Chapter 3
Symmetric Key Crypto
Sep 1st

Security of DES

- Security of DES depends a lot on S-boxes
  - Everything else in DES is linear
- Thirty years of intense analysis has revealed no "back door"
- Attacks today use exhaustive key search
- **Inescapable conclusions**
  - Designers of DES knew what they were doing
  - Designers of DES were ahead of their time

Block Cipher Notation

- P = plaintext block
- C = ciphertext block
- Encrypt P with key K to get ciphertext C
  - $C = E(P, K)$
- Decrypt C with key K to get plaintext P
  - $P = D(C, K)$
- Note that
  - $P = D(E(P, K), K)$ and $C = E(D(C, K), K)$

Triple DES

- Today, 56 bit DES key is too small
- But DES is everywhere: What to do?
- **Triple DES** or **3DES** (112 bit key)
  - $C = E(D(E(P,K_1),K_2),K_1)$
  - $P = D(E(D(C,K_1),K_2),K_1)$
- Why use Encrypt-Decrypt-Encrypt (EDE) with 2 keys?
  - Backward compatible: $E(D(E(P,K),K),K) = E(P,K)$
  - And 112 bits is enough

3DES

- Why not $C = E(E(P,K),K)$?
  - Still just 56 bit key
- Why not $C = E(E(P,K_1),K_2)$?
- A (semi-practical) known plaintext attack
  - Precompute table of $E(P,K_1)$ for every possible key K_1 (resulting table has 2^{56} entries)
  - Then for each possible K_2 compute $D(C,K_2)$ until a match in table is found
  - When match is found, have $E(P,K_1) = D(C,K_2)$
  - Result is keys: $C = E(E(P,K_1),K_2)$

