

INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY BANGALORE

BASIC COMPUTATIONAL TOPOLOGY
SM 402

BCT Implementation Assignment

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Group 15

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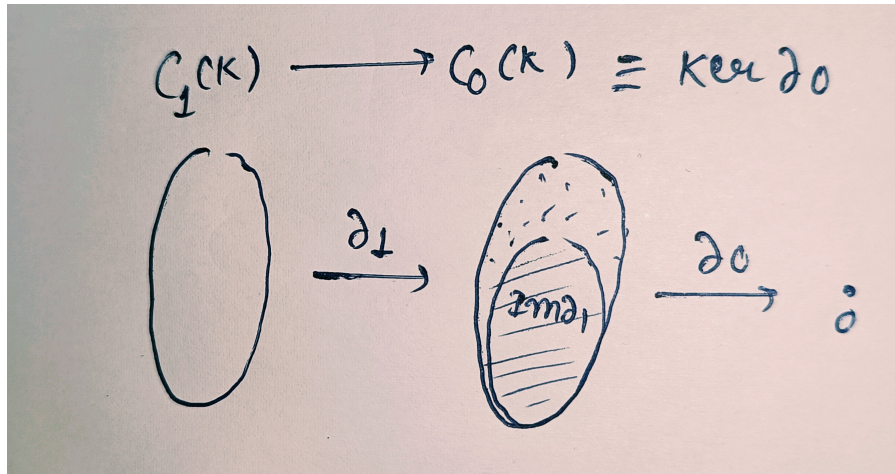


Problem Statement

Given any input simplicial complex (up to 3-dimensional), corresponding to each 0-hole compute a representative 0-cycle and visualize all the representative 0-cycles.

Algorithm

We have used the concept explained below in our code to calculate 0-Holes:



$$H_0(K) = \ker(\partial_0)/\text{Im}(\partial_1)$$

$$H_0(K) = C_0(K)/\text{Im}(\partial_1)$$

To calculate the representative 0-cycles of a k dimensional simplicial complex, our program first calculates boundary matrix ∂_1 , let's call it *MatrixA*. Then it creates a matrix of C_0 chains, let's call it *MatrixB*.

We then join matrix A and B and take Row Reduced Form of the combined matrix.

SET $P1$: The Pivots obtained from the row reduced form of the **Combined matrix**.

SET $P2$: The Pivots obtained from **Boundary matrix** part of the **Combined matrix**.

$P1 - P2$: will give us the representative zero cycles.

Implementation Steps

1. Create Boundary Matrix (∂_1), call it Matrix A and a diagonal Matrix of $C_0(K)$, call it Matrix B and join them to form Matrix $A|B$.
2. Take RREF of the combined Matrix AB .
3. Find Pivot Columns in this matrix and put them in an array.
4. Now take the pivots that belongs to Matrix B from Matrix AB .
5. Subtract no. of edges from those pivots so that they represent a vertex from all connected components.
6. The resultant points we get are called representative 0-cycles.

Steps to run the code

1. run `g++ main.cpp Matrix.cpp`
2. run `./a.out gts-files/tetrahedron.gts` (depending on the test case one may want to run)

NOTE:- Here the code may take time to calculate the result for very large data (depending on the system hardware specifications).

Demo Results

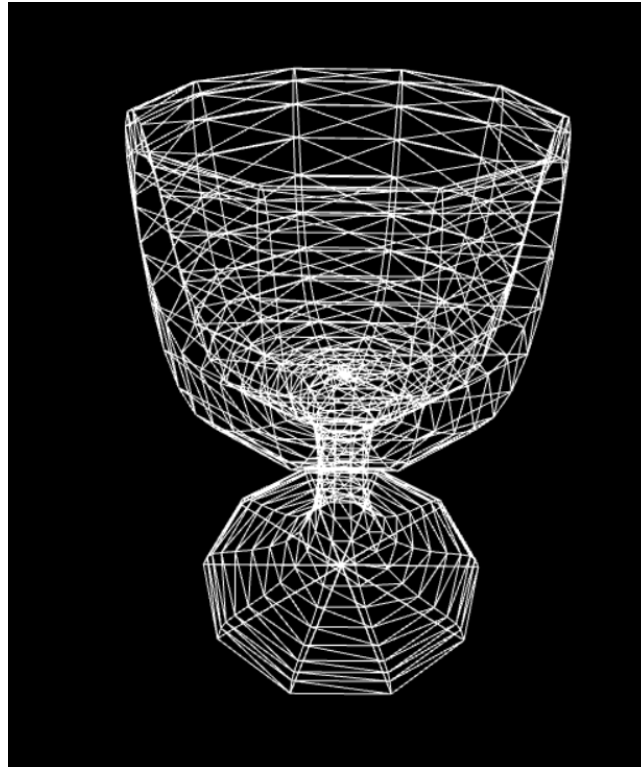


Figure 1: Testcase 1:- goblet

```
iiitb/topo/implementation-project took 16s  
> ./a.out gts-files/goblet.gts  
Number of 0-Homology Groups Formed: 1  
Representative 0 Cycles:  
{1}  
Time Taken is = 4
```

