Step1: Calculate mean of Each Column separately.

Step2: Create a covariance matrix

$$cov(x,y) = \sum_{i=1}^{n} \frac{(x_i - \bar{x}) \cdot (y_i - \bar{y})}{n-1}$$

Ex: 2.5 - 1.01

& 2.4 - 1.91

Calculate the matrix

$$C = \begin{bmatrix} cov(x,x) & cov(x,y) \\ cov(y,x) & cov(y,y) \end{bmatrix}$$

x	$x - \bar{x}$	$(x-\bar{x})(x-\bar{x})$
2.5	0.69	0.476
0.5	-1.31	1.7161
⋮	⋮	⋮

Sum = 5.5490

cov(x,x)

y	$y - \bar{y}$	$(y-\bar{y})(y-\bar{y})$
2.4	0.49	0.2401
0.7	-1.21	1.4641
		⋮

Sum = 6.449

cov(y,y)

x	y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})(y-\bar{y})$
2.5	2.4	0.69	0.49	0.33
0.5	0.7	-1.31	-1.21	1.5851
⋮				

Sum : 5.5390

Note! Covariance matrix depend on size of input

2-d 2×2

3-d 3×3

$$C = \begin{array}{c|ccc} & x & y & z \\ \hline x & xx & xy & xz \\ y & yx & yy & yz \\ z & zx & zy & zz \end{array}$$

} cov matrix

$$C = \begin{bmatrix} 0.6165 & 0.6154 \\ 0.6154 & 0.7165 \end{bmatrix}$$

Eq

Step 3: $C - \lambda I = 0$

$$\begin{bmatrix} 0.6165 & 0.6154 \\ 0.6154 & 0.7165 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0.6165 - \lambda & 0.6154 \\ 0.6154 & 0.7165 - \lambda \end{bmatrix} = 0$$

$$\Rightarrow \lambda^2 - 1.333\lambda + 0.0630 = 0$$

$$\lambda_1 = 0.0490$$
$$\lambda_2 = 1.2840$$

$\Rightarrow$ for each Eigen value we need to generate a Eigen vector

$$\Rightarrow \quad CV = \lambda V$$

$$\begin{bmatrix} 0.6165 & 0.6154 \\ 0.6154 & 0.7165 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 0.0490 \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

$$0.6165\, x_1 + 0.6154\, y_1 = 0.0490\, x_1$$
$$0.6154\, x_1 + 0.7165\, y_1 = 0.0490\, y_1$$

$$0.5674\, x_1 = -0.6154\, y_1 \quad - (1)$$
$$0.6154\, x_1 = -0.6674\, y_1 \quad - (2)$$

Eq (1) / Eq (2)

$$x_1 = -1.0845\, y_1$$

$x_1 = -1.0845\, y_1$

let put $y_1 = 1$ (temporary basis)

$$\begin{bmatrix} -1.0845 \\ 1 \end{bmatrix} \rightarrow$$ Square them, add them & take their root

$$1.17614 + 1$$
$$= \sqrt{2.17614}$$
$$= 1.47517$$

$\Rightarrow$ divide our previous value with this value

$$\begin{bmatrix} -1.0845 / 1.47517 \\ 1 / 1.47517 \end{bmatrix} \Rightarrow \begin{bmatrix} -0.7351 \\ 0.6778 \end{bmatrix}$$

final vector

Similary put value of λ_2 & got the answer

$x_2 = 0.92194\, y_2$

$$\begin{bmatrix} 0.92194 \\ 1 \end{bmatrix} = 0.8499 + 1$$
$$= \sqrt{1.8499}$$
$$= 1.3601$$

$x_1 = -1.0845\, y_1$

let put $y_1 = 1$ (temporary basis)

$$\begin{bmatrix} -1.0845 \\ 1 \end{bmatrix} \rightarrow$$ Square them
add them
& take their root

$1.17614 + 1$

$= \sqrt{2.17614}$

$= 1.47517$

$\Rightarrow$ divide our previous value with this value

$$\begin{bmatrix} -1.0845 / 1.47517 \\ 1 / 1.47517 \end{bmatrix} \Rightarrow \begin{bmatrix} -0.7351 \\ 0.6778 \end{bmatrix}$$

final vector

Similary put value of λ_2 & got the answer

$x_2 = 0.92194\, y_2$

$$\begin{bmatrix} 0.92194 \\ 1 \end{bmatrix} = 0.8499 + 1$$

$= \sqrt{1.8499}$

$= 1.3601$

$$\begin{bmatrix} 0.6778 \\ 0.7351 \end{bmatrix}$$