EE3900 : Gate Assignment-4

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Download all latex-tikz codes from

https://github.com/Rahul27n/EE3900/blob/main/Gate_Assignment_4/Gate_Assignment_4.tex

1 QUESTION: GATE EC 1998 Q1.4

The trigonometric Fourier series of a periodic time function can have only:

(A) cosine terms
(B) sine terms
(C) cosine and sine terms
(D) d.c. and cosine terms

2 SOLUTION

The trigonometric Fourier series of a periodic function $x(t)$ with period $T = \frac{2\pi}{\omega}$ is given by:

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos k\omega t + \sum_{k=1}^{\infty} b_k \sin k\omega t \quad (2.0.1)$$

where a_0 is the d.c component of the signal and a_k and b_k are Fourier coefficients. The Fourier series of some example functions are given by:

1) We have a even periodic function having period 2π defined in $[-\pi, \pi]$ as follows :

$$x(t) = \begin{cases} \frac{\pi}{2} + t, & \text{if } -\pi \le t \le 0 \\ \frac{\pi}{2} - t, & \text{if } 0 < t \le \pi \end{cases}$$

The Fourier series of $x(t)$ is given by :

$$x(t) = \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{\cos(2k-1)t}{(2k-1)^2} \quad (2.0.2)$$

2) We have a odd periodic function having period 2π defined in $(0, 2\pi)$ as follows :

$$x(t) = \begin{cases} 1, & \text{if } 0 < t < \pi \\ 0, & \text{if } t = 0, \pi, 2\pi \\ -1, & \text{if } \pi < t < 2\pi \end{cases}$$

The Fourier series of $x(t)$ is given by :

$$x(t) = \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{\sin(2k-1)t}{2k-1} \quad (2.0.3)$$

3) We have a neither even nor odd periodic function having period 2π defined in $(-\pi, \pi)$ as follows :

$$x(t) = \begin{cases} -\pi, & \text{if } -\pi < t \le 0 \\ t, & \text{if } 0 < t < \pi \end{cases}$$

The Fourier series of $x(t)$ is given by :

$$x(t) = -\frac{\pi}{4} - \frac{2}{\pi} \sum_{k=1}^{\infty} \frac{\cos(2k-1)t}{(2k-1)^2} + \sum_{k=1}^{\infty} \frac{2(-1)^k + 1}{k} \sin kt \quad (2.0.4)$$

4) We have a even periodic function having period 2π defined in $(0, 2\pi)$ as follows :

$$x(t) = \begin{cases} t, & \text{if } 0 < t \le \pi \\ 2\pi - t, & \text{if } \pi \le t < 2\pi \end{cases}$$

The Fourier series of $x(t)$ is given by :

$$x(t) = \frac{\pi}{2} - \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{\cos(2k-1)t}{(2k-1)^2} \quad (2.0.5)$$

Hence the correct answer is option (C).