Business Scenario

A store is planning to buy EAS systems of company . The company claims No more than 5% of all the consumers would say that they would never shop in a store again if the store subjected them to a false alarm.

The store hires Archie’s company to study the claim and check the validity of this claim.

Approach :
The claim made by the EAS system selling company is an assumption that need to be tested. The claims can be tested by constructing Hypothesis Tests. Archie needs to get a complete understanding of how to construct Hypothesis Test and provide final conclusion.

Use Case Discussion – EAS Systems

A company that sells and installs EAS systems claims:

No more than 5% of all the consumers would say that they would never shop in a store again if the store subjected them to a false alarm.

A store considering the purchase of such a system uses **HYPOTHESIS TESTING** to provide extremely strong evidence that the claim is not true.

Hypothesis Test

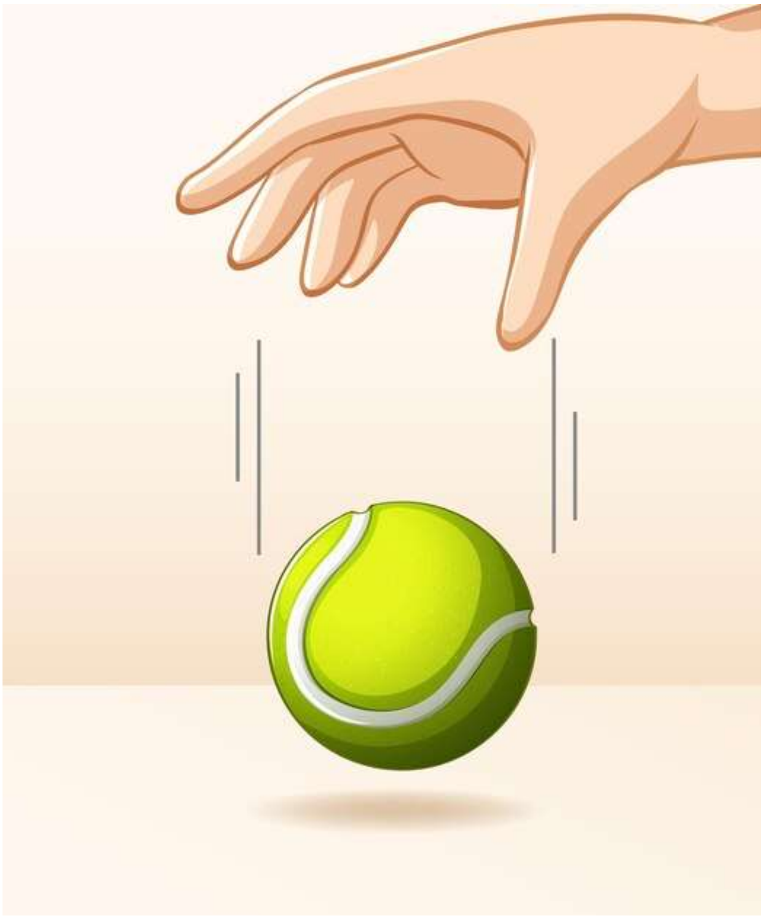
Hypothesis Testing

A hypothesis is a potential explanation for something that happens or is observed and thought to be true. Generally, business hypotheses are educated assumptions.

Hypothesis testing is a method of statistical inference to access whether the statement (called a hypothesis) made about the population is consistent with the observed data (sample).

Hypothesis Testing – A Scientific Approach

In science, the basic idea is that something cannot be proved to be True. You cannot prove that things fall when dropped.

