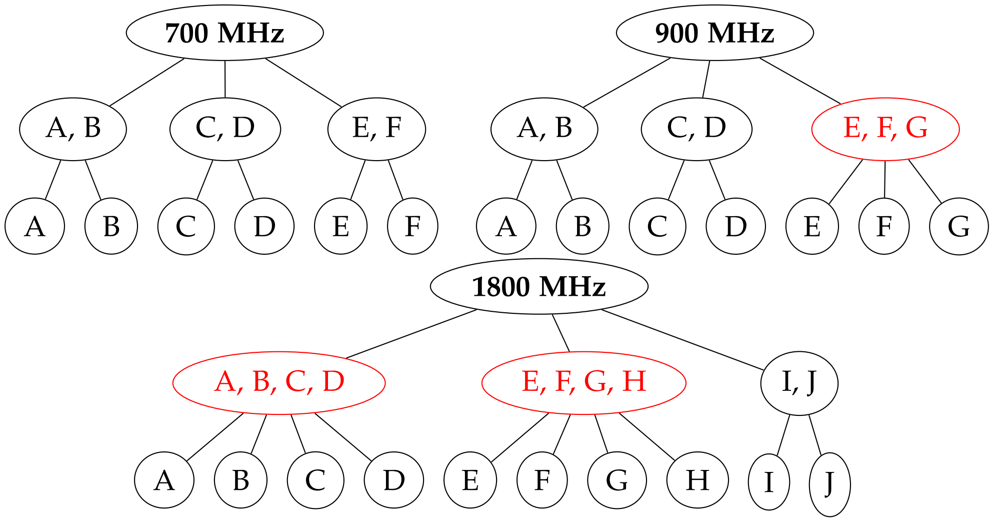

Figure 4.2: HPB Hierarchy "Competition", packages with bonus in red

hierarchy affects the bidding and thus efficiency and revenue, I created two slightly different hierarchy models. As the 900 MHz band had a spectrum cap, the bidders were limited on how many items they can acquire (here: maximum of three blocks). The other bands had no such limitations, thus each bidder would try to acquire the most items possible under the constraints of item valuations and maximizing payoff. To research the impact of the hierarchies on the bidding behaviour, I devised two different hierarchies that had different pre-packaging in the 900 MHz band, because bidders were only allowed to acquire up to three blocks, a combination that had super-additive valuation for the bidders (section 3.2) and I assumed the competition to be high in this band. Figure 4.2 shows the item hierarchy called $H_{competition}$, where one package of three items (reflecting a desired bundling) exists in the structure. The hierarchy is called "competition" as only one of the desired bundling exists.

As the 900 MHz band consists of seven items, it has enough items for incorporating two bundles of three items each. I assumed that following the formulation and the bonuses of the value model (see section 3.2) at least two bidders will try to acquire bonus enabling packages. This is what H_{VM} in Figure 4.3 incorporates. Note, that bidders were still able to gain super-additive valuation by bidding on single items (in different packages) and still be able to successfully secure three items in the 900 MHz band. In the HPB simulation, the advantage for bidders was that they could bid on packages, signalling an interest in a certain item combination. When bidding on the package, the bidder either acquires all the items in its desired packaging or none, thus resulting in a reduced exposure risk for the bidders as it prevents him from potentially

