

# EE3900 : Gate Assignment-1

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Download all python codes from

[https://github.com/Rahul27n/EE3900/blob/main/Gate\\_Assignment\\_1/Gate\\_Assignment\\_1.py](https://github.com/Rahul27n/EE3900/blob/main/Gate_Assignment_1/Gate_Assignment_1.py)

and latex-tikz codes from

[https://github.com/Rahul27n/EE3900/blob/main/Gate\\_Assignment\\_1/Gate\\_Assignment\\_1.tex](https://github.com/Rahul27n/EE3900/blob/main/Gate_Assignment_1/Gate_Assignment_1.tex)

and

$$\mathbf{X} = \begin{pmatrix} 12 \\ 2j \\ 0 \\ -2j \end{pmatrix} \quad (2.0.5)$$

Now we need to find  $\mathbf{X}_1$  satisfying the relation :

$$\mathbf{X}_1 = \mathbf{W}_{12}\mathbf{x}_1 \quad (2.0.6)$$

where

$$\mathbf{x}_1 = \begin{pmatrix} 3 \\ 0 \\ 0 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 3 \\ 0 \\ 0 \\ 4 \\ 0 \\ 0 \end{pmatrix} \quad (2.0.7)$$

## 1 QUESTION: Q.33 EC-GATE-2016

The Discrete Fourier Transform (DFT) of the 4 point sequence  $x[n] = \{3, 2, 3, 4\}$  is given by  $X[k] = \{12, 2j, 0, -2j\}$ . If  $X_1[k]$  is the DFT of the 12 point sequence  $x_1[n] = \{3, 0, 0, 2, 0, 0, 3, 0, 0, 4, 0, 0\}$ , the value of  $\left| \frac{X_1[8]}{X_1[11]} \right|$  is :

## 2 SOLUTION

The 4-point DFT matrix is given by:

$$\mathbf{W}_4 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^1 & \omega^2 & \omega^3 \\ 1 & \omega^2 & \omega^4 & \omega^6 \\ 1 & \omega^3 & \omega^6 & \omega^9 \end{pmatrix} \quad (2.0.1)$$

$$= \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{pmatrix} \quad (2.0.2)$$

where  $\omega = e^{\frac{-2\pi j}{4}} = -j$ . Now from the given information we can write:

$$\mathbf{X} = \mathbf{W}_4\mathbf{x} \quad (2.0.3)$$

where

$$\mathbf{x} = \begin{pmatrix} 3 \\ 2 \\ 3 \\ 4 \end{pmatrix} \quad (2.0.4)$$

and  $\mathbf{W}_{12}$  is the 12-point DFT matrix which is given by:

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \Omega & \Omega^2 & -j & -j\Omega & -j\Omega^2 & -1 & -\Omega & -\Omega^2 & j & j\Omega & j\Omega^2 \\ 1 & \Omega^2 & -j\Omega & -1 & -\Omega^2 & j\Omega & 1 & \Omega^2 & -j\Omega & -1 & -\Omega^2 & j\Omega \\ 1 & -j & -1 & j & 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & -j\Omega & -\Omega^2 & 1 & -j\Omega & -\Omega^2 & 1 & -j\Omega & -\Omega^2 & 1 & -j\Omega & -\Omega^2 \\ 1 & -j\Omega^2 & j\Omega & -j & -\Omega^2 & \Omega & -1 & j\Omega^2 & -j\Omega & j & \Omega^2 & -\Omega \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -\Omega & \Omega^2 & j & -j\Omega & j\Omega^2 & -1 & \Omega & -\Omega^2 & -j & j\Omega & -j\Omega^2 \\ 1 & -\Omega^2 & -j\Omega & 1 & -\Omega^2 & -j\Omega & 1 & -\Omega^2 & -j\Omega & 1 & -\Omega^2 & -j\Omega \\ 1 & j & -1 & -j & 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & j\Omega & -\Omega^2 & -1 & -j\Omega & \Omega^2 & 1 & j\Omega & -\Omega^2 & -1 & -j\Omega & \Omega^2 \\ 1 & j\Omega^2 & j\Omega & j & -\Omega^2 & -\Omega & -1 & -j\Omega^2 & -j\Omega & -j & \Omega^2 & \Omega \end{pmatrix}$$