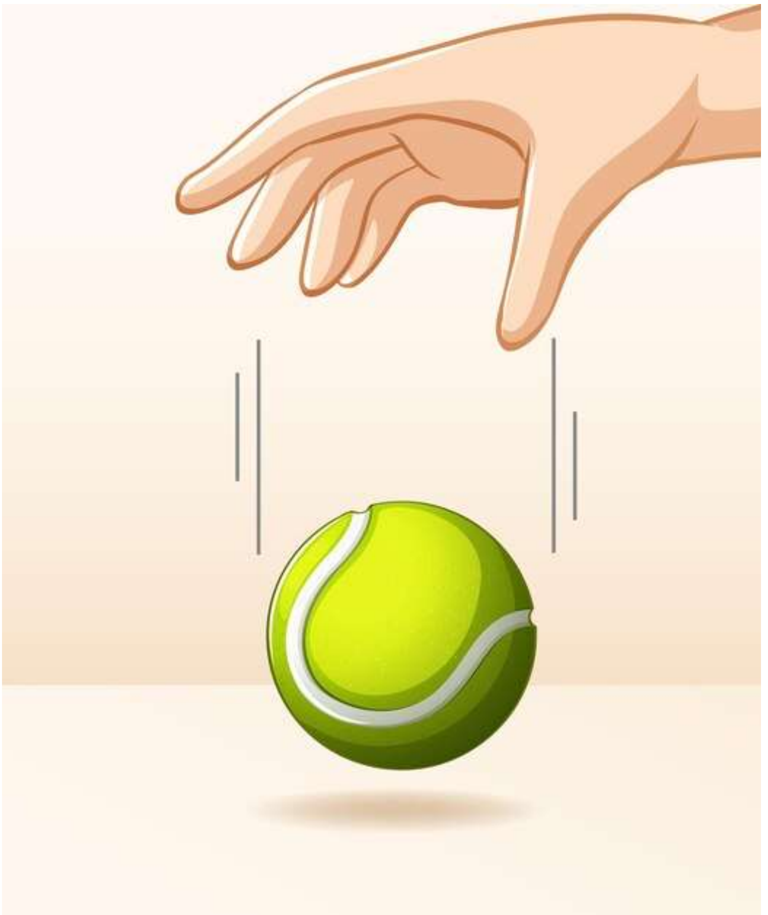


Science provides evidence that something is not True.

Hypothesis Testing - Intuition

It cannot be proved that a ball will always fall down.

Instead, it can be disproved that if a ball is dropped, it will not fall to the ground and stay at the same place by providing evidence against this idea by actually dropping the ball. This event can be repeated to provide even more evidence.



Forming Hypothesis – Null Hypothesis

The first step in a hypothesis test is to formalize it by specifying the null hypothesis.

It is a statement about the value of population parameter that holds true unless there are sufficient sample evidence to conclude otherwise.

It is represented by H_0. It is formed by expressing a condition on population parameters that would lead to status-quo.

Alternate Hypothesis or Research Hypothesis

It is denoted by H_a and it is the negation of the null hypothesis.

It is formed by expressing a condition on population parameters that would lead to change.

Note

H_0 and H_a are mutually exclusive and collectively exhaustive statements.

Example: Setting up H_0 and H_a

A manufacturer of golf balls claims that the variance in the weights of the company's golf balls is controlled to be within .0028 oz^2

How to set up the H_0 and H_a to test this claim?

H_0: variance <= .0028 oz^2

H_a: variance > .0028 oz^2

Let’s summarize

Case – I

Case II

Type 1 and Type 2 Errors

Error in Hypothesis Testing

Court of law always assumes the accused to be INNOCENT until PROVEN GUILTY.

$H_{0:}$ Accused = Innocent

$H_{a:}$ Accused = Guilty

Prosecution must provide enough evidence to reject innocence and conclude guilty.

		VERDICT	
		NOT GUILTY	GUILTY
TRUTH	INNOCENT	Correct	ERROR
	GUILTY	ERROR	Correct

Statistical Significance

Significance level (α) is the probability of rejecting the null hypothesis when it is true.

1. It is the probability of Type 1 error.
   The standard value of α are 10%, 5% and 1%. It is the risk of False positive results
2. Probability of type 2 error is denoted by β. It is the risk of False negative results.
3. 1- β is called as the power of test

		DECISION	
		Don't reject null	Reject null
STATE OF NATURE	Ho True	Correct Decision	TYPE 1 ERROR
	Ho False	TYPE 2 ERROR	Correct Decision

Typical Hypothesis Test

The Situation	Evidence of phenomenon or behavior
A hardware franchise has almost 1,000 retail out lets. Monthly sales of a particular tool averaged 612 units for each franchise. The sales manager believed a competitor's new price on a similar item may have an impact on sales. Based on a random sample of 64 franchises, the sales manager wishes to test if the observed sample mean differs from the benchmark figure.	Benchmark Vs Observed Value Test of significance QUESTIONS TO BE ANSWERED 1. Is the difference between the benchmark and the observed values statistically significant? 2. Does the magnitude of the increase(or decrease) in the phenomenon justify a change in business strategy?