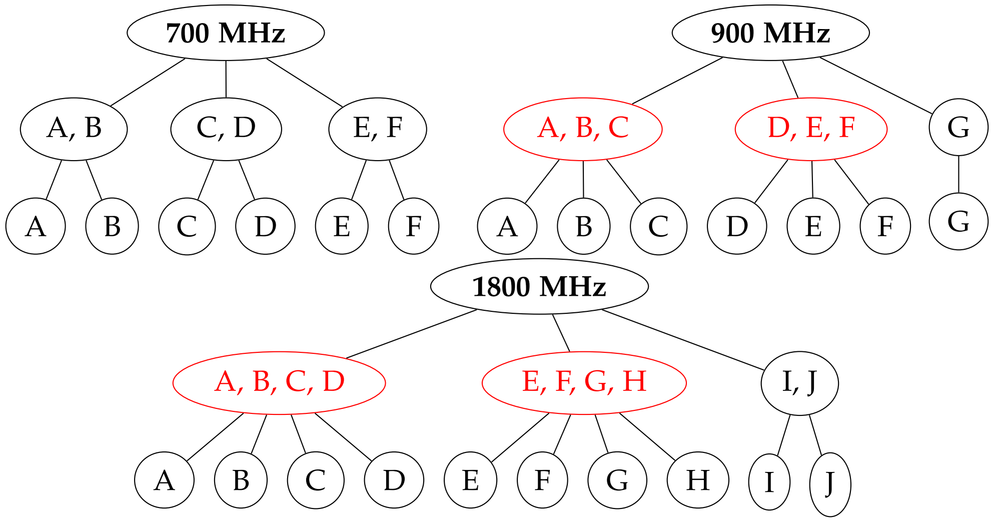

Figure 4.3: HPB Hierarchy "VM Prediction", packages with bonus in red

bidding above an item's valuation in order to realise a certain item combination.

4.2.2 Strategy and Selector Model

In the simulation, the participating agents needed to emulate a certain bidding behaviour. This is realised by the strategy model. In this experiment, the bidders followed a **straightforward bidding** behaviour, essentially meaning that they always bid according to the minimum prices in each round [Mil00, p. 270.]. This can be described as a function $p(I)$ that returns the bidding price of an item according to minimum prices, where β is the current bid price [Wel+08, p. 6 ff.].

$$p(I) = \begin{cases} \beta & \text{if item is already won} \\ \beta + increment & \text{otherwise} \end{cases} \tag{4.2}$$

The straightforward bidding is also called *"myopic best response"* [BDD00]. Additionally, the strategy used in the simulation is payoff maximizing.

As explained in the section above, value models return the valuation of certain item combinations and selectors return optimal bidding vectors $\vec{\theta}$. The strategy model translates the results into an according action, e.g. a bid based on a payoff maximizing strategy.

One could argue that once the simulations are run on a computer, finding the best bidding combination should be done via an exhaustive iteration over every single possible combination. However, using this approach showed to be an ineffective use of time and to be computationally expensive. The number of items in the simulation was 23. Lining up all the items in fixed order and using a binary vector to display if each respective item should be included into a bid creates the vector $\vec{\theta} \in \{0,1\}^{23}$. With simple combinatorics it can be deduced, that computing every possible allocation takes $2^{23} \approx 8.388 \times 10^6$ iterations. Remembering that 700 MHz had 6 blocks, 900 MHz 7 blocks and 1800 MHz 10 blocks and taking into account, that bidders were only allowed to acquire up to three blocks in the 900 MHz band, the number of iteration can be reduced to

$$2^6 * \sum_{i=1}^{3} \binom{n}{k} * 2^{10} = 2^6 * (2^6 - 1) * 2^{10} = 2^{16} * (2^6 - 1) = 2^{22} - 2^{16} \approx 4.1 \times 10^6$$

Due to the current implementation in the project, this has to be done twice for each of the three bidders, requiring an amount of $2 \times 3 \times 2^{22} - 2^{16} \approx 25.1 \times 10^6$ computation steps each auction round. Clearly, a different approach was needed to select the optimal bidding vector.

Instead, a **mixed integer linear program** (MILP) was used to compute the optimized bidding vector $\vec{\theta}$. For that, I extended existing code for an additive payoff selector from the main project and added the specific bonuses and caps to the LP. Additionally, the bidders had to incorporate bidding on packages, when the payoff for single items and package bids were the same. This behaviour was integrated by adding a *very small* bonus for using packages to the objective function (Epsilon-Method). When a bidder would bid on a package, the payoff would be marginally higher than when bidding on the single items, but by only a very small number which does not affect the overall allocation. The MILP formulation is very similar to the one in section 3.3, but this time only computes the best allocation for a single bidder. Also, the bidder's objective function was extended by the small "bidding-on-HPB-package-bonus", so that the selector MILP can take the existing package hierarchy into account. So for each super-additive package $p \in Hierarchy$ the objective function increases by a very small amount (in comparison to the sum of all the valuations) if the package was bid on. This leads to the following MILP formulation, where $Hierarchy_{900}, Hierarchy_{1800} \in Hierarchy$ are the respective subsets of packages in the 900 and 1800 MHz bands and $Hierarchy_{900bonus} \in Hierarchy_{900}, Hierarchy_{1800bonus} \in Hierarchy_{1800}$ the respective subsets with bonus packages. Additionally, $bonusV_{900}$ and $bonusV_{1800}$ stand for the bidder's valuation of the bonuses, they correspond to the $V_{MNO}^{900_3}$ and $V_{MNO}^{1800_4}$ from section 3.2 where MNO equals the respective bidder.

