

11.9.3

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QUESTION:

1. How many terms of G.P. $3, 3^2, 3^3, \dots$ are needed to give the sum 120 ?

SOLUTION:

Parameter	Description	Value
n	No. of terms in the G.P	4
$x(0)$	first term in the G.P	3
r	common ratio in the G.P	3
$x(n)$	n^{th} term in G.P	$x(0)r^n u(n)$

TABLE I
VARIABLES

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \quad |z| > |r| \quad (1)$$

$$= \frac{3}{1 - 3z^{-1}} \quad (2)$$

$$U(z) = \frac{1}{1 - z^{-1}} \quad |z| > 1 \quad (3)$$

$$y(n) = x(n) * u(n) \quad (4)$$

$$Y(z) = X(z)U(z) \quad (5)$$

$$= \left(\frac{3}{1 - 3z^{-1}} \right) \left(\frac{1}{1 - z^{-1}} \right) \quad |z| > 3 \quad (6)$$

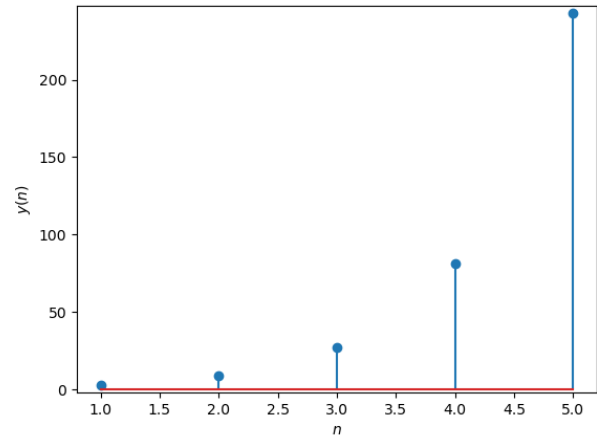


Fig. 1. Stem plot of y(n)

Using Contour integration

$$y(n) = \frac{1}{2\pi j} \oint_C \frac{3z^2}{(z-1)(z-3)} z^{n-1} dz \quad (7)$$

$$= \frac{1}{2\pi j} \oint_C \frac{3}{2} \left(\frac{1}{z-3} - \frac{1}{z-1} z^{n+1} \right) dz \quad (8)$$

$$120 = \frac{3}{2} (3^n - 1) \quad (9)$$

$$n = 4 \quad (10)$$