EE3900 : Gate Assignment-1

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Download all python codes from

```
https://github.com/Rahul27n/EE3900/blob/main/
    Gate_Assignment_1/Gate_Assignment_1.py
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and latex-tikz codes from

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https://github.com/Rahul27n/EE3900/blob/main/
    Gate_Assignment_1/Gate_Assignment_1.tex
```

1 QUESTION: Q.33 EC-GATE-2016

The Discrete Fourier Transform (DFT) of the 4 point sequence $x[n] = \{3, 2, 3, 4\}$ is given by $X[k] = \{12, 2j, 0, -2j\}$. If $X_1[k]$ is the DFT of the 12 point sequence $x_1[n] = \{3, 0, 0, 2, 0, 0, 3, 0, 0, 4, 0, 0\}$, the value of $\left|\frac{X_1[8]}{X_1[11]}\right|$ is :

2 SOLUTION

The 4-point DFT matrix is given by:

$$\mathbf{W_4} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^1 & \omega^2 & \omega^3 \\ 1 & \omega^2 & \omega^4 & \omega^6 \\ 1 & \omega^3 & \omega^6 & \omega^9 \end{pmatrix} \tag{2.0.1}$$

$$= \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{pmatrix} \tag{2.0.2}$$

where $\omega = e^{\frac{-2\pi j}{4}} = -j$. Now from the given information we can write:

$$\mathbf{X} = \mathbf{W_4}\mathbf{x} \tag{2.0.3}$$

where

$$\mathbf{x} = \begin{pmatrix} 3 \\ 2 \\ 3 \\ 4 \end{pmatrix} \tag{2.0.4}$$

and

$$\mathbf{X} = \begin{pmatrix} 12 \\ 2j \\ 0 \\ -2j \end{pmatrix} \tag{2.0.5}$$

Now we need to find $\mathbf{X_1}$ satisfying the relation :

$$\mathbf{X_1} = \mathbf{W_{12}}\mathbf{x_1} \tag{2.0.6}$$

where

$$\mathbf{x_1} = \begin{pmatrix} 3 \\ 0 \\ 0 \\ 2 \\ 0 \\ 0 \\ 3 \\ 0 \\ 0 \\ 4 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.7}$$

and $\mathbf{W_{12}}$ is the 12-point DFT matrix which is given by:

$$\begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & \Omega & \Omega^2 & -j & -j\Omega & -j\Omega^2 & -1 & -\Omega & -\Omega^2 & j & j\Omega & j\Omega^2 \\
1 & \Omega^2 & -j\Omega & -1 & -\Omega^2 & j\Omega & 1 & \Omega^2 & -j\Omega & -1 & -\Omega^2 & j\Omega \\
1 & -j & -1 & j & 1 & -j & -1 & j & 1 & -j & -1 & j \\
1 & -j\Omega & -\Omega^2 & 1 & -j\Omega & -\Omega^2 & 1 & -j\Omega & -\Omega^2 & 1 & -j\Omega & -\Omega^2 \\
1 & -j\Omega^2 & j\Omega & -j & -\Omega^2 & \Omega & -1 & j\Omega^2 & -j\Omega & j & \Omega^2 & -\Omega \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & -\Omega & \Omega^2 & j & -j\Omega & j\Omega^2 & -1 & \Omega & -\Omega^2 & -j & j\Omega & -j\Omega^2 \\
1 & -\Omega^2 & -j\Omega & 1 & -\Omega^2 & -j\Omega & 1 & -\Omega^2 & -j\Omega & 1 & -\Omega^2 & -j\Omega \\
1 & j & -1 & -j & 1 & j & -1 & -j & 1 & j & -1 & -j \\
1 & j\Omega & -\Omega^2 & -1 & -j\Omega & \Omega^2 & 1 & j\Omega & -\Omega^2 & -1 & -j\Omega & \Omega^2 \\
1 & j\Omega^2 & j\Omega & j & -\Omega^2 & -\Omega & -1 & -j\Omega^2 & -j\Omega & -j & \Omega^2 & \Omega
\end{pmatrix}$$

where

$$\Omega = e^{\frac{-2\pi j}{12}} \tag{2.0.8}$$

$$= \frac{\sqrt{3} - j}{2} \tag{2.0.9}$$

We can express $\mathbf{x_1}$ in terms of $\mathbf{x}$ as follows :

$$\mathbf{x_1} = \mathbf{A}\mathbf{x} \tag{2.0.10}$$

where

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.11}$$

$$\implies \mathbf{A} = \begin{pmatrix} \mathbf{e_1} & \mathbf{e_4} & \mathbf{e_7} & \mathbf{e_{10}} \end{pmatrix} \tag{2.0.12}$$

where $\mathbf{e_1}, \mathbf{e_4}, \mathbf{e_7}, \mathbf{e_{10}}$ represent the unit basis vectors in a subspace of $\mathbb{R}^{12}$. Now from (2.0.6) have :

$$\mathbf{X_1} = (\mathbf{W_{12}}\mathbf{A})\mathbf{x} \tag{2.0.13}$$

Only the red coloured columns in $\mathbf{W_{12}}$ give non-zero output when multiplied with $\mathbf{A}$. We can express the matrix $\mathbf{W_{12}}$ as a block matrix in the following way:

