DIFFERENTIAL EQUATIONS

CHAPTER 10

DIFFERENTIAL EQUATIONS

DIFFERENTIAL EQUATION:

An equation containing independent variables, dependent variables and derivatives of dependent variables with respect to independent variables is called a differential equation.

ORDINARY DIFFERENTIAL EQUATION:

A differential equation involving derivative or derivatives of the dependent variable with respect to only one independent variable is called ordinary differential equation.

ORDER OF A DIFFERENTIAL EQUATION:

The order of a differential equation is the order of the highest order derivative appearing in the equation.

DEGREE OF A DIFFERENTIAL EQUATION:

The highest power (positive integral index) of the highest order derivative involved in a differential equation in terms of polynomial form, is called degree of a differential equation.

FORMATION OF A DIFFERENTIAL EQUATION:

STEP 1 Write the given equation involving independent variable x(say), dependent variable y(say) and the arbitrary constants.

STEP 2 Obtain the number of arbitrary constants in step 1. Let there be n arbitrary constants.

STEP 3 Diffentiate the relation in step 1 n times with respect to x.

STEP 4 Eliminate arbitrary constants with the help of n equations involving differential coefficients obtained in step 3 and an equation in step 1. The equation so obtained is the desired differential equation.

SOLUTION OF A DIFFERENTIAL EQUATION:

A relation between the dependent and independent variables that satisfies the differential equation is called a solution of that differential equation.

GENERAL SOLUTION OF A DIFFERENTIAL EQUATION:

A solution that contains as many arbitrary constants as the order of the differential equation is called general solution of that differential equation.

PARTICULAR SOLUTION OF A DIFFERENTIAL EQUATION:

A solution obtained by giving particular values to the arbitrary constants in the general solution of the differential equation is called a particular solution of that differential equation.

DIFFERENTIAL EQUATION IN VARIABLE SEPARABLE FORM:

Suppose a first order and first-degree differential equation, $\frac{dy}{dx} = f(x, y)$, is such that $f(x, y)$ can be written as $g(x).h(y)$

Then, expressed the given differential equation as $\frac{dy}{dx} = h(y).g(x)$

If $h(y) \neq 0$, then separating the variables, equation can be written as $\frac{1}{h(y)} dy = g(x)dx$

On integrating both sides, we get the required solution of given differential equation.

HOMOGENEOUS DIFFERENTIAL EQUATION:

If a first-order first degree differential equation is expressible in the form $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$

where $f(x, y)$ and $g(x, y)$ are homogeneous functions of the same degree, then it is called a homogeneous differential equation.

SOLUTION OF HOMOGENEOUS DIFFERENTIAL EQUATION:

STEP 1 Put the differential equation in the form $\frac{dy}{dx} = \frac{\varphi(x,y)}{\omega(x,y)}$

STEP 2 Put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in the equation in step 1 and cancel out x from the right hand side. The equation reduces to the form $v + x\frac{dv}{dx} = F(v)$.

STEP 3 Shift v on RHS and separate the variables v and x.

STEP 4 Integrate both sides to obtain the solution in terms of v and x.

Step 5 Replace v by $\frac{y}{x}$ in the solution obtained in step 4 to obtain the solution in terms of x and y.

LINEAR DIFFERENTIAL EQUATION:

A first order and first-degree differential equation in which the degree of dependent variable and its derivative is one and they do not get multiplied together, is called a linear differential equation. There are two types of linear differential equations.

TYPE 1 If differential equation is of the form $\frac{dy}{dx} + Py = Q$, where P and Q are constants or functions of x. Then, its solution is $y.(I.F.) = \int Q \times (I.F.)dx + C$ where I.F. $= e^{\int P\,dx}$.

TYPE 2 If differential equation is of the form $\frac{dy}{dx} + Px = Q$, where P and Q are constants or functions of y. Then, its solution is $x.(I.F.) = \int Q \times (I.F.)dy + C$ where I.F. $= e^{\int P\,dy}$.

Differential Equations

Definition

An equation involving derivatives of the dependent variable with respect to independent variable (variables) is called a differential equation. If there is only one independent variable, then we call it as an ordinary differential equation. eg: $2\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)=0$.

Order of a Differential Equation

It is the order of the highest order derivative occurring in the differential equation
eg: The order of $\frac{dy}{dx}=e^x$ is one and order of $\frac{d^2y}{dx^2}+x=0$ is two.

Degree of a Differential Equation

It is defined if the differential equation is a polynomial equation in its derivatives, and is defined as the highest power (positive integer only) of the highest order derivative.

eg: The degree of $\left(\frac{d^2y}{dx^2}\right)^3+\frac{dy}{dx}=0$ is three

Order and degree (if defined) of a differential equation are always positive integers.

Solution of a Differential Equation

A function which satisfies the given differential equation is called its solution. The solution which contains as many arbitrary constants as the order of the differential equation is called a general solution and the solution free from arbitrary constants is called particular solution.

eg: $y=e^x+1$ is a solution of $y''-y'=0$.
Since $y'=e^x$ and $y''=e^x \Rightarrow y''-y'=e^x-e^x=0$.

(Order and family of curves)

The order of a differential equation representing a family of curves is same as the number of arbitrary constants present in the equation corresponding to the family of curves.
eg: Let the family of curves be
$y=mx$, $m=$ constant, then, $y'=m$
$y=y'x \Rightarrow y=\frac{dy}{dx}x \Rightarrow x\frac{dy}{dx}-y=0$.

Homogeneous Differential Equations

A differential equation which can be expressed in the form $\frac{dy}{dx}=f(x,y)$ or $\frac{dx}{dy}=g(x,y)$,
where, $f(x,y)$ and $g(x,y)$ are homogeneous functions of degree zero is called a homogenous differential equation
eg : $(x^2+xy)dy=(x^2+y^2)dx$
To solve this, we substitute $y=vx$, and $\frac{dy}{dx}=v+x\frac{dv}{dx}$

Linear Differential Equations

The differential equation of the form $\frac{dy}{dx}+Py=Q$, where P, Q are constants or functions of 'x' only is called a first order linear differential equation. Its solution is given as $ye^{\int P.dx}=\int Q.e^{\int P.dx}dx+c$. $eg: \frac{dy}{dx}+3y=2x$ has solution $ye^{\int 3.dx}=\int 2x.e^{\int 3.dx}dx+c \Rightarrow ye^{3x}=2\int xe^{3x}dx+c$.

Variable Separation Method

It is used to solve such an equation in which variables can be separated completely.
eg : $ydx = xdy$ can be solved as $\frac{dx}{x}=\frac{dy}{y}$;
Integrating both sides
$\log x=\log y+\log c \Rightarrow \frac{x}{y}=c \Rightarrow x=cy$,
is the solution.

