Linear Least Squares Regression:

1] Fitting polynomials with linear least squares:

↳ input variable = x_i
Target variable = y_i } $i = 1, 2, ---, m.$

↳ Polynomial $f(x)$

↳ $f(x) = a_d x^d + a_{d-1} x^{d-1} + --- + a_1 x + a_0$

$$\therefore f(x) = a_0 + a_1 x^1 + a_2 x^2 + --- + a_d x^d \quad \text{①}$$

We use ① to minimize,

$$\frac{1}{m} \sum_{i=1}^{m} (y_i - f(x_i))^2$$

$$= \frac{1}{m} \sum_{i=1}^{m} \left(y_i - (a_0 + a_1 x_i + a_2 x_i^2 + --- + a_d x_i^d) \right)^2 \quad \text{②}$$

↳ The partial derivative of above is given as;

$$\frac{\partial}{\partial a_0} = \frac{-2}{m} \sum_{i=1}^{m} \left(y_i - (a_0 + a_1 x_i + --- + a_d x_i^d) \right) = 0$$

$$\frac{\partial}{\partial a_1} = \frac{-2}{m} \sum_{i=1}^{m} \left(y_i - (a_0 + a_1 x_i + --- + a_d x_i^d) \right) x_i = 0$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\frac{\partial}{\partial a_d} = \frac{-2}{m} \sum_{i=1}^{m} \left(y_i - (a_0 + a_1 x_i + --- + a_d x^d) \right) x_i^d = 0$$

↳ we can write the above equation as,

- $$\sum_{i=1}^{m} y_i - \sum_{i=1}^{m} (a_0 + a_1 x_i + \dots + a_d x_i^d) = 0$$

$$\Rightarrow a_0 m + a_1 \sum x_i + \dots + a_d \sum x_i^d = \sum y_i$$

- $$\sum_{i=1}^{m} y_i x_i - \sum_{i=1}^{m} (a_0 + a_1 x_i + \dots + a_d x_i^d) x_i = 0$$

$$\Rightarrow a_0 \sum x_i + a_1 \sum x_i^2 + \dots + a_d \sum x_i^{d+1} = \sum y_i x_i$$

- $\vdots$

$$\sum_{i=1}^{m} y_i x_i^d - \sum_{i=1}^{m} (a_0 + a_1 x_i + \dots + a_d x_i^d) x_i^d = 0$$

$$\Rightarrow a_0 \sum x_i^d + a_1 \sum x_i^{d+1} + \dots + a_d \sum x_i^{d+d} = \sum y_i x_i^d$$

↳ Writing Above All eqn in Matrix Form:

$$\begin{bmatrix} m & \sum x_i & \cdots & \sum x_i^d \\ \sum x_i & \sum x_i^2 & \cdots & \sum x_i^{d+1} \\ \vdots & \vdots & & \vdots \\ \sum x_i^d & \sum x_i^{d+1} & \cdots & \sum x_i^{2d} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_d \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum y_i x_i \\ \vdots \\ \sum y_i x_i^d \end{bmatrix}$$

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we can write above Matrix is this form also, for least square fitting..

$$\begin{bmatrix} 1 & x_1 & \cdots & x_1^d \\ 1 & x_2 & \cdots & x_2^d \\ \vdots & \vdots & & \vdots \\ 1 & x_m & \cdots & x_m^d \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_d \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}$$

here,

x is our input variable (given)

y is target variable

& $(a_0, a_1, \ldots, a_d)$ is polynomial coefficient.

