Question 1

<u>Informal Description</u>:

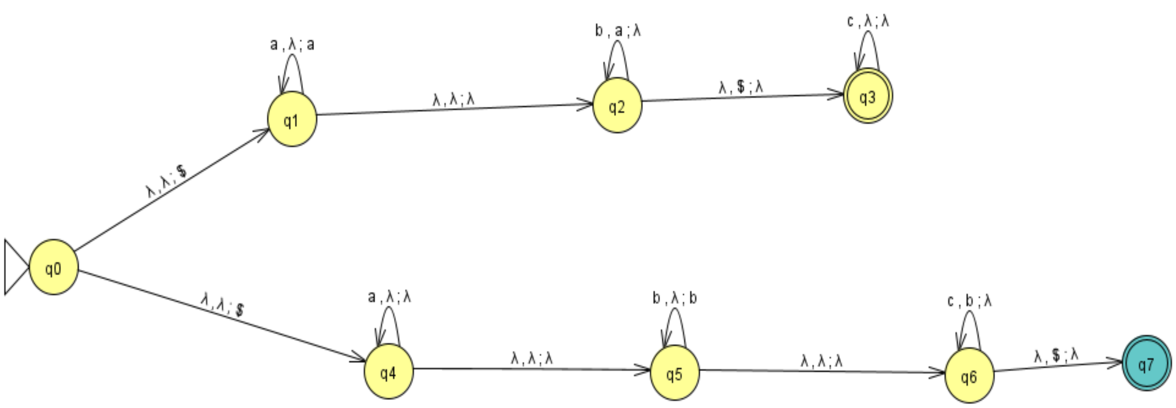
Here the language is the union of the two languages,

The informal description of the PDA that recognizes the language A is the combination of both the languages A1 & A2.

I. Read & Push a`s.
II. Read b`s while popping a`s.
III. If b`s finish when stack is empty, skip c`s on input and accept.
IV. Skip a`s on input.
V. Read & Push b`s.
VI. Read c`s, while popping b`s.
VII. If c`s finish when stack is empty, then accept.

If the string A1 = $\{a^i, b^j, c^k \mid i=j\}$, then the PDA takes the branch q0 to q1.

If the string A2 = $\{a^i, b^j, c^k \mid j=k\}$, then the PDA takes the branch q0 to q4.



Question 2

Assume towards contradiction Let C be a context free language. There exists a number p such that for every

w ∈ C, if |w|>=p then w may be divided into five parts, w= uvxyz satisfying:

1. for each i >= 0, it holds that uv^ixy^iz ∈ C.
2. |vy| > 0
3. |vxy| <= p

Take w = $1^p2^p3^p4^p$ w ∈ C with |w|>=p

By the pumping lemma, there exist a partition w = uvxyz where |vy| > 0,|vxy| <= p for each I, it holds that uv^ixy^iz ∈ C.

Now, we have to prove by contradiction for all cases if the conditions satisfy.

Case 1:

If vxy contains only 1's, then $uv^2 xy^2 z$ does not belong to C since it cannot have same number of 1's and 2's since from condition 3 where the length of vxy must be within length p. As a result, it cannot contain any 2's.

Case 2:

If vxy contains only 2's, then $uv^2 xy^2 z$ does not belong to C since it cannot have same number of 1's and 2's since from condition 3 where the length of vxy must be within length p. As a result, it cannot contain any 1's.

Case 3:

If vxy contains only 3's, then $uv^2 xy^2 z$ does not belong to C since it cannot have same number of 3's and 4's since from condition 3 where the length of vxy must be within length p. As a result, it cannot contain any 4's.