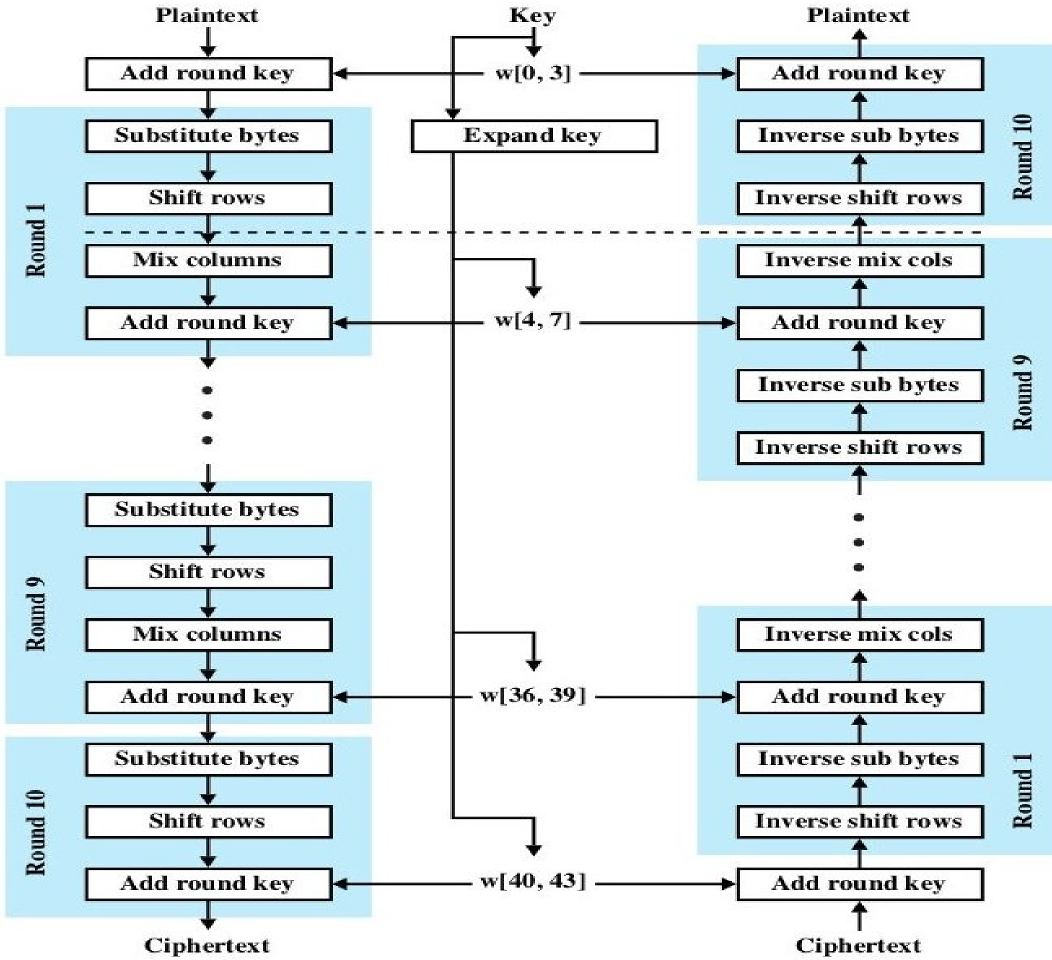
Advanced Encryption Standard

- Replacement for DES
- AES competition (late 90's)
  - NSA openly involved
  - Transparent process
  - Many strong algorithms proposed
  - Rijndael Algorithm ultimately selected
    - Pronounced like "Rain Doll" or "Rhine Doll"
- Iterated block cipher (like DES)
- Not a Feistel cipher (unlike DES)

AES Overview

- **Block size:** 128, 192 or 256 bits
- **Key length:** 128, 192 or 256 bits (independent of block size)
- 10 to 14 rounds (depends on key length)
- Each round uses 4 functions (in 3 "layers")
  - ByteSub (nonlinear layer)
  - ShiftRow (linear mixing layer)
  - MixColumn (nonlinear layer)
  - AddRoundKey (key addition layer)