Figure 2: Testcase 1:- goblet code

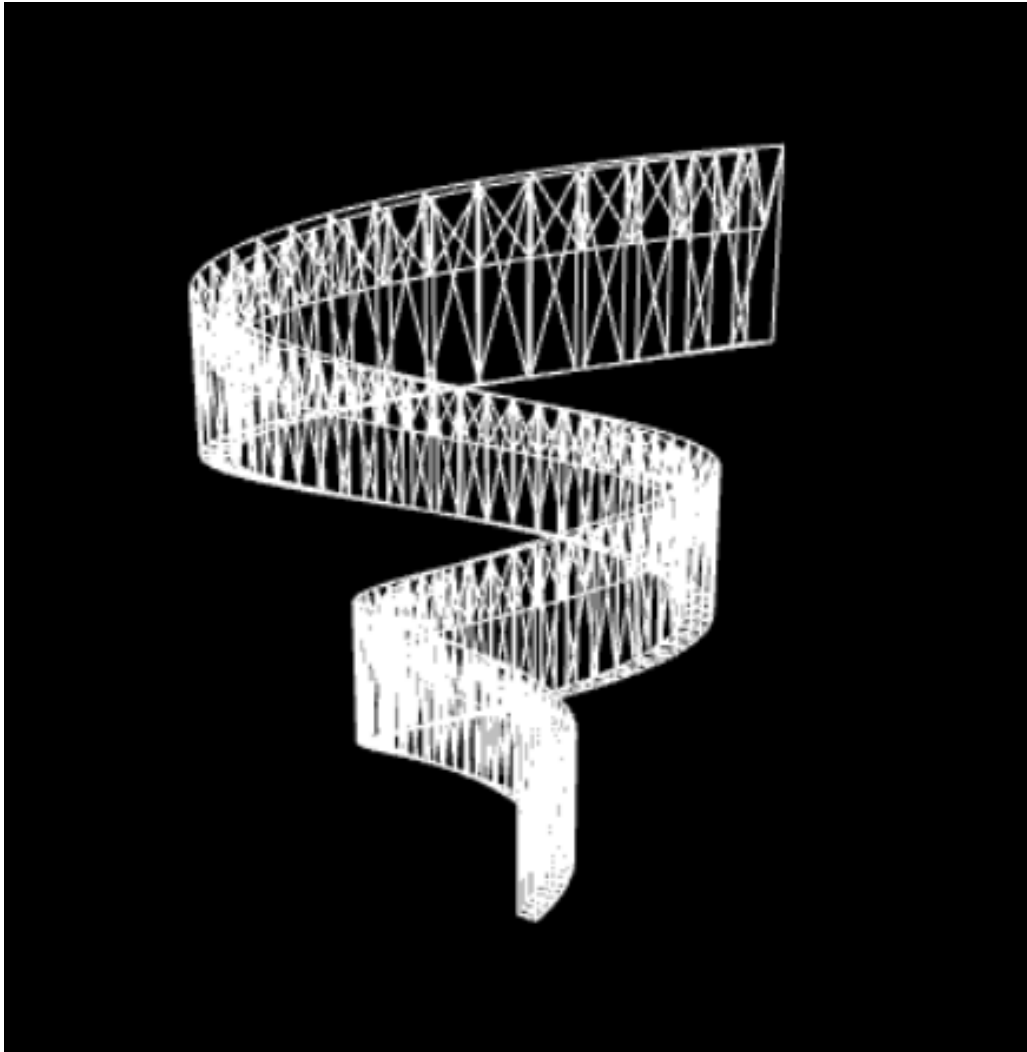


Figure 3: Testcase 2:- helix

```
iiitb/topo/implementation-project took 4s  
> ./a.out gts-files/helix2.gts  
Number of 0-Homology Groups Formed: 1  
Representative 0 Cycles:  
{1}  
Time Taken is = 4
```

