INFERENTIAL STATISTICS - GRADED PROJECT

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Problem Statement:

This project involves the application of inferential statistical methods to real-world datasets. The objective is to analyse data related to manufacturing materials and medical implants using probability theory, hypothesis testing, and ANOVA to derive meaningful conclusions.

Introduction to Data Sets:

Dataset 1: (For Problem 3)

The given data set has 75 rows and 2 columns. Both columns are of data type float64

	Unpolished	Treated and Polished
0	164.481713	133.209393
1	154.307045	138.482771
2	129.861048	159.665201
3	159.096184	145.663528
4	135.256748	136.789227

Numerical columns: Unpolished, Treated and Polished

Below are the summary statistics for numerical variables.

	count	mean	std	min	25%	50%	75%	max
Unpolished	75.0	134.110527	33.041804	48.406838	115.329753	135.597121	158.215098	200.161313
Treated and Polished	75.0	147.788117	15.587355	107.524167	138.268300	145.721322	157.373318	192.272856

Dataset 2: (For Problem 4)

This dataset has 90 rows and 5 columns. All five columns are of data type int64.

	Dentist	Method	Alloy	Temp	Response
0	1	1	1	1500	813
1	1	1	1	1600	792
2	1	1	1	1700	792
3	1	1	2	1500	907
4	1	1	2	1600	792

Numerical columns: Dentist, Method, Alloy, Temp and Response.

Below are the summary statistics for numerical variables.

	count	mean	std	min	25%	50%	75%	max
Dentist	90.0	3.000000	1.422136	1.0	2.0	3.0	4.0	5.0
Method	90.0	2.000000	0.821071	1.0	1.0	2.0	3.0	3.0
Alloy	90.0	1.500000	0.502801	1.0	1.0	1.5	2.0	2.0
Temp	90.0	1600.000000	82.107083	1500.0	1500.0	1600.0	1700.0	1700.0
Response	90.0	741.777778	145.767845	289.0	698.0	767.0	824.0	1115.0

Few Concepts and Statistical Tests Used:

ttest_1samp:
It is one sample t-test. It tests whether the mean of a single group is equal to known value or not.
Example: Is the average weight of students is less than 70?

norm.cdf:
Stands for cumulative distribution function for the normal distribution.
It helps in calculating the probability that a value is less than or equal to a given number.
It is useful for finding probabilities under a normal curve.

ttest_ind:
Independent two-sample t-test is used to compare the means of two independent groups.
Example: Are weights of students of section A and section B different?

Hypothesis Testing:
Hypothesis testing is a way to make decisions using data.
You start with a **null hypothesis (H_0)** and check whether the data provides enough evidence to **reject it** in favor of an **alternative hypothesis (H_1)**.
Example:
H_0: There is no difference in performance between two teaching methods.
H_1: There is a difference.

Shapiro-Wilk's Test:
Used to check if a dataset follows a normal distribution.
Why it is important: Many statistical tests assume normality.
Interpretation: If p-value > 0.05 → Data is likely normal. If p-value < 0.05 → Data is not normally distributed.

Levene Test:
Used to test whether two or more groups have equal variances.
Why it is needed: Some tests (like ANOVA and t-tests) assume equal variances.
Interpretation: If p-value < 0.05 → Variances are not equal.

Tukey's HSD:
It is a post-hoc test used after ANOVA if the result is significant.
It tells us which specific group pairs are significantly different.
It goes step deeper and breaks down the result of ANOVA into clear pairwise differences.

One Way ANOVA:
Tests if three or more group means are significantly different.
One factor, multiple groups (e.g., 3 teaching methods).
H_0: All group means are equal
H_1: At least one group mean is different

Two Way ANOVA:
Tests the effect of two factors and their interaction on a response.
Here two factors are independent variables and our response is a dependent variable.
Always independent variables are categorical and dependent variable is continuous .
Example: In our case effect of Dentist and Method on implant hardness.
What checks are done:

1) Main effect of each factor
2) Interaction effect between them

Analysis:

PROBLEM 1:

Problem 1

A physiotherapist with a male football team is interested in studying the relationship between foot injuries and the positions at which the players play from the data collected.

	Striker	Forward	Attacking Midfielder	Winger	**Total**
Players Injured	45	56	24	20	**145**
Players Not Injured	32	38	11	9	**90**
Total	**77**	**94**	**35**	**29**	**235**

Based on the above data, answer the following questions.

1.1 What is the probability that a randomly chosen player would suffer an injury?
From the table given in the question we can observe that there are 145 players injured in total and there are 235 total number of players.
So the probability will be 145 divided by 235 which is 0.617.

1.2 What is the probability that a player is a forward or a winger?
There are total of 94 forward players and 29 wingers. We also know that there are 235 total players.
So the probability will be addition of 94 divided by 235 and 29 divided by 235 which is 0.523.

1.3 What is the probability that a randomly chosen player plays in a striker position and has a foot injury?
There are 45 injured strikers, 77 total strikers and 235 total players.
So probability will be multiplication of total strikers divided by total players and injured strikers divided by total players which is 0.191.

1.4 What is the probability that a randomly chosen injured player is a striker?
There are 45 injured strikers and 145 total injured players.
So the probability will be 45 divided by 145, which is 0.310.

PROBLEM 2:

The breaking strength of gunny bags used for packaging cement is normally distributed with a mean of 5 kg per sq. centimeter and a standard deviation of 1.5 kg per sq. centimeter. The quality team of the cement company wants to know the following about the packaging material to better understand wastage or pilferage within the supply chain; Answer the questions below based on the given information; (Provide an appropriate visual representation of your answers, without which marks will be deducted)

2.1 What proportion of the gunny bags have a breaking strength of less than 3.17 kg per sq cm?

To calculate this, we used the norm.cdf() function which returns the cumulative probability under the normal distribution curve. The function requires three inputs: the value of interest (3.17), the population mean (5), and the standard deviation (1.5).After inputting these values, we find that the proportion of gunny bags with a breaking strength less than 3.17 kg per sq. cm is approximately 0.11.

2.2 What proportion of the gunny bags have a breaking strength of atleast 3.6 kg per sq cm?

If we look at the question it is telling us to find the gunny bags which have breaking strength greater than or equal to 3.6. For that what we can do is to subtract the value that we get when we input our value, mean and sigma in norm.cdf function from 1. The output of this will be our required proportion. Proportion of the gunny bags having a breaking strength of atleast 3.6 kg per sq cm is 0.82.

2.3 What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq cm?

We used norm.cdf function on two strengths given and subtracted the smaller from the larger. Then the proportion of the gunny baga having a breaking strength between 5 and 5.5 is 0.13.

2.4 What proportion of the gunny bags have a breaking strength not between 3 and 7.5 kg per sq cm?

For this we have first used norm.cdf function on two strengths given and subtracted the smaller from the larger. After getting that value we subtracted it from 1. By doing this the proportion of gunny bags having a breaking strength not between 3 and 7.5 is 0.14.

Note: I have rounded of the proportion values in all the questions to two decimal places.