The eqn for polynomial,

$\hookrightarrow \quad y = Xa$

To solve this, we will multiply both side by X^T

$\Rightarrow X^T y = X^T X a$

$\Rightarrow (X^T X)^{-1} X^T y = a$

$\Rightarrow \underline{a = (X^T X)^{-1} X^T y}$

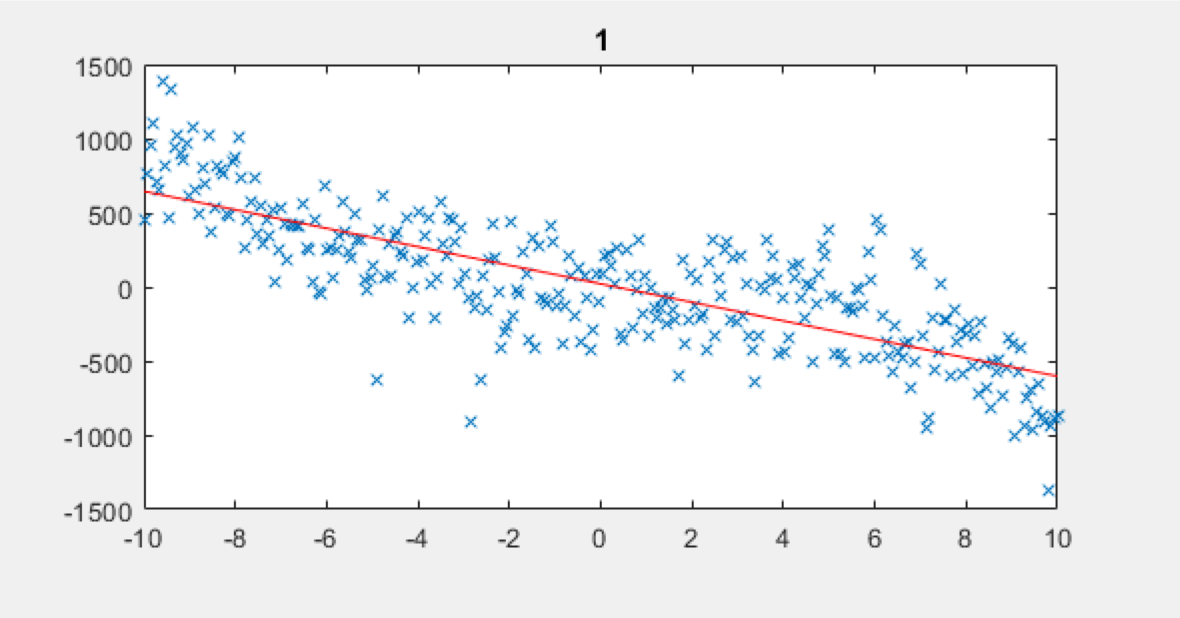
- using above equation we can find the co-efficient values of given polynomial of d-degree.
- & the above eqn is solution for linear least square regression.

2] Implementation of least square regression to fit polynomial of degree d = 1, 3, 5, 7.

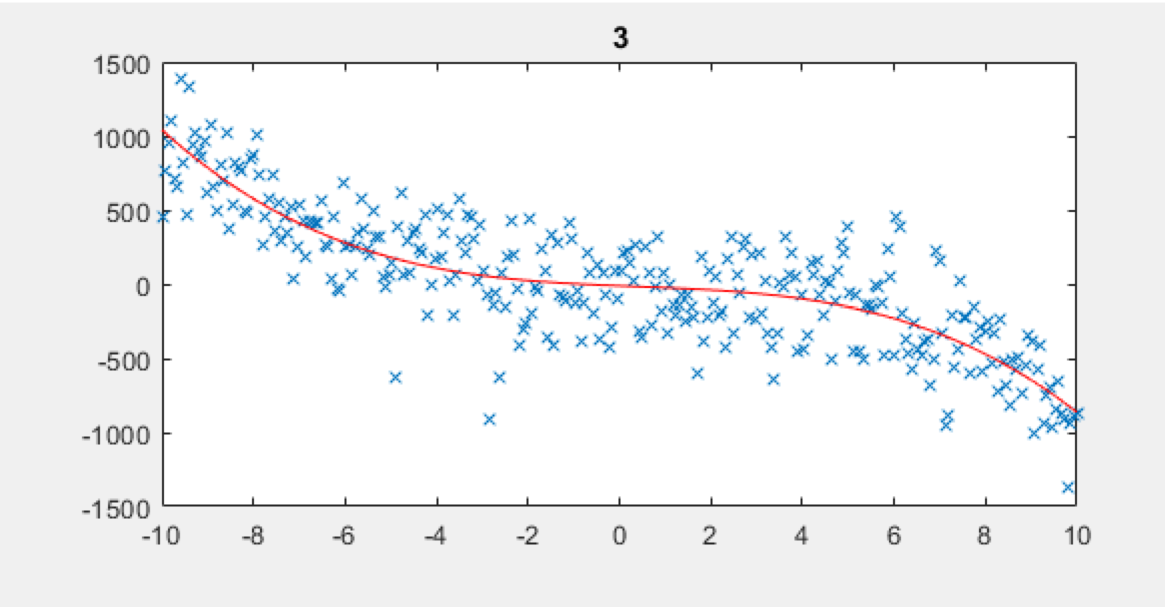
```
load('hw1data1.mat')
input_var = x';
target_var = y';
% disp(input_var);
% disp(target_var);

len = length(target_var);
X = [ones(len, 1), input_var];
calculation(X, input_var, target_var, 1)

X = [ones(len, 1), input_var, input_var.^2, input_var.^3];
calculation(X, input_var, target_var, 3)

X = [ones(len, 1), input_var, input_var.^2, input_var.^3, input_var.^4,
                                                           input_var.^5];
calculation(X, input_var, target_var, 5)

X = [ones(len, 1), input_var, input_var.^2, input_var.^3, input_var.^4,
                               input_var.^5, input_var.^6, input_var.^7];
calculation(X, input_var, target_var, 7)

function calculation(matx, input_var, target_var, order)
    parameters = inv(matx'*matx) * matx' * target_var;
    y = matx * parameters;
    disp(parameters)
    y_2 = sum((target_var - y).^2) ;
    disp(y_2)
    plotting(input_var, target_var, y, order)
end

function plotting(input_var1, target_var1, y1, degree)
    sub = floor(degree/2) + 1;
    subplot(2,2,sub)
    plot(input_var1, target_var1, 'x')
    hold on
    plot(input_var1, y1, 'red')
    title(degree)
end
```

