AI1103 : Assignment-2

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Download all python codes from

https://github.com/Rahul27n/Assignment_2/blob/main/Assignment_2.py

and latex-tikz codes from

https://github.com/Rahul27n/Assignment_2/blob/main/Assignment_2.tex

1 QUESTION: Q.33 EE-GATE-2016-SET-2

Let the probability density function of random variable,X,be given as:

$f_x(x) = \frac{3}{2}e^{-3x}u(x) + ae^{4x}u(-x)$

where u(x) is the unit step function.Then the value of a and Prob$\{X \leq 0\}$, respectively,are:

(A) 2,$\frac{1}{2}$

(B) 4,$\frac{1}{2}$

(C) 2,$\frac{1}{4}$

(D) 4,$\frac{1}{4}$

2 SOLUTION:

We know that,

$$\int_{-\infty}^{\infty} f_x(x)\,dx = 1. \tag{2.0.1}$$

$$\int_{-\infty}^{0} f_x(x)\,dx + \int_{0}^{\infty} f_x(x)\,dx = 1 \tag{2.0.2}$$

$$\int_{-\infty}^{0} ae^{4x}\,dx + \int_{0}^{\infty} \frac{3}{2}e^{-3x}\,dx = 1 \tag{2.0.3}$$

The expression (2.0.3) was written from (2.0.2) since,

$$u(x) = \begin{cases} 1, & \text{for } x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Simplifying (2.0.3) we have:

$$\int_{-\infty}^{0} ae^{4x}\,dx + \int_{0}^{\infty} \frac{3}{2}e^{-3x}\,dx = 1$$

$$\implies a\left[\frac{e^{4x}}{4}\right]_{-\infty}^{0} + \frac{3}{2}\left[\frac{e^{-3x}}{-3}\right]_{0}^{\infty} = 1 \tag{2.0.4}$$

$$\implies a\left[\frac{1}{4} - 0\right] - \frac{1}{2}\left[0 - 1\right] = 1 \tag{2.0.5}$$

$$\implies \frac{a}{4} + \frac{1}{2} = 1 \implies a = 2 \tag{2.0.6}$$

Therefore,

$$f_x(x) = \begin{cases} \frac{3}{2}e^{-3x}, & \text{for } x \geq 0 \\ 2e^{4x}, & \text{for } x < 0 \end{cases} \tag{2.0.7}$$

The plot for PDF of X can be observed at figure 0

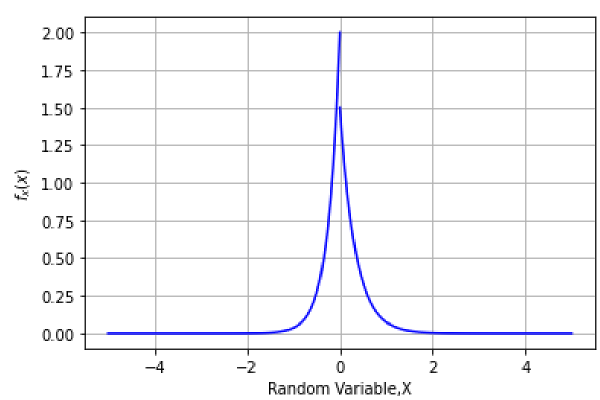

Fig. 0: The PDF of X

The CDF of X is defined as follows:

$$F_X(x) = \Pr(X \leq x) \tag{2.0.8}$$

Now for $x < 0$,

$$\Pr(X \le x) = \int_{-\infty}^{x} f_x(x)\,dx \tag{2.0.9}$$
$$= \int_{-\infty}^{x} 2e^{4x}\,dx \tag{2.0.10}$$
$$= 2\left[\frac{e^{4x}}{4}\right]_{-\infty}^{x} \tag{2.0.11}$$
$$= 2\left[\frac{e^{4x}}{4} - 0\right] \tag{2.0.12}$$
$$= \frac{e^{4x}}{2} \tag{2.0.13}$$

Similarly for $x \ge 0$,

$$\Pr(X \le x) = \int_{-\infty}^{x} f_x(x)\,dx \tag{2.0.14}$$
$$= \int_{-\infty}^{0} 2e^{4x}\,dx + \int_{0}^{x} \frac{3}{2}e^{-3x}\,dx \tag{2.0.15}$$
$$= 2\left[\frac{e^{4x}}{4}\right]_{-\infty}^{0} + \left[\frac{-e^{-3x}}{2}\right]_{0}^{x} \tag{2.0.16}$$
$$= 2\left[\frac{1}{4} - 0\right] - \frac{1}{2}\left[e^{-3x} - 1\right] \tag{2.0.17}$$
$$= 1 - \frac{e^{-3x}}{2} \tag{2.0.18}$$

The CDF of X is as below:

$$F_X(x) = \begin{cases} 1 - \frac{e^{-3x}}{2}, & \text{for } x \ge 0 \\ \frac{e^{4x}}{2}, & \text{for } x < 0 \end{cases} \tag{2.0.19}$$

The plot for CDF of X can be observed at figure 0.

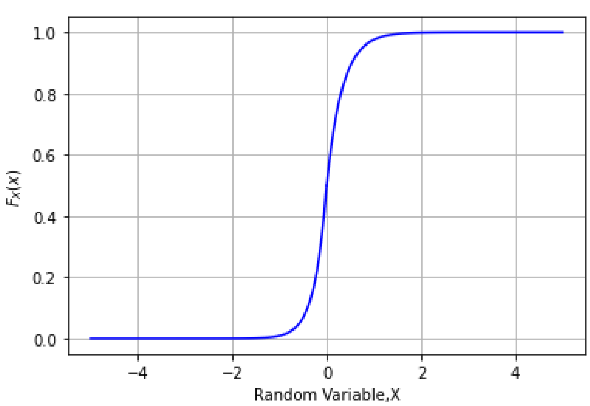

Fig. 0: The CDF of X

$$\therefore \Pr(X \le 0) = F_X(0) = \frac{1}{2} \tag{2.0.20}$$

Hence the correct answer is option (A).