Visualisations Assosciated with the 4 sub questions of the problem:

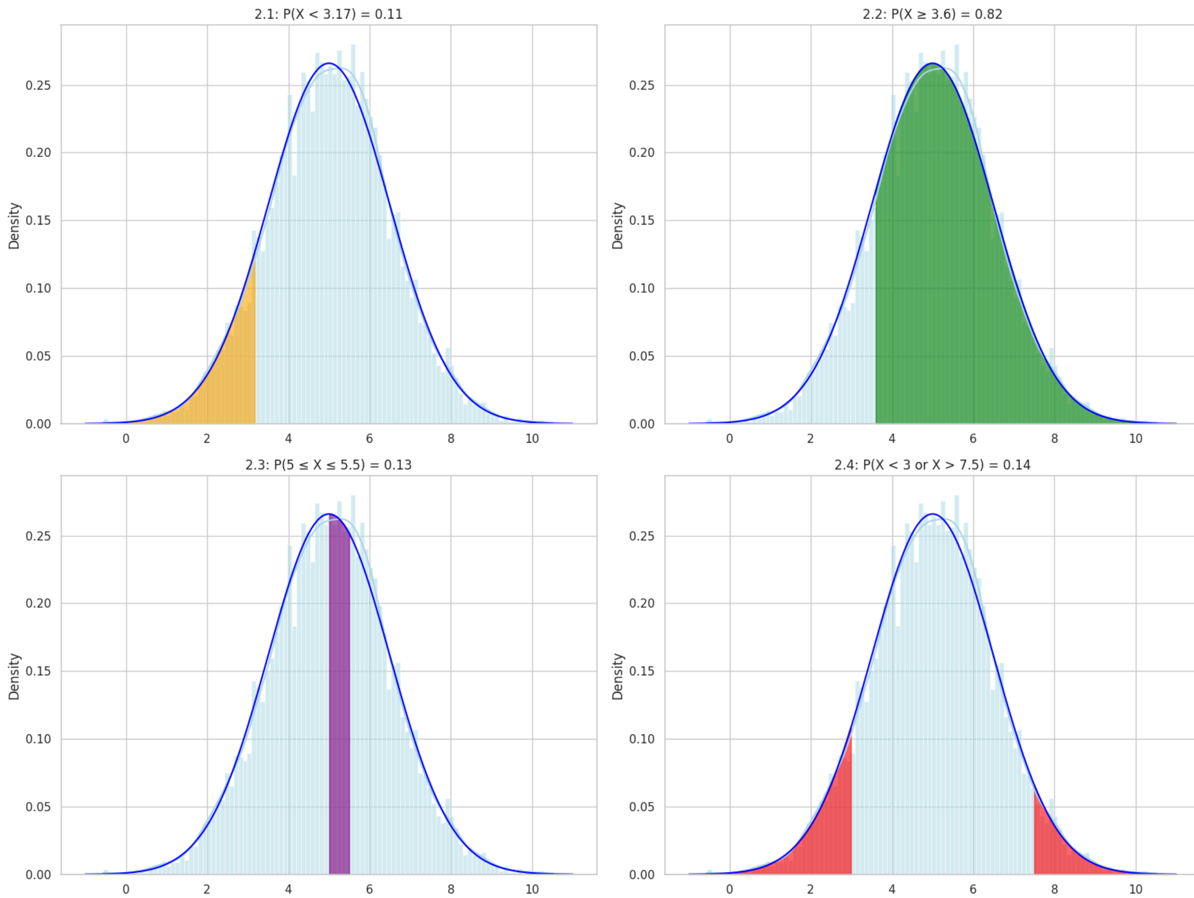

Fig 1: Visual representation of the area under the curve for four sub-questions

PROBLEM 3:

Zingaro stone printing is a company that specializes in printing images or patterns on polished or unpolished stones. However, for the optimum level of printing of the image, the stone surface has to have a Brinell's hardness index of at least 150. Recently, Zingaro has received a batch of polished and unpolished stones from its clients. Use the data provided to answer the following (assuming a 5% significance level)

3.1 Zingaro has reason to believe that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?

Formulating Null and Alternate Hypothesis:
Null Hypothesis - Brinell's hardness index is greater than or equal to 150
Alternate Hypothesis - Brinell's hardness index is less than 150
Given level of significance is 0.05

The statistical test used here is ttest_1samp.
The function used here is ttest_1samp(). It takes three parameters first is the data that we want to consider, second being the given mean and third parameter is alternative where less than or greater than or two sided can be passed.
In our case, I have passed the unpolished column, mean given and lesser. By doing this is got p value as 0.00004 which is less than level of significance 0.05. So we reject the null hypothesis.
The conclusion is that there is strong statistical evidence to say that Brinell's hardness index is less than 150.

3.2 Is the mean hardness of the polished and unpolished stones the same?

Formulating Null and Alternate Hypothesis:
Null Hypothesis - Mean hardness of unpolished stones is same as mean hardness of polished stones.
Alternate Hypothesis - Mean hardness of unpolished stones is different from mean hardness of polished stones.

Given level of significance is 0.05
The statistical test used here is ttest_ind
The function used here is ttest_ind(). It takes three parameters. First two parameters are unpolished and treated and polished columns. Third parameter is alternative is equal to two-sided.
By doing this I got p value as 0.00146 which is less than level of significance 0,05. So we reject the null hypothesis.
The conclusion is that there is strong statistical evidence to say that mean hardness of polished and unpolished stones is different.

PROBLEM 4:

The hardness of metal implants in dental cavities depends on multiple factors, such as the method of implant, the temperature at which the metal is treated, the alloy used as well as the dentists who may favor one method above another and may work better in his/her favorite method. The response is the variable of interest.

4.1 How does the hardness of implants vary depending on dentist?

We compare how the hardness of implants vary depending on a dentist for two alloys. There are two alloys used.
Considering Alloy 1
I started this question by plotting a boxplot using the seaborn library to help me understand the spread of the response variable for each of the dentists. I did this to know the mean responses

of the hardness of implants for each of the dentists. There are 5 dentists in total. Here is the visual representation.

