

CS 380/525: Intro to Crypto  
Fall 2022  
Assignment 5, due 11/22, before class

Please see class written notes and class slides for examples. Total: 100 points.

1. (25 points) Find discrete logarithm using Baby-Step-Giant-Step algorithm. Show your work:
  - (a) Given cyclic group  $\mathbb{Z}_{29}^*$ , and  $2^x \bmod 29 = 27$ . Find  $x = \log_2 27$  in  $\mathbb{Z}_{29}^*$ .
  - (b) Given cyclic group  $\mathbb{Z}_{37}^*$ , and  $2^x \bmod 37 = 6$ . Find  $x = \log_2 6$  in  $\mathbb{Z}_{37}^*$ .
  - (c) Given cyclic group  $\mathbb{Z}_{17}^*$ , and  $3^x \bmod 17 = 7$ . Find  $x = \log_3 7$  in  $\mathbb{Z}_{17}^*$ .
2. (25 points) Find discrete logarithm using Pohlig-Hellman algorithm. Show your work:
  - (a) Given cyclic group  $\mathbb{Z}_{11}^*$ , and  $2^x \bmod 11 = 10$ . Find  $x = \log_2 10$  in  $\mathbb{Z}_{11}^*$ .
  - (b) Given cyclic group  $\mathbb{Z}_{31}^*$ , and  $3^x \bmod 31 = 12$ . Find  $x = \log_3 12$  in  $\mathbb{Z}_{31}^*$ .
  - (c) Given cyclic group  $\mathbb{Z}_{23}^*$ , and  $5^x \bmod 23 = 15$ . Find  $x = \log_5 15$  in  $\mathbb{Z}_{23}^*$ .
3. (25) Consider a group  $\mathbb{Z}_{23}^*$ , and a message  $M = 10$ . Encrypt  $M$  using ElGamal encryption scheme (you'll have to pick the  $PK, SK$  before encryption) to obtain ciphertext  $C$ . Now decrypt  $C$  to verify you get  $M$  back. Show your steps.
4. (25 points) Compute  $4^{23} \bmod 187$ , and  $9^{36} \bmod 101$ , using square-and-multiply method.