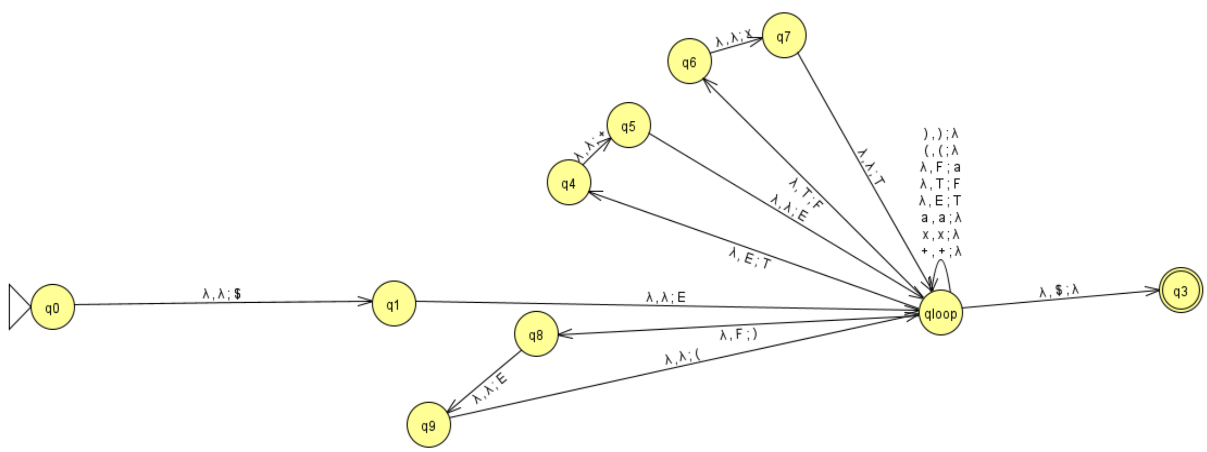
Case 4:

If vxy contains only 4's, then $uv^2 xy^2 z$ does not belong to C since it cannot have same number of 3's and 4's since from condition 3 where the length of vxy must be within length p. As a result, it cannot contain any 3's.

Case 5:

If i is considered as 0 from condition 1, then x will have the pumping length p with uxz but it also contracts with condition 2 where length of vy must be greater than 0. As a result, there is again a contradiction, and therefore C is not Context free language.

Question 3



Question 4

q10 , Uq2U, UUqaccept.

Question 5

1##1

q11##1 -> Xq3##1 -> X#q5#1 -> X##qreject1

Question 6

On input string W:

Step 1:

Start scanning the tape for the first unmarked 1, and then mark it. If no unmarked 1 is found, then proceed to step 5, otherwise place the head at the start of the tape.

Step 2:

Scan the unmarked 0, if found in the tape then mark it, otherwise reject it.

Step 3:

Scan the tape again till the unmarked 0's is found. If found mark it, otherwise reject it.

Step 4:

Move the head back to the front of the tape and go back to step 1.

Step 5:

Place the head back to the front of the tape to scan if there is any unmarked 0's to be found. If there is no unmarked 0's, then accept it, otherwise reject it.

Question7. A. Concatenation:

For any two decidable languages L1 and L2. Let M1 and M2 be the Turing machines that decide them. We construct a turing machine TM Mo that chooses concatenation of L1 and L2.

Mo = "on input w "

1. Split w into two parts w1,w2 such that w = w1w2.
2. Run M1 on the w1. If M1 rejected then reject.
3. Else run M2 on w2. If M2 rejected then reject.
4. Else accept.

Mo accepts w if M1 accepts the first part and M2 accepts the second part. Else Mo rejects as w don't belong to L1.L2(concatenation of languages).

So, decidable languages are closed under concatenation.

B. Complementation:

For any decidable languages L. Let M be the TM that decide it. We construct a TM Mo that decides the complement of L.

Mo = "ON input w"

1. Accepts if M rejects.
2. Else reject.

Mo do complement to M. It decides the complement of L. So, decidable languages are closed under complementation.

Question 8

For any Turing-recognizable language L, Let M be the turing machine that recognize it. We construct a turing machine Mo that recognizes the L*.

Mo=On input w:

1. on input w divide w into parts w1,w2….wn.

2.Run M on wi for I = 1,2,3,…n.

3. If M accepts all , aceept.

4. else reject.
Hence, the collection of turing recognizable language is closed under star operation.

Question 9

Statement: Intuitive notion of algorithms equals Turing machine algorithms.
The definition came in the 1936 papers of Alonzo Church and Alan Turing. Church used a notational system called the λ-calculus to define algorithms. Turing did it with his “machines.” These two definitions were shown to be equivalent. This connection between the informal notion of algorithm and the precise definition has come to be called the Church–Turing thesis.

Theorem is a statement which have a proof, church Turing don’t have any proof so it is not a mathematical theorem.