$$\begin{aligned}
\text{maximize} \quad & \sum_{i \in Items} ass_i * V_i + b_{900} * bonusV_{900} + b_{1800} * bonusV_{1800} \\
& + \varepsilon * \left[\sum_{p \in Hierarchy_{900bonus}} m_p + \sum_{p \in Hierarchy_{1800bonus}} m_p \right] \\
\text{subject to} \quad & \sum_{i \in Items_{900}} ass_i \geq 3 * b_{900} && (\textit{Bonus in 900 MHz}) \\
& \sum_{i \in Items_{1800}} ass_i \geq 4 * b_{1800} && (\textit{Bonus in 1800 MHz}) \\
& \sum_{i \in Items_{900}} ass_i \leq 3 && (\textit{Caps in 900 MHz}) \\
& \sum_{i \in p} ass_i \geq |p| * m_p \quad \forall p \in Hierarchy_{900bonus} \cup Hierarchy_{1800bonus} \\
& && (\textit{Preemptive bundle selection}) \\
\text{where} \quad & V_i, bonusV_{900}, bonusV_{1800} \in \mathbb{N}, \quad \forall i \in Items \\
& ass_i, b_{900}, b_{1800} \in \{0,1\} \\
& m_p \in \{0,1\}, \quad \forall p \in Hierarchy
\end{aligned}$$

4.3 Results

Now, with the project incorporating all the characteristics of the German Auction I was able to simulate the SMRA and HPB auctions. To research the sensitivity of efficiency and revenue towards the change of the set-up parameters I ran simulations iterating over sequences of the parameters $\beta_{900}, \beta_{1800}$ and s_{DT}, s_{VOD} for both Hierarchies $H_{competition}, H_{vm-prediction}$. Because of the high number of possible parameter combinations I will focus on the most important results below. To answer the question *"How well does HPB perform in comparison to SMRA in the context of the German Auction?"*, my analysis was mainly guided by the following questions:

- How sensitive are *efficiency* and *revenue* to *bonus valuations*?
- What is the impact of the *HPB hierarchy* on both metrics?
- How does *efficiency* and *revenue* change in relation to the environment of *bidder strengths*?

4.3.1 Efficiency of the Formats

A *"well performing"* auction results in a (near-)optimal allocation of the bidding items to the bidders, meaning reaching a high efficiency is desired. As described in section 3.3, efficiency is described as the ratio of the total welfare achieved in relation to the optimal welfare, which is calculated by running a sealed-bid model on the parameters. Therefore, the efficiency of the sealed-bid model is always equal to one and the other formats are judged on this basis.

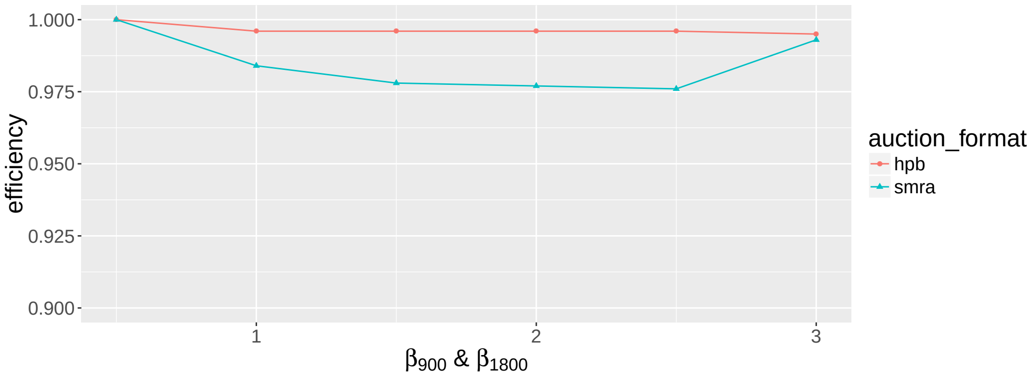

Figure 4.4: Sensitivity of efficiency against $\beta_{900} = \beta_{1800}$ in $H_{competition}$

