ASSIGNMENT 2

Question1. Give the regular expressions of the language:

a. { w | w begins with a 1 and ends with a 0 }.

$= 1 (0 \cup 1)^* 0$

b. { w | w contains at least three 1s }

$= (0 \cup 1)^*1(0 \cup 1)^*1(0 \cup 1)^*1(0 \cup 1)^*$

c. { w| w contains a substring 0101 }

$= (0 \cup 1)^* 0101(0 \cup 1)^*$

d. { w| w has length at least 3 and its third symbol is 0 }

$= (0 \cup 1)(0 \cup 1)0(0 \cup 1)^*$

e. { w | w starts with 0 and has odd length, or start with 1 and has even length }

$= 0 ((0 \cup 1)(0 \cup 1))^* \cup 1(0 \cup 1)((0 \cup 1)(0 \cup 1))^*$

f. { w | w doesn't contain the substring 110 }

$= 0^*(10^+)^*1^*$

g. { w |the length of w is at most 5 }

$= \in \cup (0 \cup 1) \cup (0 \cup 1)^2 \cup (0 \cup 1)^3 \cup (0 \cup 1)^4 \cup (0 \cup 1)^5$

or

$(\in \cup \sum)^5$

h. { w | w is any string except 11 and 111 }

$= \in \cup (0 \cup 1) \cup 0(0 \cup 1) \cup 10 \cup 0(0 \cup 1)(0 \cup 1) \cup 10(0 \cup 1) \cup 110 \cup (0 \cup 1)^3(0 \cup 1)^+$

i. { w |every odd position of w is a 1. }

$= (1(0U1))^*(\in U1)$

j. { w | w contains at least two 0s and at most one 1 }

$= 00^*00^*(\in U1)U00^*(\in U1)00^*U(\in U1)00^*00^*$

k. $\{ \in , 0 \}$

$= 0 \, U \in$

l. { w | w contains an even number of 0s, or contains exactly two 1s }

$= 1^*(01^*01^*)U0^*10^*10^*$

m. The empty set

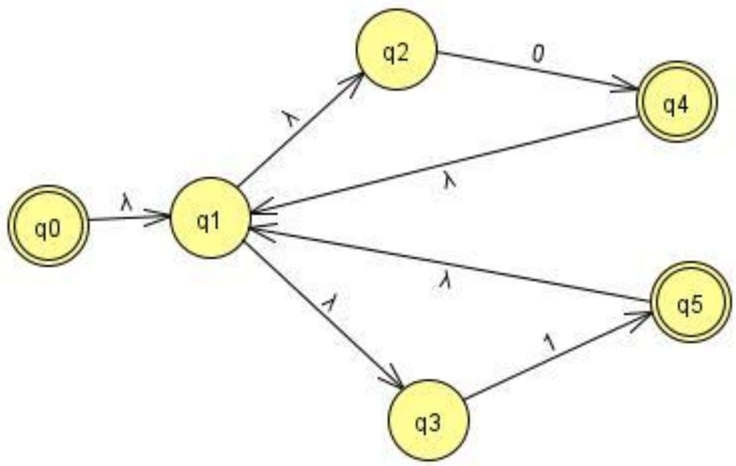
$= \varnothing$

n. All the strings except the empty string.

$= (0U1)^2$

Question 2. Convert RE to NFA: Exercise 1.19(a)

(0 U 1)*



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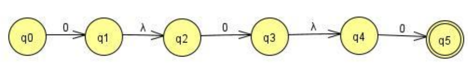
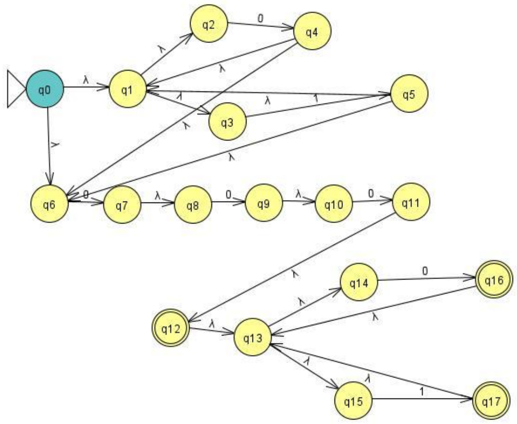
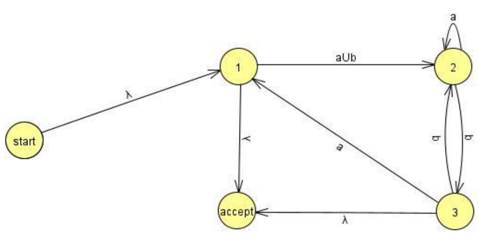