where

$$\Omega = e^{\frac{-2\pi j}{12}} \quad (2.0.8)$$

$$= \frac{\sqrt{3} - j}{2} \quad (2.0.9)$$

We can express  $\mathbf{x}_1$  in terms of  $\mathbf{x}$  as follows :

$$\mathbf{x}_1 = \mathbf{A}\mathbf{x} \quad (2.0.10)$$

where

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.11)$$

$$\Rightarrow \mathbf{A} = (\mathbf{e}_1 \ \mathbf{e}_4 \ \mathbf{e}_7 \ \mathbf{e}_{10}) \quad (2.0.12)$$

where  $\mathbf{e}_1, \mathbf{e}_4, \mathbf{e}_7, \mathbf{e}_{10}$  represent the unit basis vectors in a subspace of  $\mathbb{R}^{12}$ . Now from (2.0.6) have :

$$\mathbf{X}_1 = (\mathbf{W}_{12}\mathbf{A})\mathbf{x} \quad (2.0.13)$$

Only the red coloured columns in  $\mathbf{W}_{12}$  give non-zero output when multiplied with  $\mathbf{A}$ . We can express the matrix  $\mathbf{W}_{12}$  as a block matrix in the following way:

$$\mathbf{W}_{12} = (\mathbf{c}_1 \ \mathbf{c}_2 \ \mathbf{c}_3 \ \mathbf{c}_4 \ \mathbf{c}_5 \ \mathbf{c}_6 \ \mathbf{c}_7 \ \mathbf{c}_8 \ \mathbf{c}_9 \ \mathbf{c}_{10} \ \mathbf{c}_{11} \ \mathbf{c}_{12}) \quad (2.0.14)$$

where  $\mathbf{c}_i$  is the  $i^{th}$  column matrix of  $\mathbf{W}_{12}$ . Now we have:

$$\begin{aligned} \mathbf{X}_1 &= (\mathbf{c}_1 \ \mathbf{c}_2 \ \mathbf{c}_3 \ \mathbf{c}_4 \ \mathbf{c}_5 \ \mathbf{c}_6 \ \mathbf{c}_7 \ \mathbf{c}_8 \ \mathbf{c}_9 \ \mathbf{c}_{10} \ \mathbf{c}_{11} \ \mathbf{c}_{12})\mathbf{A}\mathbf{x} \\ \Rightarrow \mathbf{X}_1 &= (\mathbf{c}_1 \ \mathbf{c}_4 \ \mathbf{c}_7 \ \mathbf{c}_{10})\mathbf{x} \end{aligned} \quad (2.0.15)$$

We can also express  $\mathbf{W}_4$  as block matrix as follows:

$$\mathbf{W}_4 = (\mathbf{w}_1 \ \mathbf{w}_2 \ \mathbf{w}_3 \ \mathbf{w}_4) \quad (2.0.16)$$

where  $\mathbf{w}_i$  is the  $i^{th}$  column matrix of  $\mathbf{W}_4$ .

We can importantly note that :

$$\mathbf{c}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \otimes \mathbf{w}_1 = \begin{pmatrix} \mathbf{w}_1 \\ \mathbf{w}_1 \\ \mathbf{w}_1 \end{pmatrix} \quad (2.0.17)$$

$$\mathbf{c}_4 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \otimes \mathbf{w}_2 = \begin{pmatrix} \mathbf{w}_2 \\ \mathbf{w}_2 \\ \mathbf{w}_2 \end{pmatrix} \quad (2.0.18)$$

$$\mathbf{c}_7 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \otimes \mathbf{w}_3 = \begin{pmatrix} \mathbf{w}_3 \\ \mathbf{w}_3 \\ \mathbf{w}_3 \end{pmatrix} \quad (2.0.19)$$

$$\mathbf{c}_{10} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \otimes \mathbf{w}_4 = \begin{pmatrix} \mathbf{w}_4 \\ \mathbf{w}_4 \\ \mathbf{w}_4 \end{pmatrix} \quad (2.0.20)$$

where  $\otimes$  represents the **Kronecker Product**.

