CS510 Fall 2021 Assignment #2 solution

1) Give regular expression generating the language: **Exercise 1.18**.

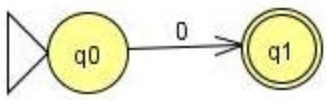
Solution:
Let $\Sigma = \{0, 1\}$. Note: There could be more than one correct answer.

a. $1\Sigma^*0$
b. $\Sigma^*1\Sigma^*1\Sigma^*1\Sigma^*$
c. $\Sigma^*0101\Sigma^*$
d. $\Sigma\Sigma 0\Sigma^*$
e. $(0 \cup 1\Sigma)(\Sigma\Sigma)^*$
f. $(0 \cup (10)^*)^*1^*$ Another solution: $0^*(10^+)^*1^*$
g. $(\varepsilon \cup \Sigma)(\varepsilon \cup \Sigma)(\varepsilon \cup \Sigma)(\varepsilon \cup \Sigma)(\varepsilon \cup \Sigma)$
h. $\Sigma^*0\Sigma^* \cup 1111\Sigma^* \cup 1 \cup \varepsilon$
i. $(1\Sigma)^*(1 \cup \varepsilon)$
j. $0^*(100 \cup 010 \cup 001 \cup 00)\, 0^*$
k. $\varepsilon \cup 0$
l. $(1^*01^*01^*)^* \cup 0^*10^*10^*$
m. $\emptyset$
n. Σ^+

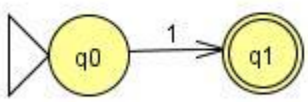
2) Convert RE to NFA: **Exercise 1.19(a)** (use JFLAP to draw the automaton)

Solution:
The following are steps in the procedure:

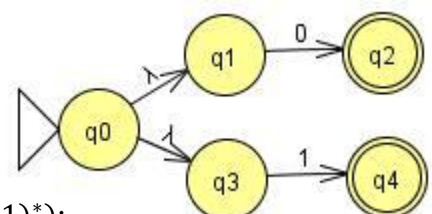
The following NFA recognizes L(0):



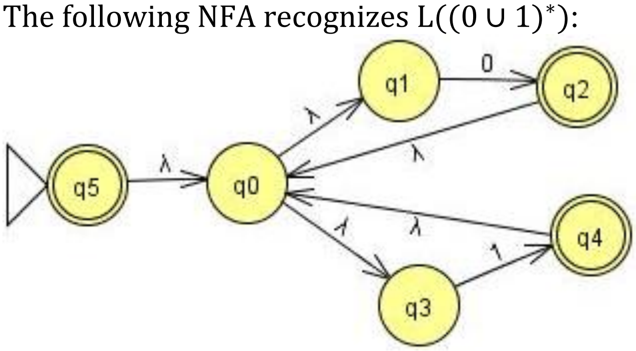
The following NFA recognizes L(1):



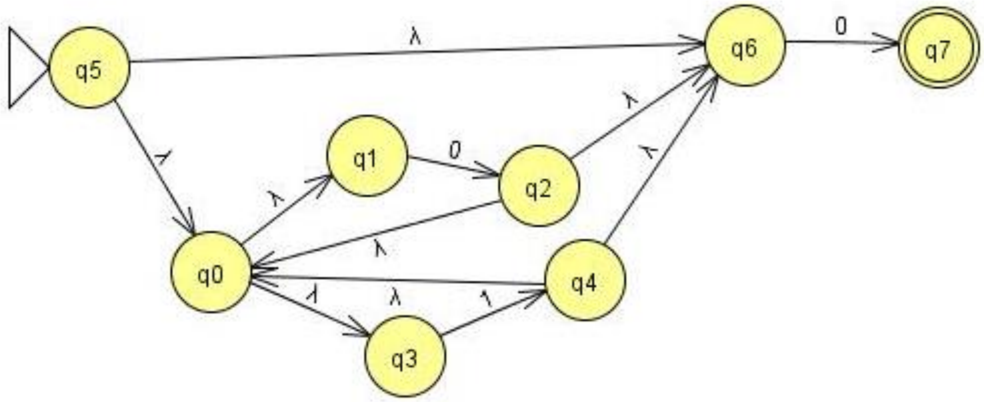
The following NFA recognizes $L(0 \cup 1)$:



