EE3900 : Quiz-2

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Download all python codes from

```
https://github.com/Rahul27n/EE3900/blob/main/Quiz_2/Quiz_2.py
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and all latex-tikz codes from

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https://github.com/Rahul27n/EE3900/blob/main/Quiz_2/Quiz_2.tex
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1 Question: 3.19(b)

For the following pair of input z-transform $X(z)$ and system function $H(z)$ determine the region of convergence for the output z-transform $Y(z)$:

$$X(z) = \frac{1}{1-2z^{-1}}, \quad |z| < 2$$
$$H(z) = \frac{1}{1-\frac{1}{3}z^{-1}}, \quad |z| > \frac{1}{3}$$

2 Solution

We know that if $X(z) = \frac{1}{1-az^{-1}}$:

$$x[n] = \begin{cases} a^n u[n], & \text{if } |z| > |a| \\ -a^n u[-n-1], & \text{if } |z| < |a| \end{cases}$$

Therefore we have :

$$x[n] = -2^n u[-n-1] \tag{2.0.1}$$
$$h[n] = \left(\frac{1}{3}\right)^n u[n] \tag{2.0.2}$$

The z- transform expansion of $x[n]$ is given by:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} \tag{2.0.3}$$
$$= \sum_{n=-\infty}^{-1} 2^n z^{-n} \tag{2.0.4}$$
$$= \sum_{m=1}^{\infty} \left(\frac{z}{2}\right)^m \tag{2.0.5}$$

Clearly the above geometric series converges only when we have :

$$|2^{-1}z| < 1 \implies |z| < 2 \tag{2.0.6}$$

Therefore the ROC of $x[n]$ is given by (2.0.6).
The z- transform expansion of $h[n]$ is given by:

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n} \tag{2.0.7}$$
$$= \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^{-n} \tag{2.0.8}$$
$$= \sum_{n=0}^{\infty} \left(\frac{1}{3z}\right)^n \tag{2.0.9}$$

Clearly the above geometric series converges only when we have :

$$|3z| > 1 \implies |z| > \frac{1}{3} \tag{2.0.10}$$

Therefore the ROC of $h[n]$ is given by (2.0.10).
We know from the convolution theorem that :

$$Y(z) = X(z)H(z) \tag{2.0.11}$$
$$Y(z) = \frac{1}{(1-2z^{-1})(1-\frac{1}{3}z^{-1})} \tag{2.0.12}$$
$$Y(z) = \frac{3z^2}{(z-2)(3z-1)} \tag{2.0.13}$$

We also know that :

$$R_{x_1[n]*x_2[n]} = R_{x_1[n]} \cap R_{x_2[n]} \tag{2.0.14}$$

where $R_{x[n]}$ represents the ROC of $X(z)$.
We have :

$$y[n] = x[n] * h[n] \tag{2.0.15}$$

Therefore from (2.0.6) and (2.0.10) the ROC of $Y(z)$ is given by:

$$R_{y[n]} = \frac{1}{3} < |z| < 2 \tag{2.0.16}$$

The plots of $x[n]$, $h[n]$ and the ROC of $Y(z)$ are given by:

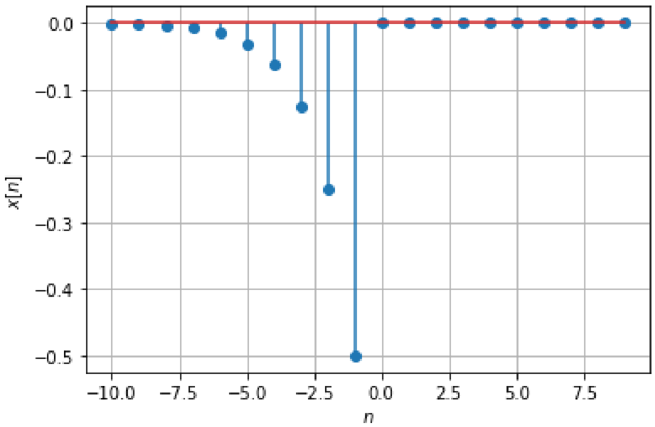

Fig. 0: $x[n]$

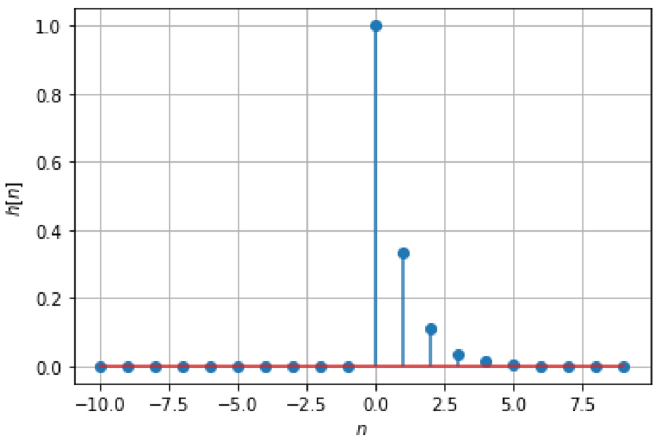

Fig. 0: $h[n]$

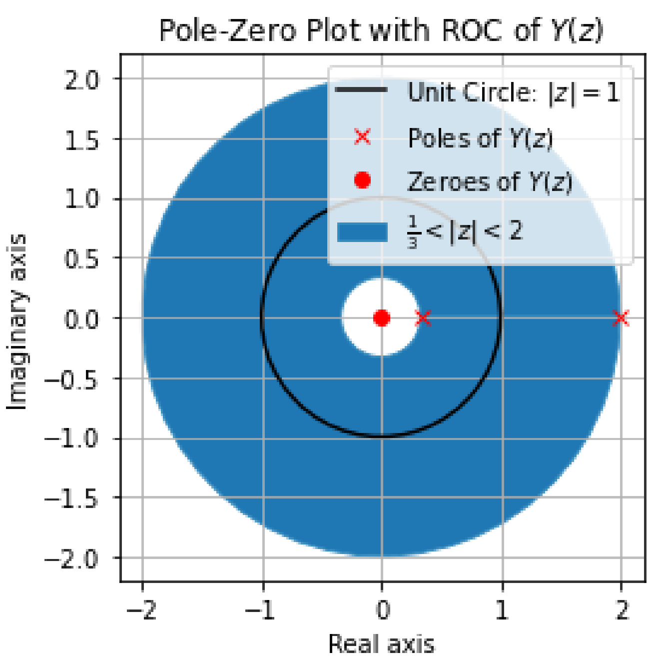

Fig. 0: ROC of $Y(z)$