EE3900 : Gate Assignment-4

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Download all latex-tikz codes from

https://github.com/Rahul27n/EE3900/blob/main/Gate_Assignment_4/Gate_Assignment_4.tex

1 QUESTION: GATE EC 1998 Q1.4

The trigonometric Fourier series of a periodic time function can have only:

(A) cosine terms
(B) sine terms
(C) d.c. ,cosine and sine terms
(D) d.c. and cosine terms

2 SOLUTION

The trigonometric Fourier series of a periodic function $x(t)$ with period T is given by:

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos\frac{2\pi kt}{T} + \sum_{k=1}^{\infty} b_k \sin\frac{2\pi kt}{T} \tag{2.0.1}$$

where a_0 is the d.c component of the signal and a_k and b_k are Fourier coefficients. The Fourier series of some example functions are given by:

1) We have a even periodic function having period 2π defined in $[-\pi, \pi]$ as follows :

$$x(t) = \begin{cases} \frac{\pi}{2} + t, & \text{if } -\pi \le t \le 0 \\ \frac{\pi}{2} - t, & \text{if } 0 < t \le \pi \end{cases}$$

The Fourier series of $x(t)$ is determined as follows:

$$\begin{aligned} a_0 &= \frac{1}{2\pi}\left(\int_{-\pi}^{0} x(t)\,dt + \int_{0}^{\pi} x(t)\,dt\right) = 0 \\ a_k &= \frac{1}{\pi}\left(\int_{-\pi}^{0} x(t)\cos kt\,dt + \int_{0}^{\pi} x(t)\cos kt\,dt\right) \\ &= \frac{2(1-(-1)^k)}{\pi k^2} \\ b_k &= \frac{1}{\pi}\int_{-\pi}^{\pi} x(t)\sin kt\,dt = 0 \end{aligned}$$

$$x(t) = \frac{4}{\pi}\sum_{k=1}^{\infty} \frac{\cos(2k-1)t}{(2k-1)^2} \tag{2.0.2}$$

2) We have a odd periodic function having period 2π defined in $[0, 2\pi]$ as follows :

$$x(t) = \begin{cases} 1, & \text{if } 0 < t < \pi \\ 0, & \text{if } t = 0, \pi, 2\pi \\ -1, & \text{if } \pi < t < 2\pi \end{cases}$$

The Fourier series of $x(t)$ is determined as follows:

$$\begin{aligned} a_0 &= \frac{1}{2\pi}\int_{-\pi}^{\pi} x(t)\,dt = 0 \\ a_k &= \frac{1}{\pi}\int_{-\pi}^{\pi} x(t)\cos kt\,dt = 0 \\ b_k &= \frac{1}{\pi}\left(\int_{-\pi}^{0} x(t)\sin kt\,dt + \int_{0}^{\pi} x(t)\sin kt\,dt\right) \\ &= \frac{2(1-(-1)^k)}{\pi k} \end{aligned}$$

$$x(t) = \frac{4}{\pi}\sum_{k=1}^{\infty} \frac{\sin(2k-1)t}{2k-1} \tag{2.0.3}$$

3) We have a neither even nor odd periodic function having period 2π defined in $(-\pi, \pi)$ as follows :

$$x(t) = \begin{cases} -\pi, & \text{if } -\pi < t < 0 \\ -\frac{\pi}{2}, & \text{if } t = 0 \\ t, & \text{if } 0 < t < \pi \end{cases}$$

The Fourier series of $x(t)$ is determined as follows:

$$\begin{aligned} a_0 &= \frac{1}{2\pi}\left(\int_{-\pi}^{0} x(t)\,dt + \int_{0}^{\pi} x(t)\,dt\right) = -\frac{\pi}{4} \\ a_k &= \frac{1}{\pi}\left(\int_{-\pi}^{0} x(t)\cos kt\,dt + \int_{0}^{\pi} x(t)\cos kt\,dt\right) \\ &= \frac{(-1)^k - 1}{\pi k^2} \end{aligned}$$

$$b_k = \frac{1}{\pi}\left(\int_{-\pi}^{0} x(t)\sin kt\,dt + \int_{0}^{\pi} x(t)\sin kt\,dt\right)$$
$$= \frac{2(-1)^k + 1}{k}$$
$$x(t) = -\frac{\pi}{4} - \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{\cos(2k-1)t}{(2k-1)^2} + \sum_{k=1}^{\infty}\frac{2(-1)^k + 1}{k}\sin kt \tag{2.0.4}$$

4) We have a even periodic function having period 2π defined in $(0, 2\pi)$ as follows :

$$x(t) = \begin{cases} t, & \text{if } 0 < t \le \pi \\ 2\pi - t, & \text{if } \pi \le t < 2\pi \end{cases}$$

The Fourier series of $x(t)$ is determined as follows:

$$a_0 = \frac{1}{2\pi}\left(\int_{-\pi}^{0} x(t)\,dt + \int_{0}^{\pi} x(t)\,dt\right) = \frac{\pi}{2}$$
$$a_k = \frac{1}{\pi}\left(\int_{-\pi}^{0} x(t)\cos kt\,dt + \int_{0}^{\pi} x(t)\cos kt\,dt\right)$$
$$= \frac{2((-1)^k - 1)}{\pi k^2}$$
$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} x(t)\sin kt\,dt = 0$$
$$x(t) = \frac{\pi}{2} - \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\cos(2k-1)t}{(2k-1)^2} \tag{2.0.5}$$

Hence the correct answer is option (C).