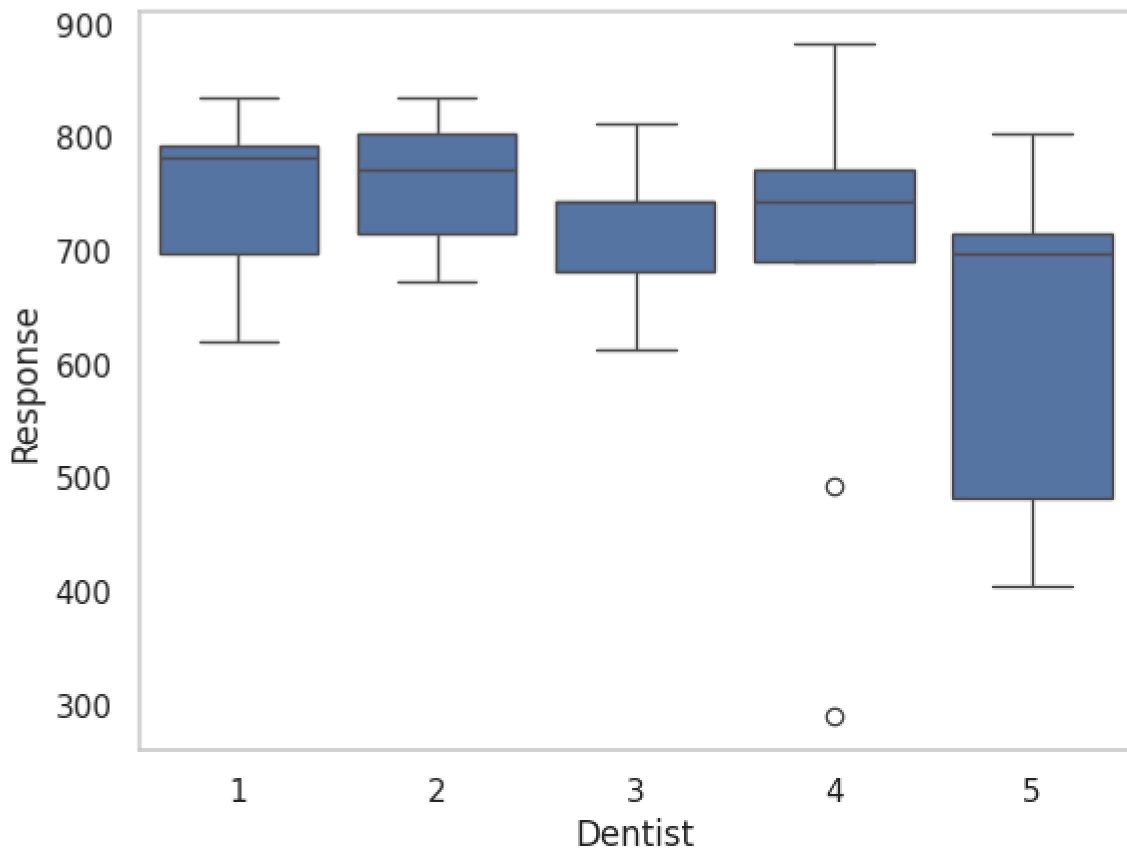

Fig 2: Dentist vs Response

Observations:

For four dentists, we could see that the distribution of responses is more or less the same. For the fifth dentist, the distribution of responses is too varied.

Formulating Null and Alternate Hypothesis

Null Hypothesis - The mean hardness of implants is the same for all the dentists.
Alternate Hypothesis - The mean hardness of implants is different for at least one of the dentists.

The level of significance is 0.05
The Statistical test used is one-way ANOVA.

In a one-way ANOVA test, we compare the means from several populations to test if there is any significant difference between them. The results from an ANOVA test are most reliable when the assumptions of normality and equality of variances are satisfied.

- For testing of normality, Shapiro-Wilk's test is applied to the response variable.
- For equality of variance, the Levene test is applied to the response variable.

Considering Shapiro-Wilk's Test
Null Hypothesis - Response follows normal distribution
Alternate Hypothesis - Response does not follow normal distribution

The p-value I got is 0.000001 and is less than level of significance which is 0.05. We reject the null hypothesis.
The conclusion is that we have enough statistical evidence to say that the response does not follow the normal distribution.

Considering Levene Test
Null Hypothesis is all response variances are equal for the dentists.
Alternate Hypothesis is at least for one dentist, the response variance is different from the rest.

Here, when performing the levene test we consider association of each of the dentists with respect to response. We subgroup the data frame with respect to dentist and response and pass it into the levene function.
The p-value that I got is 0.256, which is less than 0.05. We reject the null hypothesis.
The conclusion is that we have statistical evidence to say that at least for one dentist group response variance is different from the rest.

Performing One way ANOVA
We use the f_oneway function to obtain p-value. Here we pass the subgroups of the data.
Obtained p-value is 0.116 which is greater than level of significance 0.05. We fail to reject the null hypothesis.
The conclusion is that there is not enough statistical evidence to say that the mean hardness of implants differs across dentists.

Considering Alloy 2
I started this question by plotting a boxplot using the seaborn library to help me understand the spread of the response variable for each of the dentists. I did this to know the mean responses of the hardness of implants for each of the dentists. Here is the visual representation.

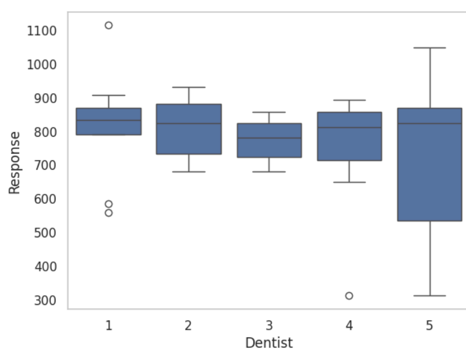

Fig 3: Dentist vs Response

Observation:
Dentist 5 has a varied response for hardness of implants when compared to other dentists.

Formulating Null and Alternate Hypothesis
Null Hypothesis - The mean hardness of implants is the same for all the dentists.
Alternate Hypothesis - The mean hardness of implants is different for at least one of the dentists.

The level of significance is 0.05
The Statistical test used is one-way ANOVA.

Considering Shapiro-Wilk's Test
Null Hypothesis - Response follows normal distribution
Alternate Hypothesis - Response does not follow normal distribution

The p-value I got is 0.0004 and is less than level of significance which is 0.05. We reject the null hypothesis.
The conclusion is that we have enough statistical evidence to say that the response does not follow the normal distribution.

Considering Levene Test
Null Hypothesis is all response variances are equal for the dentists.
Alternate Hypothesis is at least for one dentist, the response variance is different from the rest.

Here, when performing the levene test we consider association of each of the dentists with respect to response. We subgroup the data frame with respect to dentist and response and pass it into the levene function.

The p-value that I got is 0.236, which is less than 0.05. We reject the null hypothesis.
The conclusion is that we have statistical evidence to say that at least for one dentist group response variance is different from the rest.

Performing One way ANOVA
We use the f_oneway function to obtain p-value. Here we pass the subgroups of the data.
Obtained p-value is 0.718 which is greater than level of significance 0.05. We fail to reject the null hypothesis.
The conclusion is that there is not enough statistical evidence to say that the mean hardness of implants differs across dentists.

4.2 How does the hardness of implants vary depending on methods?

Considering Alloy 1
I started this question by plotting a boxplot using the seaborn library to help me understand the spread of the response variable for each of the methods. There are 3 methods used in total.
Here is the visual representation.

