CUET PRATICE PAPER 02

1. The order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right] = \frac{d^2y}{dx^2}$ are:

   a) $2, \frac{3}{2}$

   b) 2,3

   c) 2,1

   d) 3,4

2. If $f(x) = ax^2 + 6x + 5$ attains its minimum value at x=1, then the value of a is

   a) 0

   b) 5

   c) 3

   d) -3

3. The tangent to the curve $y = ax^2 + bx$ at (2,-8) is parallel to x-axis. Then

   a) $a = 2, b = -2$

   b) $a = 2, b = -4$

   c) $a = 2, b = -8$

   d) $a = 4, b = -4$

4. The differential equation of the family of curves $y = Ae^{3x} + Be^{5x}$, where A and B are arbitrary constants, is

   a) $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 15y = 0$

   b) $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 0$

   c) $\frac{d^2y}{dx^2} - \frac{dy}{dx} + y = 0$

   d) None of these

5. If x is real then find the minimum value of (x+5)(x+7)

    a) 0

    b) -1

    c) 1

    d) 2

6. The equation of normal to the curve $3x^2 - y^2 = 8$ which is parallel to the line $x + 3y = 8$ is

    a) $3x - y = 8$

    b) $3x + y + 8 = 0$

    c) $x + 3y \pm 8 = 0$

    d) $x + 3y = 0$

7. The function $f(x) = x^2 - 4x, x \in [0,4]$ attains minimum value at

    a) $x = 0$

    b) $x = 1$

    c) $x = 2$

    d) $x = 4$

8. $\int \frac{3x^2+1}{x} dx$

    a) $2x^3 = 2\sqrt{x^2} + c$

    b) $x^3 + \sqrt{x^2} + c$

    c) $2x^3 \log x + c$

    d) $x^3 + logx + c$

9. The area under the curve $y = x^2$ between the lines $x = 2$ and $x = 3$ is:

    a) $\frac{19}{3}$

    b) $\frac{1}{9}$

    c) $\frac{9}{19}$

    d) $\frac{19}{8}$

10. The maximum value of $\left(\frac{1}{x}\right)^x$ is:

a) e

b) e^e

c) $e^{\frac{1}{e}}$

d) $\left(\frac{1}{e}\right)^{1/e}$

11. Evaluate $\int tan^3 x sec^2 x\, dx$

a) $sec^2 x + c$

b) $\frac{tan^4 x}{4} + c$

c) $\frac{tan^4 x}{2} + c$

d) $2 \tan x \sec x + c$

12. $\int \frac{x^9}{(4x^2+1)^6} dx$ is equal to

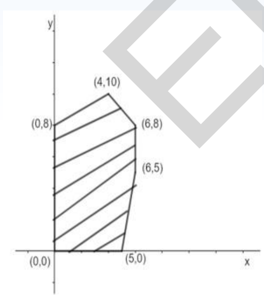
a) $\frac{1}{5x}\left(4+\frac{1}{x^2}\right)^{-5} + c$

b) $\frac{1}{5}\left(4+\frac{1}{x^2}\right)^{-5} + c$

c) $\frac{1}{10x}(1+4)^{-5} + c$

d) $\frac{1}{10}\left(4+\frac{1}{x^2}\right)^{-5} + c$

13. The feasible solution for an LPP is shown in figure. Let Z = 3x-4y be the objective function. (Maximum value of Z +Minimum value of Z) is equal to:



a) 13

b) 1

c) -13

d) -17

14. Six coins are tossed simultaneously. What is the probability of getting exactly 2 head

a) $\frac{49}{64}$

b) $\frac{1}{64}$

c) $\frac{3}{8}$

d) $\frac{15}{64}$

15. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to:

a) A

b) $I + A$

c) $I - A$

d) I

16. Which of the following is the principal value branch of $\csc^{-1} x$?

a) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

b) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

d) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

17. x and y be two variables such that x>0 and xy=1, then the minimum value of (x+y) is

a) 2

b) 3

c) 4

d) 0

18. If the sum of the matrices $\begin{bmatrix} x \\ y \\ z \end{bmatrix}, \begin{bmatrix} y \\ y \\ z \end{bmatrix}$ and $\begin{bmatrix} z \\ 0 \\ 0 \end{bmatrix}$ is the matrix $\begin{bmatrix} 10 \\ 5 \\ 5 \end{bmatrix}$ then what is the value of y?

