**Question 1**

Let $EQ_{DFA,\ REX}$ = {<A, R> | A is a DFA, R is a regular expression and L(A) = L(R)}. The following TM E decides $EQ_{DFA,\ REX}$.

E = " On input <A, R>:

1.Convert regular expression, R to an equivalent DFA B.

2.Use the TM F for deciding $EQ_{DFA}$ on input <A, B>.

3.If F accepts then accept. Else if F rejects then reject."

**Question 2**

There can be two ways to show $ALL_{DFA}$ is decidable.

Answer a).

There is a Turing Machine TM decides $ALL_{DFA}$.

S= "on input ‹A›, A is a DFA.

1. Construct a DFA D such that L(D) is complement of L(A).
2. Run the Turing machine TM on input <D>.
3. If TM accepts accept, else reject.

Answer b).

S= "on input ‹A›, A is a DFA.

1. Construct a DFA D such that L(D)=∑*.
2. Run the Turing machine Tm on input <A,D>.
3. If TM accepts accept,else reject.

**Question 3**

Construct an algorithm A that decides AεCFG.

For the given input <G>, First A converts the grammar G into CNF G' such that L(G) = L(G').

As the variables that other than the new start variable S0 cannot have ε rules in G', M checks if any rule of S0 is an ε rule. If M accepts <G> else M reject. So that M halt on all inputs and checks whether the grammar generate ε and it decides language AεCFG.

**Question 4:**

(b) f is not onto because there does not exist x ϵ X such that f(1) = f(3) = f(5) = 6.f(2) = f(4) = 7. And those elements 8, 9 and 10 in set Y are left out without any elements in x ∈ X such that y = f(x).So, therefore the function is not an onto function.

(c) f is not some correspondence since f is not one-to-one or onto.

(e) g is onto.

(f) g is some correspondence since g is one-to-one or onto.

**Question 5:**

Prove by contradiction. We assume that A is countable, i.e., we can give an enumeration $f_1$, $f_2$, $f_3$, … of A. To come to a contradiction, we construct a new function f' as $f'(x) = f_x(x) + 1$ for $x \in N$. The function f' is constructed from the diagonal of the function values of $f_i \in A$ as represented in the figure below. For each x, f' differs from $f_x$ on input x. Hence, f' does not appear in the given enumeration. However, f' is a function and f': N → N. Such an f' can be given for any chosen enumeration. This leads to a contradiction. Therefore, A cannot be enumerated. Hence, A is uncountable.

| | | | | |
|---|---|---|---|---|
| $f_1$ | $f_1(1)$ | $f_1(2)$ | $f_1(3)$ | ... |
| $f_2$ | $f_2(1)$ | $f_2(2)$ | $f_2(3)$ | ... |
| $f_3$ | $f_3(1)$ | $f_3(2)$ | $f_3(3)$ | ... |
| ... | ... | ... | ... | ... |

**Question 6:**

A set is countable if it is a subset of another countable set.

Here, T is the subset of Natural Numbers set N.

Consider the natural numbers, N = { 1, 2, 3, 4, 5,….} an Let T will be {(1, 1, 2),(2, 3, 4),(1, 2, 4),…..}

Natural numbers are countable because, Let F : N -> N, then f(x) = x, which gives a one to one relationship. For a countable set, there will be a one-to-one relationship f from N to T. Another proof is cardinality of countable set can be finite number. Here, consider the triplet T = { 1, 2, 4} which has |T| = 3, which is a finite number, which is listable. Hence T is countable.

**Question 7:**

We observe that L(R) belongs to L(S) if and only if L(R) intersects L'(S)(complement) is null. The following TM X decides A.

X = "On input <R,S> where R and S are regular expressions:

1.Construct DFA E such that L(E) = L(R) intersects L'(S).

2.Run TM T on <E>, where T decides $E_{DFA}$.

3.If T accepts then accept. Else if T rejects, then reject."