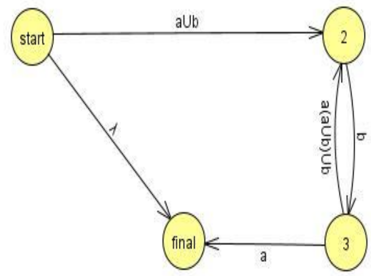
(0 U 1)* 000 (0 U 1)*



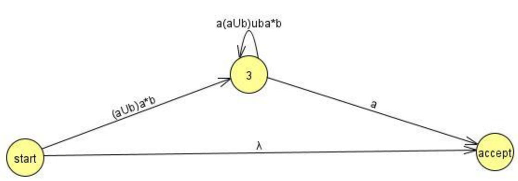
Question 3. Convert FA to RE. Problem 1.21 (b)
Step 1: add new start state with epsilon transition from the new start state to old start state, add new accept start with epsilon transition from new accept state to old accept state.



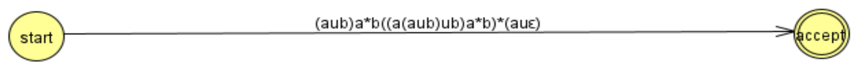
Eliminate state 1:



Eliminate state 2:



Final Diagram:



Hence the regular expression for the given DFA is

(a U b)a*b((a(a U b) U b) a* b)*(a U ε)

Question 4: A2 = {www| w ∈ {a, b}∗}

Assume L is regular.
Let w = $a^n b^n$
|www| = | $a^n b^n a^n b^n a^n b^n$| = 6n >n
Divide the language into 3 parts xyz
W = xyz , such that

i) |xy| <= n
ii) |y| > 0
iii) xy^iz i > 0

so, x = a^{n-1} , y = a , z = $b^n a^n b^n a^n b^n$
|xy| = | a^{n-1}a| =|a^n| =n → |xy| <= n
|a| → |y| > 0
Consider : i=2
xy^iz = a^{n-1} $a^i b^n a^n b^n a^n b^n$
= a^{n-1} $a^2 b^n a^n b^n a^n b^n$

$= a^{n+1}b^n a^n b^n a^n b^n \notin L$

∴ Our assumption is wrong

∴ The given language is not regular.

Question 5. Show that the language is regular: $B_n = \{a^k \mid K \text{ is a multiple of } n\}$.(Exercise 1.36.)

Ans.) Suppose, K = ni, where i is any positive integer.

Let`s consider, i = 1

If i = 1 and n = 1

$B1 = \{a^k\} = \{a^{n*i}\} = \{a^{1*1}\} = \{a\}$

Now, let i = 1 and n = 2.

So, $B2 = \{a^k\} = \{a^{n*i}\} = \{a^{2*1}\} = \{a^2\} = \{aa\}$

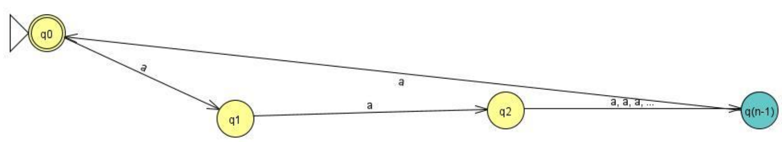
Again, let i = 1 and n = 3.

So, $B3 = \{a^k\} = \{a^{n*i}\} = \{a^{3*1}\} = \{a^3\} = \{aaa\}$

Therefore, $B_1, B_2, B_3, \ldots, B_n$.

{a}, {aa}, {aaa}, ….

Finite automata of regular expression is:



Question 6. Give CFG that generates the language. Exercise 2.4(b), (c), (e), (f).
Ans.) 2.4 b) {w| w starts and ends with the same symbol}

G = (V,∑,R,S)
CFG for the language will be
S $\rightarrow$ 0A0|1A1|0|1
A $\rightarrow$ 0A|1A|ε
V={A,S}
∑ - {0,1}
R –
S $\rightarrow$ 0A0|1A1|0|1
A $\rightarrow$ 0A|1A|ε
S – S

2.4 c) {w| the length of w is odd}

G = (V,∑,R,S)
CFG for given language
S $\rightarrow$ 0|1|0S0|0S1|1S0|1S1
V={S}
∑ - {0,1}
R – S $\rightarrow$ 0|1|0S0|0S1|1S0|1S1
S – S