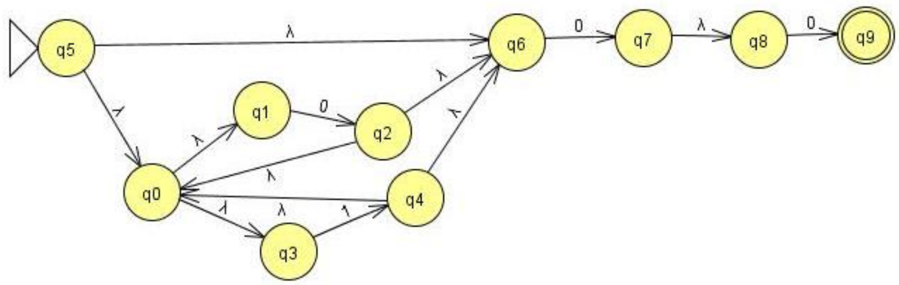
The following NFA recognizes $L((0 \cup 1)^*)$:



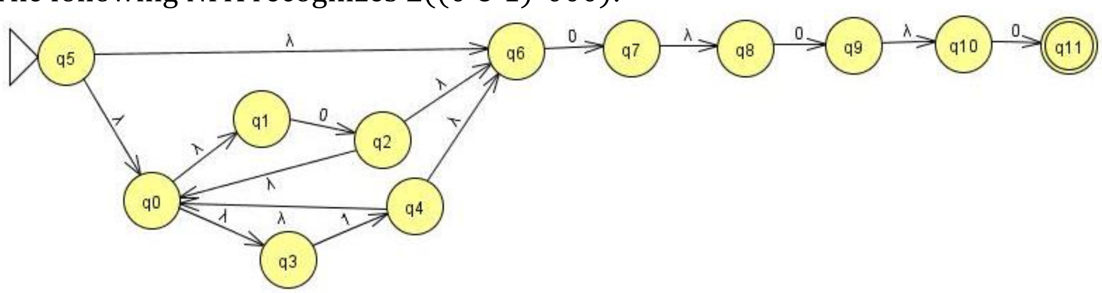
The following NFA recognizes L((0 ∪ 1)*0):



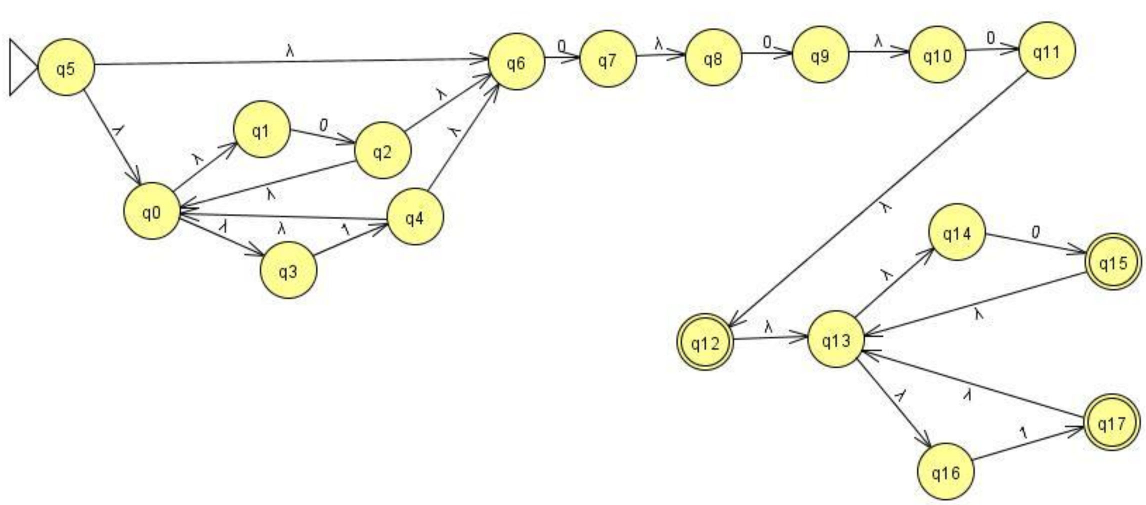
The following NFA recognizes L((0 ∪ 1)*00):