Figure 4: Testcase 2:- helix code

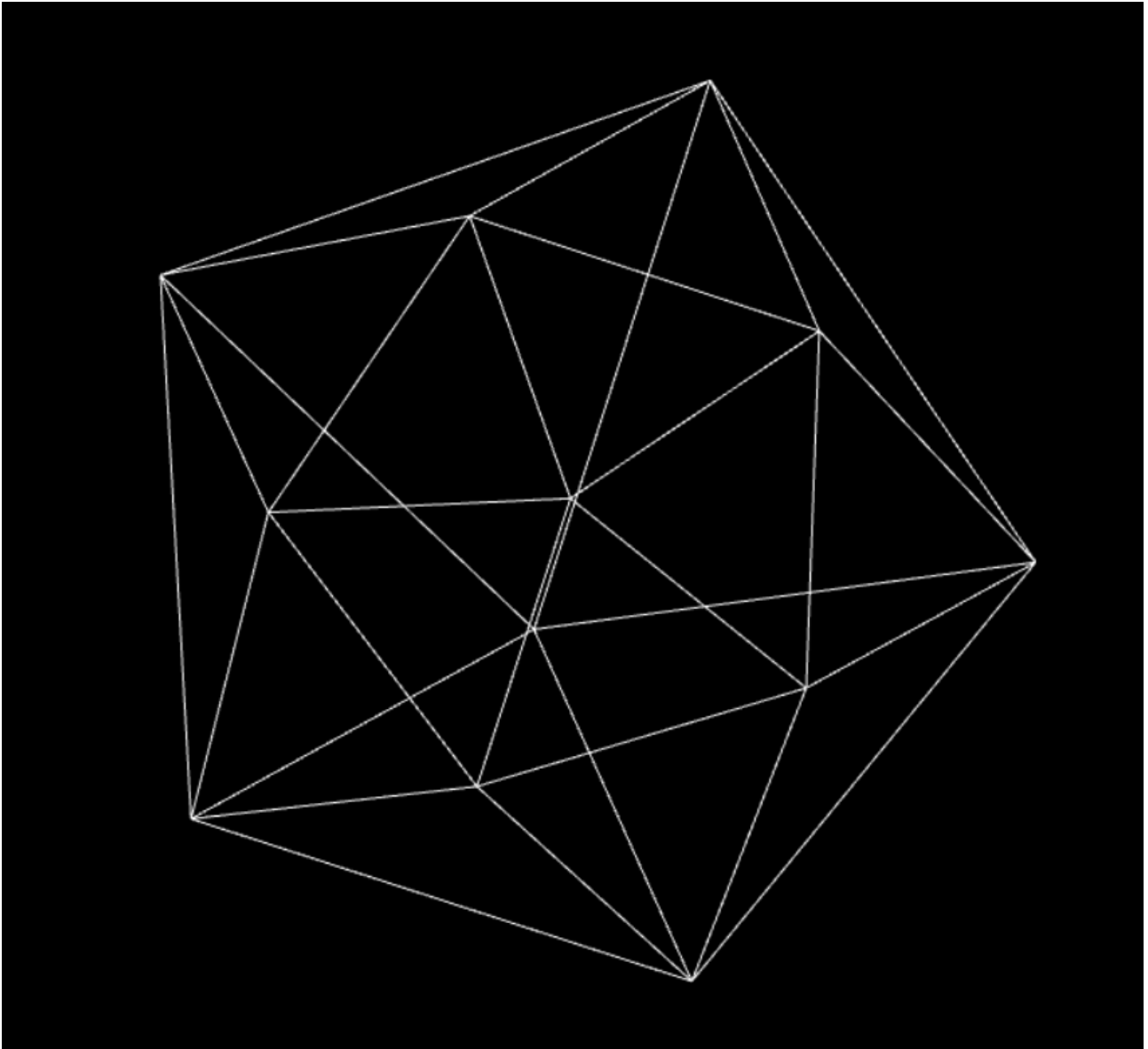


Figure 5: Testcase 3:- icosahedron

```
iiitb/topo/implementation-project took 4s  
> ./a.out gts-files/icosa.gts  
Number of 0-Homology Groups Formed: 1  
Representative 0 Cycles:  
{1}  
Time Taken is = 0
```

Figure 6: Testcase 3:- icosahedron code)