2.4 e) {w| w = w^R, that is, w is a palindrome}

G = (V,∑,R,S)
CFG for given language
S $\rightarrow$ 0S0|1S1|0|1|ε
V={S}
∑ - {0,1}
R $\rightarrow$ 0S0|1S1|0|1|ε
S – S

2.4 f) The empty set

$G = (V,\sum,R,S)$
$S \rightarrow S$
$V=\{S\}$
$\sum$ - {0,1}
R - $S \rightarrow S$
S -S

7. Convert the following DFA into an equivalent CFG.
Ans.) Make variable R_i for each state q_i of DFA

V – $\{R_0, R_1, R_2\}$
S – R_0
$\sum$ – { a,b}

R -
R_0 ----- > $aR_0 \mid bR_1$

R_1 ------- > $aR_0 \mid bR_2$

R_2 ------ > $aR_0 \mid bR_2 \mid \varepsilon$

V – final set of variables

$\sum$– finite set, disjoint from V called terminals.

R – rules.

S – start variables.

Question 8. Convert the CFG to CNF:

$\mathbf{S \rightarrow XY}$
$\mathbf{X \rightarrow abb \mid aXb \mid \varepsilon}$
$\mathbf{Y \rightarrow c \mid cY}$

Ans.) Step 1: add new start state.

$\mathbf{S_0 \rightarrow S}$
$\mathbf{S \rightarrow XY}$
$\mathbf{X \rightarrow abb \mid aXb \mid \varepsilon}$
$\mathbf{Y \rightarrow c \mid cY}$

Step 2: eliminate ‘**ε**’ productions.

$\mathbf{S_0 \rightarrow S}$
$\mathbf{S \rightarrow XY \mid Y}$
$\mathbf{X \rightarrow abb \mid aXb \mid ab}$
$\mathbf{Y \rightarrow c \mid cY}$

Step 3: Remove unit rules.

$\underline{S \rightarrow Y}$

$\mathbf{S_0 \rightarrow S}$
$\mathbf{S \rightarrow XY \mid c \mid cY}$
$\mathbf{X \rightarrow abb \mid aXb \mid ab}$
$\mathbf{Y \rightarrow c \mid cY}$

$\underline{S_0 \rightarrow S}$

$\mathbf{S_0 \rightarrow XY \mid c \mid cY}$
$\mathbf{S \rightarrow XY \mid c \mid cY}$
$\mathbf{X \rightarrow abb \mid aXb \mid ab}$
$\mathbf{Y \rightarrow c \mid cY}$

Step 4: Convert each rule with 3 or more symbols in RHS

$\mathbf{S_0 \rightarrow XY \mid c \mid cY}$
$\mathbf{X \rightarrow aY_b \mid aX_1 \mid a\,U_b}$
$\mathbf{X_1 \rightarrow Xb}$
$\mathbf{U_b \rightarrow b}$
$\mathbf{Y_b \rightarrow U_b\ U_b}$
$\mathbf{Y \rightarrow c \mid cY}$

Then,

$\mathbf{S_0 \rightarrow XY \mid c \mid UcY}$

$\mathbf{Uc \rightarrow c}$

$\mathbf{X \rightarrow UaY_b \mid UaX_1 \mid U_aU_b}$

$\mathbf{U_a \rightarrow a}$

$\mathbf{Y \rightarrow c \mid UcY}$

Question 9. Ambiguity: Consider the following two CFGs:

a) $S \rightarrow Ab \mid A$

$A \rightarrow b \mid bA$

Ans.) Let W = bbb

<u>LMD1</u>:

$\mathbf{S \rightarrow Ab}$

$\mathbf{S \rightarrow Abb}$

$\mathbf{S \rightarrow bbb}$

<u>LMD2</u>:

$\mathbf{S \rightarrow A}$

$\mathbf{S \rightarrow bA}$

$\mathbf{S \rightarrow bbA}$

$\mathbf{S \rightarrow bbb}$

As the string have two LMD's the grammar is ambiguous.

b) $S \rightarrow aS \mid Sa \mid b$

Ans.) Let W = aba

<u>LMD1</u>:

$\mathbf{S \rightarrow Sa}$

$\mathbf{S \rightarrow aSa}$

$\mathbf{S \rightarrow aba}$

<u>LMD2</u>:

$\mathbf{S \rightarrow aS}$

$\mathbf{S \rightarrow aSa}$

$\mathbf{S \rightarrow aba}$

As the string have two LMD's the grammar is ambiguous.