Hypothesis Testing Procedure

Determine if the deviation between the sample mean and its expected value(hypothesized mean) would have occurred by chance alone if the statistical hypothesis were true

↑

Take an actual sample and calculate the sample mean(or any other appropriate parameter)

↑

Assess the sampling distribution of the mean if hypothesis were a true statement of the nature of population

↑

Determine Statistical hypothesis

The Audi R-18 e-Tron Quattro Case

Audi R-18 e-Tron Quattro is equipped with a top-class brake system with astonishing braking distance.

Braking distance is the distance required to bring the vehicle to a complete stop from a speed of 60 mph.

(Imagine) One of the competitors is advertised to achieve an average braking distance of 20 m. Audi would like to claim in the new television advertisement that Audi R-18 e-Tron Quattro achieves a better braking distance.

The Audi R-18 e-Tron Quattro Case

According to the protocols, this advertisement can be broadcasted only if Audi convinces that its average braking distance is less than 20 m.

If a random sample of 70 Audi R-16 have average stopping distance of 19.5 m, will National Motors be allowed to advertise the claim? Population standard deviation is 1.5 m.

Solution Steps: Decide H_0 , H_a and α

Step 1: Set up H_0 and H_a

H_0 : μ (Average braking distance) >= 20
H_a: μ (Average braking distance) < 20

Step 2: Ascertain significance level(α)

- Use standard value or establish it based on business requirements.
- Take 5%

Calculate Test Statistic

Step 3: Utilize test statistic

$$\text{test statistic} = \frac{\bar{X} - \mu}{\sigma_{\bar{x}}}$$

$$z = {}^{(19.5-20)}/_{1.5/\sqrt{70}} = -2.78$$

- The test statistic measures the distance between sample mean($\bar{X}$) and hypothesized mean(μ)
- Division by $\sigma_{\bar{X}}$ says that this distance is measured in the units of the standard deviation of all possible means

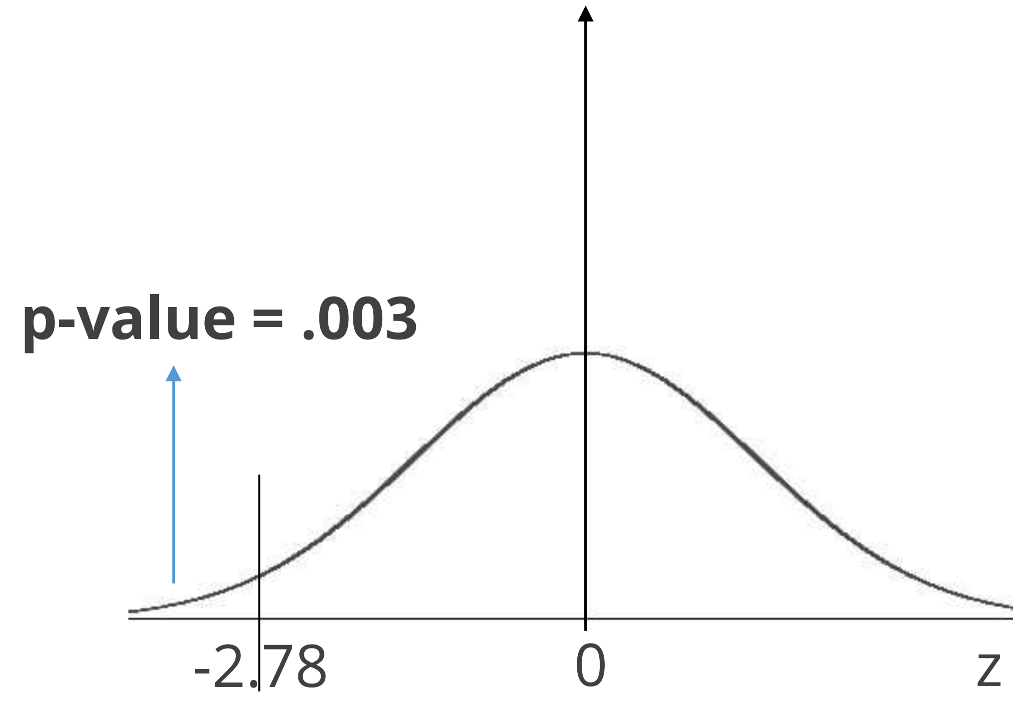
Note

- If population standard deviation(σ) is given, test statistic = z, and $\sigma_{\bar{X}} = \sigma/\sqrt{n}$
- If population standard deviation is not given, test statistic = t, and $\sigma_{\bar{X}} = s/\sqrt{n}$, where s is the standard deviation of the sample used to test the hypothesis, assuming normal population distribution.

