

EE3900 : Gate-Assignment

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Download all python codes from

https://github.com/Rahul27n/EE3900/blob/main/Gate_Assignment/Gate_Assignment.py

and

and latex-tikz codes from

https://github.com/Rahul27n/EE3900/blob/main/Gate_Assignment/Gate_Assignment.tex

$$\mathbf{X} = \begin{pmatrix} 12 \\ 2j \\ 0 \\ -2j \end{pmatrix} \quad (2.0.5)$$

Now we need to find \mathbf{X}_1 satisfying the relation :

$$\mathbf{X}_1 = \mathbf{W}_{12}\mathbf{x}_1 \quad (2.0.6)$$

where

$$\mathbf{x}_1 = \begin{pmatrix} 3 \\ 0 \\ 0 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 3 \\ 0 \\ 0 \\ 4 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (2.0.7)$$

1 QUESTION: Q.33 EC-GATE-2016

The Discrete Fourier Transform (DFT) of the 4 point sequence $x[n] = \{3, 2, 3, 4\}$ is given by $X[k] = \{12, 2j, 0, -2j\}$. If $X_1[k]$ is the DFT of the 12 point sequence $x_1[n] = \{3, 0, 0, 2, 0, 0, 3, 0, 0, 4, 0, 0\}$, the value of $\left| \frac{X_1[8]}{X_1[11]} \right|$ is :

2 SOLUTION

The 4-point DFT matrix is given by:

$$\mathbf{W}_4 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^1 & \omega^2 & \omega^3 \\ 1 & \omega^2 & \omega^4 & \omega^6 \\ 1 & \omega^3 & \omega^6 & \omega^9 \end{pmatrix} \quad (2.0.1)$$

$$= \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{pmatrix} \quad (2.0.2)$$

where $\omega = e^{\frac{-2\pi j}{4}} = -j$. Now from the given information we can write:

$$\mathbf{X} = \mathbf{W}_4\mathbf{x} \quad (2.0.3)$$

where

$$\mathbf{x} = \begin{pmatrix} 3 \\ 2 \\ 3 \\ 4 \end{pmatrix} \quad (2.0.4)$$

and \mathbf{W}_{12} is the 12-point DFT matrix which is given by:

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \Omega & \Omega^2 & -j & -j\Omega & -j\Omega^2 & -1 & -\Omega & -\Omega^2 & j & j\Omega & j\Omega^2 \\ 1 & \Omega^2 & -j\Omega & -1 & -\Omega^2 & j\Omega & 1 & \Omega^2 & -j\Omega & -1 & -\Omega^2 & j\Omega \\ 1 & -j & -1 & j & 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & -j\Omega & -\Omega^2 & 1 & -j\Omega & -\Omega^2 & 1 & -j\Omega & -\Omega^2 & 1 & -j\Omega & -\Omega^2 \\ 1 & -j\Omega^2 & j\Omega & -j & -\Omega^2 & \Omega & -1 & j\Omega^2 & -j\Omega & j & \Omega^2 & -\Omega \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -\Omega & \Omega^2 & j & -j\Omega & j\Omega^2 & -1 & \Omega & -\Omega^2 & -j & j\Omega & -j\Omega^2 \\ 1 & -\Omega^2 & -j\Omega & 1 & -\Omega^2 & -j\Omega & 1 & -\Omega^2 & -j\Omega & 1 & -\Omega^2 & -j\Omega \\ 1 & j & -1 & -j & 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & j\Omega & -\Omega^2 & -1 & -j\Omega & \Omega^2 & 1 & j\Omega & -\Omega^2 & -1 & -j\Omega & \Omega^2 \\ 1 & j\Omega^2 & j\Omega & j & -\Omega^2 & -\Omega & -1 & -j\Omega^2 & -j\Omega & -j & \Omega^2 & \Omega \end{pmatrix}$$

where

$$\Omega = e^{\frac{-2\pi j}{12}} \quad (2.0.8)$$

$$= \frac{\sqrt{3} - j}{2} \quad (2.0.9)$$

Only the red coloured columns in \mathbf{W}_{12} give non-zero output when multiplied with \mathbf{x}_1 . We can express the matrix \mathbf{W}_{12} as a block matrix in the following way:

$$\mathbf{W}_{12} = (\mathbf{c}_1 \mathbf{c}_2 \mathbf{c}_3 \mathbf{c}_4 \mathbf{c}_5 \mathbf{c}_6 \mathbf{c}_7 \mathbf{c}_8 \mathbf{c}_9 \mathbf{c}_{10} \mathbf{c}_{11} \mathbf{c}_{12}) \quad (2.0.10)$$

where \mathbf{c}_i is the i^{th} column matrix of \mathbf{W}_{12} . Now we have:

$$\mathbf{X}_1 = (\mathbf{c}_1 \mathbf{c}_2 \mathbf{c}_3 \mathbf{c}_4 \mathbf{c}_5 \mathbf{c}_6 \mathbf{c}_7 \mathbf{c}_8 \mathbf{c}_9 \mathbf{c}_{10} \mathbf{c}_{11} \mathbf{c}_{12}) \begin{pmatrix} 3 \\ 0 \\ 0 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 3 \\ 0 \\ 0 \\ 4 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \mathbf{X}_1 = 3\mathbf{c}_1 + 2\mathbf{c}_4 + 3\mathbf{c}_7 + 4\mathbf{c}_{10} \quad (2.0.11)$$