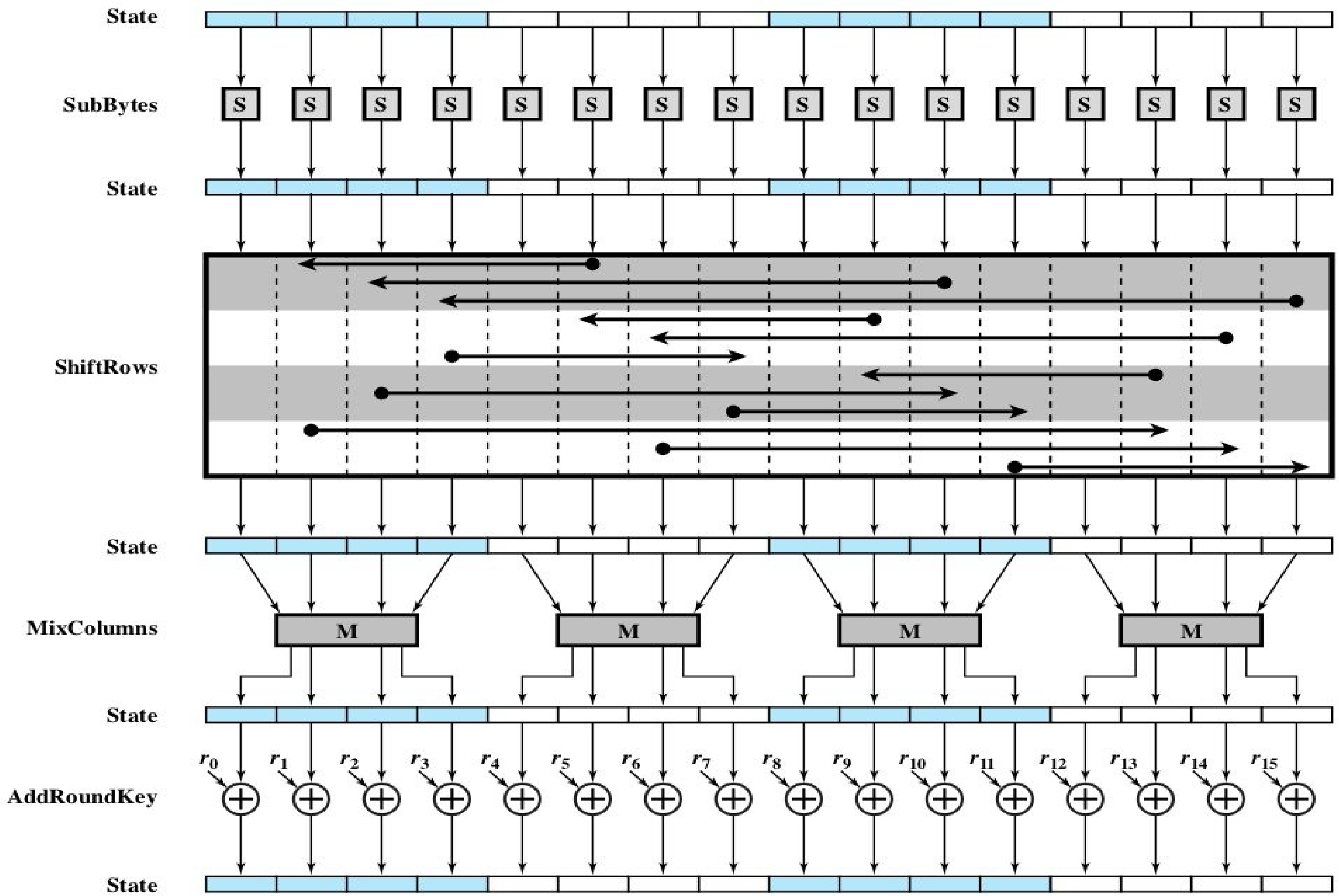
AES Overview



(a) Encryption (b) Decryption

From Computer Security, Priciples and Practice by William Stallings

AES Encryption Round



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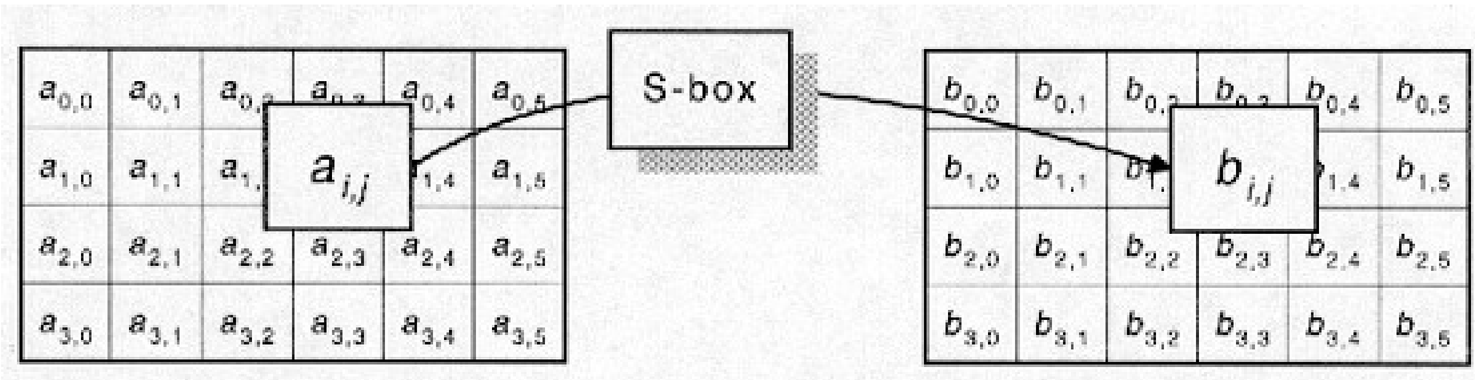
AES ByteSub

- Assume 192 bit block, 4x6 bytes



- ByteSub is AES's "S-box"
- Can be viewed as nonlinear (but invertible) composition of two math operations
- Resilient to known attacks (low correlation between input and output bits)

AES "S-box"