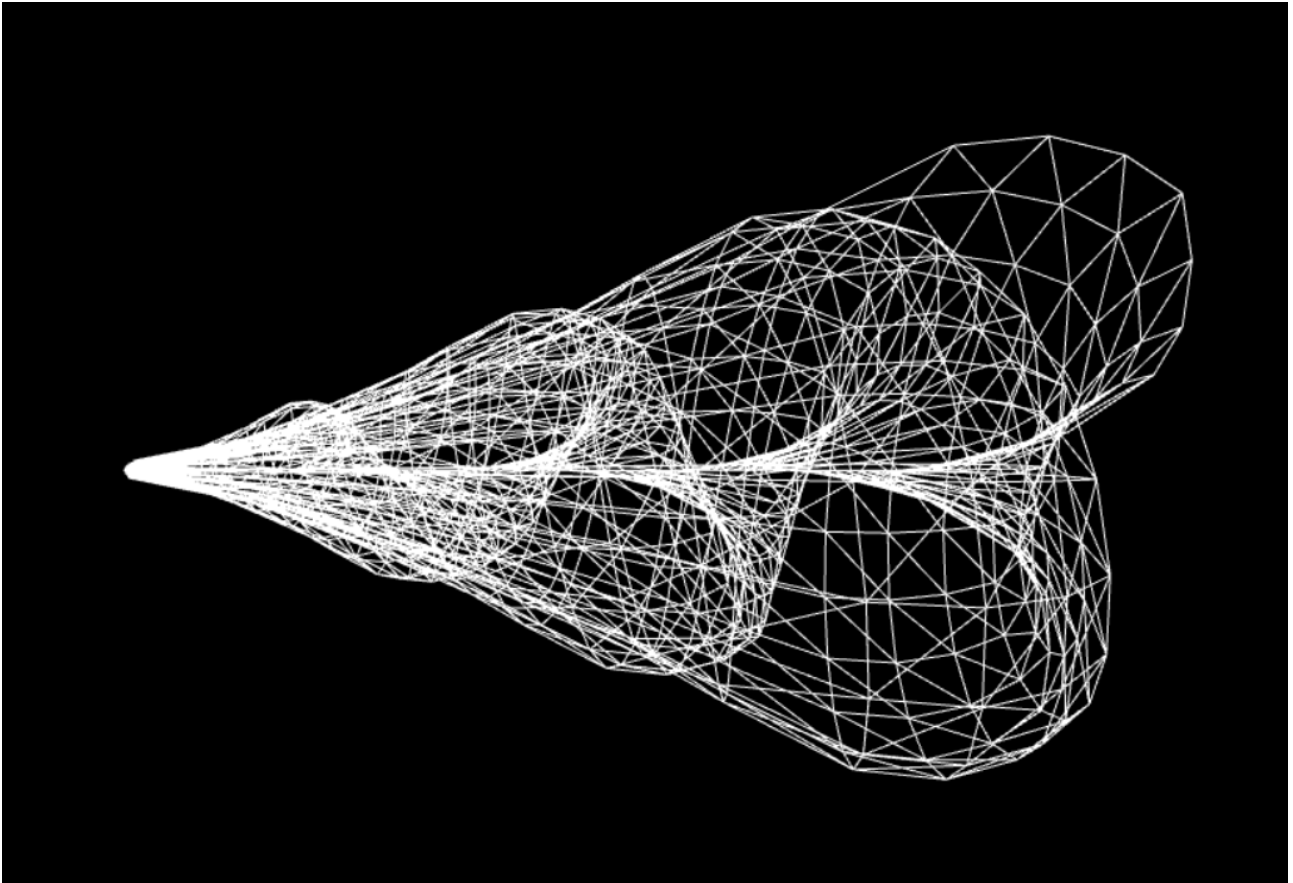


Figure 7: Testcase 4:- seashell

```
iiitb/topo/implementation-project  
> ./a.out gts-files/seashell.gts  
Number of 0-Homology Groups Formed: 1  
Representative 0 Cycles:  
{1}  
Time Taken is = 27
```

