Notes: (1) These questions require thought, but do not require long answers. Please be as concise as possible. (2) For problems that require programming, please include in your submission a printout of your code (with comments) and any figures that you are asked to plot. You also need to submit your code and figures electronically to BLS before the due date. (3) For programming assignment, you should implement your own version of the main algorithm, and should not call a built-in library function.

Linear least squares regression:

(a) Fitting polynomials with linear least squares. Given an input variable x_i and a target variable y_i, $i = 1, \cdots, m$, our goal is to fit a polynomial function of degree d:

$$f(x) = a_d x^d + a_{d-1} x^{d-1} + \cdots + a_1 x + a_0,$$

to minimize $\frac{1}{m} \sum_{i=1}^{m} (y_i - f(x_i))^2$. Write this problem in the form of a linear least squares regression, specifying the matrix and vectors in the objective function.

(b) On the BLS system, the file `hw1data1.mat` contain the input variable `x` and target variable `y`, respectively. Implement least squares regression to fit polynomials of degree $d = 1, 3, 5$ and 7 to this data set. Your code should output (i) the parameters of the polynomial, i.e., $a_0, a_1, \cdots, a_d$ and (ii) the optimal fitting error $\frac{1}{m} \sum_{i=1}^{m} (y_i - f(x_i))^2$.

(c) Plot the each of the regression results in a separate figure, specifying the order of the model in the title of the figure (using the `title` function). In each figure, first plot the raw data, your axes should be x and y, corresponding to the input and target variables (using the `plot` function). Then in the same figure, plot the output of the regression function obtained for the input data using a different symbol.