Trace the Mind Map
- First Level
- Second Level
- Third Level

PRACTICE QUESTIONS

1. The order and degree of the differential equation of the family of parabolas having vertex at origin and axis along positive x-axis is are
   a) 1,1
   b) 1,2
   c) 2,1
   d) 2,2
2. The general solution of the differential equation $e^x dy + (ye^x + 2x)dx = 0$ is
   a) $xe^y + x^2 = C$
   b) $xe^y + y^2 = C$
   c) $ye^x + x^2 = C$
   d) $ye^y + x^2 = C$
3. The general solution of the differential equation of the type $\frac{dy}{dx} + P_1 x = Q_1$ is
   a) $ye^{\int P_1 dy} = \int \{Q_1 e^{\int P_1 dy}\} dy + C$
   b) $ye^{\int P_1 dx} = \int \{Q_1 e^{\int P_1 dx}\} dx + C$
   c) $xe^{\int P_1 dy} = \int \{Q_1 e^{\int P_1 dy}\} dy + C$
   d) $xe^{\int P_1 dx} = \int \{Q_1 e^{\int P_1 dx}\} dx + C$
4. The general solution of the differential equation $\frac{ydx - xdy}{y} = 0$, is
   a) $xy = C$
   b) $x = Cy^2$
   c) $y = Cx$
   d) $y = Cx^2$

5. The integrating factor of the differential equation $(1-y^2)\frac{dx}{dy}+yx=ay\ (-1<y<1)$ is

   a) $\frac{1}{y^2-1}$

   b) $\frac{1}{\sqrt{y^2-1}}$

   c) $\frac{1}{1-y^2}$

   d) $\frac{1}{\sqrt{1-y^2}}$

6. The integrating factor of the differential equation $x\frac{dy}{dx}-y=2x^2$

   a) e^{-x}

   b) e^{-y}

   c) 1/x

   d) x

7. Which of the following is a homogeneous differential equation?

   a) $(4x+6y+5)dy-(3y+2x+4)dx=0$

   b) $xydx-(x^3+y^3)dy=0$

   c) $(x^3+2y^2)dx+2xydy=0$

   d) $y^2dx+(x^2-xy-y^2)dy=0$

8. A homogeneous differential equation of the form $\frac{dx}{dy}=h\left(\frac{x}{y}\right)$ can be solved by making the substitution

   a) $y=vx$

   b) $v=yx$

   c) $x=vy$

   d) $x=v$

9. The general solution of the differential equation $\frac{dy}{dx}=e^{x+y}$, is

   a) $e^x+e^{-y}=C$

   b) $e^x+e^y=C$

   c) $e^{-x}+e^y=C$

   d) $e^{-x}+e^{-y}=C$

10. Which of the following differential equations has y=x as one of its particular solution?

a) $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = x$

b) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = x$

c) $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0$

d) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = 0$

11. The solution of the differential equation $(1+y^2)\tan^{-1}x\ dx + y(1+x^2)dy = 0$ is

a) $\log\left(\frac{\tan^{-1}x}{x}\right) + y(1+x^2) = c$

b) $\log(1+y^2) + (\tan^{-1}x)^2 = c$

c) $\log(1+x^2) + \log(\tan^{-1}y) + c$

d) $(\tan^{-1}x)(1+y^2) + c = 0$

12. The solution of $dy = \cos x\,(2 - y\operatorname{cosec} x)dx$, where $y = \sqrt{2}$, when $x = \pi/4$ is

a) $y = \sin x + \frac{1}{2}\operatorname{cosec} x$

b) $y = \tan(x/2) + \cot(x/2)$

c) $y = \left(1/\sqrt{2}\right)\sec(x/2) + \sqrt{2}\cos(x/2)$

d) None of the above

13. The slope of a curve at any point is the reciprocal of twice the ordinate at the point and it passes through the point (4, 3). The equation of the curve is

a) $x^2 = y + 5$

b) $y^2 = x - 5$

c) $y^2 = x + 5$

d) $x^2 = y - 5$

14. The equation of the curve satisfying the differential equation $y_2(x^2+1) = 2xy_1$ passing through the point (0,1) and having slope of tangent at $x = 0$ as 3 is

a) $y = x^3 + 3x + 1$

b) $y = x^3 - 3x + 1$

c) $y = x^2 + 3x + 1$

d) $y = x^2 - 3x + 1$

15. A function $y = f(x)$ has a second order derivative $f'' = 6(x-1)$. If its graph passes through the point (2,1) and at point the tangent to the graph is $y = 3x - 5$ then the function is

   a) $(x-1)^2$
   b) $(x-1)^3$
   c) $(x+1)^3$
   d) $(x+1)^2$

16. The degree of the differential equations $x = 1 + \left(\frac{dy}{dx}\right) + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{1}{3!}\left(\frac{dy}{dx}\right)^3 + \cdots$

   a) 3
   b) 2
   c) 1
   d) Not defined

17. The equation of the curve in which subnormal varies as the square of the ordinate is (λ is constant of proportionality)

   a) $y = C\, e^{2\lambda x}$
   b) $y = C\, e^{\lambda x}$
   c) $\frac{y^2}{2} + \lambda x = C$
   d) $y^2 + \lambda x^2 = C$

18. The order of the differential equation whose general solution is given by $y = (c_1 + c_2)\cos(x + c_3) - c_4 e^{x + c_5}$ where c_1, c_2, c_3, c_4 and c_5 are arbitrary constants is

   a) 5
   b) 6
   c) 3
   d) 2

19. The solution of $\frac{dy}{dx} = \cos(x+y) + \sin(x+y)$ is

   a) $\log\left[1 + \tan\left(\frac{x+y}{2}\right)\right] + c = 0$
   b) $\log\left[1 + \tan\left(\frac{x+y}{2}\right)\right] = x + c$
   c) $\log\left[1 - \tan\left(\frac{x+y}{2}\right)\right] = x + c$
   d) None of these

