Automata Assignment : 1

1. 1a.) Let A be the set {x,y,z} and B be the set {x,y}.

    a. Is A a subset of B?

    Ans: No, A is not a subset of B, since A contains an extra element than B.

    b. Is B a subset of A?

    Ans: Yes, B is not a subset of A, since all elements of B exists in A.

    c. What is A U B?

    Ans: { x, y, z}

    d. What is A $\cap$ B?

    Ans: { x, y }.

    e. What is A X B?

    Ans: { (x, x), (x,y), (y,x), (y,y), (z,x), (z,y)}

    f. What is the power set of B?

    Ans: { $\varnothing$, (x), (y), (x , y) }.

1b.) If A has a elements and B has b elements, how many elements are in A X B?

Ans: We will have (a x b) elements. According to cartesian product each element of A will be paired with each element of B, so the total number of elements will be the multiplication of (a x b).

For instance, let us consider A = 2 and B = 9.

So here, A X B = 18.

1c.) If C is a set with c elements, how many elements are in the power set of C?

Ans: Power set can be calculated using the formula 2^(no.of elements in the set). So C have c elements

Therefore, in the power set of C there will be 2^c elements.

2.) Let S (n) = 1 + 2 + 3 + - - - - - - + n be the sum of the first n natural numbers and let C(n) = $1^3 + 2^3 + 3^3 + \cdots + n^3$ be the sum of the first n cubes. Prove the following equalities by induction on n, to arrive at the curious conclusion that C(n) = S^2(n) for every n.

a. S(n)= $1/2n(n+1)$

b. S(n) = ¼$(n^4 + 2n^3 + n^2) = 1/4n^2(n+1)^2$

a.) Solution:

considering, the first equation we have to use the induction method to solve it. The main purpose of the mathematical induction is to prove the equality of both sides.

In proof by induction we will have two steps.

i). Basis ii) Inductive step.

Basis: To show s(1) is true.

Now, we substitute n = 1 in the first equation.

S(1) = ½$\big(1(1+1)\big)$

= ½$\big(1(2)\big)$

= ½(2)

=1

Therefore, L.H.S = R.H.S

Hence, proved.

Inductive:

Assume s(n) is true for n=k for some integer k,k>=1.

i.e, s(k) $\longrightarrow$ s(k+1)

s(k) is true means:

1+2+3+……k= ½ k (k+1)

Then, we have

n with k+1. Here, k = k+1.

S(k+1): 1+2_....+k+1=1/2(k+1)(k+2)

Now,

L.H.S

S(k+1)= 1+2+…k+k+1

= s(k)+k+1

= k(k+1)/2 + k+1

= k^2+3k+2/2

Hence L.H.S = RHS

Therefore, s(k+1) is true for s(k) is true.

b.) Solution:

Considering the above solution, we must clearly use mathematical induction method.

$$C_n = 1/4n^2(n+1)^2$$

In proof by induction we will have two steps.

i). Basis ii) Inductive step.

Basis: To show s(1) is true.

now substitute n = 1 in the above equation.

$$1^3 = \frac{1}{4}\ 1^2 * (1 + 1)^2$$

$$1 = \frac{1}{4}\ (4)$$

$$1 = 1$$

Therefore, L.H.S = R.H.S

Inductive:

Assume s(n) is true for n=k for some integer k,k>=1.

i.e, s(k) $\longrightarrow$ s(k+1)

s(k) is true means:

$1^3+2^3+\ldots\ldots\ldots\ldots k^3 = \frac{1}{4}\ k^2(k+1)^2$

Then, we have

n with k+1. Here, k = k+1.

$1^3+2^3+\ldots\ldots\ldots\ldots(k+1)^3 = = \frac{1}{4}\ (k+1)^2(K+1+1)^2$

$C(k+1) = 1^3+2^3+\ldots\ldots\ldots\ldots k^3 + (k+1)^3$

$$= \frac{1}{4}\ k^2(k+1)^2 + (k+1)^3$$

$$= \left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3$$

$$= (k+1)^2\ (\frac{k^2}{4} + k + 1)$$

$$= (k+1)^2(k^2 + 4k + 4/4)$$

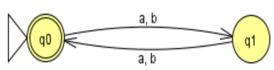
$$= (\frac{(k+1)(k+2)}{2})^2$$

Hence L.H.S = R.H.S

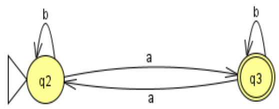
Therefore, s(k+1) is true for s(k) is true.