Figure 8: Testcase 4:- seashell code

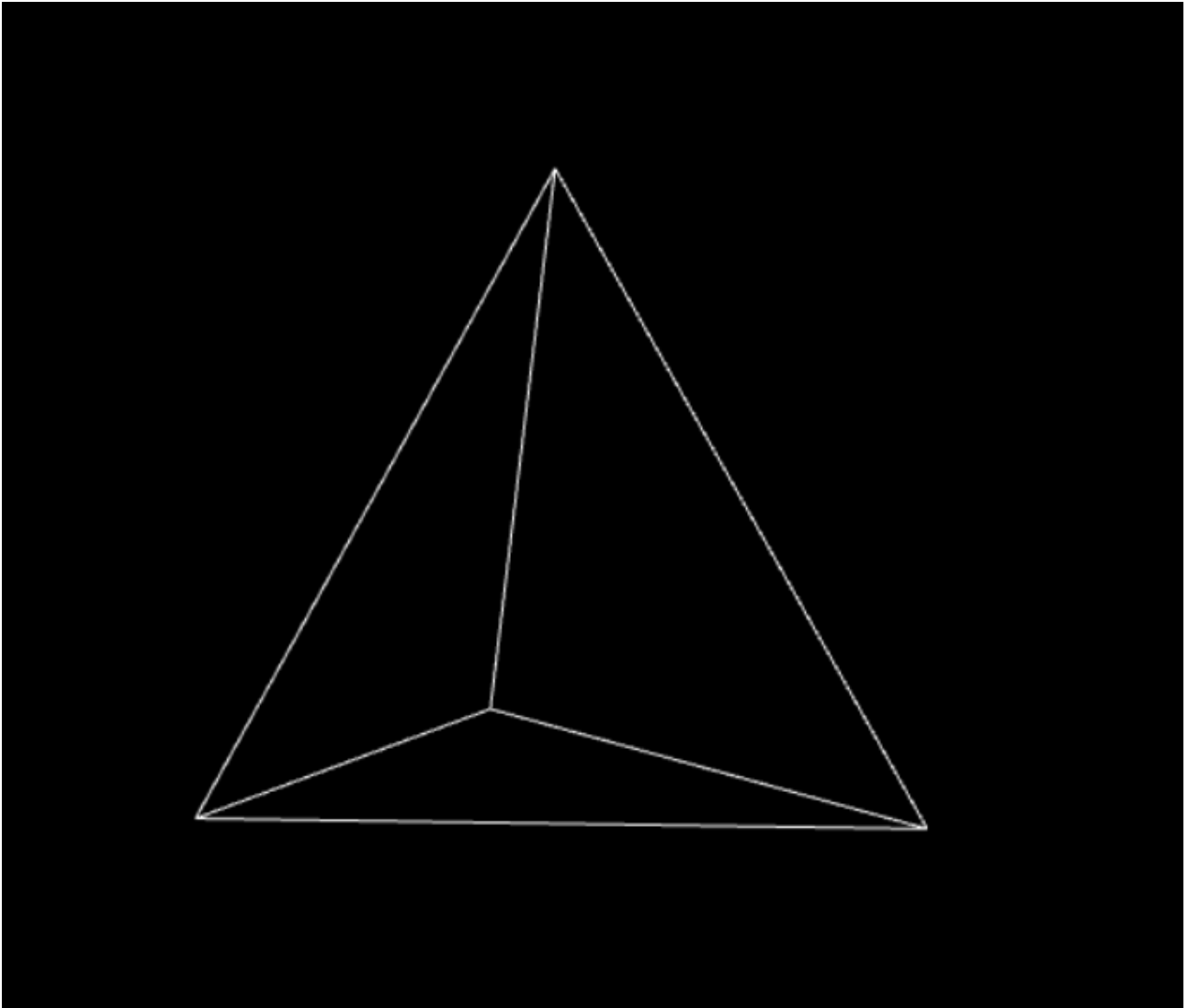


Figure 9: Testcase 6:- tetrahedron

```
iiitb/topo/implementation-project took 15m50s  
> ./a.out gts-files/tetrahedron.gts  
Number of 0-Homology Groups Formed: 1  
Representative 0 Cycles:  
{1}  
Time Taken is = 0
```