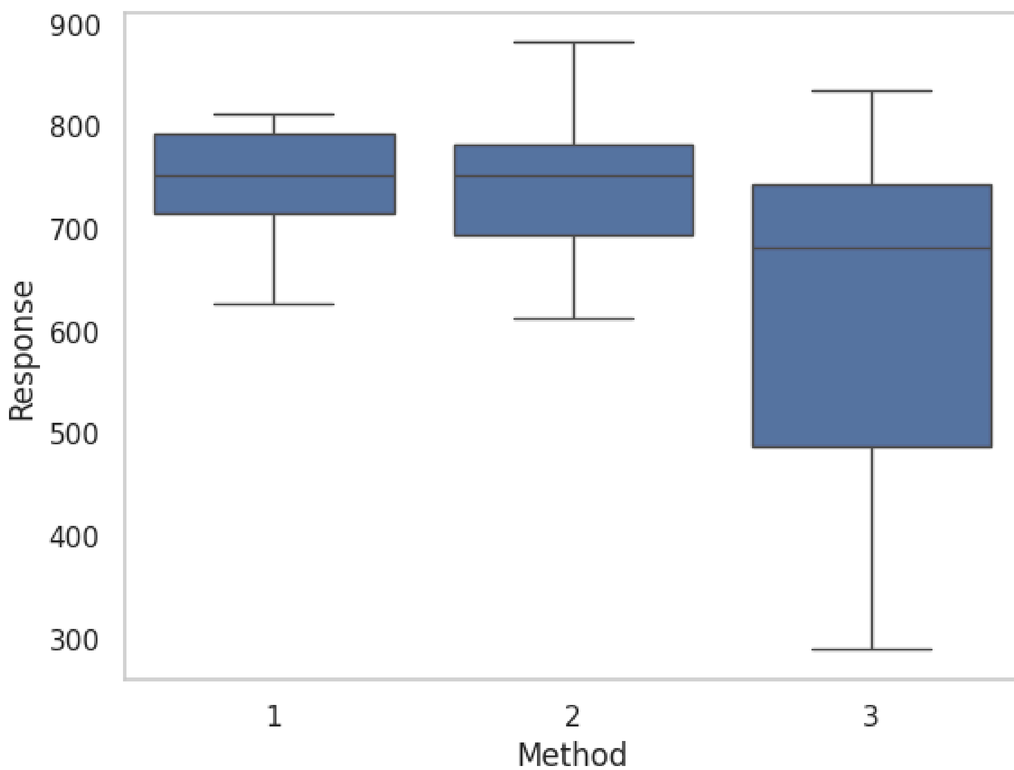

Fig 4: Method vs Response

Observations:

1) From the first two methods, we could see that the distribution of responses is nearly the same for the hardness of the implant.
2) The third method yielded varied distribution of responses for the hardness of the implants.

Formulating the Null and Alternate Hypothesis
Null Hypothesis - The mean hardness of implants is the same for all the methods.
Alternate Hypothesis - The mean hardness of implants is different for atleast one of the methods.

As dependent variable is same as 4.1 we can say that response is not following normal distribution. (Shapiro Test)

Considering Levene Test
Null Hypothesis - All response variances are equal for all the methdods
Alternate Hypothesis - Atleast for one of the methods response variance is different from the rest.

Here, when performing the levene test we consider association of each of the methods with respect to response. We subgroup the data frame with respect to method and response and pass it into the levene function.

The p-value that I got is 0.003, which is less than 0.05. We reject the null hypothesis.
The conclusion is that we have statistical evidence to say that at least for one method response variance is different from the rest.

Performing One way ANOVA
We use the f_oneway function to obtain p-value. Here we pass the subgroups of the data. Obtained p-value is 0.004 which is less than level of significance 0.05. We reject the null hypothesis.

The conclusion is that there is enough statistical evidence to say that the mean hardness of implants differs across methods.

Performing Tukey's HSD Test for alloy 1

```
Tukey HSD results for alloy 1:
Multiple Comparison of Means - Tukey HSD, FWER=0.05
=======================================================
group1 group2  meandiff p-adj    lower    upper   reject
-------------------------------------------------------
     1      2   -6.1333  0.987  -102.714  90.4473  False
     1      3    -124.8 0.0085 -221.3807 -28.2193   True
     2      3 -118.6667 0.0128 -215.2473  -22.086   True
-------------------------------------------------------
```

Table 1: Tukey HSD Test for alloy 1

If we see True in the reject column that means we are rejecting the null hypothesis and there is significant effect of that particular method on hardness of implants.
From this we can conclude that method 3 plays significant role in hardness of implants.

Considering Alloy 2
I started this question by plotting a boxplot using the seaborn library to help me understand the spread of the response variable for each of the methods. There are 3 methods used in total. Here is the visual representation.

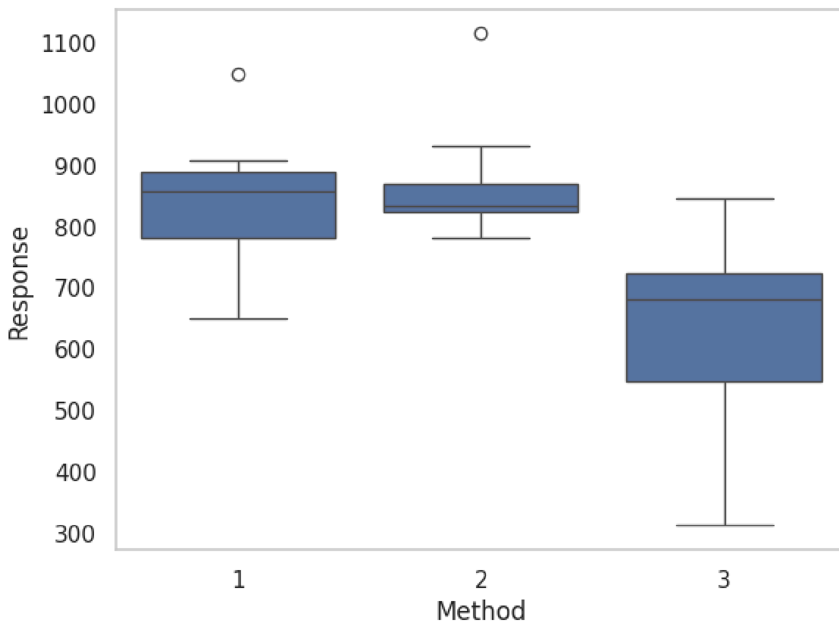

Fig 5: Method vs Response

Observation:
Method 3 has given varied response on hardness of implants when comparedto other two methods.

Formulating Null and Alternate Hypothesis
Null Hypothesis - The mean hardness of implants is the same for all the methods.
Alternate Hypothesis - The mean hardness of implants is different for atleast one of the methods.

Considering Levene
Null Hypothesis - All response variances are equal for all the methdods
Alternate Hypothesis - Atleast for one of the methods response variance is different from the rest.

Here, when performing the levene test, we consider the association of each of the methods with respect to response. We subgroup the data frame with respect to method and response and pass it into the levene function.

The p-value that I got is 0.04, which is less than 0.05. We reject the null hypothesis.
The conclusion is that we have statistical evidence to say that at least for one method response variance is different from the rest.

Performing One-way ANOVA
We use the f_oneway function to obtain p-value. Here we pass the subgroups of the data. Obtained p-value is 0.000005 which is less than level of significance 0.05. We reject the null hypothesis.
The conclusion is that there is enough statistical evidence to say that the mean hardness of implants differs across methods.

Performing Tukey's HSD Test for alloy 2

Multiple Comparison of Means - Tukey HSD, FWER=0.05

group1	group2	meandiff	p-adj	lower	upper	reject
1	2	27.0	0.8212	-82.4546	136.4546	False
1	3	-208.8	0.0001	-318.2546	-99.3454	True
2	3	-235.8	0.0	-345.2546	-126.3454	True

Table 2: Tukey HSD Test for alloy 2

Conclusion: Method 3 is significantly differ from other methods in terms of response of hardness of implants.

