

Step 1: Calculate mean of Each Column separately.

Step 2: Create a covariance matrix

$$\text{Cov}(x, y) = \frac{\sum_{i=1}^n (x_i - \bar{x}) \cdot (y_i - \bar{y})}{n-1}$$

Ex: $2.5 - 1.01$

& $2.4 - 1.91$

Calculate the matrix

$$C = \begin{bmatrix} \text{Cov}(x, x) & \text{Cov}(x, y) \\ \text{Cov}(y, x) & \text{Cov}(y, y) \end{bmatrix}$$

| x | $x - \bar{x}$ | $(x - \bar{x})(x - \bar{x})$ | y | $y - \bar{y}$ | $(y - \bar{y})(y - \bar{y})$ |
|--------------------|---------------|------------------------------|--------------------|---------------|------------------------------|
| 2.5 | 0.69 | 0.4761 | 2.4 | 0.49 | 0.2401 |
| 0.5 | -1.31 | 1.7161 | 0.7 | -1.21 | 1.4641 |
| : | : | : | : | : | : |
| | | Sum = 5.5490 | | | Sum = 6.999 |
| $\text{Cov}(x, x)$ | | | $\text{Cov}(y, y)$ | | |

| x | y | $x - \bar{x}$ | $y - \bar{y}$ | $(x - \bar{x}) \cdot (y - \bar{y})$ |
|--------------|-----|---------------|---------------|-------------------------------------|
| 2.5 | 2.4 | 0.69 | 0.49 | 0.33 |
| 0.5 | 0.7 | -1.31 | -1.21 | 1.5851 |
| ⋮ | | | | |
| Sum : 5.5390 | | | | |

Note! Covariance matrix depend on size of input

2-d 2×2
3-d 3×3

$$C = \begin{array}{c|ccc} & x & y & z \\ \hline x & xx & xy & xz \\ y & yx & yy & yz \\ z & zx & zy & zz \end{array} \quad \left. \vphantom{\begin{array}{c|ccc} & x & y & z \\ \hline x & xx & xy & xz \\ y & yx & yy & yz \\ z & zx & zy & zz \end{array}} \right\} \text{cov matrix}$$

$$C = \begin{bmatrix} 0.6165 & 0.6154 \\ 0.6154 & 0.7165 \end{bmatrix}$$

$$\begin{bmatrix} 0.6165 & 0.6154 \\ 0.6154 & 0.7165 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0.6165 - \lambda & 0.6154 \\ 0.6154 & 0.7165 - \lambda \end{bmatrix} = 0$$

$$\Rightarrow \lambda^2 - 1.333\lambda + 0.0630 = 0$$

$$\lambda_1 = 0.0490$$

$$\lambda_2 = 1.2840$$

\Rightarrow for each Eigen value we need to generate a Eigen vector

$$\Rightarrow CV = \lambda V$$

$$\begin{bmatrix} 0.6165 & 0.6154 \\ 0.6154 & 0.7165 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 0.0490 \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

$$\begin{aligned} 0.6165 x_1 + 0.6154 y_1 &= 0.0490 x_1 \\ 0.6154 x_1 + 0.7165 y_1 &= 0.0490 y_1 \end{aligned}$$

$$0.5674 x_1 = -0.6154 y_1 \quad - (1)$$

$$0.6154 x_1 = -0.6674 y_1 \quad - (2)$$

$\text{Eq (1)} / \text{Eq (2)}$

$$\boxed{x_1 = -1.0845 y_1}$$

$$x_1 = -1.0845 y_1$$

let put $y_1 = 1$ (temporary basis)

$$\begin{bmatrix} -1.0845 \\ 1 \end{bmatrix} \rightarrow \text{Square them} \\ \text{add them} \\ \& \text{ take their root}$$

$$1.17614 + 1$$

$$= \sqrt{2.17614}$$

$$= 1.47517$$

\Rightarrow divide our previous value with this value

$$\begin{bmatrix} -1.0845 / 1.47517 \\ 1 / 1.47517 \end{bmatrix} \Rightarrow \begin{bmatrix} -0.7351 \\ 0.6778 \end{bmatrix}$$

final vector

similarly put
value of λ_2 & got
the answer

$$x_2 = 0.92194 y_2$$

$$\begin{bmatrix} 0.92194 \\ 1 \end{bmatrix} = 0.8499 + 1$$

$$= \sqrt{1.8499}$$

$$= 1.3601$$

$$x_1 = -1.0845 y_1$$

let put $y_1 = 1$ (temporary basis)

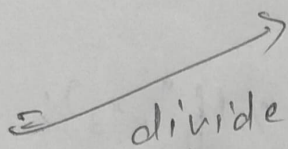
$$\begin{bmatrix} -1.0845 \\ 1 \end{bmatrix} \rightarrow \begin{array}{l} \text{Square them} \\ \text{add them} \\ \text{\& take their root} \end{array}$$



$$1.17614 + 1$$

$$= \sqrt{2.17614}$$

$$= 1.47517$$



divide our previous value with this value

$$\begin{bmatrix} -1.0845 / 1.47517 \\ 1 / 1.47517 \end{bmatrix} \Rightarrow$$

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$$\begin{bmatrix} 0.6778 \\ 0.7351 \end{bmatrix}$$