a) -5

b) 0

c) 5

d) 10

19. If f(x) is an invertible function, what is $f^{-1}(x)$ if $f(x) = \frac{3x-2}{5}$

a) $\frac{3x-2}{5}$

b) $\frac{3x+2}{5}$

c) $\frac{5x+2}{3}$

d) $\frac{5x-2}{3}$

20. The position vector of the point which divides the join of points with position vectors $\vec{a}+\vec{b}$ and $2\vec{a}-\vec{b}$ in the ratio 1:2 is

a) $\frac{3\vec{a}+2\vec{b}}{3}$

b) $\vec{a}$

c) $\frac{5\vec{a}-\vec{b}}{3}$

d) $\frac{4\vec{a}+\vec{b}}{3}$

21. Find the value of b if $\int \frac{dx}{\sqrt{9-x^2}} = \sin^{-1}\frac{x}{b} + C$

a) 2

b) 3

c) 4

d) 5

22. If A and B are square matrices of the same order and AB=3I, then A^{-1} is equal to

a) $3B$

b) $\frac{1}{3}B$

c) $3B^{-1}$

d) $\frac{1}{3}B^{-1}$

23. If the rolle's theorem holds for the function $f(x) = x^4 + ax^3 + bx$, in $-1 \leq x \leq 1$ and $f'\left(\frac{1}{2}\right) = 0$ then ab=

a) -4

b) -64

c) -1

d) -8

24. Evaluate $\int_0^1 \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}} dx$

a) $e - 1$

b) $e^{\frac{\pi}{2}} - 1$

c) $e^{\frac{\pi}{2}} - e$

d) $-e^{\frac{\pi}{2}} - 1$

25. The function $f: R \rightarrow R$ defined as $f(x) = x^3$ is:

a) One-one but not onto

b) Not one-one but onto

c) Neither one-one nor onto

d) One-one and onto

26. If a relation R on the set {1,2,3} be defined by R={(1,2)}, then R is

a) Reflexive

b) Transitive

c) Symmetric

d) None of these

27. Find the value of the $\int_{-\pi}^{\pi} \cos x \, dx$

a) 0

b) 1

c) -1

d) 2

28. If $x = t^2 - 1$ and $y = t^2 + 1$, then $\frac{dy}{dx} =$?

a) $\frac{1}{2t}$

b) $2t$

c) $1 + \frac{1}{2t}$

d) None of these

29. If $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{x}{3}\right) = \frac{\pi}{4}$, where $0 < x < 6$, then what is x equal to?

a) 1

b) 2

c) 3

d) 5

30. If $y = \log \log x$, then $e^y \frac{dy}{dx} =$

a) $\frac{1}{x \log x}$

b) $\frac{1}{x}$

c) $\frac{1}{logx}$

d) e^y

31. If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$ then the values of k, a and b respectively are

a) -6,-12,-18

b) -6,-4,-9

c) -6,4,9

d) -6,12,18

32. Given that A is a non-singular matrix of order 3 such that $A^2 = 2A$, then value of $|2A|$ is

a) 4

b) 8

c) 64

d) 16

33. The domain of $\sin^{-1} 2x$ is

a) [0,1]

b) [-1,1]

c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

d) [-2,2]

34. The plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1}(\alpha)$ with x-axis. The value of α is equal to

a) $\frac{\sqrt{3}}{2}$

b) $\frac{\sqrt{3}}{3}$

c) $\frac{2}{7}$

d) $\frac{3}{7}$

35. Let $f(x) = \begin{cases} 3x - 4, 0 \leq x \leq 2 \\ 2x + l,\ 2 < x \leq 9 \end{cases}$. If f is continuous at x=2, then what is the value of l?