20. The function $f(\theta) = \frac{d}{d\theta}\int_0^{\theta} \frac{dx}{1-\cos\theta\cos x}$ satisfies the differential equation

a) $\frac{df}{d\theta} + 2f(\theta) = 0$

b) $\frac{df}{d\theta} - 2f(\theta) = 0$

c) $\frac{df}{d\theta} - 2f(\theta) = \tan\theta$

d) $\frac{df}{d\theta} + 2f(\theta)\cot\theta = 0$

21. If $y = f(x)$ is the equation of the curve an its differential equation is given by $\frac{dy}{dx} = \frac{x+2}{y+3}$, then the equation of the curve, if it passes through (2, 2), is

a) $x^2 - y^2 + 4x - 6y + 4 = 0$

b) $x^2 - y^2 + 4x + 6y = 0$

c) $x^2 - y^2 - 4x - 6y = 0$

d) $x^2 - y^2 - 4x - 6y - 4 = 0$

22. The order and degree of the following differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{5/2} = \frac{d^3y}{dx^3}$ are respectively

a) 3,2

b) 3,10

c) 2,3

d) 3,5

23. $y = ae^{mx} + be^{-mx}$ satisfies which of the following differential equation?

a) $\frac{dy}{dx} - my = 0$

b) $\frac{dy}{dx} + my = 0$

c) $\frac{d^2y}{dx^2} + m^2y = 0$

d) $\frac{d^2y}{dx^2} - m^2y = 0$

24. The solution of the differential equation $\frac{x+y\frac{dy}{dx}}{y-x\frac{dy}{dx}} = x^2 + 2y^2 + \frac{y^4}{x^2}$ is

a) $\frac{y}{4} + \frac{1}{x^2+y^2} = c$

b) $\frac{y}{x} - \frac{1}{x^2+y^2} = c$

c) $\frac{x}{y} - \frac{1}{x^2+y^2} = c$

d) None of these

25. If c_1, c_2, c_3, c_4, c_5 and c_6 are constants, then the order of the differential equation whose general solution is given by

$y = c_1 \cos(x + c_2) + c_3 \sin(x + c_4) + c_5 e^x + c_6$ is

a) 6

b) 5

c) 4

d) 3

26. The solution of the differential equation $\left\{\frac{1}{x} - \frac{y^2}{(x-y)^2}\right\} dx + \left\{\frac{x^2}{(x-y)^2} - \frac{1}{y}\right\} dy = 0$ is

a) In $\left|\frac{x}{y}\right| + \frac{xy}{(x-y)} = c$

b) In $|xy| + \frac{xy}{(x-y)} = c$

c) $\frac{xy}{(x-y)} = ce^{\frac{x}{y}}$

d) $\frac{xy}{(x-y)} = ce^{xy}$

27. A curve $y = f(x)$ passes through the point $P(1, 1)$. The normal to the curve at point P is $a(y-1) + (x-1) = 0$. If the slope of the tangent at any point on the curve is proportional to the ordinate at that point, then the equation of the curve is

a) $y = e^{ax} - 1$

b) $y = e^{ax} + 1$

c) $y = e^{ax} - a$

d) $y = e^{a(x-1)}$

28. The differential equation of the family of parabolas with focus at the origin and the x-axis as axis, is

a) $y\left(\frac{dy}{dx}\right)^2 + 4x\frac{dy}{dx} = 4y$

b) $-y\left(\frac{dy}{dx}\right)^2 = 2x\frac{dy}{dx} - y$

c) $y\left(\frac{dy}{dx}\right)^2 + y = 2xy\frac{dy}{dx}$

d) $y\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx} + y = 0$

29. The real value of n for which the substitution $y = u^n$ will transform the differential equation $2x^4y\frac{dy}{dx} + y^4 = 4x^6$ into a homogenous equation is

a) 1/2

b) 1

c) 3/2

d) 2

30. The equation of the curve for which the square of the ordinate is twice the rectangle contained by the abscissa and the intercept of the normal on x-axis and passing through (2, 1) is

a) $x^2 + y^2 - x = 0$

b) $4x^2 + 2y^2 - 9y = 0$

c) $2x^2 + 4y^2 - 9x = 0$

d) $4x^2 + 2y^2 - 9x = 0$

31. The differential equation of all straight lines touching the circle $x^2 + y^2 = a^2$ is

a) $\left(y - \frac{dy}{dx}\right)^2 = a^2\left[1 + \left(\frac{dy}{dx}\right)^2\right]$

b) $\left(y - x\frac{dy}{dx}\right)^2 = a^2\left[1 + \left(\frac{dy}{dx}\right)^2\right]$

c) $\left(y - x\frac{dy}{dx}\right) = a^2\left[1 + \frac{dy}{dx}\right]$

d) $\left(y - \frac{dy}{dx}\right) = a^2\left[1 - \frac{dy}{dx}\right]$

32. If $x = A\cos 4t + B\sin 4t$, then $\frac{d^2x}{dt^2}$ is equal to

a) $-16x$

b) $16x$

c) x

d) $-x$

33. The solution of the differential equation $y_1y_3 = 3y_2^2$ is

a) $x = A_1y^2 + A_2y + A_3$

b) $x = A_1y + A_2$

c) $x = A_1y^2 + A_2y$

d) None of these

34. A particle starts at the origin and moves along the x-axis in such a way that its velocity at the point $(x, 0)$ is given by the formula $\frac{dx}{dt} = \cos^2 \pi x$. Then, the particle never reaches the point on

a) $x = \frac{1}{4}$

b) $x = \frac{3}{4}$

c) $x = \frac{1}{2}$

d) $x = 1$

35. The order and degree of the differential equation $\sqrt{y + \frac{d^2y}{dx^2}} = x + \left(\frac{dy}{dx}\right)^{3/2}$ are

a) 2,2

b) 2,1

c) 1,2

d) 2,3

36. The differential equation of all circles in the first quadrant which touch the coordinate axes is of order

a) 1

b) 2

c) 3

d) None of these

37. The order and degree of the differential equation $\frac{d^2y}{dx^2} = \sqrt[3]{1 - \left(\frac{dy}{dx}\right)^4}$ are respectively

a) 2,3

b) 3,2

c) 2,4

d) 2,2

38. Observe the following statements.

I. IF $dy + 2xy\, dx = 2e^{-x^2}\, dx$, then $ye^{x^2} = 2x + c$

II. II.IF $ye^{x^2} - 2x = c$, then $dx = \left(2e^{-x^2} - 2xy\right)dy$

Which is/are correct statements?