Figure 10: Testcase 6:- tetrahedron code

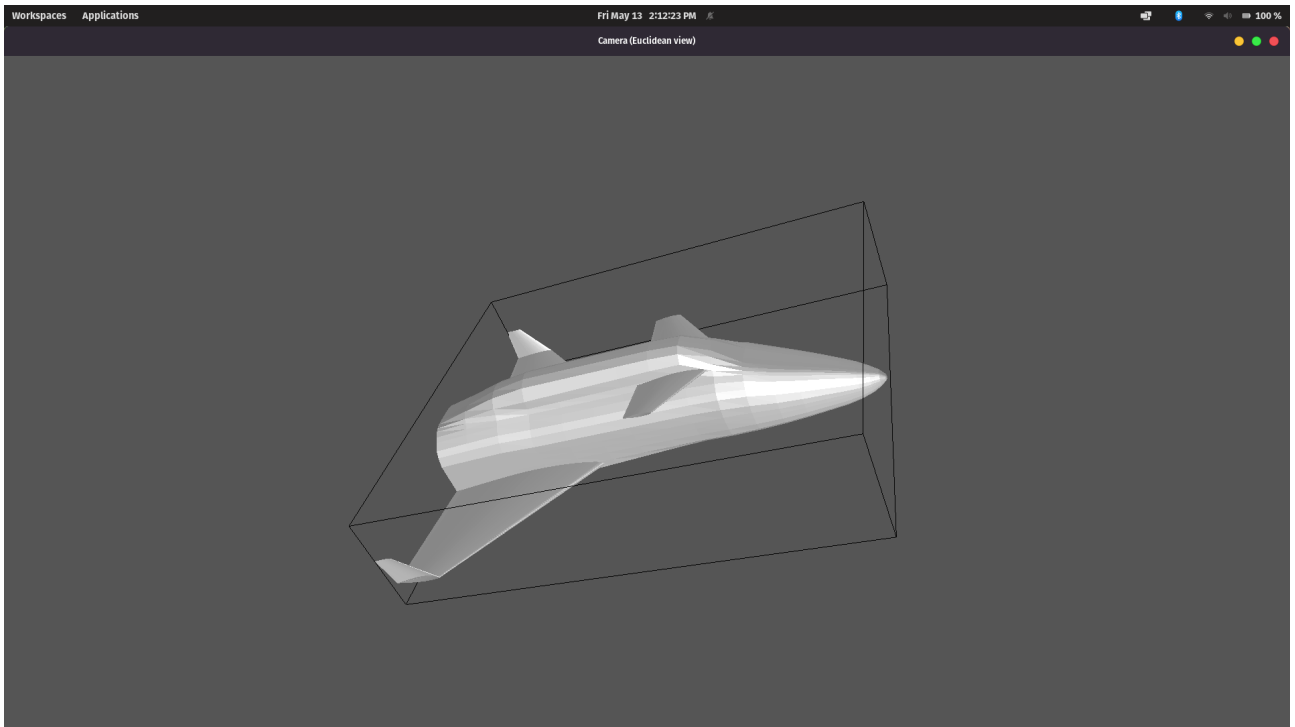


Figure 11: Testcase 6:- space shuttle

```
iiitb/topo/implementation-project took 15m50s  
> ./a.out gts-files/tetrahedron.gts  
Number of 0-Homology Groups Formed: 1  
Representative 0 Cycles:  
{1}  
Time Taken is = 0
```