4.3 What is the interaction effect between the dentist and method on the hardness of dental implants for each type of alloy?
Our goal for this question is to create an interaction plot between Method and Dentist on Response for each kind of alloy
Interaction plot between Method and Dentist on Response for alloy 1

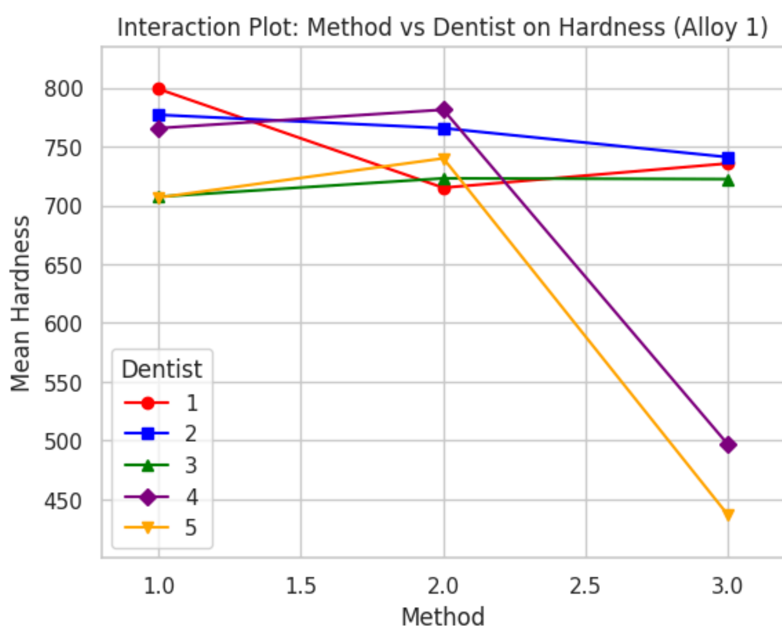

Fig 6: Interaction plot of Method and Dentist on Response of hardness of implant for alloy 1

Observations:

1) Dentist 5 shows a sharp drop in hardness from Method 2 to Method 3, indicating a strong interaction effect.
2) Dentist 1 exhibits the highest hardness with Method 1 among all combinations.
3) Dentists 2 and 3 show minimal variation in hardness across methods, suggesting consistency.
4) Dentist 4's hardness drops significantly from Method 2 to Method 3.
5) Method 3 generally shows reduced hardness for most dentists, especially 4 and 5.

Interaction plot between Method and Dentist on Response for alloy 2

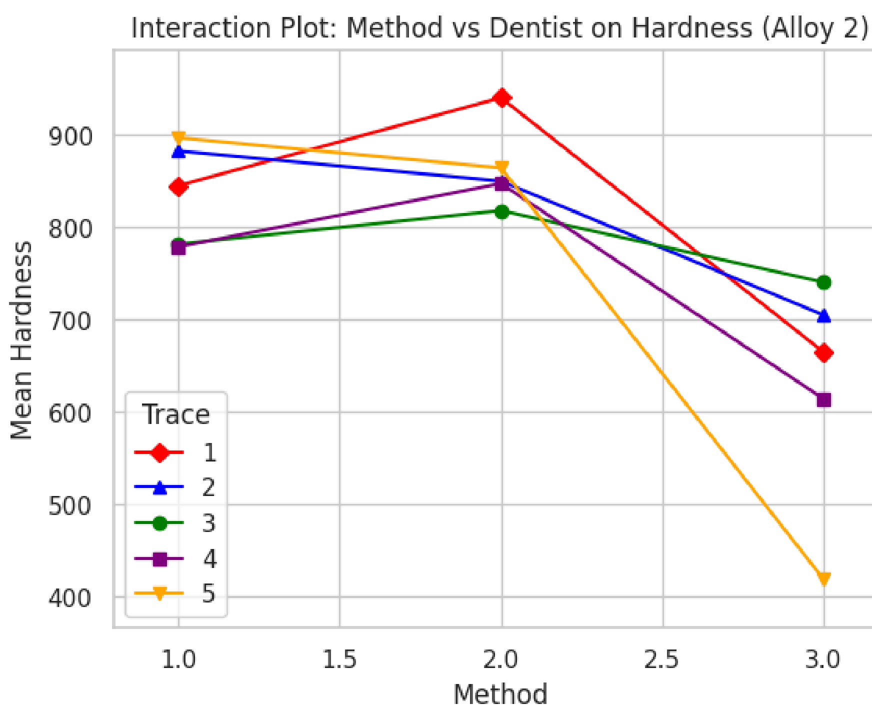

Fig 7: Interaction plot of Method and Dentist on Response of hardness of implant for alloy 2

Observations:

1) Dentist 1 achieves the highest hardness using Method 2 among all combinations.
2) Dentist 5 shows a dramatic drop in hardness from Method 2 to Method 3, indicating strong interaction.
3) Dentist 3 maintains the most stable trend, with minor variation across methods.
4) Method 3 results in significantly lower hardness for most dentists, especially 4 and 5.
5) Dentists 2 and 4 follow a similar pattern, peaking at Method 2 and dropping at Method 3.

4.4 How does the hardness of implants vary depending on dentists and methods together?

Global Formulation of Null and Alternate hypothesis

- Null Hypothesis is there is no significant effect of Dentists, methods or their interaction on hardness of implants.
- Alternate Hypothesis is there is effect of atleast one of the Dentists, methods or their interaction on hardness of implants.

Formulating Null and Alternate Hypothesis for Dentist

- Null Hypothesis is there is no significant effect of Dentist on hardness of implants.
- Alternate Hypothesis is there is significant effect of Dentist on hardness of implants.

Formulating Null and Alternate Hypothesis for Method

- Null Hypothesis is there is no significant effect of Method on hardness of implants.
- Alternate Hypothesis is there is significant effect of Method on hardness of implants.

Formulating Null and Alternate Hypothesis for interaction between Dentist and Method

- Null Hypothesis is there is no significant interaction between Dentist and Method on hardness of implants.
- Alternate Hypothesis is there is significant interaction between Dentist and Method on hardness of implants.

Statistical test used is Two way ANOVA
Significance level given is 0.05

Considering Alloy 1
For two way test we have to check for some assumptions same as how we did for One way ANOVA but difference here is when doing shapiro test we consider residual.

Considering Shapiro-Wilk's Test

Formulating Null and Alternate Hypothesis
Null hypothesis is residuals are normally distributed
Alternate hypothesis is residuals are not normally distributed

After doing the shapiro test p value obtained is 0.090 which is greater than level of significance 0.05. We fail to reject the null hypothesis

Conclusion is that we don't have enough statistical evidence to say that residuals are not normally distributed.

Considering Levene Test

Formulating Null and Alternate Hypothesis
Null Hypothesis is All group variances are equal
Alternate Hypothesis is Atleast one group has a different variance

After doing the levene test p value obtained is 0.312 which is greater than level of significance 0.05. We fail to reject the null hypothesis

Conclusion is that we don't have enough statistical evidence to say that atleast one group has a different variance.