Therefore from (2.0.15) we have:

$$\mathbf{X}_1 = \begin{pmatrix} \mathbf{w}_1 & \mathbf{w}_2 & \mathbf{w}_3 & \mathbf{w}_4 \\ \mathbf{w}_1 & \mathbf{w}_2 & \mathbf{w}_3 & \mathbf{w}_4 \\ \mathbf{w}_1 & \mathbf{w}_2 & \mathbf{w}_3 & \mathbf{w}_4 \end{pmatrix} \mathbf{x} \quad (2.0.21)$$

$$= \begin{pmatrix} \mathbf{X} \\ \mathbf{X} \\ \mathbf{X} \end{pmatrix} \quad (2.0.22)$$

$$= \begin{pmatrix} 12 \\ 2j \\ 0 \\ -2j \\ 12 \\ 2j \\ 0 \\ -2j \\ 12 \\ 2j \\ 0 \\ -2j \end{pmatrix} \quad (2.0.23)$$

Therefore we have:

$$\left| \frac{\mathbf{X}_1[8]}{\mathbf{X}_1[11]} \right| = \left| \frac{12}{-2j} \right| = |6j| = 6 \quad (2.0.24)$$

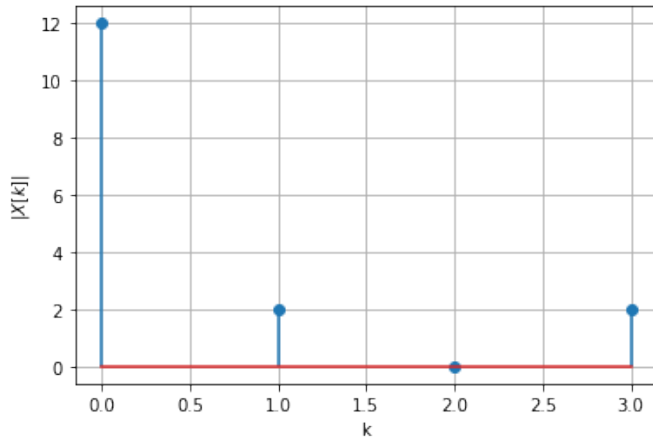


Fig. 0: Magnitude of  $\mathbf{X}[k]$  vs  $k$

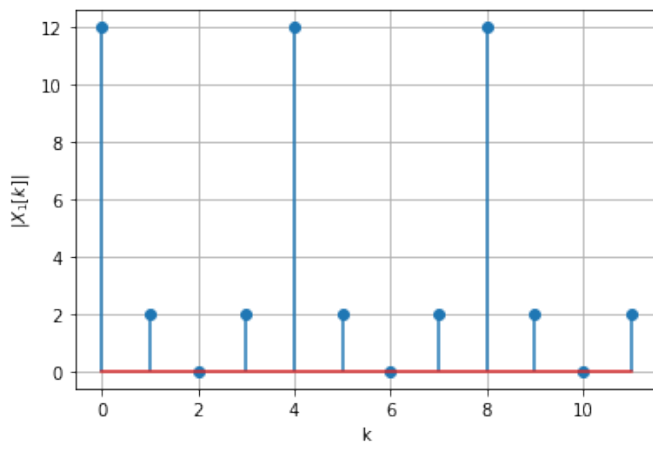


Fig. 0: Magnitude of  $\mathbf{X}_1[k]$  vs  $k$