

# EE3900 : Gate Assignment-4

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Download all latex-tikz codes from

[https://github.com/Rahul27n/EE3900/blob/main/Gate\\_Assignment\\_4/Gate\\_Assignment\\_4.tex](https://github.com/Rahul27n/EE3900/blob/main/Gate_Assignment_4/Gate_Assignment_4.tex)

## 1 QUESTION: GATE EC 1998 Q1.4

The trigonometric Fourier series of a periodic time function can have only:

- (A) cosine terms
- (B) sine terms
- (C) cosine and sine terms
- (D) d.c. and cosine terms

## 2 SOLUTION

The trigonometric Fourier series of a periodic function  $x(t)$  with period  $T = \frac{2\pi}{\omega}$  is given by:

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos k\omega t + \sum_{k=1}^{\infty} b_k \sin k\omega t \quad (2.0.1)$$

where  $a_0$  is the d.c component of the signal and  $a_k$  and  $b_k$  are Fourier coefficients. The Fourier series of some example functions are given by:

- 1) We have a even periodic function having period  $2\pi$  defined in  $[-\pi, \pi]$  as follows :

$$x(t) = \begin{cases} \frac{\pi}{2} + t, & \text{if } -\pi \leq t \leq 0 \\ \frac{\pi}{2} - t, & \text{if } 0 < t \leq \pi \end{cases}$$

The Fourier series of  $x(t)$  is given by :

$$x(t) = \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{\cos(2k-1)t}{(2k-1)^2} \quad (2.0.2)$$

- 2) We have a odd periodic function having period  $2\pi$  defined in  $(0, 2\pi)$  as follows :

$$x(t) = \begin{cases} 1, & \text{if } 0 < t < \pi \\ 0, & \text{if } t = 0, \pi, 2\pi \\ -1, & \text{if } \pi < t < 2\pi \end{cases}$$

The Fourier series of  $x(t)$  is given by :

$$x(t) = \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{\sin(2k-1)t}{2k-1} \quad (2.0.3)$$

- 3) We have a odd periodic function having period  $2\pi$  defined in  $[-\pi, \pi]$  as follows :

$$x(t) = \begin{cases} -1, & \text{if } -\pi \leq t \leq -\frac{\pi}{2} \\ 0, & \text{if } -\frac{\pi}{2} < t \leq \frac{\pi}{2} \\ 1, & \text{if } \frac{\pi}{2} < t \leq \pi \end{cases}$$

The Fourier series of  $x(t)$  is given by :

$$x(t) = \frac{2}{\pi} \sum_{k=1}^{\infty} \frac{1}{k} \left( \cos \frac{k\pi}{2} - \cos k\pi \right) \sin kt \quad (2.0.4)$$

- 4) We have a even periodic function having period  $2\pi$  defined in  $(0, 2\pi)$  as follows :

$$x(t) = \begin{cases} t, & \text{if } 0 < t \leq \pi \\ 2\pi - t, & \text{if } \pi \leq t < 2\pi \end{cases}$$

The Fourier series of  $x(t)$  is given by :

$$x(t) = \frac{\pi}{2} - \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{\cos(2k-1)t}{(2k-1)^2} \quad (2.0.5)$$

Hence the correct answer is option (C).