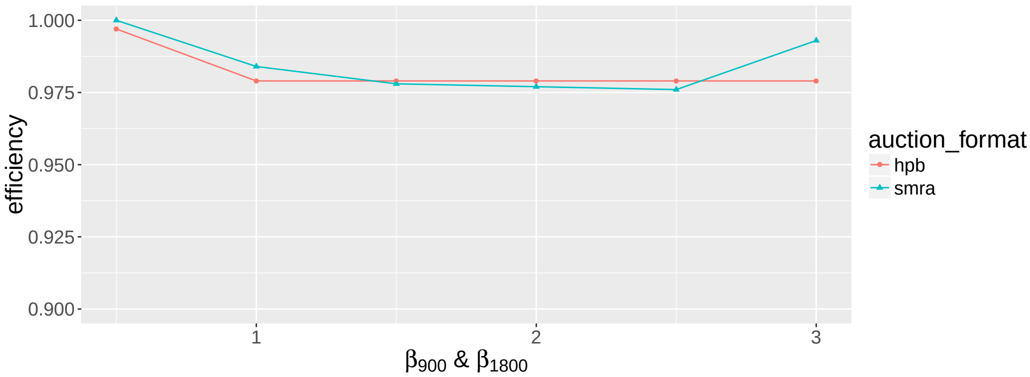

Figure 4.5: Sensitivity of efficiency against $\beta_{900} = \beta_{1800}$ in $H_{vm-prediction}$

When running simulations iterating over the bonuses valuations, differences between both hierarchies became clear. Figure 4.4 shows the development of efficiency in relation to the bonuses valuations in the $H_{competition}$ hierarchy and Figure 4.5 for $H_{vm-prediction}$ respectively. Despite the fact that the efficiency levels of HPB showed to be more stable than SMRA, there were differences in the results between the two hierarchies.

In $H_{competition}$ HPB showed to be superior to SMRA with $HPB \succ SMRA$. Also, HPB showed to be much more stable in resulting in a high efficiency level, which was near-optimal. In comparison, the overall allocation in SMRA seems to be more susceptible to the bonuses valuations, but still resulting in a high efficiency.

From the data it seems that in HPB the valuation of the bonus inducing packages has little effect on the overall allocation and efficiency of the auction, making it a versatile tool to auction items with such characteristics. But the correct choice of the structure appears to be an important question. The findings indicate, that a competitive HPB hierarchy might help in reaching a better overall allocation in comparison SMRA. But this needs to be verified by further experiments (see chapter 5).

4.3.2 Revenue of the Formats

While reaching an optimal allocation is especially relevant to the participating bidders of an auction, maximizing revenue - to an appropriate level - mostly concerns the auctioneer. As revenue has no upper bound (like efficiency), there is no *perfect* revenue. To analyse the sensitivity of revenue I will at first have a look on the influence of bonus valuations and the hierarchies. Afterwards, I will establish an understanding of the competition environment of the auction formats, meaning the sensitivity towards the distribution of bidder strength.

The influence of bonus valuations and hierarchies. The general tendency is that β_{900} and β_{1800} have a positive impact on the development of revenues. With higher bonuses valuations, the overall revenue rises, as can be seen in Figure 4.6 and Figure 4.3. Interestingly, the effect of the different HPB hierarchies can be observed. When using $H_{competition}$ in most cases HPB $\succ$ SMRA, where the HPB format produced slightly higher revenues in comparison to SMRA. This is contrary to the effect seen in Figure 4.3, where HPB $\prec$ SMRA and through beta-space HPB generated prominently lower revenues. Note that in $H_{vm-prediction}$ two bonuses inducing pre-packaged bundles in the 900 MHz band exist, instead of only one in $H_{competition}$. This is an indicator, that HPB might help bidders to coordinate earlier in the auction if the respective hierarchies enable them to, but simultaneously leading to lower revenue. Whereas a more competitive HPB hierarchy tends to drive competition and thus revenue.

The competition environment. In this section, I will focus on the findings using the $H_{competition}$ hierarchy as it produced the higher efficiency (see subsection 4.3.1) and thus higher social welfare, which I argue to be an optimization goal of an auctioneer.

Figure 4.8 shows the competition environment of the HPB format in the simulation. Each tile shows the revenue generated according to the set-up of the bidder strengths.