a) 0

b) 2

c) -2

d) -1

36. If $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, then find $\frac{dy}{dx}$

a) $\frac{1}{1+x^2}$

b) $\frac{2}{1+x^2}$

c) $\frac{2}{2+x^2}$

d) None of these

37. If $\tan^{-1} x + \tan^{-1} y = \frac{4\pi}{5}$, then $\cot^{-1} x + \cot^{-1} y$ equals

a) $\frac{\pi}{5}$

b) $\frac{2\pi}{5}$

c) $\frac{3\pi}{5}$

d) π

38. If $y = 2^x + x \log x$, then find $\frac{dy}{dx}$:

a) $2^x \log 2 - \log x - 1$

b) $2^x \log 2 + \log x + 1$

c) $2^x \log 2 - \log x + 1$

d) $2^x \log 2 + \log x - 1$

39. The area of a triangle with vertices A(3,0),B(7,0) and C(8,4) is:

a) 14

b) 8

c) 28

d) 6

40. Evaluate $\int \frac{dx}{x^2+4}$

a) $\frac{1}{4}\tan^{-1}\frac{x}{4} + C$

b) $\frac{1}{2}\tan^{-1}\frac{x}{2} + C$

c) $\tan^{-1}\frac{x}{4} + C$

d) $\tan^{-1}\frac{x}{2} + C$

41. If A and B are two independent events with $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$, then $P(A' \cap B')$ equals

a) $\frac{4}{15}$

b) $\frac{8}{45}$

c) $\frac{1}{3}$

d) $\frac{2}{9}$

42. The radius of a circle is changing at the rate of $\frac{dr}{dt} = 0.01m/sec$. The rate of change of its area $\frac{dA}{dt}$, when the radius of the circle is 4m, is

a) $16\pi \frac{m^2}{sec}$

b) $0.16\pi \frac{m^2}{sec}$

c) $0.08\pi \frac{m^2}{sec}$

d) $0.04\pi \frac{m^2}{sec}$

43. Find the value of the $\int_0^{\pi/2} \frac{\tan x}{\tan x + \cot x} dx$

a) $\frac{\pi}{4}$

b) $\frac{\pi}{7}$

c) $\frac{\pi}{2}$

d) $\frac{\pi}{8}$

44. Which of the following is not a homogeneous function of x and y.

a) $x^2 + 2xy$

b) $2x - y$

c) $cos^2\left(\frac{y}{x}\right) + \frac{y}{x}$

d) $\sin x - \cos y$

45. Let $f: R \rightarrow R$ be the function defined by $f(x) = \frac{2x-1}{2}$ and $g: Q \rightarrow R$ be another function defined by g(x)=x+2. Then (gof)$\frac{3}{2}$ is

a) 1
b) – 1
c) 7/2
d) None of these

Direction: Based on the following information, answer the following questions:

Volume of the container given as the function of length is $V(x) = -x^2 + 25x + 7500$.

46. What will be the length (in m) when the volume is maximum?

a) 10
b) 11.5
c) 12.5
d) 9

47. What is the maximum volume of the container (in m^3)?

a) 7656.25
b) 7968.75
c) 7432.25
d) 7864.75

48. In which interval, the volume function is strictly increasing?

a) (7.5,13.5)
b) (12.5,∞)
c) (0,12.5)
d) None of these

49. What will be the volume of the container (in m^3) when the length is 4m?

a) 7744
b) 7832
c) 7256
d) 7584

50. What will be the length (in m) when the volume of the container is $7650m^3$?

a) 10

b) 11

c) 12

d) 13

HINTS AND SOLUTIONS

1. Given: $\left[1+\left(\frac{dy}{dx}\right)^2\right]=\frac{d^2y}{dx^2}$

   Here, highest order derivative is $\frac{d^2y}{dx^2}$

   The order is 2.

   The power of the highest order derivative is 1.

   The degree is 1.

2. Given: $f(x)=ax^2+6x+5=0$

   $\frac{df(x)}{dx}=2ax+6=0$

   $\Rightarrow x=-\frac{3}{a}$

   Given that x=1 has maxima, put x=1 in above equation we get a= - 3

   Check $\frac{d^2y}{dx^2}=2a=-6<0$

   ⇒x =1 has maxima is correct.

