Q1.) Naive Bayesian Approach.

1. Estimate the conditional probabilities for P(A|+), P(B|+), P(C|+), P(A|-), P(B|-), P(C|-).

Ans.) P(A|+) => P(0|+) = 2/7 = 0.285.

P(1|+) = 5/7 = 0.714.

P(A|-) => P(0|-) = 3/3 = 1.

P(1|-) = 0/3 = 0.

P(B|+) => P(0|+) = 6/7 = 0.857.

P(1|+) = 1/7 = 0.142.

P(B|-) => P(0|-) = 1/3 = 0.333.

P(1|-) = 2/3 = 0.666.

P(C|+) => P(0|+) = 1/7 = 0.142.

P(1|+) = 6/7 = 0.857.

P(C|-) => P(0|-) = 0/3 = 0.

P(1|-) = 3/3 = 1.

1. Use the estimate of conditional probabilities given in the previous question to predict the class label for a test sample (A=0, B=1, C=0) using the naive Bayes approach. Work out the solution manually and show the detailed manual calculation steps.

Ans.) For test sample (A=0, B=1, C=0)

On probability ‘+’, P(A=0, B=1, C=0 | +)

P(A=0 | +) \* P(B=1 | +) \* P(C=0 | +) \* P(+) (since, from above problem we know the probabilities)

0.285 \* 0.142 \* 0.142 \* 0.7 = 0.004022.

On probability ‘-‘, P(A=0, B=1, C=0 | -)

P(A=0 | -) \* P(B=1 | -) \* P(C=0 | -) \* P(-)

1 \* 0.666 \* 0 \* 0.3 = 0.

Therefore, P(A=0, B=1, C=0 | +) i.e., ‘+’ is the class label.

1. Estimate the conditional probabilities using the m-estimate approach, with p = 1/2 and m = 4.

Ans.) We know that m-estimate is, **P(Ai = ai | C = cj ) = (Nij + m \* p) / Nj + m.**

P(A=0 | +) = (2 + 2) / 11 = 0.363.

P(A=0 | -) = (3 + 2) / 7 = 0.714.

P(B=1 | +) = (1 + 2) / 11 = 0.272.

P(B=1 | -) = (2 + 2) / 7 = 0.571.

P(C=0 | +) = (1 + 2) / 11 = 0.272.

P(C=0 | -) = (0 + 2) / 7 = 0.285.

1. Repeat part (b) using the conditional probabilities given part (c).

Ans.) For test sample (A=0, B=1, C=0)

On probability ‘+’, P(A=0, B=1, C=0 | +)

P(A=0 | +) \* P(B=1 | +) \* P(C=0 | +) \* P(+) (since, from problem ‘c’ we know the probabilities)

0.363 \* 0.272 \* 0.272 \* 0.7 = 0.018799.

On probability ‘-‘, P(A=0, B=1, C=0 | -)

P(A=0 | -) \* P(B=1 | -) \* P(C=0 | -) \* P(-)

0.714 \* 0.571 \* 0.285 \* 0.3 = 0.034857.

Therefore, P(A=0, B=1, C=0 | -) i.e., ‘-’ is the class label.

Q2.) Linear Model.

1. Demonstrate how the perceptron model can be used to represent the following Boolean functions **A OR B**. (Hint: draw a table with all the possible values of A and B and the result of A OR B. Then, derive a linear equation that can separate the two different classes.)

Ans.) Logical Table for ‘**A OR B**’ is:

|  |  |  |
| --- | --- | --- |
| A | B | A OR B |
| 0 | **0** | **0** |
| 0 | **1** | **1** |
| 1 | **0** | **1** |
| 1 | **1** | **1** |

We know that for perceptron, the equation is **w1 \* x1 + w2 \* x2 + w0**.

* The weighted sum has to be more than or equal to 0 when the output is 1.
* Based on the OR function output for various sets of inputs, we solved the weights based on those conditions, i.e., w0 = -1 , w1 = 2 , w2 = 2 and we got a line which is linearly separating the different classes.

1. Is the problem **NOT A AND C** linearly separable? Explain why. (Hint: draw a table with all the possible values of A and C and the result of NOT A AND C. Then, check and see whether a linear equation that can separate the two different classes exists or not.)

Ans.) Logical Table for ‘**NOT A AND C**’ is:

|  |  |  |
| --- | --- | --- |
| A | C | NOT A AND C |
| 0 | **0** | **0** |
| 0 | **1** | **1** |
| 1 | **0** | **0** |
| 1 | **1** | **0** |

We know that for perceptron, the equation is **w1 \* x1 + w2 \* x2 + w0**.

* The weighted sum has to be more than or equal to 0 when the output is 1.
* Based on the NOT function output for various sets of inputs, we solved the weights based on those conditions, and we didn`t get a line that linearly separates the different classes.