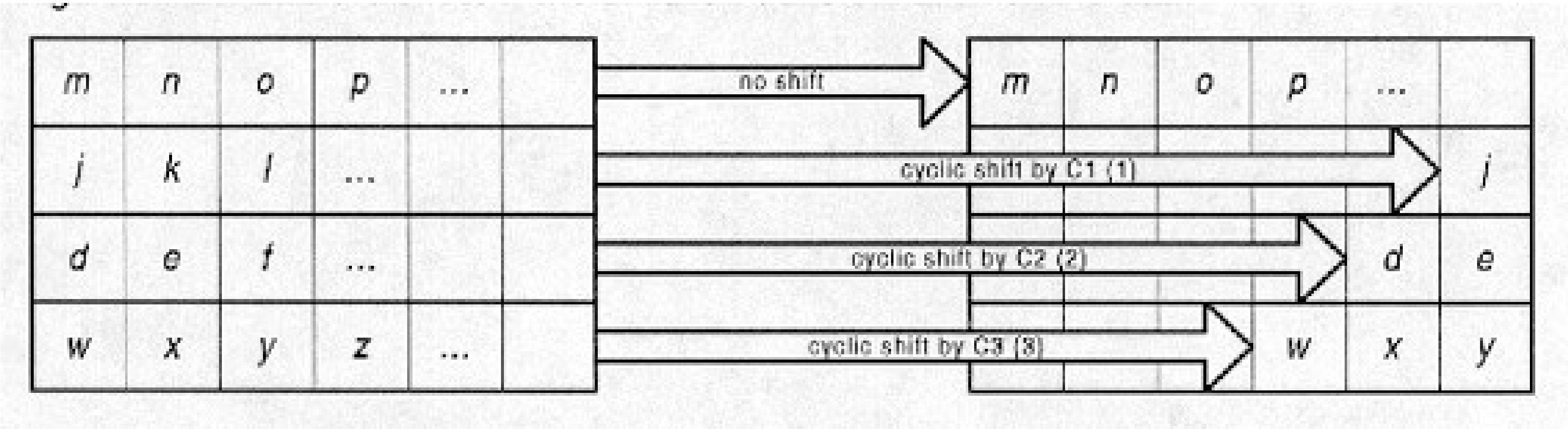
Last 4 bits of input

First 4 bits of input

	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
1	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
2	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
3	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
4	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
5	53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
6	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
7	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
8	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
9	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
a	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
b	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
c	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
d	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
e	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
f	8c	a1	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16

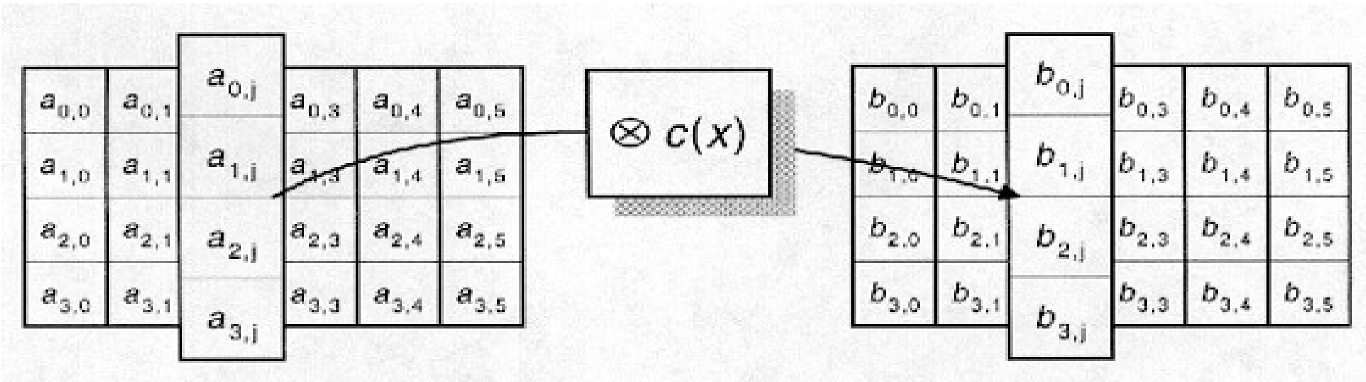
AES ShiftRow

- Cyclic shift rows



AES MixColumn

- Nonlinear, invertible operation applied to each column (just as S-box it is based on finite fields again)



- Implemented as a (big) lookup table
- Each byte of a column is function off all bytes in the column
- Together with shift row => after few rounds all output bits depend on all input bits

AES AddRoundKey

- XOR subkey with block

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} & a_{04} & a_{05} \\ a_{10} & a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{20} & a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{30} & a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \end{bmatrix} \oplus \begin{bmatrix} k_{00} & k_{01} & k_{02} & k_{03} & k_{04} & k_{05} \\ k_{10} & k_{11} & k_{12} & k_{13} & k_{14} & k_{15} \\ k_{20} & k_{21} & k_{22} & k_{23} & k_{24} & k_{25} \\ k_{30} & k_{31} & k_{32} & k_{33} & k_{34} & k_{35} \end{bmatrix} = \begin{bmatrix} b_{00} & b_{01} & b_{02} & b_{03} & b_{04} & b_{05} \\ b_{10} & b_{11} & b_{12} & b_{13} & b_{14} & b_{15} \\ b_{20} & b_{21} & b_{22} & b_{23} & b_{24} & b_{25} \\ b_{30} & b_{31} & b_{32} & b_{33} & b_{34} & b_{35} \end{bmatrix}$$

Block Subkey

- RoundKey (subkey) determined by **key schedule** algorithm

AES Decryption

- To decrypt, process must be invertible
- Inverse of MixAddRoundKey is easy, since $\oplus$ is its own inverse
- MixColumn is invertible (inverse is also implemented as a lookup table)
- Inverse of ShiftRow is easy (cyclic shift the other direction)
- ByteSub is invertible (inverse is also implemented as a lookup table)