3. $y=ax^2+bx$

   $\Rightarrow \frac{dy}{dx}=2ax+b$

   $\Rightarrow \left(\frac{dy}{dx}\right)_{(2,-8)}=4a+b$

   Tangent is parallel to x-axis

   $\therefore \frac{dy}{dx}=0$

   $\Rightarrow b=-4a$

   Now point (2,-8) is on the curve of $y=ax^2+bx$,

   $\therefore -8=4a+2b$

   $\Rightarrow a=2, b=-8$

4. Given $y = Ae^{3x} + Be^{5x}$.....(i)

On differentiating we get

$\frac{dy}{dx} = 3Ae^{3x} + 5Be^{5x}$ ....(ii)

Again on differentiating we get

$\frac{d^2y}{dx^2} = 9Ae^{3x} + 25Be^{5x}$ ......(iii)

Now, on applying (iii)-8(ii)+15(i), we get

$\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 9Ae^{3x} + 25Be^{5x} - 8(3Ae^{3x} + 5Be^{5x}) + 15(Ae^{3x} + Be^{5x})$

$\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 0$

5. y = (x+5)(x+7)

⇒$y = x^2 + 12x + 35$.....(i)

Differentiating equation(i)

dy/dx=0

⇒2x+12=0.....(ii)

⇒x=-6

Now double differentiating equation (ii) $\frac{d^2y}{dx^2} = 2 > 0$

That means, the minimum value of equation (i) is at x=-6

Minimum value of equation (i)

y=-1

6. Given curve $3x^2 - y^2 = 8$

Differentiating w.r.t x

$6x - 2y\,y' = 0 \Rightarrow y' = \frac{3x}{y}$

The slope of normal = $\frac{3x}{y}$. Given line is x+3y=8 ⇒$y = -\frac{1}{3}x + \frac{8}{3}$

In the form y=mx+cSlope of line =-1/3

Given the slope of normal to the curve is parallel to the line $-\frac{y}{3x} = -\frac{1}{3}$

Substituting the result in the equation of the curve, we get:

$3x^2 - x^2 = 8 \Rightarrow 2x^2 = 8$

Hence $x = \pm 2$ and $y = \pm 2$.Hence the point of intersection is $(\pm 2, \pm 2)$We have the point and slope, the required equation is $x + 3y \pm 8 = 0$.

7. Given: $f(x) = x^2 - 4x$

   Differentiating w.r.t x, we get

   $\Rightarrow f'(x) = 2x - 4$

   For minimum value, $f'(x) = 0$

   $2x - 4 = 0 \Rightarrow x = 2$

   Again differentiating w.r.t x we get

   $\Rightarrow f''(x) = 2 > 0$

   Hence f(x) attains minimum value at x=2.

8. We have, $\int \frac{3x^2+1}{x} dx$

   $\Rightarrow 3\int x^2 dx + \int \frac{1}{x} dx \Rightarrow 3 \times \frac{x^3}{3} + \log x + C$

   $x^3 + \log x + C$

9. Given the curve $y = x^2$ between the lines x=2 and x=3.

   $\Rightarrow$Area under the curve = $\int_2^3 x^2 dx = \left[\frac{x^3}{3}\right]_2^3$

   $= \frac{19}{3}$

10. Given: $f(x) = \left(\frac{1}{x}\right)^x$

    Taking log on both sides.

    $\Rightarrow \log f(x) = \log\left(\frac{1}{x}\right)^x \Rightarrow \log f(x) = x \log \frac{1}{x}$

    $\Rightarrow \log f(x) = -x \log x$

    Differentiating the above equation with respect to x.

    $\frac{f'(x)}{f(x)} = -x \times \frac{1}{x} - \log x$

    $f'(x) = -f(x)(1 + \log x)$

    Substituting f(x) in the above equation.