We can also express \mathbf{W}_4 as block matrix as follows:

$$\mathbf{W}_4 = (\mathbf{w}_1 \mathbf{w}_2 \mathbf{w}_3 \mathbf{w}_4) \quad (2.0.12)$$

where \mathbf{w}_i is the i^{th} column matrix of \mathbf{W}_4 . Now from (2.0.3) we can write:

$$\begin{pmatrix} 12 \\ 2j \\ 0 \\ -2j \end{pmatrix} = (\mathbf{w}_1 \mathbf{w}_2 \mathbf{w}_3 \mathbf{w}_4) \begin{pmatrix} 3 \\ 2 \\ 3 \\ 4 \end{pmatrix} \quad (2.0.13)$$

$$\Rightarrow 3\mathbf{w}_1 + 2\mathbf{w}_2 + 3\mathbf{w}_3 + 4\mathbf{w}_4 = \begin{pmatrix} 12 \\ 2j \\ 0 \\ -2j \end{pmatrix} \quad (2.0.14)$$

We can importantly note that :

$$\mathbf{c}_1 = \begin{pmatrix} \mathbf{w}_1 \\ \mathbf{w}_1 \\ \mathbf{w}_1 \end{pmatrix} \quad (2.0.15)$$

$$\mathbf{c}_4 = \begin{pmatrix} \mathbf{w}_2 \\ \mathbf{w}_2 \\ \mathbf{w}_2 \end{pmatrix} \quad (2.0.16)$$

$$\mathbf{c}_7 = \begin{pmatrix} \mathbf{w}_3 \\ \mathbf{w}_3 \\ \mathbf{w}_3 \end{pmatrix} \quad (2.0.17)$$

$$\mathbf{c}_{10} = \begin{pmatrix} \mathbf{w}_4 \\ \mathbf{w}_4 \\ \mathbf{w}_4 \end{pmatrix} \quad (2.0.18)$$

Therefore from (2.0.11) and (2.0.14) we can write :

$$\mathbf{X}_1 = \begin{pmatrix} 3\mathbf{w}_1 + 2\mathbf{w}_2 + 3\mathbf{w}_3 + 4\mathbf{w}_4 \\ 3\mathbf{w}_1 + 2\mathbf{w}_2 + 3\mathbf{w}_3 + 4\mathbf{w}_4 \\ 3\mathbf{w}_1 + 2\mathbf{w}_2 + 3\mathbf{w}_3 + 4\mathbf{w}_4 \end{pmatrix} = \begin{pmatrix} 12 \\ 2j \\ 0 \\ -2j \\ 12 \\ 2j \\ 0 \\ -2j \\ 12 \\ 2j \\ 0 \\ -2j \end{pmatrix} \quad (2.0.19)$$

Therefore we have:

$$\left| \frac{\mathbf{X}_1[8]}{\mathbf{X}_1[11]} \right| = \left| \frac{12}{-2j} \right| = |6j| = 6 \quad (2.0.20)$$

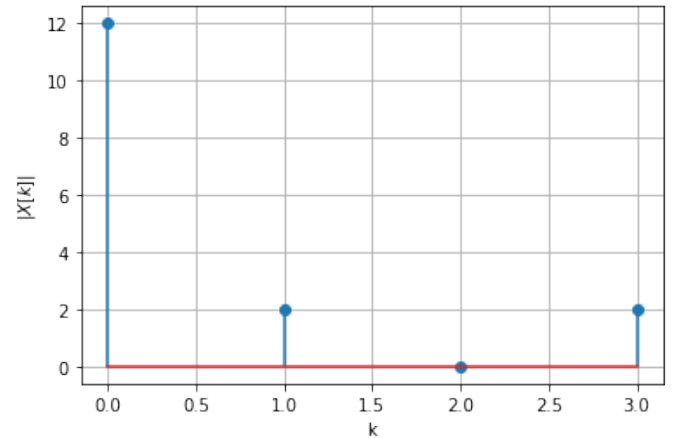


Fig. 0: Magnitude of $\mathbf{X}[k]$ vs k

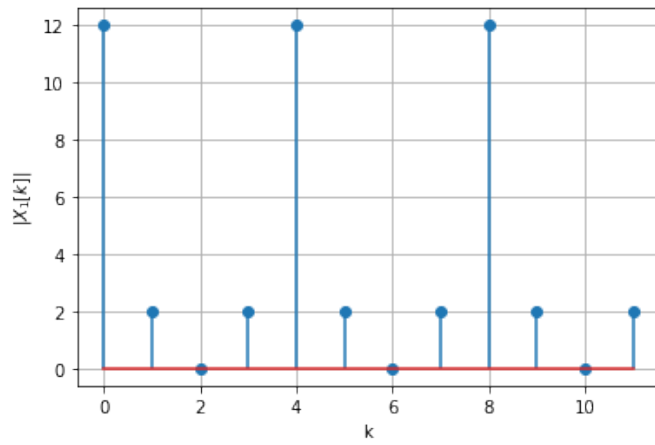


Fig. 0: Magnitude of $X_1[k]$ vs k