We fit an ANOVA model for Alloy 1 using Ordinary Least Squares (OLS):

```
ols('Response ~ C(Dentist) + C(Method) + C(Dentist):C(Method)', data=df1_alloy1).fit()
```

Here we are seeing how response is affected with respect to Dentist, Method and Dentist and Method combined interaction.
We use anova_lm to check how each factor (and their interaction) significantly affects the response.

ANOVA Table 1:

	sum_sq	df	F	PR(>F)
C(Dentist)	106683.688889	4.0	3.899638	0.011484
C(Method)	148472.177778	2.0	10.854287	0.000284
C(Dentist):C(Method)	185941.377778	8.0	3.398383	0.006793
Residual	205180.000000	30.0	NaN	NaN

Table 3: ANOVA Table for alloy 1

Observations for Alloy 1:

1) If we consider Dentist, as we can observe that p value is less than 0.05 we have statistical evidence to say that different dentists have significant effect on response. (We reject null hypothesis)
2) If we consider Method, as we can observe that p value is less than 0.05 we have statistical evidence to say that different methods have significant effect on reponse. (We reject null hypothesis)
3) If we consider interaction between Dentist and Method we can observe that p value is less than 0.05 and we have statistical evidence to say that different combination of Dentist and Method have significant effect on response. (We reject null hypothesis)

Tukey HSD Test
Here, I created a new column called group, which is of form x_y
Here, x means the dentist involved, and y means the method used. X can take 5 values and y can take 3 values.
After passing this column along with the response and dataframe, I got to know which dentist and method groups have made a significant impact on the hardness of implants.

```
Tukey Test 1:
Multiple Comparison of Means - Tukey HSD, FWER=0.05
==========================================================
group1 group2  meandiff p-adj    lower     upper   reject
----------------------------------------------------------
   1_1    1_2     -84.0 0.9933 -332.8283  164.8283  False
   1_1    1_3  -63.3333 0.9996 -312.1617   185.495  False
   1_1    2_1     -22.0    1.0 -270.8283  226.8283  False
   1_1    2_2  -33.3333    1.0 -282.1617   215.495  False
   1_1    2_3     -58.0 0.9999 -306.8283  190.8283  False
   1_1    3_1  -91.6667 0.9853  -340.495  157.1617  False
   1_1    3_2     -76.0 0.9975 -324.8283  172.8283  False
   1_1    3_3  -76.6667 0.9972  -325.495  172.1617  False
   1_1    4_1  -33.3333    1.0 -282.1617   215.495  False
   1_1    4_2  -17.6667    1.0  -266.495  231.1617  False
   1_1    4_3 -302.6667  0.007  -551.495  -53.8383   True
   1_1    5_1  -92.3333 0.9844 -341.1617   156.495  False
   1_1    5_2     -59.0 0.9998 -307.8283  189.8283  False
   1_1    5_3 -362.6667 0.0007  -611.495 -113.8383   True
   1_2    1_3   20.6667    1.0 -228.1617   269.495  False
   1_2    2_1      62.0 0.9997 -186.8283  310.8283  False
   1_2    2_2   50.6667    1.0 -198.1617   299.495  False
   1_2    2_3      26.0    1.0 -222.8283  274.8283  False
   1_2    3_1   -7.6667    1.0  -256.495  241.1617  False
   1_2    3_2       8.0    1.0 -240.8283  256.8283  False
   1_2    3_3    7.3333    1.0  -241.495  256.1617  False
   1_2    4_1   50.6667    1.0 -198.1617   299.495  False
   1_2    4_2   66.3333 0.9994  -182.495  315.1617  False
   1_2    4_3 -218.6667 0.1324  -467.495   30.1617  False
   1_2    5_1   -8.3333    1.0 -257.1617   240.495  False
   1_2    5_2      25.0    1.0 -223.8283  273.8283  False
   1_2    5_3 -278.6667 0.0173  -527.495  -29.8383   True
   1_3    2_1   41.3333    1.0  -207.495  290.1617  False
   1_3    2_2      30.0    1.0 -218.8283  278.8283  False
   1_3    2_3    5.3333    1.0  -243.495  254.1617  False
   1_3    3_1  -28.3333    1.0 -277.1617   220.495  False
   1_3    3_2  -12.6667    1.0  -261.495  236.1617  False
   1_3    3_3  -13.3333    1.0 -262.1617   235.495  False
   1_3    4_1      30.0    1.0 -218.8283  278.8283  False
   1_3    4_2   45.6667    1.0 -203.1617   294.495  False
```

1_3	4_3	-239.3333	0.0688	-488.1617	9.495	False
1_3	5_1	-29.0	1.0	-277.8283	219.8283	False
1_3	5_2	4.3333	1.0	-244.495	253.1617	False
1_3	5_3	-299.3333	0.0079	-548.1617	-50.505	True
2_1	2_2	-11.3333	1.0	-260.1617	237.495	False
2_1	2_3	-36.0	1.0	-284.8283	212.8283	False
2_1	3_1	-69.6667	0.999	-318.495	179.1617	False
2_1	3_2	-54.0	0.9999	-302.8283	194.8283	False
2_1	3_3	-54.6667	0.9999	-303.495	194.1617	False
2_1	4_1	-11.3333	1.0	-260.1617	237.495	False
2_1	4_2	4.3333	1.0	-244.495	253.1617	False
2_1	4_3	-280.6667	0.016	-529.495	-31.8383	True
2_1	5_1	-70.3333	0.9989	-319.1617	178.495	False
2_1	5_2	-37.0	1.0	-285.8283	211.8283	False
2_1	5_3	-340.6667	0.0016	-589.495	-91.8383	True
2_2	2_3	-24.6667	1.0	-273.495	224.1617	False
2_2	3_1	-58.3333	0.9999	-307.1617	190.495	False
2_2	3_2	-42.6667	1.0	-291.495	206.1617	False
2_2	3_3	-43.3333	1.0	-292.1617	205.495	False
2_2	4_1	0.0	1.0	-248.8283	248.8283	False
2_2	4_2	15.6667	1.0	-233.1617	264.495	False
2_2	4_3	-269.3333	0.0243	-518.1617	-20.505	True
2_2	5_1	-59.0	0.9998	-307.8283	189.8283	False
2_2	5_2	-25.6667	1.0	-274.495	223.1617	False
2_2	5_3	-329.3333	0.0025	-578.1617	-80.505	True
2_3	3_1	-33.6667	1.0	-282.495	215.1617	False
2_3	3_2	-18.0	1.0	-266.8283	230.8283	False
2_3	3_3	-18.6667	1.0	-267.495	230.1617	False
2_3	4_1	24.6667	1.0	-224.1617	273.495	False
2_3	4_2	40.3333	1.0	-208.495	289.1617	False
2_3	4_3	-244.6667	0.0576	-493.495	4.1617	False
2_3	5_1	-34.3333	1.0	-283.1617	214.495	False
2_3	5_2	-1.0	1.0	-249.8283	247.8283	False
2_3	5_3	-304.6667	0.0065	-553.495	-55.8383	True
3_1	3_2	15.6667	1.0	-233.1617	264.495	False
3_1	3_3	15.0	1.0	-233.8283	263.8283	False
3_1	4_1	58.3333	0.9999	-190.495	307.1617	False
3_1	4_2	74.0	0.9981	-174.8283	322.8283	False
3_1	4_3	-211.0	0.166	-459.8283	37.8283	False
3_1	5_1	-0.6667	1.0	-249.495	248.1617	False
3_1	5_2	32.6667	1.0	-216.1617	281.495	False
3_1	5_3	-271.0	0.0229	-519.8283	-22.1717	True
3_2	3_3	-0.6667	1.0	-249.495	248.1617	False
3_2	4_1	42.6667	1.0	-206.1617	291.495	False
3_2	4_2	58.3333	0.9999	-190.495	307.1617	False
3_2	4_3	-226.6667	0.1035	-475.495	22.1617	False
3_2	5_1	-16.3333	1.0	-265.1617	232.495	False