$$\mathbf{W_{12}} = \begin{pmatrix} \mathbf{c_1} & \mathbf{c_2} & \mathbf{c_3} & \mathbf{c_4} & \mathbf{c_5} & \mathbf{c_6} & \mathbf{c_7} & \mathbf{c_8} & \mathbf{c_9} & \mathbf{c_{10}} & \mathbf{c_{11}} & \mathbf{c_{12}} \end{pmatrix} \tag{2.0.14}$$

where $\mathbf{c_i}$ is the i^{th} column matrix of $\mathbf{W_{12}}$. Now we have:

$$\mathbf{X_1} = \begin{pmatrix} \mathbf{c_1} & \mathbf{c_2} & \mathbf{c_3} & \mathbf{c_4} & \mathbf{c_5} & \mathbf{c_6} & \mathbf{c_7} & \mathbf{c_8} & \mathbf{c_9} & \mathbf{c_{10}} & \mathbf{c_{11}} & \mathbf{c_{12}} \end{pmatrix} \mathbf{A}\mathbf{x}$$

$$\implies \mathbf{X_1} = \begin{pmatrix} \mathbf{c_1} & \mathbf{c_4} & \mathbf{c_7} & \mathbf{c_{10}} \end{pmatrix} \mathbf{x} \tag{2.0.15}$$

We can also express $\mathbf{W_4}$ as block matrix as follows:

$$\mathbf{W_4} = \begin{pmatrix} \mathbf{w_1} & \mathbf{w_2} & \mathbf{w_3} & \mathbf{w_4} \end{pmatrix} \tag{2.0.16}$$

where $\mathbf{w_i}$ is the i^{th} column matrix of $\mathbf{W_4}$.

We can importantly note that :

$$\mathbf{c_1} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \otimes \mathbf{w_1} \tag{2.0.17}$$

$$\mathbf{c_4} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \otimes \mathbf{w_2} \tag{2.0.18}$$

$$\mathbf{c_7} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \otimes \mathbf{w_3} \tag{2.0.19}$$

$$\mathbf{c_{10}} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \otimes \mathbf{w_4} \tag{2.0.20}$$

where $\otimes$ represents the **Kronecker Product**. Therefore from (2.0.15) we have:

$$\mathbf{X_1} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} \mathbf{w_1} & \mathbf{w_2} & \mathbf{w_3} & \mathbf{w_4} \end{pmatrix} \mathbf{x} \tag{2.0.21}$$

$$= \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \otimes \mathbf{X} \tag{2.0.22}$$

$$= \begin{pmatrix} 12 \\ 2j \\ 0 \\ -2j \\ 12 \\ 2j \\ 0 \\ -2j \\ 12 \\ 2j \\ 0 \\ -2j \end{pmatrix} \tag{2.0.23}$$

Therefore we have:

$$\left| \frac{\mathbf{X_1}[8]}{\mathbf{X_1}[11]} \right| = \left| \frac{12}{-2j} \right| = \left| 6j \right| = 6 \tag{2.0.24}$$

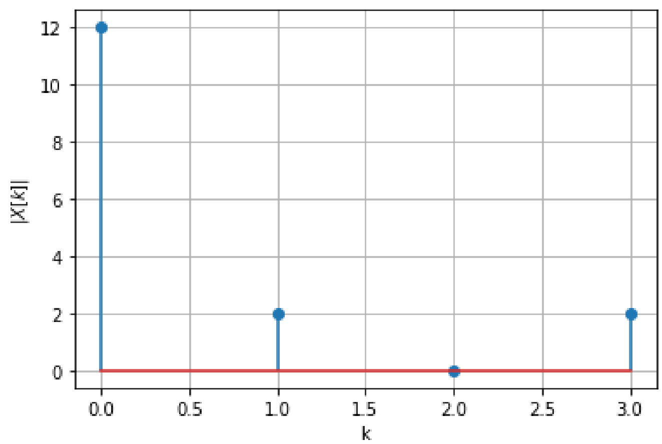

Fig. 0: Magnitude of $\mathbf{X}[k]$ vs k

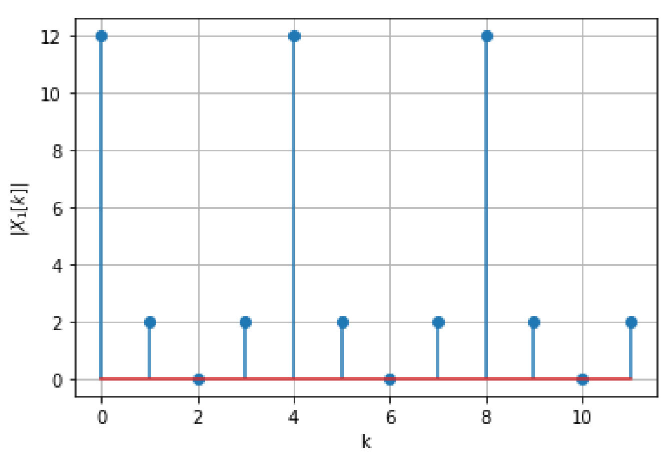

Fig. 0: Magnitude of $\mathbf{X_1}[k]$ vs k