3) (Exercise 1.4 g):

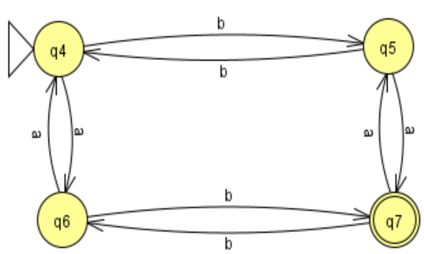
$\Sigma = \{a, b\}$.
Language = {w | w has even length and an odd number of a's}
So, assume,
Language_1 = {w | w has even length}
Language_2 = {w | w has odd number of a's}
So, Language_1:



Language$_2$:



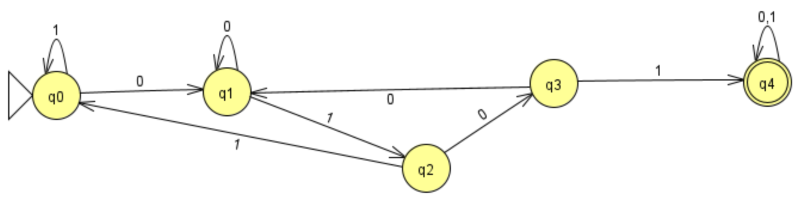
As a result, Language = Language$_1$ + Language$_2$
Therefore, Language:



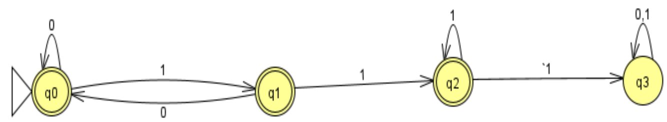
4.)

$\Sigma = \{a, b\}$.

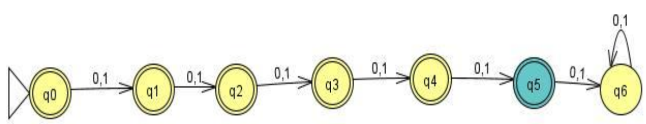
1.6 c) {w| w contains the substring 0101 (i.e., w = x0101y for some x and y)}



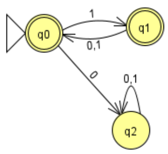
1.6 f {w| w doesn't contain the substring 110}



1.6g. {w| the length of w is at most 5}



1.6i {w| every odd position of w is a 1}



Question 5

Use the construction in the proof of Theorem 1.45 to give the state diagrams of NFAs recognizing the union of the languages described in a.

b. Exercises 1.6c and 1.6f

Proof

Let N1 = (Q1, Σ, δ1, q1, F1) recognize A1,

and N2 = (Q2, Σ, δ2, q2, F2) recognize A2.

Construct N = (Q, Σ, δ, q0, F) to recognize A1 ∪ A2.

1. Q = {q0} ∪ Q1 ∪ Q2. The states of N are all the states of N1 and N2, with the addition of a new start state q0.

2. The state q0 is the start state of N.

3. The set of accept states F = F1 ∪ F2. The accept states of N are all the accept states of N1 and N2. That way, N accepts if either N1 accepts or N2 accepts.

4. Define δ so that for any q ∈ Q and any a ∈ Σε,

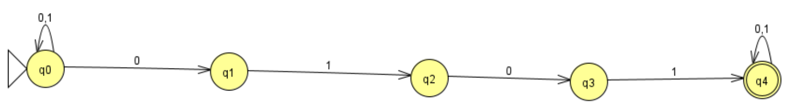
δ(q, a) = δ1(q, a) q ∈ Q1

$\delta 2(q, a)$ $q \in Q2$

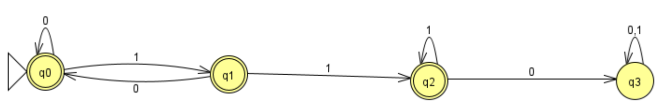
$\{q1, q2\}$ $q = q0$ and $a = \varepsilon$

$\emptyset$ $q = q0$ and $a \neq \varepsilon$

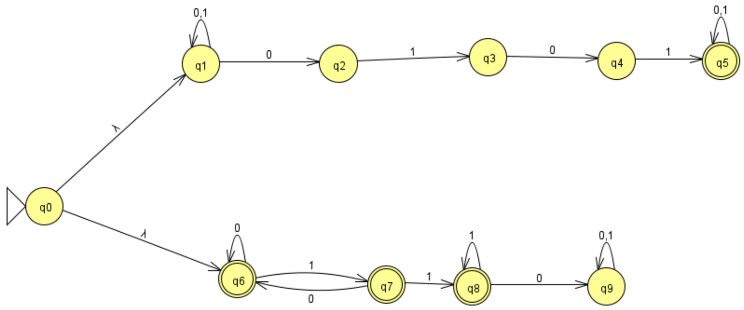
L1 =



L2 =



Therefore L1 U L2



Question 6

Use the construction in the proof of Theorem 1.47 to give the state diagrams of NFAs recognizing the concatenation of the languages described in a.

Exercises 1.6g and 1.6i.

Proof:

Let N1 = (Q1, Σ, δ1, q1, F1) recognize A1,

and N2 = (Q2, Σ, δ2, q2, F2) recognize A2.

Construct N = (Q, Σ, δ, q1, F2) to recognize A1 ◦ A2.