a) Both I and II are true

b) Neither I nor II is true

c) I is true, II is false

d) I is false, II is true

39. A particles moves in a straight line with a velocity given by $\frac{dx}{dt} = x + 1$ (x is the distance described). The time taken by a particle to traverse a distance of 99 metres is

a) $\log_{10} e$

b) $2 \log_e 10$

c) $2 \log_{10} e$

d) $\frac{1}{2} \log_{10} e$

40. The differential equation of the family $y = ae^x + bx\, e^x + cx^2\, e^x$ of curves, where a, b, c are arbitrary constants, is

a) $y''' + 3y'' + 3y' + y = 0$

b) $y''' + 3y'' - 3y' - y = 0$

c) $y''' - 3y'' - 3y' + y = 0$

d) $y''' - 3y' + 3y' - y = 0$

41. The difference equation of the family of circles with fixed radius r and with centre on y-axis is

a) $y^2(1+y_1^2)=r^2y_1^2$

b) $y^2=r^2y_1+y_1^2$

c) $x^2(1+y_1^2)=r^2y_1^2$

d) $x^2=r^2y_1+y_1^2$

42. The solution of the differential equation $x\,dy-y\,dx=\sqrt{x^3+y^2}\,dx$, is

a) $x+\sqrt{x^2+y^2}=Cx^2$

b) $y-\sqrt{x^2+y^2}=Cx$

c) $x-\sqrt{x^2+y^2}=Cx$

d) $y+\sqrt{x^2+y^2}=Cx^2$

43. The solution of the differential equation $y'=1+x+y^2+xy^2, y(0)=0$ is

a) $y^2=\exp\left(x+\frac{x^2}{2}\right)-1$

b) $y^2=1+C\exp\left(x+\frac{x^2}{2}\right)$

c) $y=\tan(C+x+x^2)$

d) $y=\tan\left(x+\frac{x^2}{2}\right)$

44. The differential equation of all 'Simple Harmonic Motions' of given period $\frac{2\pi}{n}$, is

a) $\frac{d^2x}{dt^2}+nx=0$

b) $\frac{d^2x}{dt^2}+n^2x=0$

c) $\frac{d^2x}{dt^2}-n^2x=0$

d) $\frac{d^2x}{dt^2}+\frac{1}{n^2}x=0$

45. The order and degree of the differential equation

$5\left(\frac{d^2y}{dx^2}\right)^5+4\left(\frac{d^3y}{dx^3}\right)^2+\left(\frac{dy}{dx}\right)^3+2y+x^3=0$ are respectively

a) 2,5

b) 3,2

c) 1,3

d) 2,3

46. The equation of the curve whose tangent at any point (x, y) makes an angle $\tan^{-1}(2x+3y)$ with x-axis and which passes through (1,2) is

a) $6x+9y+2=26e^{3(x-1)}$

b) $6x-9y+2=26e^{3(x-1)}$

c) $6x+9y-2=26e^{3(x-1)}$

d) $6x-9y-2=26e^{3(x-1)}$

47. $y=Ae^{x}+Be^{2x}+Ce^{3x}$ satisfies the differential equation

a) $y'''-6y'+11y'-6y=0$

b) $y'''+6y''+11y'+6y=0$

c) $y'''+6y''-11y'+6y=0$

d) $y'''-6y''-11y'+6y=0$

48. Observe the following statements A:Interating factor of $\frac{dy}{dx}+y=x^2$ is e^x

R: Integrating factor of $\frac{dy}{dx}+P(x)y=Q(x)$ is $e^{\int P(x)dx}$.Then, the true statement among the following is

a) A is true, R is false

b) A is false, R is true

c) A is true, R is true

d) A is false, R is false

49. The solution of the differential equation $\frac{dy}{dx}=e^{x-y}+x^2e^{-y}$ is

a) $y=e^{x-y}-x^2e^{-y}+c$

b) $e^y-e^x=\frac{1}{3}x^3+c$

c) $e^x+e^y=\frac{1}{3}x^3+c$

d) $e^x-e^y=\frac{1}{3}x^3+c$

50. The solution of differential equation $t=1+(ty)\frac{dy}{dt}+\frac{(ty)^2}{2!}\left(\frac{dy}{dx}\right)^2+\ldots\infty$ is

a) $y=\pm\sqrt{(\log t)^2+c}$

b) $ty=t^y+c$

c) $y=\log t+c$

d) $y=(\log t)^2+c$

ANSWER KEY

1. A
2. C
3. C
4. C
5. D
6. C
7. D
8. C
9. A
10. C
11. B
12. A
13. C
14. A
15. B
16. C
17. B
18. C
19. B
20. D
21. A
22. A
23. D
24. B
25. C
26. A
27. D
28. B
29. C
30. D
31. B
32. A
33. A
34. C
35. A
36. A
37. A
38. C
39. B
40. D
41. C
42. D
43. D
44. B
45. B
46. A
47. A
48. C
49. B
50. A

HINTS AND SOLUTIONS

1. Order=1

   Degree=1

2. $e^x dy + (ye^x + 2x)dx = 0$

   $e^x dy = -(ye^x + 2x)dx \Rightarrow \frac{dy}{dx} = \frac{-(ye^x+2x)}{e^x}$

   $\frac{dy}{dx} = -y - 2xe^{-x} \Rightarrow \frac{dy}{dx} + y = -2xe^{-x}$

   I.F.= $e^{\int dx} = e^x$

   $e^x\left(\frac{dy}{dx} + y\right) = -2xe^{-x}e^x \Rightarrow \frac{d}{dx}e^x y = -2x$

   $\int de^x y = -2\int xdx \Rightarrow e^x y = -2\frac{x^2}{2} + C \Rightarrow ye^x + x^2 = C$

3. $\frac{dy}{dx} + P_1 x = Q_1$

   I.F. = $e^{\int Pdx}$

   $e^{\int Pdx}\left(\frac{dy}{dx} + P_1 x\right) = e^{\int Pdx}Q_1 \Rightarrow xe^{\int P_1 dy} = \int\left\{Q_1 e^{\int P_1 dy}\right\}dy + C$

4. $\frac{ydx - xdy}{y} = 0 \Rightarrow \frac{y - x\frac{dy}{dx}}{\frac{y}{dx}} = 0$

   $y - x\frac{dy}{dx} = 0 \Rightarrow y = x\frac{dy}{dx}$

   $\frac{dx}{x} = \frac{dy}{y} \Rightarrow \int\frac{dx}{x} = \int\frac{dy}{y}$

   $\log x + \log C = \log y \Rightarrow C = \frac{y}{x} \Rightarrow y = Cx$

5. $(1 - y^2)\frac{dx}{dy} + yx = ay$

   Divide equation by $(1 - y^2)$

   $\frac{dx}{dy} + \frac{y}{1-y^2}x = \frac{ay}{1-y^2}$

   On comparing we get P=$\frac{y}{1-y^2}$

   I.F. = $e^{\int\frac{y}{1-y^2}dy}$ $\Rightarrow$I.F.=$e^{\log\left|\frac{1}{\sqrt{1-y^2}}\right|}$ $\Rightarrow$I.F.= $\frac{1}{\sqrt{1-y^2}}$

