11.9.3

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QUESTION:

1.How many terms of G.P.3,3^2,3^3, ... are needed to give the sum 120 ?

SOLUTION:

Parameter	Description	Value
n	No. of terms in the G.P	4
$x(0)$	first term in the G.P	3
r	common ratio in the G.P	3
$x(n)$	n^{th} term in G.P	$x(0)r^n u(n)$

TABLE I
VARIABLES

$$x(z) = \frac{x(0)}{1 - rz^{-1}} \qquad |z| > |r| \tag{1}$$

$$= \frac{3}{1 - 3z^{-1}} \tag{2}$$

$$U(z) = \frac{1}{1 - z^{-1}} \qquad |z| > 1 \tag{3}$$

$$y(n) = x(n) * u(n) \tag{4}$$

$$Y(z) = X(z)U(z) \tag{5}$$

$$= \left(\frac{3}{1 - 3z^{-1}}\right)\left(\frac{1}{1 - z^{-1}}\right) \qquad |z| > 3 \tag{6}$$

Using Contour integration

$$y(n) = \frac{1}{2\pi j}\oint_C \frac{3z^2}{(z-1)(z-3)} z^{n-1} dz \tag{7}$$

$$= \frac{1}{2\pi j}\oint_C \frac{3}{2}\left(\frac{1}{z-3} - \frac{1}{z-1} z^{n+1}\right) dz \tag{8}$$

$$= \frac{3}{2}\left(\left(\lim_{z\to 3} \frac{z^{n+1}}{z-3}(z-3)\right) - \left(\lim_{z\to 1} \frac{z^{n+1}}{z-1}(z-1)\right)\right) \tag{9}$$

$$= \frac{3}{2}(3^n - 1) \tag{10}$$

$$120 = \frac{3}{2}(3^n - 1) \tag{11}$$

$$n = 4 \tag{12}$$

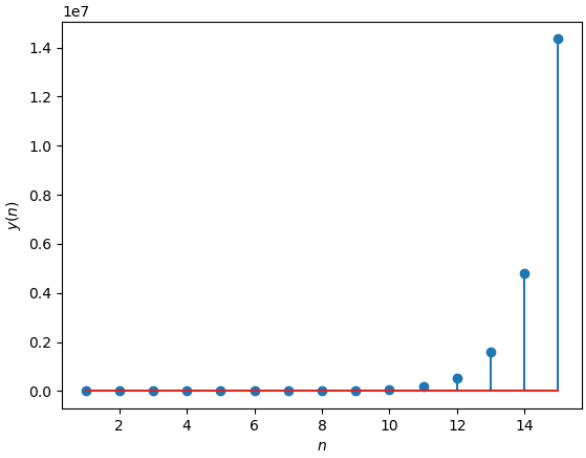

Fig. 1. Stem plot of y(n)