The following NFA recognizes L((0 ∪ 1)*000):



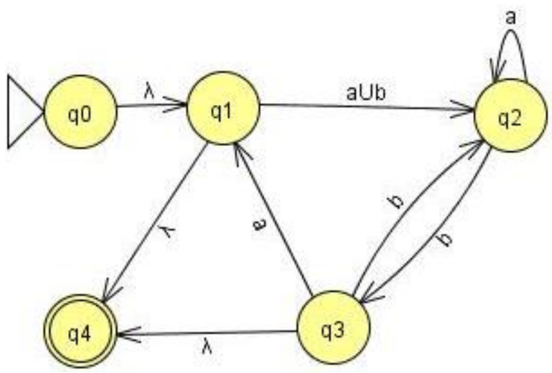
The following NFA recognizes L((0 ∪ 1)*000(0 ∪ 1)*):



3) Convert FA to RE: **Exercise 1.21(b)**

Solution:

We first add a new start state, a new accept state, and replace label "a, b" with "a∪b":



Several correct solutions are possible. The answer depends on the order in which the states are removed. If the states are removed in the order 1, 2, then 3, then the following expression is obtained:

$\varepsilon \cup ((a \ \cup \ b)a^*b((b \ \cup \ a(a \ \cup \ b)) \ a^*b)^*(\varepsilon \cup a))$

4) Use Pumping lemma to show the language is not regular: **Exercise 1.29(b)**

Solution:

Let $A2 = \{www|\ w \in \{a,b\}*\}$. We show that $A2$ is nonregular using the pumping lemma. Assume to the contrary that $A2$ is regular. Let p be the pumping length given by the pumping lemma. Let s be the string $a^p ba^p ba^p b$. Because s is a member of $A2$ and s has length more than p, the pumping lemma guarantees that s can be split into three pieces, $s = xyz$, satisfying the three conditions of the lemma. Conditions 2 and 3 imply that both x and y must consist only of a's, and y must consist of 1 or more a's. Then, string $xyyz$ will be of the form $a^{p+k}ba^p ba^p b$, where k≥1. Therefore, $xyyz \notin A2$ and condition 1 is violated. Therefore $A2$ is nonregular.

5) Show that the language is regular: **Exercise 1.36**

Solution:

For each n ≥ 1, we build a DFA M with the n states $q_0, q_1, \dots, q_{n-1}$ to count the number of consecutive a's modulo n read so far. For each character a that is input, the counter increments by 1 and jumps to the next state in M. It accepts the string if and only if the machine stops at q_0. That means the length of the string consists of all a's and its length is a multiple of n.
More formally, the set of states of M is $Q = \{q_0, q_1, \dots, q_{n-1}\}$. The state q_0 is the start state and the only accept state. Define the transition function as: $\delta(q_i, \text{a}) = q_j$ where $j = i + 1$ mod n.