A Few Other Block Ciphers

- Briefly…
  - IDEA
  - Blowfish
  - RC6
- More detailed…
  - TEA

IDEA

- Invented by James Massey
  - One of the giants of modern crypto
- IDEA has 64-bit block, 128-bit key
- IDEA uses **mixed-mode arithmetic**
- Combine different math operations to produce the necessary nonlinearity
  - IDEA the first to use this approach
  - Frequently used today

Blowfish

- Blowfish encrypts 64-bit blocks
- Key is variable length, up to 448 bits
- Invented by Bruce Schneier
- Almost a Feistel cipher

$$R_i = L_{i-1} \oplus K_i$$

$$L_i = R_{i-1} \oplus F(L_{i-1} \oplus K_i)$$

- The round function F uses 4 S-boxes
  - Each S-box maps 8 bits to 32 bits
- **Key-dependent S-boxes**
  - S-boxes determined by the key

RC6

- Invented by Ron Rivest (RC4, RSA, MD5)
- Variables
  - Block size
  - Key size
  - Number of rounds
- An AES finalist
- Uses **data dependent rotations**
  - Unusual to rely on data as part of algorithm

Tiny Encryption Algorithm

- 64 bit block, 128 bit key
- Assumes 32-bit arithmetic
- Number of rounds is variable (32 is considered secure)
- Uses "weak" round function, so large number rounds required

TEA Encryption

Assuming 32 rounds:

(K[0],K[1],K[2],K[3]) = 128 bit key

(L,R) = plaintext (64-bit block)

delta = 0x9e3779b9

sum = 0

for i = 1 to 32

sum += delta

L += ((R<<4)+K[0]) ⊕(R+sum)⊕((R>>5)+K[1])

R += ((L<<4)+K[2]) ⊕(L+sum)⊕((L>>5)+K[3])

next i

ciphertext = (L,R)

TEA Decryption

Assuming 32 rounds:

(K[0],K[1],K[2],K[3]) = 128 bit key

(L,R) = ciphertext (64-bit block)

delta = 0x9e3779b9

sum = delta << 5

for i = 1 to 32

$R \mathrel{-}= ((L\ll 4)+K[2]) \oplus (L+sum) \oplus ((L\gg 5)+K[3])$

$L \mathrel{-}= ((R\ll 4)+K[0]) \oplus (R+sum) \oplus ((R\gg 5)+K[1])$

sum −= delta

next i

plaintext = (L,R)

TEA comments

- **Almost** a Feistel cipher
  - Uses + and - instead of $\oplus$ (XOR)
- Simple, easy to implement, fast, low memory requirement, etc.
- Possibly a related key attack
- eXtended TEA (XTEA) eliminates related key attack (slightly more complex)
- Simplified TEA (STEA) — insecure version used as an example for cryptanalysis

Block Cipher Modes

Multiple Blocks

- How to encrypt multiple blocks?
- A new key for each block?
  - As bad as (or worse than) a one-time pad!
- Encrypt each block independently?
- Make encryption depend on previous block(s), i.e., "chain" the blocks together?
- How to handle partial blocks?

Modes of Operation

- Many modes of operation — we discuss three
- Electronic Codebook (**ECB**) mode
  - Obvious thing to do
  - Encrypt each block independently
  - There is a serious weakness
- Cipher Block Chaining (**CBC**) mode
  - Chain the blocks together
  - More secure than ECB, virtually no extra work
- Counter Mode (**CTR**) mode
  - Acts like a stream cipher
  - Popular for random access

ECB Mode

- Notation: $C=E(P,K)$
- Given plaintext $P_0,P_1,\ldots,P_m,\ldots$
- Obvious way to use a block cipher is

Encrypt	**Decrypt**
$C_0 = E(P_0, K)$,	$P_0 = D(C_0, K)$,
$C_1 = E(P_1, K)$,	$P_1 = D(C_1, K)$,
$C_2 = E(P_2, K),\ldots$	$P_2 = D(C_2, K),\ldots$

- For a fixed key K, this is an electronic version of a codebook cipher
- A new codebook for each key

ECB Cut and Paste Attack

- Suppose plaintext is

  `Alice digs Bob. Trudy digs Tom.`

- Assuming 64-bit blocks and 8-bit ASCII:

  P_0 = "`Alice di`", P_1 = "`gs Bob. `",

  P_2 = "`Trudy di`", P_3 = "`gs Tom. `"

- Ciphertext: C_0,C_1,C_2,C_3
- Trudy cuts and pastes: C_0,C_3,C_2,C_1
- Decrypts as