Figure 12: Testcase 6:- space shuttle code

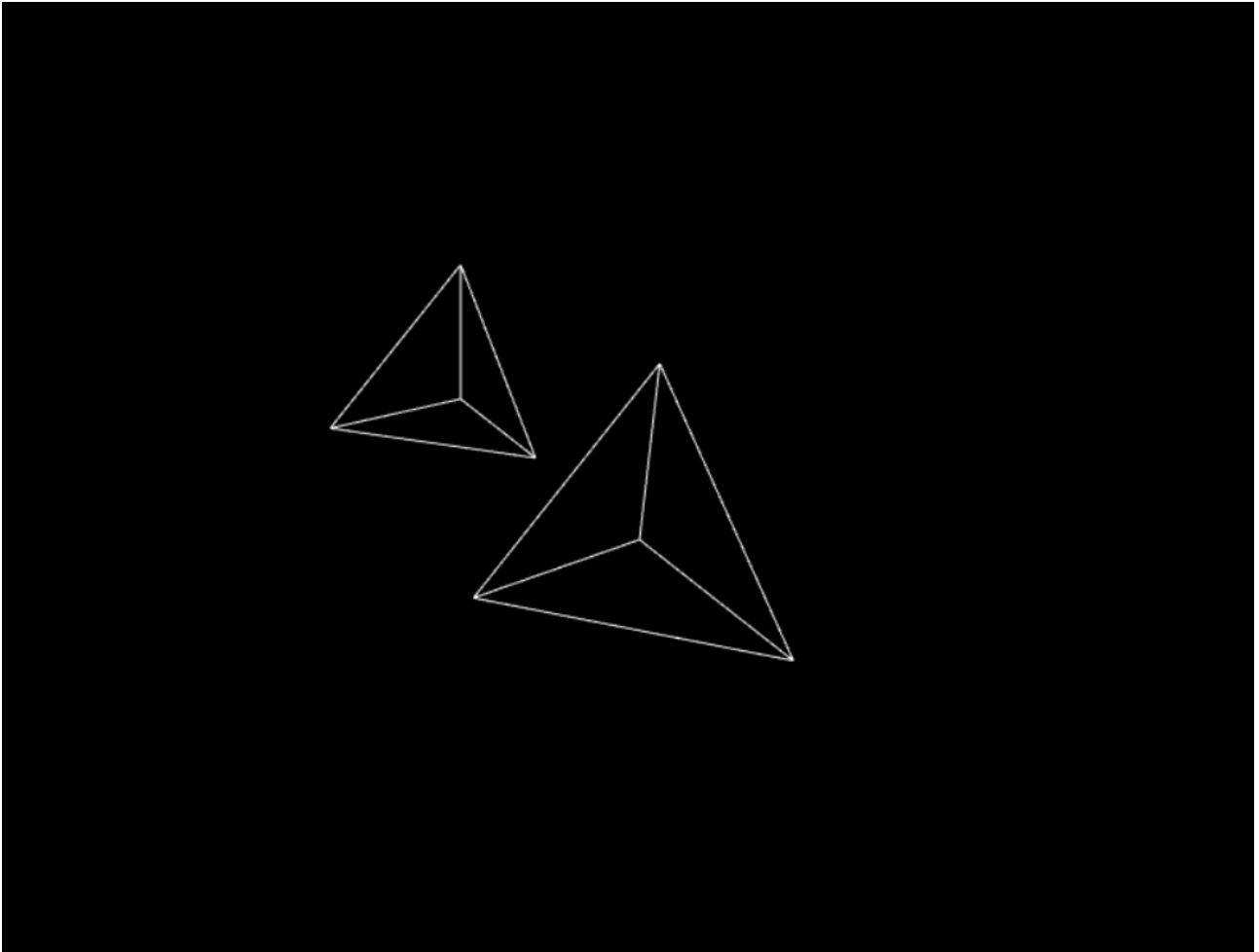


Figure 13: Testcase 7:- Custom test case(two tetrahedrons, 1 edge and 1 point as 4 separate components)

```
iiitb/topo/implementation-project  
> ./a.out gts-files/custom3D.gts  
Number of 0-Homology Groups Formed: 4  
Representative 0 Cycles:  
{1} {4} {7} {8}  
Time Taken is = 0
```

Figure 14: Testcase 7:- Custom test case code

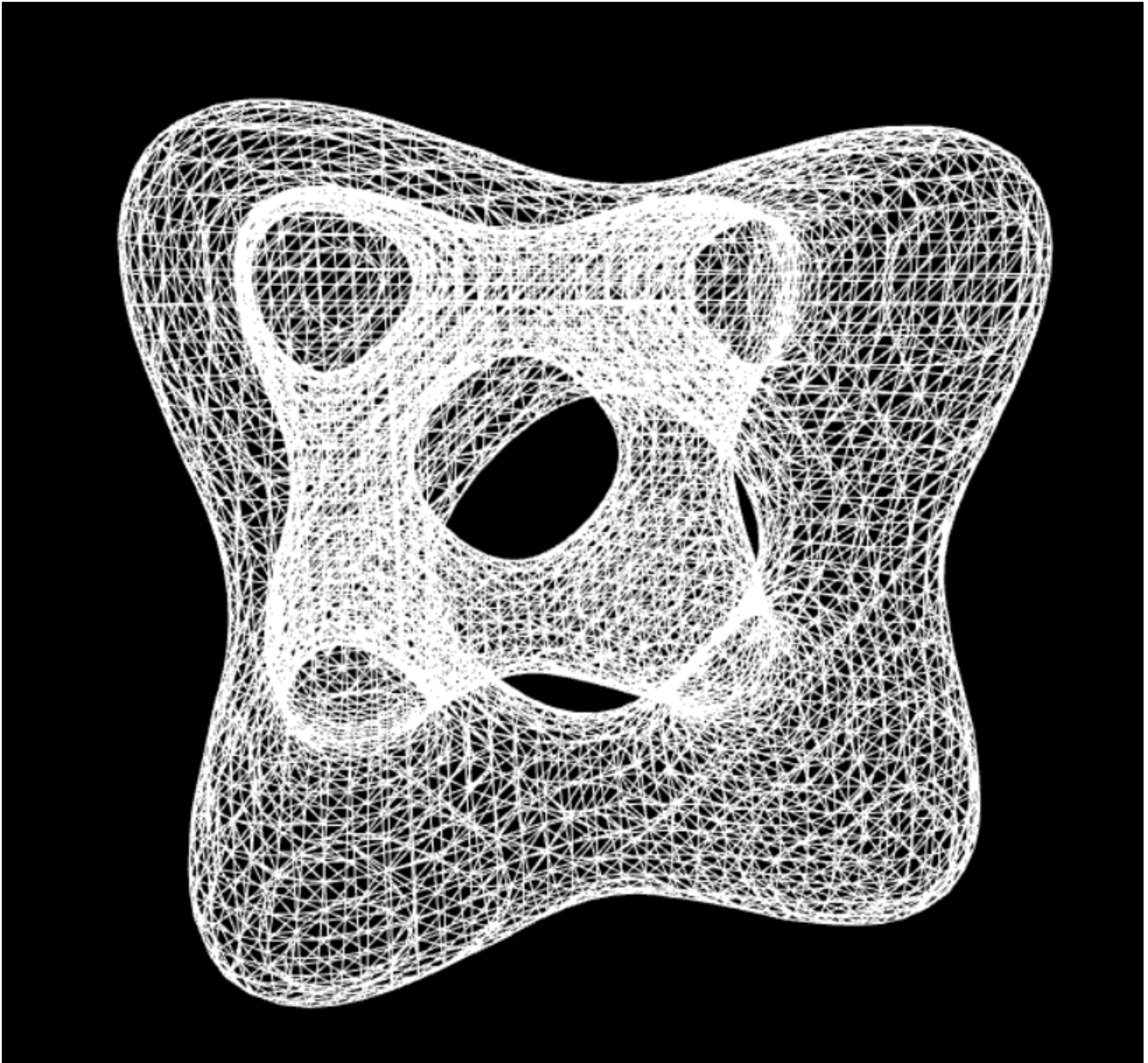


Figure 15: Testcase 8:- tangle.png

```
iiitb/topo/implementation-project  
> ./a.out gts-files/tangle.gts  
Number of 0-Homology Groups Formed: 1  
Representative 0 Cycles:  
{1}  
Time taken : 1849
```