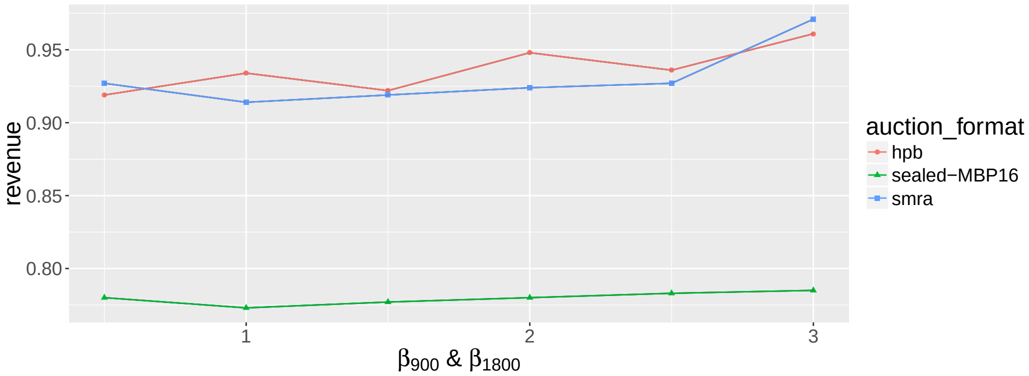

Figure 4.6: Sensitivity of revenue against $\beta_{900} = \beta_{1800}$ in $H_{competition}$

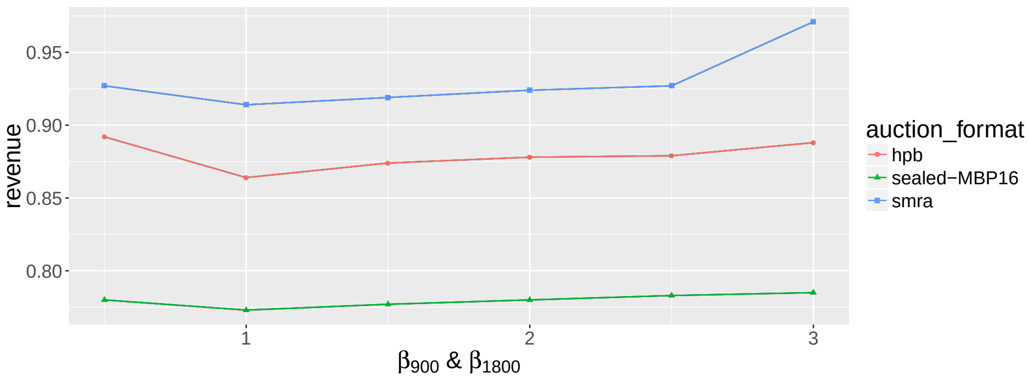

Figure 4.7: Sensitivity of revenue against $\beta_{900} = \beta_{1800}$ in $H_{vm-prediction}$

As can be seen from the figure, when all bidders have the same strength ($s_{TEF} = s_{DT} = s_{VOD} = 1$) the highest revenues were generated due to the fact, that each bidder had to bid very close to its valuation. In an environment with one very strong bidder, revenues tend to yield much lower level as the strong bidder can much earlier outbid its competitors and thus decrease overall revenue. As the revenues from both formats were very similar when using the competitive hierarchy (see Figure 4.6), the competition environment also looks very similar. To compare both environments, Figure 4.9 depicts the difference between the revenues yielded in HPB and SMRA. The figure suggests, that there is no systematic relation in the superiority or inferiority of an auction format when dealing with different bidder strength environments.

To compare both formats further, Figure 4.10 and Figure 4.11 show the development of revenue in relation to two different bidder strength. As both depict that revenue decreases with increasing bidder strengths, Figure 4.10 shows that in less unequally