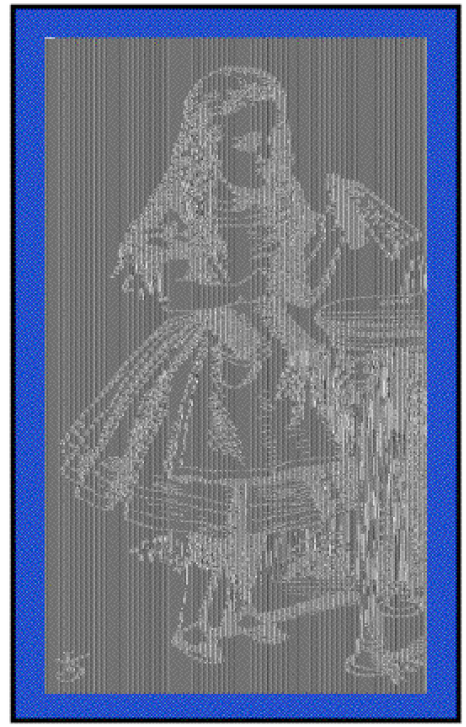
  `Alice digs Tom. Trudy digs Bob.`

ECB Weakness

- Suppose $P_i = P_j$
- Then $C_i = C_j$ and Trudy knows $P_i = P_j$
- This gives Trudy some information, even if she does not know P_i or P_j
- Trudy might know P_i
- Is this a serious issue?

Alice Hates ECB Mode

- Alice's uncompressed image, Alice ECB encrypted (TEA)

- Why does this happen?
- Same plaintext block $\Rightarrow$ same ciphertext!

CBC Mode

- Blocks are "chained" together
- A random initialization vector, or IV, is required to initialize CBC mode
- IV is random, but need not be secret

Encryption

$C_0 = E(IV \oplus P_0, K),$

$C_1 = E(C_0 \oplus P_1, K),$

$C_2 = E(C_1 \oplus P_2, K),\ldots$

Decryption

$P_0 = IV \oplus D(C_0, K),$

$P_1 = C_0 \oplus D(C_1, K),$

$P_2 = C_1 \oplus D(C_2, K),\ldots$

CBC Mode

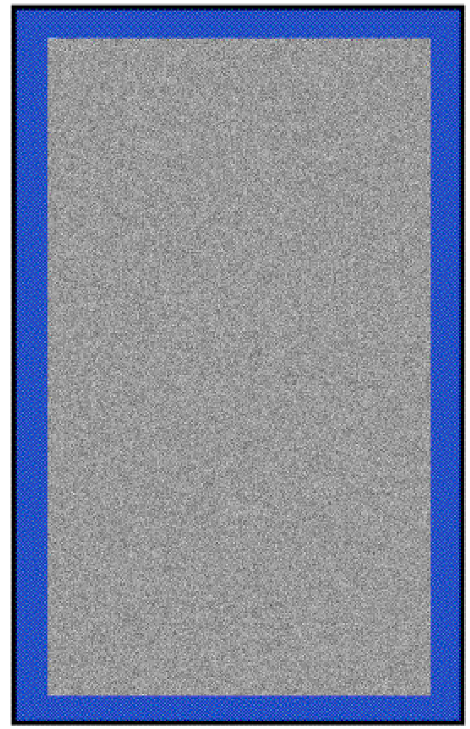
- Identical plaintext blocks yield different ciphertext blocks
- Cut and paste is still possible, but more complex (and will cause garbles)
- If C_1 is garbled to, say, G then

$$P_1 \neq C_0 \oplus D(G, K),\ P_2 \neq G \oplus D(C_2, K)$$

- But $P_3 = C_2 \oplus D(C_3, K),\ P_4 = C_3 \oplus D(C_4, K), \ldots$
- Automatically recovers from errors!

Alice Likes CBC Mode

- Alice's uncompressed image, Alice CBC encrypted (TEA)



- Why does this happen?
- Same plaintext yields different ciphertext!

Counter Mode (CTR)

- CTR is popular for random access
- Use block cipher like stream cipher

Encryption **Decryption**

$C_0 = P_0 \oplus E(IV, K),$ $P_0 = C_0 \oplus E(IV, K),$

$C_1 = P_1 \oplus E(IV+1, K),$ $P_1 = C_1 \oplus E(IV+1, K),$

$C_2 = P_2 \oplus E(IV+2, K),\ldots$ $P_2 = C_2 \oplus E(IV+2, K),\ldots$

Next …
Integrity
(& Public Key Cryptography)