

- 1. First, observe that the given PDA meets the three requirements for simplicity:
 - (a) It has one final state.
 - (b) It empties its stack before accepting.
 - (c) Each transition either consists of a single push or a single pop.
- 2. The start variable of the grammar is A_{02} , which indicates that our PDA must begin in q_0 and end up in q_2 .
- 3. Introduce grammar rules corresponding to moves of the PDA of the form "push, compute, pop":

$$A_{02} \to A_{11}$$
 $A_{11} \to 0A_{11}1$

4. Introduce grammar rules corresponding to moves of the PDA which drive it from states p to r and then from r to q:

$$\begin{split} A_{00} &\to A_{00} A_{00} | A_{01} A_{10} | A_{02} A_{20} \\ A_{01} &\to A_{00} A_{01} | A_{01} A_{11} | A_{02} A_{21} \\ A_{02} &\to A_{00} A_{02} | A_{01} A_{12} | A_{02} A_{22} \\ A_{10} &\to A_{10} A_{00} | A_{11} A_{10} | A_{12} A_{20} \\ A_{11} &\to A_{10} A_{01} | A_{11} A_{11} | A_{12} A_{21} \\ A_{12} &\to A_{10} A_{02} | A_{11} A_{12} | A_{12} A_{22} \\ A_{20} &\to A_{20} A_{00} | A_{21} A_{10} | A_{22} A_{20} \\ A_{21} &\to A_{20} A_{01} | A_{21} A_{12} | A_{22} A_{22} \\ A_{22} &\to A_{20} A_{02} | A_{21} A_{12} | A_{22} A_{22} \end{split}$$

5. Introduce grammar rules for ϵ :

$$A_{00} \to \varepsilon$$

$$A_{11} \to \varepsilon$$

$$A_{22} \to \varepsilon$$