

# Linear Forms Q2.18

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# Question

## Linear Forms Q2.18

Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point  $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ .

# Solution

## General Equation of a line

The general equation of a line can be written as :

$$\mathbf{n}^T \mathbf{x} = c \quad (1)$$

where  $\mathbf{n}$  is the normal to the line.

The standard basis vectors in 2D plane are given by:

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2)$$

$$\mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (3)$$

Let the line (1) cut the x and y co-ordinate axes at A and B respectively. They can be written as :

$$A = \frac{ce_1}{n^T e_1} \quad (4)$$

$$B = \frac{ce_2}{n^T e_2} \quad (5)$$

It is given that the line cuts off equal intercepts on the co-ordinate axes. Hence from (4) and (5) we have:

$$n^T e_1 = n^T e_2 \quad (6)$$

which is equivalent to:

$$n^T (e_1 - e_2) = 0 \quad (7)$$

$$n^T \begin{pmatrix} 1 \\ -1 \end{pmatrix} = 0 \quad (8)$$

$$\implies n = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (9)$$

Hence from (1) and (9) , the equation of the line is given by:

$$(1 \ 1) x = c \quad (10)$$

It is given that  $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$  lies on the line. Hence from (10) we have:

$$c = (1 \ 1) \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 5 \quad (11)$$

Therefore the equation of the line is:

$$(1 \ 1) x = 5 \quad (12)$$

The illustration of the line is shown below :

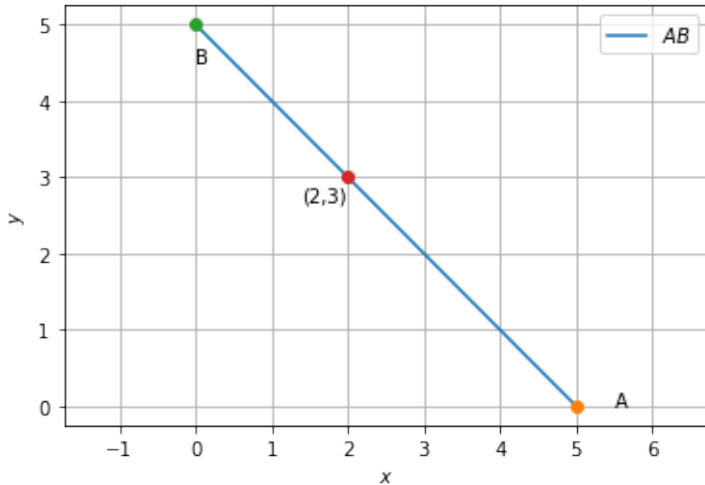


Figure: Line AB making equal intercepts on co-ordinate axes