EE3900 : Gate Assignment-2

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Download all python codes from

https://github.com/Rahul27n/EE3900/blob/main/Gate_Assignment_2/Gate_Assignment_2.py

and latex-tikz codes from

https://github.com/Rahul27n/EE3900/blob/main/Gate_Assignment_2/Gate_Assignment_2.tex

1 QUESTION: GATE EC 2007 Q.49

The frequency response of a linear, time-invariant system is given by :

$$H(f) = \frac{5}{1 + j10\pi f}$$

The step response of the system is:

(A) $5(1 - e^{-5t})u(t)$

(B) $5(1 - e^{-\frac{t}{5}})u(t)$

(C) $\frac{1}{5}(1 - e^{-5t})u(t)$

(D) $\frac{1}{5}(1 - e^{-\frac{t}{5}})u(t)$

2 SOLUTION

Lemma 2.1 (Table of Laplace Transforms)**.**

Time Function $f(t) = \mathcal{L}^{-1}\{F(s)\}$	***Laplace transform*** *of f(t)* $F(s) = \mathcal{L}\{f(t)\}$
$u(t)$	$\frac{1}{s},\ s > 0$
$e^{-at}u(t)$	$\frac{1}{(s+a)},\ (s + a) > 0$

The frequency response $H(f)$ of the system can be rewritten as follows :

$$H(s) = \frac{5}{1 + 5s} = \frac{1}{s + \frac{1}{5}} \tag{2.0.1}$$

where $s = j\omega$ and $w = 2\pi f$. Hence the step response $x(t)$ is given by the convolution of inverse Laplace transforms of $H(s)$ and $\frac{1}{s}$. Using the Lemma-2.1 we have:

$$x(t) = (e^{-\frac{t}{5}}u(t)) * u(t) \tag{2.0.2}$$

$$x(t) = \int_0^t e^{-\frac{\tau}{5}}u(\tau)u(t - \tau)\,d\tau \tag{2.0.3}$$

$$x(t) = 5(1 - e^{-\frac{t}{5}})u(t) \tag{2.0.4}$$

Hence the correct answer is option (B).

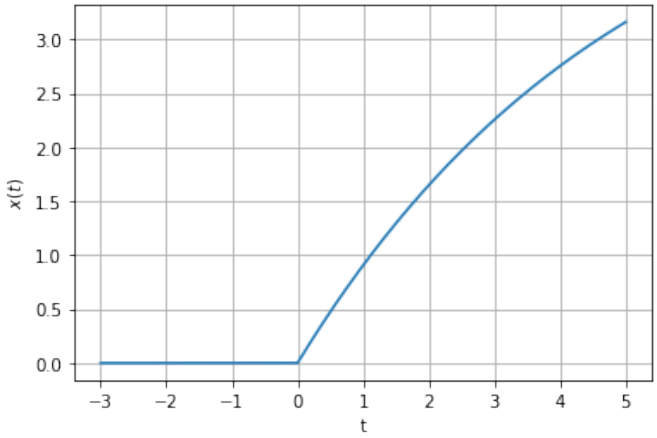

Fig. 4: Step response $x(t)$ vs t