EE3900 : Assignment-4

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Download all python codes from

```
https://github.com/Rahul27n/EE3900/blob/main/
    Assignment_4/Assignment_4.py
```

and latex-tikz codes from

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https://github.com/Rahul27n/EE3900/blob/main/
    Assignment_4/Assignment_4.tex
```

1 QUESTION: LINEAR FORMS Q2.18

Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point $\begin{pmatrix}2\\3\end{pmatrix}$.

2 SOLUTION

The general equation of a line can be written as :

$$\mathbf{n}^T\mathbf{x} = c \tag{2.0.1}$$

where $\mathbf{n}$ is the normal to the line.
The standard basis vectors in 2D plane are given by:

$$\mathbf{e_1} = \begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.2}$$

$$\mathbf{e_2} = \begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.3}$$

Let the line (2.0.1) cut the x and y co-ordinate axes at $\mathbf{A}$ and $\mathbf{B}$ respectively. They can be written as :

$$\mathbf{A} = \frac{c\mathbf{e_1}}{\mathbf{n}^T\mathbf{e_1}} \tag{2.0.4}$$

$$\mathbf{B} = \frac{c\mathbf{e_2}}{\mathbf{n}^T\mathbf{e_2}} \tag{2.0.5}$$

Hence we have :

$$\mathbf{A} = \frac{\begin{pmatrix}c\\0\end{pmatrix}}{\mathbf{n}^T\mathbf{e_1}} \tag{2.0.6}$$

$$\mathbf{B} = \frac{\begin{pmatrix}0\\c\end{pmatrix}}{\mathbf{n}^T\mathbf{e_2}} \tag{2.0.7}$$

It is given that the line cuts off equal intercepts on the co-ordinate axes. Hence from (2.0.6) and (2.0.7) we have:

$$\mathbf{n}^T\mathbf{e_1} = \mathbf{n}^T\mathbf{e_2} \tag{2.0.8}$$

which is equivalent to:

$$\mathbf{n}^T(\mathbf{e_1} - \mathbf{e_2}) = 0 \tag{2.0.9}$$

$$\mathbf{n}^T\begin{pmatrix}1\\-1\end{pmatrix} = 0 \tag{2.0.10}$$

$$\implies \mathbf{n} = \begin{pmatrix}1\\1\end{pmatrix} \tag{2.0.11}$$

Hence from (2.0.1) and (2.0.11) , the equation of the line is given by:

$$\begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x} = c \tag{2.0.12}$$

It is given that $\begin{pmatrix}2\\3\end{pmatrix}$ lies on the line. Hence from (2.0.12) we have:

$$c = \begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}2\\3\end{pmatrix} = 5 \tag{2.0.13}$$

Therefore the equation of the line is:

$$\begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x} = 5 \tag{2.0.14}$$

The illustration of the line is shown below :

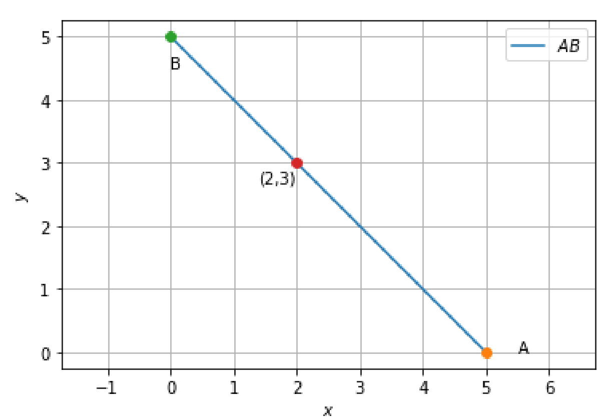

Fig. 0: Line **AB** making equal intercepts on co-ordinate axes