6. $x\frac{dy}{dx} - y = 2x^2 \Rightarrow \frac{dy}{dx} - \frac{1}{x}y = 2x^2$

P=$-\frac{1}{x}$

I.F.=$e^{\int -\frac{1}{x}}$

I.F.= e^{-logx}=$\frac{1}{x}$

7. A differential equation is homogeneous if all the terms in the equation have equal degree and it can be written in the form $\frac{dy}{dx} = \frac{g(x,y)}{f(x,y)}$

8. A homogeneous differential equation of the form $\frac{dy}{dx} = h\left(\frac{x}{y}\right)$ can be solved by making the substitution $\frac{x}{y} = v \Rightarrow x = vy$

9. $\frac{dy}{dx} = e^{x+y} \Rightarrow \int e^{-y}dy = \int e^{x}dx \Rightarrow -e^{-y} = e^{x} + C \Rightarrow e^{x} + e^{-y} = C$

10. y=x

$\frac{dy}{dx} = 1$

$\frac{d^2y}{dx^2} = 0 \Rightarrow x^2 + \frac{d^2y}{dx^2} = x^2 \Rightarrow xy + \frac{d^2y}{dx^2} - \frac{dy}{dx} \times x^2 = 0$

11. Given, $\frac{\tan^{-1}x}{1+x^2}dx + \frac{y}{1+y^2}dy = 0$

$\Rightarrow \frac{(\tan^{-1}y)^2}{2} + \frac{1}{2}\log(1+y^2) = \frac{c}{2} \quad \Rightarrow (\tan^{-1}x)^2 + \log(1+y^2) = c$

12. Given, $\frac{dy}{dx} = 2\cos x - y\cos x \ \operatorname{cosec} x \Rightarrow \frac{dy}{dx} + y\cot x = 2\cos x$

$\text{IF} = e^{\int \cot x\, dx} = e^{\log(\sin x)} = \sin x$

Solution is $y\sin x = \int 2\cos x \sin x \ dx + c \Rightarrow y\sin x = \int \sin 2x \ dx + c$

$\Rightarrow \ y\sin x = \frac{-\cos 2x}{2} + c$

At $x = \frac{\pi}{4}, y = \sqrt{2}$

$\therefore \sqrt{2}\sin\frac{\pi}{4} = \frac{-\cos 2(\pi/4)}{2} + c$

$\Rightarrow \ c = 1$

$\therefore \ y\sin x = -\frac{1}{2}\cos 2x + 1 \Rightarrow \ y = -\frac{1}{2}.\frac{\cos 2x}{\sin x} + \operatorname{cosec} x$

$\Rightarrow \ y = -\frac{1}{2\sin x}(1 - 2\sin^2 x) + \operatorname{cosec} x \Rightarrow y = \frac{1}{2}\ \operatorname{cosec} x + \sin x$

13. We have,

Slope $= \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{2y} \Rightarrow 2\,y\,dy = dx$

Integrating both sides, we get $y^2 = x + C$

This passes through (4, 3)

$\therefore 9 = 4 + C \Rightarrow C = 5$

So, the equation of the curve is $y^2 = x + 5$

14. Given, $\frac{d^2y}{dx^2}(x^2+1) = 2x\,\frac{dy}{dx} \Rightarrow \frac{\frac{d^2y}{dx^2}}{\frac{dy}{dx}} = \frac{2x}{x^2+1}$

On integrating both sides, we get

$\log\frac{dy}{dx} = \log(x^2+1) + \log c \;\Rightarrow\; \frac{dy}{dx} = c(x^2+1)$ …(i)

As at $x = 0$, $\frac{dy}{dx} = 3$

$\therefore \quad 3 = c(0+1) \Rightarrow \quad c = 3$

$\therefore$ From Eq. (i), $\frac{dy}{dx} = 3(x^2+1) \Rightarrow \quad dy = 3(x^2+1)dx$

Again, integrating both sides, we get

$$y = 3\left(\frac{x^3}{3} + x\right) + c_1$$

At point (0,1)

$$1 = 3(0+0) + c_1 \Rightarrow c_1 = 1$$

$\therefore \quad y = 3\left(\frac{x^3}{3} + x\right) + 1 \Rightarrow \quad y = x^3 + 3x + 1$

15. Since, $f''(x) = 6(x-1)$

$\Rightarrow \; f'(x) = 3(x-1)^2 + c \quad$ …(i)

Also, at the point (2,1) the tangent to graph is $y = 3x - 5$.Slope of tangent=3

$\Rightarrow \quad f'(2) = 3$

$3(2-1)^2 + c = 3 \qquad \Rightarrow \; 3 + c = 3 \Rightarrow \; c = 0$

From Eq. (i),

$f'(x) = 3(x-1)^2 \Rightarrow \quad f(x) = (x-1)^2 + k$ [integrating] …(ii)

Since, it passes through (2,1)

$\therefore \qquad 1 = (2-1)^2 + k \quad \Rightarrow k = 0$

Hence, equation of function is $f(x) = (x-1)^2$

16. $x = 1 + \frac{dy}{dx} + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{1}{3!}\left(\frac{dy}{dx}\right)^3 + \cdots.$

$\Rightarrow\ x = e^{\frac{dy}{dx}} \Rightarrow \frac{dy}{dx} = \log_e x$

$\Rightarrow$ Degree of differential equation is 1.

17. We have,

$y\frac{dy}{dx} = \lambda\, y^2 \Rightarrow \frac{dy}{dx} = \lambda\, y$

$\Rightarrow \frac{1}{y} dy = \lambda\, dx \Rightarrow \log y = \lambda\, x + \log C \Rightarrow y = Ce^{\lambda\, x}$

18. $y = (c_1 + c_2)\cos(x + c_3) - c_4 e^{x+c_5}$

$y_1 = -(c_1 + c_2)\sin(x + c_3) - c_4\, e^{x+c_5}$

$y_2 = -(c_1 + c_2)\cos(x + c_3) - c_4\, e^{x+c_5} = -y - 2c_4 e^{x+c_5}$

$y_3 = -y_1 - 2c_4 e^{x+c_5}$

$y_3 = -y_1 + y_2 - y$

$\therefore$ Differential equation is

$y_3 - y_2 + y_1 - y = 0$

Which is order 3

19. Solve on your own.

20. Solve on your own.

21. The given equation is

$(y + 3)dy = (x + 2)dx\ \Rightarrow \frac{y^2}{2} + 3y = \frac{x^2}{2} + 2x + c$

Since, it passes through (2, 2).

