**Question 1**

Consider a relation schema R (A, B, C, D, E) that satisfies the set of functional dependencies F = {BC🡪D, D🡪 E, A 🡪C, B🡪 C}. Calculate (AB)+.

**Solution:**

1. result = AB
2. result = ABC (A🡪C, B🡪C)
3. result = ABCD (BC🡪D and BC ⊆ ABC)
4. result = ABCDE (D🡪E and D ⊆ ABCD )

**Question 2**

Consider a relation schema R (A, B, C, D, E) that satisfies the set of functional dependencies F = {BC🡪D, D🡪 E, A 🡪C, B🡪 C}.

Is BC🡪E in F+ ? Please justify (Justification carries 5 pts).

**Solution:**

Yes. By **transitivity** from BC 🡪 D and D🡪E, BC🡪E is in.

**Question 3**

Consider a relation schema R (A, B, C, D, E) that satisfies the set of functional dependencies F = {BC🡪D, D🡪 E, A 🡪C, B🡪 C}.

Find all the candidate keys of R and show your steps to find them (Steps carry 15 pts).

Step 1: Calculate the closure for the individual attributes

= AC = BC

= C

= DE

= E

Step 2: Since no individual attributes are a key, find attributes closures for attributes with length 2

* =
  1. result = AB
  2. result = ABC (A🡪C, B🡪C)
  3. result = ABCD (BC🡪D and BC ⊆ ABC)
  4. result = ABCDE (D🡪E and D ⊆ ABCD )
* =

1. result = AC

* =

1. result = AD
2. result = ACDE(A🡪C,D🡪E)

* =

1. result = AE
2. result = ACE(A🡪C)

* =

1. result = BC
2. result = BCD(BC🡪D and BC ⊆ BC)
3. result = BCDE (D🡪E and D ⊆ BCD)

* =

1. result = BD
2. result = BCDE(B🡪C, D🡪E)

* =

1. result = BE
2. result = BCE(B🡪C)
3. result = BCDE(BC🡪D and BC ⊆ BCE)

* =

1. result = CD
2. result = CDE(D🡪E)

* =

1. result = CE

* =

1. result = DE

Step 3: Since includes all attributes of R, check AB is a candidate key

* Is AB is a super key? – yes (AB🡪R == )
* Is any subset of AB is a super key
  + Does A → R? == Is (A)+ ⊇ R : No
  + Does B → R? == Is (B)+ ⊇ R : No

From step 3 it is found that AB is a candidate key (minimal super key).

Step 4: Find attributes closures for attributes with length 3. Since we find out that AB is a candidate key, we do not need to look at any combinations that include AB.

* =

1. result = ACD
2. result = ACDE (D🡪E and D ⊆ ACD)

* =

1. result = ACE

* =

1. result = ADE
2. result = ACDE (A🡪C)

* =

1. result = BCD
2. result = BCDE (D🡪E)

* =

1. result = BCE
2. result = BCDE (BC🡪D and BC ⊆ BCE )

* =

1. result = BDE
2. result = BCDE (B🡪C)

* =

1. result = CDE

Step 5: Find attributes closures for attributes with length 4. Since we find out that AB is a candidate key, we do not need to look at any combinations that include AB.

* =

1. result = ACDE

* =

1. result = BCDE

Since we find out that AB as a candidate key, we do not need to check attributes with length 5. That is, ABCDE.

So the only one candidate key is AB.

**Question 4**

Consider a relation schema R (A, B, C, D) that satisfies the set of functional dependencies F = {AB🡪D, D🡪C}.

Is R in BCNF? Please justify your answer (Justification carries 8 pts).

No. R is not in BCNF. Because, either the functional dependency is trivial or the left hand side of every functional dependency in F closure is a super key.

Here AB🡪D holds in R and AB is a super key.

* =

1. result = AB
2. result = ABD (AB🡪D)
3. result = ABCD (D🡪C and D ⊆ ABD)

But D🡪C holds in R and **D is not a super key**.

* =

1. result = D
2. result = DC (D🡪C)

**Question 5**

Consider a relation schema R (A, B, C, D) that satisfies the set of functional dependencies F = {AB🡪D, D🡪C}.

Is R in 3NF? Please justify your answer (Justification carries 8 pts).

No. R is not in 3NF. Because a relation schema R is in third normal form (3NF) if for all α → β in F+ at least one of the following holds:

α → *β* is trivial (i.e., *β* ∈ α)

α is a super key for *R*

Each attribute *A* in *β* – α is contained in a candidate key for *R*

Here AB is a super key. But D is does not satisfy any of the above conditions.

That is, D🡪C is not trivial, D is not a super key and C-D = ABC is not a candidate key. AB is the only candidate key here.

**Question 6**

Consider a relation schema R (A, B, C, D) that satisfies the set of functional dependencies F = {A🡪B, BC🡪D}.

Find all the candidate keys of R and show your steps to find them (Steps carry 15 pts).

Step 1: Calculate the closure for the individual attributes

* = AB = B
* = C
* = D

Step 2: Since no individual attributes are a key, find attributes closures for attributes with length 2

* =

1. result = AB

* =

1. result = AC
2. result = ABC(A🡪B)
3. result = ABCD(BC🡪D and BC ⊆ ABC )

* =

1. result = AD
2. result = ABD(A🡪B)

* =

1. result = BC
2. result = BCD(BC🡪D)

* =

1. result = BD

* =

1. result = CD

Step 3: Since includes all attributes of R, check AC is a candidate key

* Is AC is a super key? – yes (AC🡪R == )
* Is any subset of AC is a super key
  + Does A → R? == Is (A)+ ⊇ R : No
  + Does C → R? == Is (C)+ ⊇ R : No

From step 3 it is found that AC is a candidate key (minimal super key).

Step 4: Find attributes closures for attributes with length 3. Since we find out that AC is a candidate key, we do not need to look at any combinations that include AC.

* =

1. result = ABD

* =

1. result = BCD

Since we find out that AC as a candidate key, we do not need to check attributes with length 4. That is, ABCD.

So the only one candidate key is AC.

Question 7

Consider a relation schema R (A, B, C, D) that satisfies the set of functional dependencies F = {A🡪B, BC🡪D}.

Is R in BCNF? (No need for explanation.) If not, decompose R such that all the resultant schemas are in BCNF.

No. The relation R is not in BCNF.

1. Consider non trivial functional dependency A🡪B (α=A and β=B)

* A ∪ B (α ∪ β) = R1(A,B)
* R – B (R - β) = R2 (A,C,D)

2. Consider non trivial functional dependency BC🡪D (α=BC and β=D)

* BC ∪ D (α ∪ β) = R1(B,C,D)
* R – D (R - β) = R2 (A,B,C)