Find p-value

The farther the value of the test statistic is below 0 (or the farther $\bar{x}$ is below 20), the stronger is the evidence to support rejecting H_0 in favor of H_a.

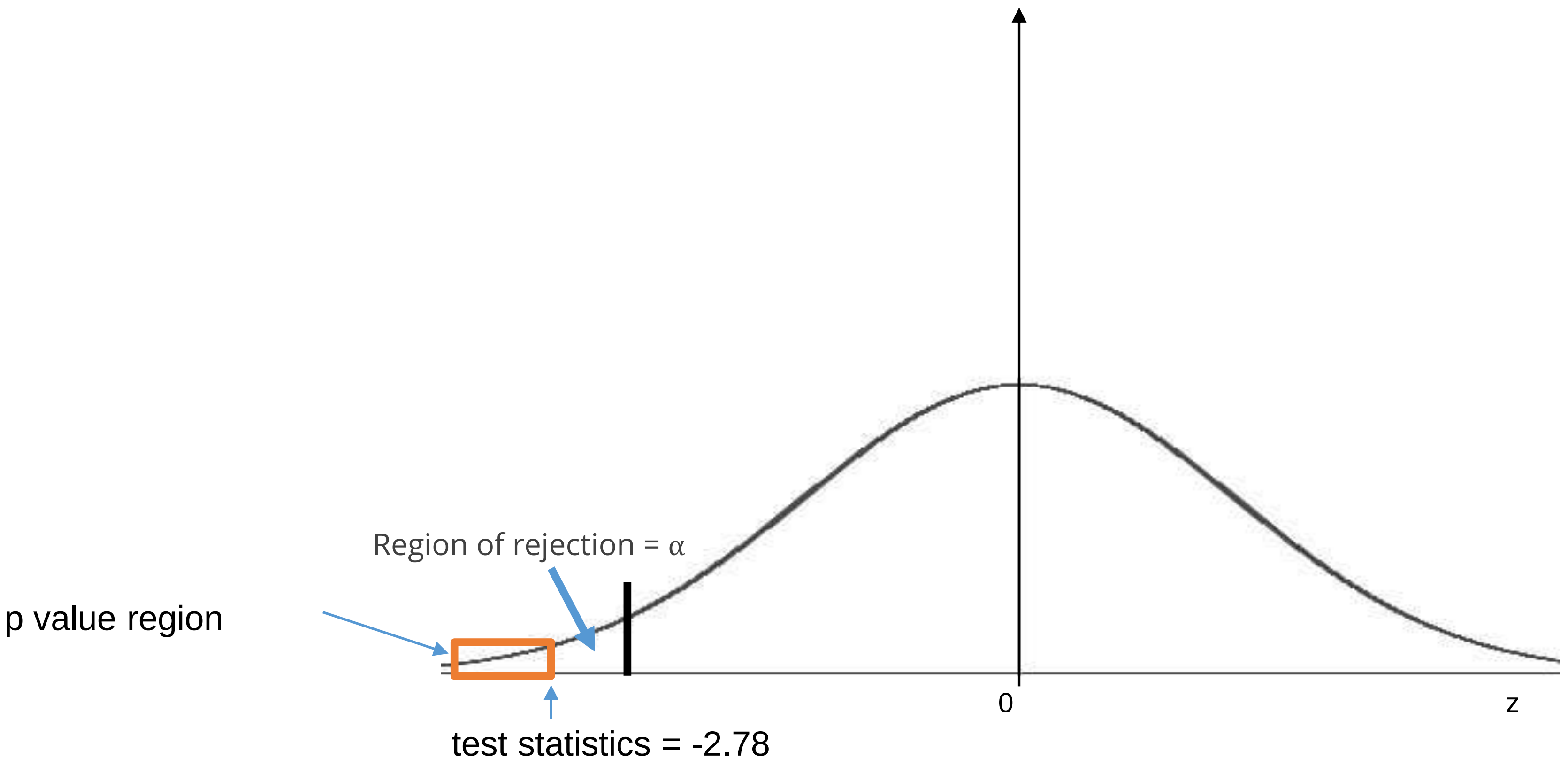
To see how small z must be in order to reject H_0. We use p-value approach. It is calculated using test statistic.



p-value is a probability that provides a measure of the evidence against the null hypothesis provided by the sample.