    $f'(x) = -\left(\frac{1}{x}\right)^x (1 + \log x)$

    To find the stationary point put $f'(x) = 0$

    $-\left(\frac{1}{x}\right)^x (1 + \log x) = 0$

    Maximum value exists at x=1/e

    Hence $f(1/e) = e^{1/e}$

11. $\int tan^3 x sec^2 x\, dx$

Substitute $u = tanx \Rightarrow \frac{du}{dx} = sec^2 x$

$= \int u^3 du$

Apply power rule:

$= \frac{u^4}{4} + C = \frac{tan^4 x}{4} + C$

12. Solve on own.

13. Corner points of the feasible region are (0,0),(5,0),(6,5),(6,8),(4,10) and (0,8)

Corner points	Z=3x-4y value
(0,0)	0
(5,0)	15
(6,5)	-2
(6,8)	-14
(4,10)	-28
(0,8)	-32

Maximum value Minimum value=-17

14. Probability of getting head = ½

Probability of not getting head =1/2

We want probability of exactly 2 heads. Using binomial distribution, we get

$P(X = 2) = {}^6C_2 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^4 = \frac{6\times5}{2\times1} \times \frac{1}{2^6} = \frac{15}{64}$

15. Given $A^2 = A$…(1)

$(I + A)^3 - 7A = I^3 + A^3 + 3A(I + A) - 7A$

From equation 1

$= I + A.A + 3A + 3A - 7A = I + A^2 + 6A - 7A = I + A - A = I$

16. The principal value of an inverse trigonometric function at point x is the value of the inverse function at point x, which lies in the range of principal branch. Principal value branch of $\csc^{-1} x$ is equal to the domain of cosecx, which is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

17. Let f(x)=x+y

According to the question,xy=1

Therefore, f(x)=x+1/x.Differentiate w.r.t x

$f'(x) = 1 - \frac{1}{x^2}$

$f''(x) = \frac{2}{x^3}$

For maxima and minima,

$1 - \frac{1}{x^2} = 0 \Rightarrow x = \pm 1$

But according to question x>0 Hence x=1.Minimum value=2

18. Matrices have same number of rows and columns, they can be added.

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} y \\ y \\ z \end{bmatrix} + \begin{bmatrix} z \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 5 \end{bmatrix}$

$\begin{bmatrix} x + y + z \\ x + y \\ y + z \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 5 \end{bmatrix}$

From above we can write $x + y + z = 10$ ….(i)

$x + y = 5$ …...(ii)

$y + z = 5$ ….(iii)

From (i) and (ii), we get

5+z=10

z=5, y=0

19. $f(x) = \frac{3x-2}{5}$

Let $f(x) = y$

$y = \frac{3x-2}{5} \Rightarrow 5y = 3x - 2 \Rightarrow x = \frac{5y+2}{3}$

20. The position vector of the point which divides the line joining the above points in the ratio 1:2 is given by

$= \frac{2(\vec{a}+\vec{b})+(2\vec{a}-\vec{b})}{3} = \frac{4\vec{a}+\vec{b}}{3}$

21. Given: $\int \frac{dx}{\sqrt{9-x^2}} = \sin^{-1}\frac{x}{b} + C$

Using the formula,

$\int \frac{dx}{\sqrt{9-x^2}} = \int \frac{dx}{\sqrt{3^2-x^2}} = \sin^{-1}\frac{x}{3} + C$ ....(i)

It is given that $\int \frac{dx}{\sqrt{9-x^2}} = \sin^{-1}\frac{x}{b} + C$...(ii)

On comparing (i) and (ii) we get b=3.

22. $AB = 3I$

Pre multiplication by A^{-1}

$\Rightarrow A^{-1}(AB) = A^{-1}(3I)$

$\Rightarrow A^{-1}AB = 3A^{-1}I$

$\Rightarrow IB = 3A^{-1}$

$\Rightarrow B = 3A^{-1}$

$\Rightarrow A^{-1} = \frac{1}{3}B$

23. Given Rolle's theorem holds for $f(x) = x^4 + ax^3 + bx$, in $-1 \leq x \leq 1$

$\Rightarrow f(-1) = f(1)$

$\Rightarrow 1 - a - b = 1 + a + b$

$\Rightarrow a + b = 0$

$\Rightarrow a = -b$….(i)

$f'(x) = 4x^3 + 3ax^2 + b$

Given $f'\left(\frac{1}{2}\right) = 0$

$f'\left(\frac{1}{2}\right) = 4 \times \left(\frac{1}{2}\right)^3 + 3 \times a \times \left(\frac{1}{2}\right)^2 + b = 0$

$3a + 4b = -2$ ….(ii)

Substituting a=-b in eq (ii)

b=-2 and a=2

ab=-4

24. Let $\sin^{-1} x = t \Rightarrow \frac{1}{\sqrt{1-x^2}} dx = dt$

$\Rightarrow I = \int_0^1 \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}} dx$

$\Rightarrow I = \int e^t dt$

$\Rightarrow I = e^t + C = e^{\sin^{-1} x} + C$

Putting the limits

$I = e^{\frac{\pi}{2}} - 1$

25. Check whether every image in R has a unique pre image in R. Also check for one-one function. Applying rules we get that function is one-one and onto.