$\therefore 2 + 6 = 2 + 4 + c\ \Rightarrow c = 2$

$\therefore \frac{y^2}{2} + 3y = \frac{x^2}{2} + 2x + 2$

$\Rightarrow y^2 + 6y = x^2 + 4x + 4 \Rightarrow x^2 + 4x - y^2 - 6y + 4 = 0$

22. Given, $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{5/2} = \frac{d^3y}{dx^3} \Rightarrow \left(\frac{d^3y}{dx^3}\right)^2 = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^5$

Here, order=3, degree=2

23. $y = ae^{mx} + be^{-mx}$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = mae^{mx} - mbe^{-mx}$$

Again, on differentiating, we get

$$\frac{d^2y}{dx^2} = m^2ae^{mx} + m^2be^{-mx}$$

$$= m^2(ae^{mx} + be^{-mx}) = m^2y$$

$$\Rightarrow \frac{d^2y}{dx^2} - m^2y = 0$$

24. Solve by yourself

25. Given, $y = c_1 \cos(x + c_2) + c_3 \sin(x + c_4) + c_5e^x + c_6$

$$y = c_1[\cos x \cos c_2 - \sin x \sin c_2]$$

$$+c_3[\sin x \cos c_4 + \cos x \sin c_4] + c_5e^x + c_6$$

$$= \cos x\,(c_1 \cos c_2 + c_3 \sin c_4) + \sin x\,(-c_1 \sin c_2 + c_3 \cos c_4) + c_5e^x + c_6$$

$$= A \cos x + B \sin x + Ce^x + D$$

Where, $A = c_1 \cos c_2 + c_3 \sin c_4$

$B = -c_1 \sin c_2 + c_3 \cos c_4$, $C = c_5, D = c_6$

Hence, order is 4

26. The given equation can be written as

$$\left(\frac{dx}{x} - \frac{dy}{y}\right) + \frac{(x^2dy - y^2dx)}{(x-y)^2} = 0$$

$$\Rightarrow \left(\frac{dx}{x} - \frac{dy}{y}\right) + \frac{\left(\frac{dy}{y^2} - \frac{dx}{x^2}\right)}{\left(\frac{1}{y} - \frac{1}{x}\right)^2} = 0 \Rightarrow \left(\frac{dx}{x} - \frac{dy}{y}\right) + \frac{\frac{dy}{y^2} - \frac{dx}{x^2}}{\left(\frac{1}{x} - \frac{1}{y}\right)^2} = 0$$

On integrating both sides, we get

$$\text{In } |x| - \text{In } |y| - \frac{1}{\left(\frac{1}{x} - \frac{1}{y}\right)} = c$$

$$\Rightarrow \text{In } \left|\frac{x}{y}\right| - \frac{xy}{(y-x)} = c \Rightarrow \text{In } \left|\frac{x}{y}\right| + \frac{xy}{(x-y)} = c$$

27. $\because$ Equation of normal at $P(1, 1)$ is

$ay + x = a + 1$ (given)

$\because$ Slope of normal at $(1, 1) = -\frac{1}{a}$

$\therefore$ Slope of tangent at $(1, 1) = a$ ...(i)

Also, given $\frac{dy}{dx} \propto y \Rightarrow \frac{dy}{dx} = ky$

$\left.\frac{dy}{dx}\right|_{(1,1)} = k = a$ [from Eq. (i)]

Then, $\frac{dy}{dx} = ay$

$\Rightarrow \frac{dy}{y} = a\,dx \Rightarrow \text{In}|y| = ax + c$

$\because$ It is passing through (1,1), then $c = -a$

$\Rightarrow \text{In}|y| = a(x-1) \Rightarrow |y| = e^{a(x-1)}$

28. Equation of family of parabolas with focus at (0, 0) and x-axis as axis is

$y^2 = 4a(x + a)$ ....(i)

On differentiating Eq. (i), we get

$2yy_1 = 4a$, putting the value of a in Eq. (i)

$\Rightarrow y^2 = 2yy_1\left(x + \frac{yy_1}{2}\right) \Rightarrow y = 2xy_1 + yy_1^2 \Rightarrow y\left(\frac{dy}{dx}\right)^2 + 2x\,\frac{dy}{dx} = y$

29. $\because y = u^n$

$\therefore \frac{dy}{dx} = nu^{n-1}\frac{du}{dx}$

On substituting the values of y and $\frac{dy}{dx}$ in the given equation, then

$2x^4 \cdot u^n \cdot nu^{n-1}\frac{du}{dx} + u^{4n} = 4x^6 \Rightarrow \frac{du}{dx} = \frac{4x^6 - u^{4n}}{2nx^4u^{2n-1}}$

Since, it is homogeneous. Then, the degree of $4x^6 - u^{4n}$ and $2nx^4u^{2n-1}$ must be same.

$\therefore 4n = 6$ and $4 + 2n - 1 = 6$

Then, we get $n = \frac{3}{2}$

30. $\because$ Equation of normal at (x, y) is

$$Y - y = \frac{dx}{dy}(X - x)$$

Put, $y = 0$

Then, $X = x + y\,\frac{dy}{dx}$

Given, $y^2 = 2x\,X$

$$\Rightarrow y^2 = 2x\left(x + y\,\frac{dy}{dx}\right) \Rightarrow \frac{dy}{dx} = \frac{y^2 - 2x^2}{2xy} = \frac{\left(\frac{y}{x}\right)^2 - 2}{2\left(\frac{y}{x}\right)}$$

Put $y = vx$, we get

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Then, $v + x\frac{dv}{dx} = \frac{v^2 - 2}{2v}$

$$\Rightarrow x\frac{dv}{dx} = -\frac{(2+v^2)}{2v} \Rightarrow \frac{2v\,dv}{(2+v^2)} + \frac{dv}{x} = 0$$

On integrating both sides, we get

$$\text{In }(2 + v^2) + \text{In}|x| = \text{In }c \Rightarrow \text{In}\left(|x|(2 + v^2)\right) = \text{In }c \Rightarrow |x|\left(2 + \frac{y^2}{x^2}\right) = c$$

$\because$ It passes through (2, 1), then

$$2\left(2 + \frac{1}{4}\right) = c \Rightarrow c = \frac{9}{2}$$

Then, $|x|\left(2 + \frac{y^2}{x^2}\right) = \frac{9}{2}$

$$\Rightarrow 2x^2 + y^2 = \frac{9}{2}|x| \Rightarrow 4x^2 + 2y^2 = 9|x|$$

31. The equation of straight line touching the given circle is

$$x\cos\theta + y\sin\theta = a \qquad \ldots\text{(i)}$$

On differentiating w.r.t. x, regarding θ as a constant

$$\Rightarrow \cos\theta + \frac{dy}{dx}\sin\theta = 0 \qquad \ldots\text{(ii)}$$