3_2	5_2	17.0	1.0	-231.8283	265.8283	False
3_2	5_3	-286.6667	0.0128	-535.495	-37.8383	True
3_3	4_1	43.3333	1.0	-205.495	292.1617	False
3_3	4_2	59.0	0.9998	-189.8283	307.8283	False
3_3	4_3	-226.0	0.1057	-474.8283	22.8283	False
3_3	5_1	-15.6667	1.0	-264.495	233.1617	False
3_3	5_2	17.6667	1.0	-231.1617	266.495	False
3_3	5_3	-286.0	0.0131	-534.8283	-37.1717	True
4_1	4_2	15.6667	1.0	-233.1617	264.495	False
4_1	4_3	-269.3333	0.0243	-518.1617	-20.505	True
4_1	5_1	-59.0	0.9998	-307.8283	189.8283	False
4_1	5_2	-25.6667	1.0	-274.495	223.1617	False
4_1	5_3	-329.3333	0.0025	-578.1617	-80.505	True
4_2	4_3	-285.0	0.0137	-533.8283	-36.1717	True
4_2	5_1	-74.6667	0.9979	-323.495	174.1617	False
4_2	5_2	-41.3333	1.0	-290.1617	207.495	False
4_2	5_3	-345.0	0.0013	-593.8283	-96.1717	True
4_3	5_1	210.3333	0.1692	-38.495	459.1617	False
4_3	5_2	243.6667	0.0596	-5.1617	492.495	False
4_3	5_3	-60.0	0.9998	-308.8283	188.8283	False
5_1	5_2	33.3333	1.0	-215.495	282.1617	False
5_1	5_3	-270.3333	0.0234	-519.1617	-21.505	True
5_2	5_3	-303.6667	0.0067	-552.495	-54.8383	True

Table 4: Pairwise Tukey HSD Test result for alloy 1

Observations:

1) In all group comparisons where the result was significant (i.e., p-value < α, and result = True), Dentist 5 using Method 3 consistently appeared, suggesting this specific combination has a notable influence on the hardness of the implant. This supports the hypothesis that the interaction between dentist and method affects implant hardness.
2) Another combination worth noting is Dentist 4 using Method 3, which also appeared in a significant group comparison, further emphasizing the potential influence of specific dentist-method pairings on implant performance.

Considering Alloy 2

Considering Shapiro-Wilk's Test

Formulating Null and Alternate Hypothesis

Null hypothesis is residuals are normally distributed
Alternate hypothesis is residuals are not normally distributed

After doing the shapiro test p value obtained is 0.094 which is greater than level of significance 0.05. We fail to reject the null hypothesis

Conclusion is that we don’t have enough statistical evidence to say that residuals are not normally distributed.

Considering Levene Test

Formulating Null and Alternate Hypothesis
Null Hypothesis is All group variances are equal
Alternate Hypothesis is Atleast one group has a different variance

After doing the levene test p value obtained is 0.783 which is greater than level of significance 0.05. We fail to reject the null hypothesis

Conclusion is that we don’t have enough statistical evidence to say that atleast one group has a different variance.

Same as for alloy 1, we check how response is affected for dentist, method and dentist and method interaction.
We get the ANOVA table same as before

ANOVA Table 2:

	sum_sq	df	F	PR(>F)
C(Dentist)	56797.911111	4.0	1.106152	0.371833
C(Method)	499640.400000	2.0	19.461218	0.000004
C(Dentist):C(Method)	197459.822222	8.0	1.922787	0.093234
Residual	385104.666667	30.0	NaN	NaN

Table 5: ANOVA Table for alloy 2

Observations:

1) If we consider Dentist, as we can observe that p value is greater than 0.05 we don't have enough statistical evidence to say that different dentists have significant effect on response. (We fail to reject null hypothesis)
2) If we consider Method, as we can observe that p value is less than 0.05 we have statistical evidence to say that different methods have significant effect on response. (We reject null hypothesis)
3) If we consider interaction between Dentist and Method we can observe that p value is greater than 0.05 and we don't have enough statistical evidence to say that different combination of Dentist and Method have significant effect on response. (We fail to reject null hypothesis)

Tukey HSD Test

We do the same as what we did for alloy 1. This tells us which all gub groups are affecting the hardness of implants.

```
Tukey Test 2:
Multiple Comparison of Means - Tukey HSD, FWER=0.05
=========================================================
group1 group2 meandiff p-adj    lower     upper   reject
---------------------------------------------------------
   1_1    1_2   95.3333  0.999 -245.5625  436.2292  False
   1_1    1_3 -180.6667 0.8085 -521.5625  160.2292  False
   1_1    2_1   37.6667    1.0 -303.2292  378.5625  False
   1_1    2_2       5.0    1.0 -335.8958  345.8958  False
   1_1    2_3 -140.3333 0.9635 -481.2292  200.5625  False
   1_1    3_1  -63.3333    1.0 -404.2292  277.5625  False
   1_1    3_2     -27.0    1.0 -367.8958  313.8958  False
   1_1    3_3 -104.6667 0.9973 -445.5625  236.2292  False
   1_1    4_1  -66.3333    1.0 -407.2292  274.5625  False
   1_1    4_2    2.3333    1.0 -338.5625  343.2292  False
   1_1    4_3 -231.3333 0.4686 -572.2292  109.5625  False
   1_1    5_1      52.0    1.0 -288.8958  392.8958  False
   1_1    5_2   19.3333    1.0 -321.5625  360.2292  False
   1_1    5_3    -427.0 0.0049 -767.8958  -86.1042   True
   1_2    1_3    -276.0 0.2169 -616.8958   64.8958  False
   1_2    2_1  -57.6667    1.0 -398.5625  283.2292  False
   1_2    2_2  -90.3333 0.9994 -431.2292  250.5625  False
   1_2    2_3 -235.6667 0.4396 -576.5625  105.2292  False
   1_2    3_1 -158.6667  0.912 -499.5625  182.2292  False
   1_2    3_2 -122.3333 0.9884 -463.2292  218.5625  False
   1_2    3_3    -200.0 0.6868 -540.8958  140.8958  False
   1_2    4_1 -161.6667 0.9005 -502.5625  179.2292  False
   1_2    4_2     -93.0 0.9992 -433.8958  247.8958  False
   1_2    4_3 -326.6667 0.0709 -667.5625   14.2292  False
   1_2    5_1  -43.3333    1.0 -384.2292  297.5625  False
   1_2    5_2     -76.0 0.9999 -416.8958  264.8958  False
   1_2    5_3 -522.3333 0.0003 -863.2292 -181.4375   True
   1_3    2_1  218.3333 0.5587 -122.5625  559.2292  False
   1_3    2_2  185.6667 0.7793 -155.2292  526.5625  False
   1_3    2_3   40.3333    1.0 -300.5625  381.2292  False
   1_3    3_1  117.3333  0.992 -223.5625  458.2292  False
   1_3    3_2  153.6667 0.9291 -187.2292  494.5625  False
   1_3    3_3      76.0 0.9999 -264.8958  416.8958  False
   1_3    4_1  114.3333 0.9937 -226.5625  455.2292  False
   1_3    4_2     183.0 0.7951 -157.8958  523.8958  False
   1_3    4_3  -50.6667    1.0 -391.5625  290.2292  False
   1_3    5_1  232.6667 0.4596 -108.2292  573.5625  False
   1_3    5_2     200.0 0.6868 -140.8958  540.8958  False
   1_3    5_3 -246.3333 0.3717 -587.2292   94.5625  False
   2_1    2_2  -32.6667    1.0 -373.5625  308.2292  False
   2_1    2_3    -178.0 0.8234 -518.8958  162.8958  False
```