26. Let A={1,2,3}

The relation R is defined by R={(1,2)}

Since (1,1)∉R

It is not reflexive

Since (1,2)∈R but (2,1)∉R

It is not symmetric. But there is no counter example to disapprove of transitive condition.

It is transitive

27. Given: $\int_{-\pi}^{\pi} \cos x \, dx$

Let f(x)=cosx

As we can see that f(-x)=cos(-x)=cosx=f(x)

So, cosx is an even function. As we know that, when f(x) is an even function then

$\Rightarrow \int_{-\pi}^{\pi} \cos x \, dx = 2\int_{0}^{\pi} cosx \, dx = 2(\sin\pi - \sin 0) = 0$

28. It is given that $x = t^2 - 1$

$\Rightarrow \frac{dx}{dt} = 2t$

And, $y = t^2 + t$

$\Rightarrow \frac{dy}{dt} = 2t + 1$

$\Rightarrow \frac{dy}{dx} = (2t+1) \times \frac{1}{2t} = 1 + \frac{1}{2t}$

29. Here, $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{x}{3}\right) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{\left(\frac{1}{2}+\frac{x}{3}\right)}{1-\frac{1}{2}\times\frac{x}{3}}\right) = \frac{\pi}{4}$

$\Rightarrow \frac{\frac{3+2x}{6}}{\frac{6-x}{6}} = \tan\frac{\pi}{4} \Rightarrow \frac{3+2x}{6-x} = 1$

$\Rightarrow 3 + 2x = 6 - x$

$\Rightarrow 3x = 3$

$x = 1$

30. If $y = \log\log x$

$\Rightarrow e^y = \log x$

Differentiating both sides with respect to x,

$\Rightarrow e^y \frac{dy}{dx} = \frac{1}{x}$

31. Given: $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$

Now $kA = \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$

By equating the corresponding elements,

k=-6. Also, 2k=3a

2(-6)=3a⇒a=-4

And 3k=2b

3(-6)=2b

2b=-18

b=-9

32. Given: $A^2 = 2A$

Taking determinants on both sides we get

$|A^2| = |2A|$

$|A^2| = 2^3|A|$

Now, $|A| = 0{,}8$

But A is a non-singular matrix

So $|A| = 8$

Now, $|2A| = 2^3|A| = 64$

33. If $\sin\theta = x \Rightarrow \theta = \sin^{-1} x$

Let $f(x) = \sin^{-1} 2x$

We know that $-1 \leq \sin^{-1} x \leq 1$

For f(x): $-1 \leq 2x \leq 1$

$\Rightarrow -\frac{1}{2} \leq x \leq \frac{1}{2}$

Domain of $\sin^{-1} 2x = \left[-\frac{1}{2}, \frac{1}{2}\right]$

34. Direction ratios of the plane = (A,B,C) =(2,3,-6) and direction ratios of x-axis = (a,b,c)=(1,0,0)

$\therefore \sin\theta = \left|\frac{aA+bB+cC}{\sqrt{a^2+b^2+c^2}\sqrt{A^2+B^2+C^2}}\right| = \left|\frac{1\times2+0\times(-3)+0\times6}{\sqrt{1+0+0}\sqrt{4+9+36}}\right| = \frac{2}{\sqrt{49}} = \frac{2}{7}$

$\theta = \sin^{-1}\left(\frac{2}{7}\right)$

$\therefore \alpha = \frac{2}{7}$

35. $f(x) = \begin{cases} 3x-4, & 0 \le x \le 2 \\ 2x+l, & 2 < x \le 9 \end{cases}$