3] Plotting of regression results with input data and target variable.

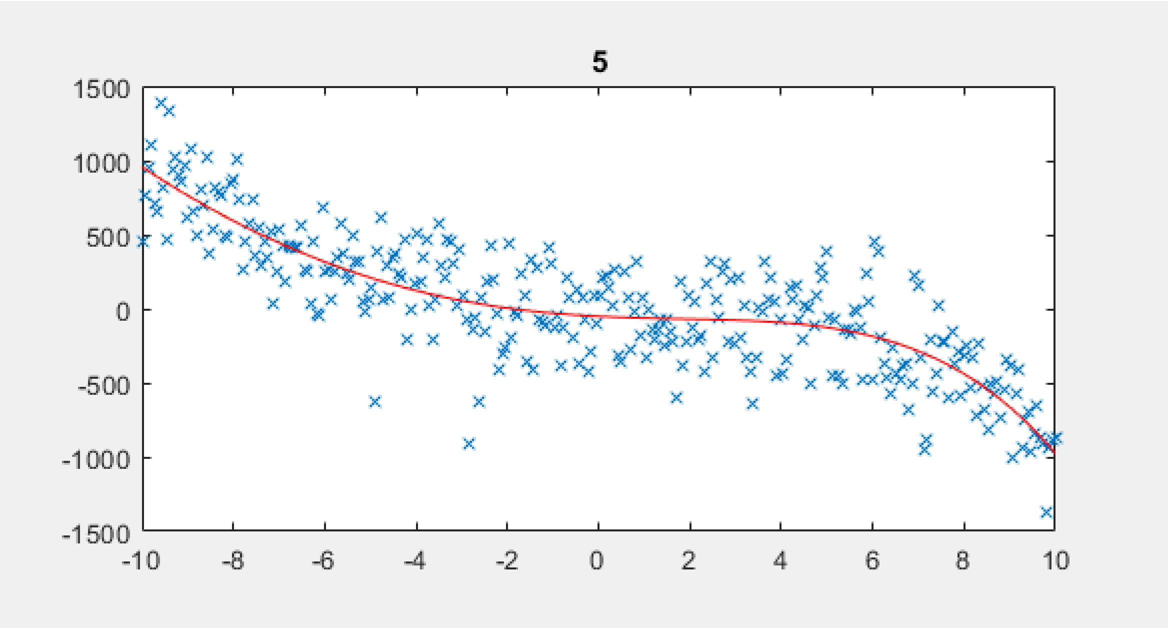
a) Model with degree = 1



b) Model with degree = 3



c) Model with degree = 5



d) Model with degree = 7