1. Q = Q1 ∪ Q2. The states of N are all the states of N1 and N2.

2. The state q1 is the same as the start state of N1.

3. The accept states F2 are the same as the accept states of N2.

4. Define δ so that for any q ∈ Q and any a ∈ Σε,

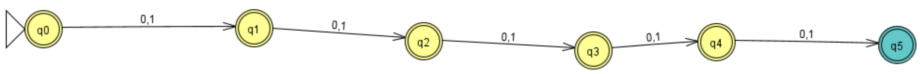
δ(q, a) = δ1(q, a) q ∈ Q1 and q ∉ F1

δ1(q, a) q ∈ F1 and a ≠ ε

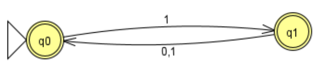
δ1(q, a) ∪ {q2} q ∈ F1 and a = ε

δ2(q, a) q ∈ Q2

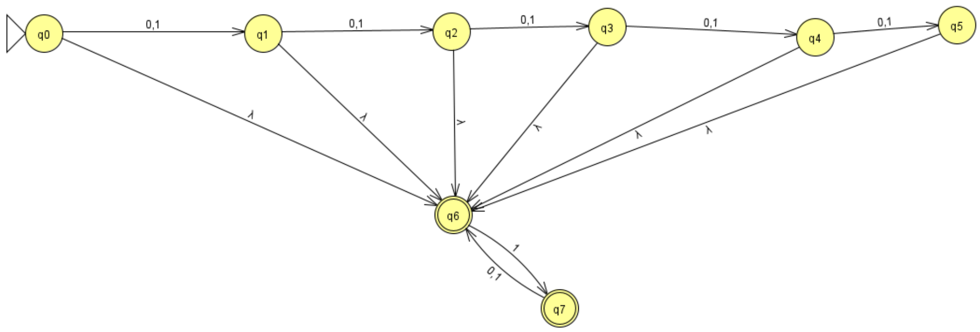
L1 =



L2 =



Therefore L1 . L2



Question 7:

Step 1:

Table for given NFA.

	a	b	ε
1	{ 3 }	Ø	{ 2 }
2	{ 1 }	Ø	Ø
3	{ 2 }	{ 2 , 3 }	Ø

Step 2:

E closure of { 1 } = { 1 , 2 }

E closure of { 2 } = { 2 }

E closure of { 3 } = { 3 }

Step 3:

E closure of (δ ({ 1, 2 } , a))

=E closure of (δ ((1 , a) U (2 , a)))

=E closure of ({ 3 } U { 1 })

={ 1 , 2 , 3 }.

E closure of (δ{ 1, 2 } , b))

=E closure of (δ ({ **Ø } U { Ø }))**

Ø

E closure of (δ ({ 2 } , a))

=E closure of ({ 1 })

={ 1 , 2 }

E closure of (δ { 2 }, b)

=E closure of ({ **Ø })**

=Ø

E closure of (δ { 3 }, a)

=E closure of ({ 2 })

={ 2 }

E closure of (δ ({ 3 }, b))

=E closure of (δ (3 , b))

=E closure of ({ 2 , 3 })

={ 2 , 3 }

E closure of (δ ({ 1 , 2 , 3 }), a)

=E closure of (δ (1 , a) U (2 , a) U (3 , a))

=E closure of ({ 3 } U { 1 } U { 2 })

={ 1 , 2 , 3 }

E closure of (δ ({ 1 , 2 , 3 } , b))

=E closure of (δ (1 , b) U (2 , b) U (3 , b))

=E closure of ({ Ø } U { Ø } U { 2 , 3 })

=E closure of (δ ({ 2 , 3 } , a))

=E closure of (δ (2 , a) U (2 , b))

=E closure of ({ 1 } U { Ø })

={ 1 , 2 }

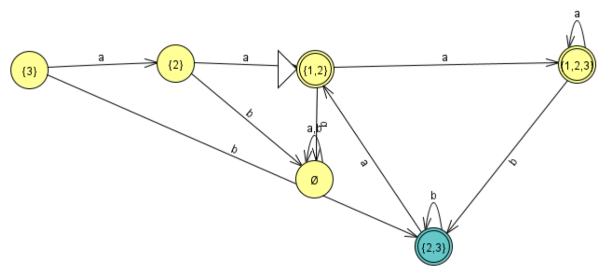
E closure of (δ ({ 2 , 3 } , b))

=E closure of (δ (Ø) U { 2 , 3 })

={ 2 , 3 }

DFA transition table:

	a	b
{ 1 , 2 }	{ 1 , 2 , 3 }	Ø
{ 2 }	{ 1 , 2 }	Ø
{ 3 }	{ 2 }	{ 2 , 3 }
{ 1 , 2 , 3 }	{ 1 , 2 , 3 }	{ 2 , 3 }
{ 2 , 3 }	{ 1 , 2 }	{ 2 , 3 }



Final DFA :