2_1	3_1	-101.0	0.9981	-441.8958	239.8958	False
2_1	3_2	-64.6667	1.0	-405.5625	276.2292	False
2_1	3_3	-142.3333	0.9594	-483.2292	198.5625	False
2_1	4_1	-104.0	0.9975	-444.8958	236.8958	False
2_1	4_2	-35.3333	1.0	-376.2292	305.5625	False
2_1	4_3	-269.0	0.2485	-609.8958	71.8958	False
2_1	5_1	14.3333	1.0	-326.5625	355.2292	False
2_1	5_2	-18.3333	1.0	-359.2292	322.5625	False
2_1	5_3	-464.6667	0.0017	-805.5625	-123.7708	True
2_2	2_3	-145.3333	0.9525	-486.2292	195.5625	False
2_2	3_1	-68.3333	1.0	-409.2292	272.5625	False
2_2	3_2	-32.0	1.0	-372.8958	308.8958	False
2_2	3_3	-109.6667	0.9958	-450.5625	231.2292	False
2_2	4_1	-71.3333	1.0	-412.2292	269.5625	False
2_2	4_2	-2.6667	1.0	-343.5625	338.2292	False
2_2	4_3	-236.3333	0.4352	-577.2292	104.5625	False
2_2	5_1	47.0	1.0	-293.8958	387.8958	False
2_2	5_2	14.3333	1.0	-326.5625	355.2292	False
2_2	5_3	-432.0	0.0043	-772.8958	-91.1042	True
2_3	3_1	77.0	0.9999	-263.8958	417.8958	False
2_3	3_2	113.3333	0.9942	-227.5625	454.2292	False
2_3	3_3	35.6667	1.0	-305.2292	376.5625	False
2_3	4_1	74.0	0.9999	-266.8958	414.8958	False
2_3	4_2	142.6667	0.9586	-198.2292	483.5625	False
2_3	4_3	-91.0	0.9994	-431.8958	249.8958	False
2_3	5_1	192.3333	0.7376	-148.5625	533.2292	False
2_3	5_2	159.6667	0.9083	-181.2292	500.5625	False
2_3	5_3	-286.6667	0.1746	-627.5625	54.2292	False
3_1	3_2	36.3333	1.0	-304.5625	377.2292	False
3_1	3_3	-41.3333	1.0	-382.2292	299.5625	False
3_1	4_1	-3.0	1.0	-343.8958	337.8958	False
3_1	4_2	65.6667	1.0	-275.2292	406.5625	False
3_1	4_3	-168.0	0.8735	-508.8958	172.8958	False
3_1	5_1	115.3333	0.9932	-225.5625	456.2292	False
3_1	5_2	82.6667	0.9998	-258.2292	423.5625	False
3_1	5_3	-363.6667	0.0279	-704.5625	-22.7708	True
3_2	3_3	-77.6667	0.9999	-418.5625	263.2292	False
3_2	4_1	-39.3333	1.0	-380.2292	301.5625	False
3_2	4_2	29.3333	1.0	-311.5625	370.2292	False
3_2	4_3	-204.3333	0.657	-545.2292	136.5625	False
3_2	5_1	79.0	0.9999	-261.8958	419.8958	False
3_2	5_2	46.3333	1.0	-294.5625	387.2292	False
3_2	5_3	-400.0	0.0105	-740.8958	-59.1042	True
3_3	4_1	38.3333	1.0	-302.5625	379.2292	False
3_3	4_2	107.0	0.9967	-233.8958	447.8958	False
3_3	4_3	-126.6667	0.9842	-467.5625	214.2292	False
3_3	5_1	156.6667	0.9191	-184.2292	497.5625	False

```
   3_3    5_2      124.0 0.9869 -216.8958  464.8958  False
   3_3    5_3 -322.3333 0.0786 -663.2292   18.5625  False
   4_1    4_2   68.6667    1.0 -272.2292  409.5625  False
   4_1    4_3    -165.0 0.8868 -505.8958  175.8958  False
   4_1    5_1  118.3333 0.9914 -222.5625  459.2292  False
   4_1    5_2   85.6667 0.9997 -255.2292  426.5625  False
   4_1    5_3 -360.6667 0.0302 -701.5625  -19.7708   True
   4_2    4_3 -233.6667 0.4529 -574.5625  107.2292  False
   4_2    5_1   49.6667    1.0 -291.2292  390.5625  False
   4_2    5_2      17.0    1.0 -323.8958  357.8958  False
   4_2    5_3 -429.3333 0.0046 -770.2292  -88.4375   True
   4_3    5_1  283.3333 0.1871  -57.5625  624.2292  False
   4_3    5_2  250.6667 0.3458  -90.2292  591.5625  False
   4_3    5_3 -195.6667 0.7158 -536.5625  145.2292  False
   5_1    5_2  -32.6667    1.0 -373.5625  308.2292  False
   5_1    5_3    -479.0 0.0011 -819.8958 -138.1042   True
   5_2    5_3 -446.3333 0.0028 -787.2292 -105.4375   True
----------------------------------------------------------
```

Table 6: Pairwise Tukey HSD Test for alloy 2

Observations

1) Dentist 5 using Method 3 (group 5_3) consistently appears in all 10 significant comparisons.
2) By this we can say that Dentist 5 using Method 3 has a significant impact on hardness of dental implants.

Conclusion:

In this project I have applied a range of inferential statistical techniques to real-world manufacturing and medical datasets to get meaningful insights.
Here we used Probability estimation, normal distribution modelling, ttest for one sample, two independent sample t-test, Hypothesis Testing, Shapiro and Levene test before proceeding with one way ANOVA, Tukey HSD Test and two way ANOVA to solve the problems and get insightful inferences.