From eqs. (i) and (ii), we get $\cos\theta = \frac{a\frac{dy}{dx}}{x\frac{dy}{dx} - y}$ and $\sin\theta = -\frac{a}{x\frac{dy}{dx} - y}$

$\therefore \qquad \cos^2\theta + \sin^2\theta = 1$

$$\therefore \qquad \frac{a^2\left(\frac{dy}{dx}\right)^2 + a^2}{\left(x\frac{dy}{dx} - y\right)^2} = 1 \Rightarrow \qquad \left(y - x\,\frac{dy}{dx}\right)^2 = a^2\left[1 + \left(\frac{dy}{dx}\right)^2\right]$$

32. Here, $x = A\cos 4t + B\sin 4t$

On differentiating w.r.t. t, we get

$$\frac{dx}{dt} = -4A\sin 4t + 4B\cos 4t$$

Again, on differentiating w.r.t. t, we get

$$\frac{d^2x}{dt^2} = -16A\cos 4t - 16B\sin 4t$$

$$= -16(A\cos 4t + B\sin 4t)$$

$$\Rightarrow \frac{d^2x}{dt^2} = -16x$$

33. We have,

$$y_1y_3 = 3\,y_2^2 \Rightarrow \frac{y_3}{y_2} = 3\frac{y_2}{y_1}$$

Integrating both sides, we get

$$\log y_2 = 3\log y_1 + \log c_1$$

$$\Rightarrow y_2 = c_1y_1^3 \Rightarrow \frac{y_2}{y_1^3} = c_1 \Rightarrow \frac{d\,y_1}{y_1^3} = c_1$$

Integrating both sides w.r.t. x, we get

$$-\frac{1}{2y_1^2} = c_1x + c_2$$

$$\Rightarrow y_1^2 = \frac{1}{(-2\,c_1)x+(-2\,c_2)} \Rightarrow y_1^2 = \frac{1}{ax+b}, \text{ where } a = -2c_1, b = -2\,c_2$$

$$\Rightarrow y_1 = \frac{1}{\sqrt{ax+b}}$$

Integrating both sides w.r.t. x, we get

$$y = \frac{2}{a}\sqrt{ax+b} + c_3 \Rightarrow \frac{ay-c_3}{2} = \sqrt{ax+b}$$

$$\Rightarrow ax + b = \left(\frac{ax - c_3}{2}\right)^2$$

$$\Rightarrow x = \frac{a}{4}y^2 - \frac{c^3}{2}y + \frac{1}{a}\left(\frac{c_3^2}{4} - b\right) \Rightarrow x = A_1y^2 + A_2y + A_3,$$

where $= A_1 = \frac{a}{4}, A_2 = -\frac{c_3}{2}$ and

$$A_3 = \frac{1}{a}\left(\frac{c_3^2}{4} - b\right)$$

34. Given, $\frac{dx}{dt} = \cos^2 \pi x$

On differentiating w.r.t. x, we get

$\frac{d^2x}{dt^2} = -2\pi \sin 2\pi x = \text{negative}$

The particle never reaches the point, it means

$\frac{d^2x}{dt^2} = 0 \Rightarrow -2\pi \sin 2\pi x = 0$

$\Rightarrow \sin 2\pi x = \sin \pi \Rightarrow 2\pi x = \pi \Rightarrow x = \frac{1}{2}$

The particle never reaches at $x = \frac{1}{2}$

35. The given differential equation can be rewritten as

$$y + \frac{d^2y}{dx^2} = \left[a + \left(\frac{dy}{dx}\right)^{3/2}\right]^2 \Rightarrow y + \frac{d^2y}{dx^2} = x^2 + \left(\frac{dy}{dx}\right)^3 + 2x\left(\frac{dy}{dx}\right)^{3/2}$$

$$\Rightarrow \left[y + \frac{d^2y}{dx^2} - x^2 - \left(\frac{dy}{dx}\right)^3\right]^2 = \left[2x\left(\frac{dy}{dx}\right)^{3/2}\right]^2$$

$\therefore$ Order and degree of the given differential equation is 2 and 2 respectively.

36. The equation of the family of circles which touch both the axes is

$(x-a)^2 + (y-a)^2 = a^2$, where a is a parameter. This is one parameter family of curves. Order=1

37. Given differential equation is

$$\frac{d^2y}{dx^2} = \sqrt[3]{1 - \left(\frac{dy}{dx}\right)^4} \Rightarrow \left(\frac{d^2y}{dx^2}\right)^3 = 1 - \left(\frac{dy}{dx}\right)^4$$

$\therefore$ Order=2, degree=3

38. I. $\frac{dy}{dx} + 2xy = 2e^{-x^2}$

$\therefore \quad \text{IF} = e^{\int 2x\,dx} = e^{x^2}$

$\therefore$ Complete solution is

$$ye^{x^2} = 2\int e^{-x^2} e^{x^2} dx + c \Rightarrow ye^{x^2} = 2x + c$$

II. $ye^{x^2} - 2x = c$

$\Rightarrow \quad ye^{x^2}.2x + e^{x^2}.\frac{dy}{dx} - 2 = 0 \Rightarrow e^{x^2}.\frac{dy}{dx} = 2 - 2xy\,e^{x^2}$

$\Rightarrow \quad \frac{dy}{dx} = 2e^{-x^2} - 2xy$

$\therefore$ I is true and II is false.

39. We have, $\frac{dx}{dt} = x + 1$

$$\Rightarrow \frac{1}{x+1} dx = dt \Rightarrow \log(x+1) = t + C$$

Putting $t = 0, x = 0$, we get

$\log 1 = C \Rightarrow C = 0$

$\therefore t = \log(x+1)$

Putting $x = 99$, we get $t = \log_e 100 = 2 \log_e 10$

40. Given, $y = ae^x + bx\,e^x + cx^2e^x$ ...(i)

On differentiating w.r.t. x, we get

$$y' = ae^x + b(xe^x + e^x) + c(x^2e^x + 2xe^x)$$

$$\Rightarrow y' = ae^x + bxe^x + cx^2e^x + be^x + 2cxe^x \Rightarrow y' = y + be^x + 2cxe^x$$

Again, differentiating w.r.t. x, we get

$$y'' = y' + be^x + 2c(xe^x + e^x) \Rightarrow y'' = y' + be^x + 2cxe^x + 2ce^x$$

$\Rightarrow \quad y'' = 2y' - y + 2ce^x$ ...(iii) [from Eq. (ii)]