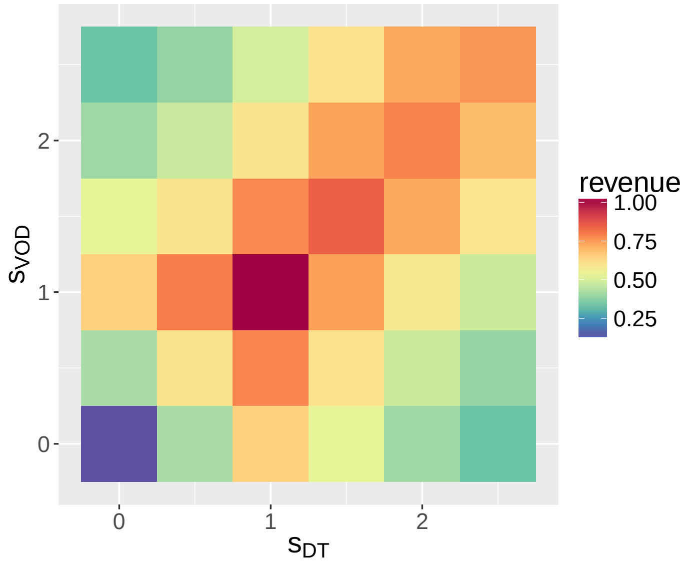

Figure 4.8: Competition environment of HPB with $H_{competition}$ & $[rev \sim s_{DT}, s_{VOD}]$

distributed bidder environments HPB $\succ$ SMRA, where HPB can yield slightly higher revenues even when bidder strengths are high. On the other hand, in an environment where one bidder is much stronger than the other participating bidders, this not only results in much lower revenues, but also slightly lower revenues by the HPB format in comparison to SMRA, even though the difference is marginal.

4.3.3 Summary of the Results

The following table summarises the findings of the simulations, where β means $\beta_{900} = \beta_{1800}$ and s means $s_{DT} = s_{VOD}$.

		β	s
Efficiency	$H_{competition}$	$HPB \succ SMRA$	$HPB \equiv SMRA$ after $s = 1.5$
	$H_{vm-predict}$	$HPB \approx SMRA$	$HPB \prec SMRA$ when $s > 1$
Revenue	$H_{competition}$	$HPB \succ SMRA$	$HPB \succ SMRA$, rapidly decreasing
	$H_{vm-predict}$	$HPB \prec SMRA$	$HPB \prec SMRA$

Table 4.2: Summary of the findings in the simulations

From the table it can be seen, that the simulations indicate that HPB might be helpful in enabling bidders to find more efficient overall allocations when choosing an appropriate bidding hierarchy. This improvement came with the reduction of overall revenue as bidder signalling improved. SMRA was superior when a less competitive HPB hierarchy was used, indicating that choosing a suitable hierarchy is key to the success of the auction.

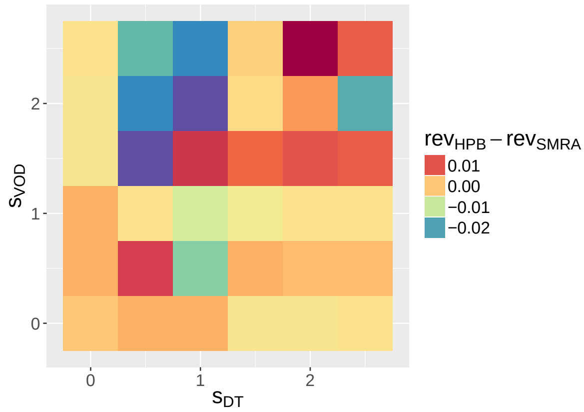

Figure 4.9: Competition environment of HPB vs. SMRA $[rev_{HPB} - rev_{VOD} \sim s_{DT}, s_{VOD}]$