Figure 16: Testcase 8:- tangle code

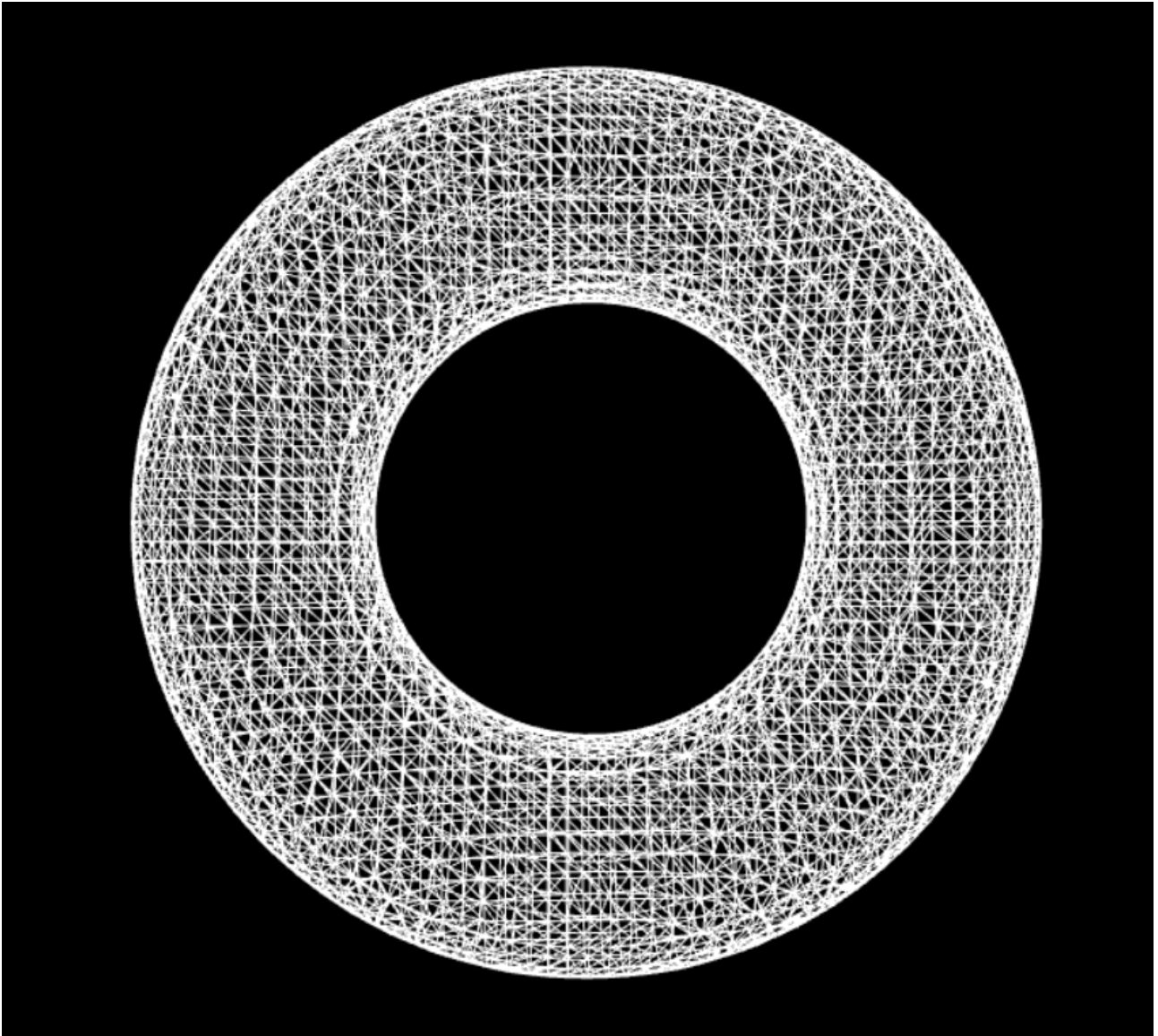


Figure 17: Testcase 9:- torus

```
iiitb/topo/implementation-project  
> ./a.out gts-files/torus.gts  
Number of 0-Homology Groups Formed: 1  
Representative 0 Cycles:  
{1}  
Time taken : 2134
```

Figure 18: Testcase 9:- torus code