Again, differentiating w.r.t. x, we get

$$y''' = 2y'' - y' + 2ce^x$$

$\Rightarrow \quad y''' = 2y'' - y' + (y'' - 2y' + y)$ [from eq. (iii)]

$$\Rightarrow y''' - 3y'' + 3y' - y = 0$$

41. The equation of the family of circles is

$x^2 + (y-k)^2 = r^2$ ...(i)

Where k is a parameter

Differentiating w.r.t. x, we get

$2x + 2(y-k)y_1 = 0 \Rightarrow y - k = -\frac{x}{y_1}$ ...(ii)

Eliminating k from (i) and (ii), we obtain

$$x^2 + \frac{x^2}{y_1^2} = r^2 \Rightarrow x^2 = \frac{r^2y_1^2}{1+y_1^2} \Rightarrow x^2(y_1^2+1) = r^2y_1^2$$

42. We have,

$$x\,dy - y\,dx = \sqrt{x^2+y^2}dx \Rightarrow \frac{dy}{dx} - \frac{y}{x} = \frac{\sqrt{x^2+y^2}}{x}$$

Putting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, we get

$$v + x\frac{dv}{dx} - v = \sqrt{1+v^2} \Rightarrow \frac{1}{\sqrt{1+v^2}}dv = \frac{dx}{x}$$

Integrating, we get

$$\log\left|v+\sqrt{v^2+1}\right| = \log x + \log C \Rightarrow v + \sqrt{v^2+1} = Cx \Rightarrow y + \sqrt{x^2+y^2} = Cx^2$$

43. We have,

$$\frac{dy}{dx} = 1 + x + y^2 + xy^2$$

$$\Rightarrow \frac{dy}{dx} = (1+x)(1+y^2) \Rightarrow \frac{1}{1+y^2}dy = (1+x)dx \Rightarrow \tan^{-1} y = \left(x + \frac{x^2}{2}\right) + C \quad \ldots \text{(i)}$$

It is given that $y(0) = 0$ $i.e.$ $y = 0$ when $x = 0$

$\therefore \tan^{-1} 0 = 0 + C \Rightarrow c = 0$

Hence, $\tan^{-1} y = x + \frac{x^2}{2} \Rightarrow y = \tan\left(x + \frac{x^2}{2}\right)$

44. The displacement x for all SHM is given by

$$x = a\cos(nt+b)$$

$$\Rightarrow \frac{dx}{dt} = -na\sin(nt+b) \Rightarrow \frac{d^2x}{dt^2} = -n^2 a\cos(nt+b)$$

$$\Rightarrow \frac{d^2x}{dt^2} = -n^2x \Rightarrow \frac{d^2x}{dt^2} = +n^2x = 0$$

45. $5\left(\frac{d^2y}{dx^2}\right)^5 + 4\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{dy}{dx}\right)^3 + 2y + x^3 = 0$

Here, highest order derivative is 3 whose degree is 2.

46. Given, $\frac{dy}{dx} = \tan\theta = 2x + 3y$

Put $2x + 3y = z \quad \Rightarrow 2 + 3\frac{dy}{dx} = \frac{dz}{dx}$

$\Rightarrow \frac{dy}{dx} = \left(\frac{dz}{dx} - 2\right)\frac{1}{3}$

$\therefore \quad \frac{dz}{dx} - 2 = 3z \;\Rightarrow \frac{dz}{3z+2} = dx$

On integrating, we get

$$\frac{\log(3z+2)}{3} = x + C$$

$$\Rightarrow \quad \frac{\log(6x+9y+2)}{3} = x + C$$

Since, it passes through (1,2).

$$\therefore \quad \frac{\log(6+18+2)}{3} = 1 + C$$

$$\Rightarrow \quad C = \frac{\log 26}{3} - 1$$

$$\therefore \quad \frac{\log(6x+9y+2)}{3} = x + \frac{\log 26}{3} - 1 \Rightarrow \log\left(\frac{6x+9y+2}{26}\right) = 3(x-1)$$

$$\Rightarrow \quad 6x + 9y + 2 = 26e^{3(x-1)}$$

47. Given, $y = Ae^x + Be^{2x} + Ce^{3x}$ ...(i)

$$\Rightarrow \quad y' Ae^x + 2Be^{2x} + 3Ce^{3x}$$

From Eq. (i),

$$Ae^x = y - Be^{2x} - Ce^{3x} \Rightarrow \; y' = y + Be^{2x} - Ce^{3x}$$

$\therefore \quad y'' = y' + Be^{2x} + 6Ce^{3x}$ ...(ii)

From Eq. (ii),

$$Be^{2x} = y' - y - 2Ce^{3x}$$

$$\therefore \quad y'' = y' + 2y' - 2y - 4Ce^{3x} + 6Ce^{3x}$$

$\Rightarrow \quad y'' = 3y' - 2y + 2Ce^{3x}$ ...(iii)

Again, differentiating w.r.t. x, we get

$$y''' = 3y'' - 2y' + 6Ce^{3x}$$

From Eq. (iii),

$$2Ce^{3x} = y'' - 3y' + 2y$$

$$\therefore \quad y''' = 3y'' - 2y' + 3(y'' - 3y' + 2y) \Rightarrow \; y''' - 6y'' + 11y' - 6y = 0$$

48. Given, $\frac{dy}{dx} + y = x^2$

$\therefore$ IF $= e^{\int 1\, dx} = e^x$

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$\therefore$ IF $= e^{\int P(x)dx}$

$\therefore$ Both statements A and B are true and R $\Rightarrow$ A

49. Given, $\frac{dy}{dx} = e^{-y}(e^x + x^2)$

$\Rightarrow \int e^y\, dy = \int e^x\, dx + \int x^2\, dx$

$\Rightarrow \quad e^y = e^x + \frac{x^3}{3} + c$

$\Rightarrow \quad e^y - e^x = \frac{x^3}{3} + c$

50. The given equation is

$$t = 1 + (ty)\left(\frac{dy}{dt}\right) + \frac{(ty)^2}{2!}\left(\frac{dy}{dt}\right)^2 + \ldots\infty$$

$\Rightarrow t = e^{ty\left(\frac{dy}{dt}\right)}$

$\Rightarrow \log t = ty\,\frac{dy}{dt}$

$\Rightarrow y\, dy = \frac{\log t}{t} dt$

On integrating both sides, we get

$\frac{y^2}{2} = \frac{(\log t)^2}{2} + k$

$\Rightarrow y = \pm\sqrt{(\log t)^2 + 2k}$

$\Rightarrow y = \pm\sqrt{(\log t)^2 + c}$