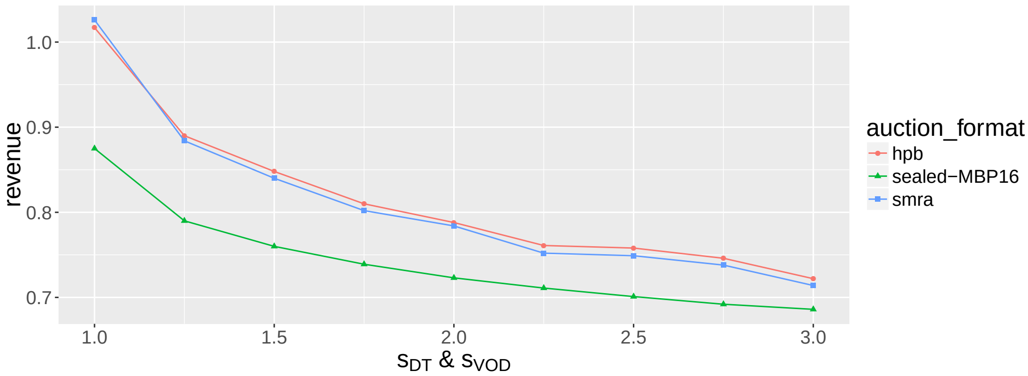

Figure 4.10: Sensitivity of revenue against bidder strength $[s_{DT} = s_{VOD}]$

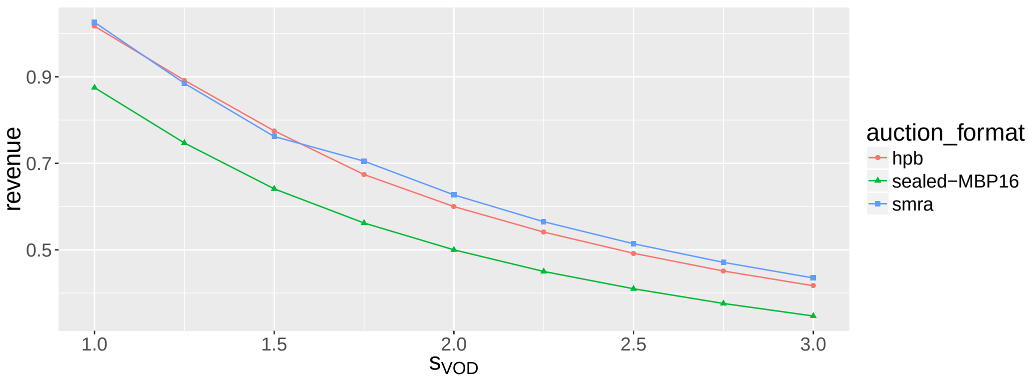

Figure 4.11: Sensitivity of revenue with one stronger bidder $[s_{TEF} = s_{DT} = 1]$

5 Conclusions and Future Work

The purpose of this thesis is to research the impact of using HPB instead of SMRA on efficiency and revenue in big spectrum auctions by running simulations based on the German Auction from 2015. A value model was developed to describe the valuations of the participating bidders, as well as a selector model that incorporated the characteristics of the German Auction like spectrum caps and super-additive item bundling. Also, the selector was extended to support package bidding behaviour of the agents and two different HPB hierarchies featuring a competitive and less competitive structure were developed.

Simulations were run on different parameter settings on the bonuses valuations for the super-additive item packages and bidder strengths for the two different HPB hierarchy structures, comparing the results to the SMRA format. The analysis indicates that when choosing an appropriate package structure, HPB might help finding more efficient overall allocations during the auction, but due to the signalling properties of the package bids it might reduce overall revenue. HPB showed to be more stable in relation to different super-additive valuations than SMRA, indicating that it might be a useful tool in auctions that have those characteristics. When using less competitive package structures, the established SMRA showed to be superior in creating higher revenues. The response of revenue to different competition environment seems to be very similar in HPB and SMRA.

Still, HPB is an auction format that is simple to implement and facilitates signalling desired item combinations. With a pre-defined package structure it reduces the need for computationally expensive combinatorial calculations and substitutes it for a simple "tax system" that hands down price increases on packages to the item level. This makes price calculations understandable in comparison to the often intricate price calculations in combinatorial auctions.

To further investigate the eligibility of using HPB in big spectrum auctions more research needs to be done. In particular, the impact of the pre-defined item packages and to what extent it might help with bidder collusion. The research showed that more competitive hierarchies might lead to more efficient auctions and less competitive hierarchies can negatively impact revenue. To evaluate if this is an inherent property of HPB, more experiments with a broad spectrum of different hierarchies need to be run to ascertain if this is only an artifact of the modelling done in this thesis. If this verifies, choosing the right HPB hierarchy has big implications on the auctions run.

Also, the simulations were run based on the value model developed in this thesis. Further work can be put into constructing a more complex model that also takes into account the six item bundle in the 1800 MHz as stated in [BGJ16] as well as incorporating the 1500 MHz frequency band, which was left out due to lack of importance, because as of writing of this thesis no devices that can utilize this frequency band exist. Also, the prices yielded in the auction were very low in comparison to the other bands.

Additionally, subsequent research needs to be done to show the effect of HPB on the exposure of bidders. This work excluded exposure by technically allowing bidders to bid over multiples of an item's valuation, which would not be feasible in real auctions. Analysing the impact would have been out of the scope of this thesis and is strongly recommended as the focus for future research on this topic.

List of Figures

List of Tables

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