$\Rightarrow LHL = \lim_{h \to 0} f(2-h)$

$= \lim_{h \to 0} 3(2-h) - 4$

$\Rightarrow LHL = 2$

Similarly RHL=4+l

Since f(x) is continuous at x=2,

LHL=RHL=f(2)

4+l=3(2)-4=2

l=-2

36. We have $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$,

Let x=tanz

$\therefore y = \cos^{-1}\left(\frac{1-tan^2 z}{1+tan^2 z}\right) = \cos^{-1}(\cos 2z) = 2z = 2\tan^{-1} x$

Differentiating w.r.t x

$\frac{dy}{dx} = \frac{d}{dx}(2\tan^{-1} x) = \frac{2}{1+x^2}$

37. $\tan^{-1} x + \tan^{-1} y = \frac{4\pi}{5}$

$\left(\frac{\pi}{2} - \cot^{-1} x\right) + \left(\frac{\pi}{2} - \cot^{-1} y\right) = \frac{4\pi}{5}$

$\pi - (\cot^{-1} x + \cot^{-1} y) = \frac{4\pi}{5}$

$\cot^{-1} x + \cot^{-1} y = \frac{\pi}{5}$

38. $y = 2^x + x\log x$

$\Rightarrow \frac{dy}{dx} = 2^x \log 2 + [(x)' \log x + x(\log x)']$

$\Rightarrow \frac{dy}{dx} = 2^x \log 2 + \log x + 1$

39. Area of triangle A = $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 3 & 0 & 1 \\ 7 & 0 & 1 \\ 8 & 4 & 1 \end{vmatrix}$

$\Rightarrow 2A = 3(0-4) - 0(7-8) + 1(28-0)$

$\Rightarrow 2A = -12 + 28$

$\Rightarrow A = \frac{16}{2} = 8\ sq.unit$

40. $\int \frac{dx}{x^2+4} = \int \frac{dx}{x^2+2^2} = \frac{1}{2}\tan^{-1}\frac{x}{2} + C$

41. Given P(A)=3/5 and P(B)=4/9

$P(A') = 1 - \frac{3}{5} = \frac{2}{5}$

$P(B') = 1 - \frac{4}{9} = \frac{5}{9}$

$P(A' \cap B') = P(A').P(B') = \frac{2}{5} \times \frac{5}{9} = \frac{2}{9}$

42. We know that the area of a circle of radius units is given by $A = \pi r^2$

$\therefore \frac{dA}{dt} = \frac{d}{dt}(\pi r^2) = 2\pi r\left(\frac{dr}{dt}\right)$

Now $\left[\frac{dA}{dt}\right]_{r=4} = 8\pi(0.01) = 0.08\pi$

43. Solve on your own

44. $f(kx, ky) = \sin kx - \cos ky \neq f(x, y)$

Hence not a homogeneous function.

45. Composition of a function.

When the function f(x) is substituted in the function g(x), the composite function formed is represented by (gof)x. It can be termed as g of f of x.

We have $f(x) = \frac{2x-1}{2}$ and g(x)=x+2.

$f\left(\frac{3}{2}\right) = \frac{2\times\frac{3}{2}-1}{2} = \frac{2}{2} = 1$

$(gof)\frac{3}{2} = g\left(f\left(\frac{3}{2}\right)\right) = g(1) = 1 + 2 = 3$

46. Given: $V(x) = -x^2 + 25x + 7500$

$\frac{dV}{dx} = -2x + 25$

For maximum volume, put $\frac{dV}{dx} = 0$

$\Rightarrow -2x + 25 = 0$

2x=25

x=12.5

47. Volume is maximum at 12.5

$V(12.5) = -(12.5)^2 + 25 \times 12.5 + 7500 = 7656.25$

48. For volume to strictly increase, dV/dx>0

-2x+25>0

x<12.5

Interval in which volume is strictly increasing is (0,12.5)

49. $V(4) = -4^2 + 25 \times 4 + 7500 = 7584$

50. V(x)=7650

$7650 = -x^2 + 25x + 7500$

$x^2 - 25x + 150 = 0$

$(x - 10)(x - 15) = 0$

x=10 or x=15