

EE3900 : Gate Assignment-4

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Download all latex-tikz codes from

[https://github.com/Rahul27n/EE3900/blob/main/
Gate_Assignment_4/Gate_Assignment_4.tex](https://github.com/Rahul27n/EE3900/blob/main/Gate_Assignment_4/Gate_Assignment_4.tex)

1 QUESTION: GATE EC 1998 Q1.4

The trigonometric Fourier series of a periodic time function can have only:

- (A) cosine terms
- (B) sine terms
- (C) d.c. ,cosine and sine terms
- (D) d.c. and cosine terms

2 SOLUTION

The trigonometric Fourier series of a periodic function $x(t)$ with period $T = \frac{2\pi}{\omega}$ is given by:

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos k\omega t + \sum_{k=1}^{\infty} b_k \sin k\omega t \quad (2.0.1)$$

where a_0 is the d.c component of the signal and a_k and b_k are Fourier coefficients. The Fourier series of some example functions are given by:

- 1) We have a even periodic function having period 2π defined in $[-\pi, \pi]$ as follows :

$$x(t) = \begin{cases} \frac{\pi}{2} + t, & \text{if } -\pi \leq t \leq 0 \\ \frac{\pi}{2} - t, & \text{if } 0 < t \leq \pi \end{cases}$$

The Fourier series of $x(t)$ is determined as follows:

$$\begin{aligned} a_0 &= \frac{1}{2\pi} \left(\int_{-\pi}^0 x(t) dt + \int_0^{\pi} x(t) dt \right) = 0 \\ a_k &= \frac{1}{\pi} \left(\int_{-\pi}^0 x(t) \cos kt dt + \int_0^{\pi} x(t) \cos kt dt \right) \\ &= \frac{2(1 - (-1)^k)}{\pi k^2} \\ b_k &= \frac{1}{\pi} \int_{-\pi}^{\pi} x(t) \sin kt dt = 0 \end{aligned}$$

$$x(t) = \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{\cos(2k-1)t}{(2k-1)^2} \quad (2.0.2)$$

- 2) We have a odd periodic function having period 2π defined in $[0, 2\pi]$ as follows :

$$x(t) = \begin{cases} 1, & \text{if } 0 < t < \pi \\ 0, & \text{if } t = 0, \pi, 2\pi \\ -1, & \text{if } \pi < t < 2\pi \end{cases}$$

The Fourier series of $x(t)$ is determined as follows:

$$\begin{aligned} a_0 &= \frac{1}{2\pi} \int_{-\pi}^{\pi} x(t) dt = 0 \\ a_k &= \frac{1}{\pi} \int_{-\pi}^{\pi} x(t) \cos kt dt = 0 \\ b_k &= \frac{1}{\pi} \left(\int_{-\pi}^0 x(t) \sin kt dt + \int_0^{\pi} x(t) \sin kt dt \right) \\ &= \frac{2(1 - (-1)^k)}{\pi k} \\ x(t) &= \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{\sin(2k-1)t}{2k-1} \quad (2.0.3) \end{aligned}$$

- 3) We have a neither even nor odd periodic function having period 2π defined in $(-\pi, \pi)$ as follows :

$$x(t) = \begin{cases} -\pi, & \text{if } -\pi < t < 0 \\ -\frac{\pi}{2}, & \text{if } t = 0 \\ t, & \text{if } 0 < t < \pi \end{cases}$$

The Fourier series of $x(t)$ is determined as follows:

$$\begin{aligned} a_0 &= \frac{1}{2\pi} \left(\int_{-\pi}^0 x(t) dt + \int_0^{\pi} x(t) dt \right) = -\frac{\pi}{4} \\ a_k &= \frac{1}{\pi} \left(\int_{-\pi}^0 x(t) \cos kt dt + \int_0^{\pi} x(t) \cos kt dt \right) \\ &= \frac{(-1)^k - 1}{\pi k^2} \end{aligned}$$

$$\begin{aligned}
b_k &= \frac{1}{\pi} \left(\int_{-\pi}^0 x(t) \sin kt \, dt + \int_0^{\pi} x(t) \sin kt \, dt \right) \\
&= \frac{2(-1)^k + 1}{k} \\
x(t) &= -\frac{\pi}{4} - \frac{2}{\pi} \sum_{k=1}^{\infty} \frac{\cos(2k-1)t}{(2k-1)^2} + \sum_{k=1}^{\infty} \frac{2(-1)^k + 1}{k} \sin kt
\end{aligned} \tag{2.0.4}$$

- 4) We have a even periodic function having period 2π defined in $(0, 2\pi)$ as follows :

$$x(t) = \begin{cases} t, & \text{if } 0 < t \leq \pi \\ 2\pi - t, & \text{if } \pi \leq t < 2\pi \end{cases}$$

The Fourier series of $x(t)$ is determined as follows:

$$\begin{aligned}
a_0 &= \frac{1}{2\pi} \left(\int_{-\pi}^0 x(t) \, dt + \int_0^{\pi} x(t) \, dt \right) = \frac{\pi}{2} \\
a_k &= \frac{1}{\pi} \left(\int_{-\pi}^0 x(t) \cos kt \, dt + \int_0^{\pi} x(t) \cos kt \, dt \right) \\
&= \frac{2((-1)^k - 1)}{\pi k^2} \\
b_k &= \frac{1}{\pi} \int_{-\pi}^{\pi} x(t) \sin kt \, dt = 0 \\
x(t) &= \frac{\pi}{2} - \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{\cos(2k-1)t}{(2k-1)^2}
\end{aligned} \tag{2.0.5}$$

Hence the correct answer is option (C).