6) Give CFG that generates the language. **Exercise 2.4 (b), (c), (e), (f).**

Solutions (Note: there are multiple correct solutions; one possible set is listed below):

2.4(b) V={S, T}, Σ={0,1}, start variable is S, R contains the following rules:

S→ 0T0 | 1T1 | 0 | 1 | ε
T→ 0T | 1T | ε

2.4(c) V={S}, Σ={0,1}, start variable is S, R contains the following rules:

S→ 0 | 1 | 00S | 01S | 10S | 11S

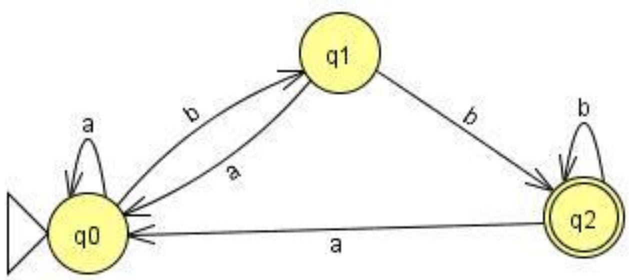
2.4(e) V={S}, Σ={0,1}, start variable is S, R contains the following rules:

S→ 0S0 | 1S1 | 0 | 1 | ε

2.4(f) V={S}, Σ={0,1}, start variable is S, R contains the following rules:

S→ S

7) **Convert the following DFA into an equivalent CFG**:



Note: CFG is a 4-tuple (V, Σ, R, S). You need to explicitly state what V is, what Σ is, what rules are in R, and what S is.

Solution:

V={R_0, R_1 R_2}, Σ={a, b}, start variable is R_0, R contains the following rules:

$R_0 \to bR_1$
$R_0 \to aR_0$
$R_1 \to bR_2$
$R_1 \to aR_0$
$R_2 \to bR_2$
$R_2 \to aR_0$
$R_2 \to \varepsilon$

8) Convert CFG to CNF: **Convert the following CFG into an equivalent CFG in Chomsky normal form, using the procedure given in Theorem 2.9:**

S → XY
X → abb | aXb | ε
Y → c | cY

Solution:

Add a new start variable S_0.

$S_0 \rightarrow S$
$S \rightarrow XY$
$X \rightarrow abb \mid aXb \mid \varepsilon$
$Y \rightarrow c \mid cY$

Eliminate ε-rules.

$S_0 \rightarrow S$
$S \rightarrow XY \mid Y$
$X \rightarrow abb \mid aXb \mid ab$
$Y \rightarrow c \mid cY$

Eliminate unit rules. Eliminate $S_0 \rightarrow S$

$S_0 \rightarrow XY \mid Y$
$S \rightarrow XY \mid Y$
$X \rightarrow abb \mid aXb \mid ab$
$Y \rightarrow c \mid cY$

Eliminate unit rules. Eliminate $S_0 \rightarrow Y$ and $S \rightarrow Y$

$S_0 \rightarrow XY \mid c \mid cY$
$S \rightarrow XY \mid c \mid cY$
$X \rightarrow abb \mid aXb \mid ab$
$Y \rightarrow c \mid cY$

Convert the remaining rules.

$S_0 \rightarrow XY \mid c \mid U_cY$
$S \rightarrow XY \mid c \mid U_cY$
$X \rightarrow U_aX_1 \mid U_aX_2 \mid U_aU_b$
$Y \rightarrow c \mid U_cY$
$X_1 \rightarrow U_bU_b$
$X_2 \rightarrow XU_b$
$U_a \rightarrow a$
$U_b \rightarrow b$
$U_c \rightarrow c$

9) Ambiguity: **Consider the following two CFGs:**

(a) $S \rightarrow Ab \mid A$
$A \rightarrow b \mid bA$
(b) $S \rightarrow aS \mid Sa \mid b$

For each of the grammars, demonstrate that it is ambiguous by providing a string that has two or more different leftmost derivations.

Solution (multiple correct solutions are possible):

(a) String bb has 2 different leftmost derivations: $S \rightarrow Ab \rightarrow bb$ and $S \rightarrow A \rightarrow bA \rightarrow bb$
(b) String aba has 2 different leftmost derivations: $S \rightarrow aS \rightarrow aSa \rightarrow aba$ and $S \rightarrow Sa \rightarrow aSa \rightarrow aba$