Comparing p-value with significance level

Since test statistic z falls into rejection region or p-value $<$ α. Hence reject the Null hypothesis.



Note

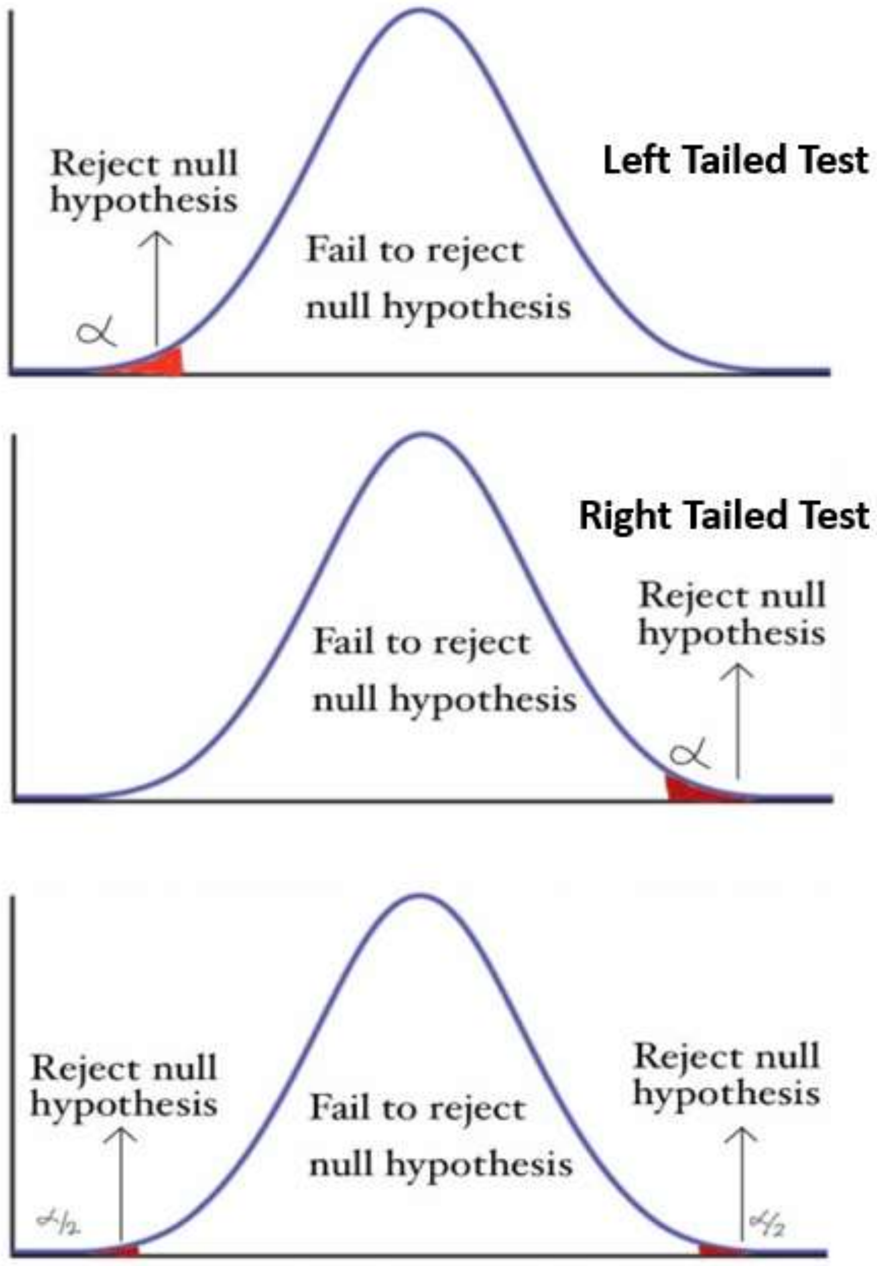
This is an example of 1 sample left tail hypothesis test.

One Sample Hypothesis Testing

Null Hypothesis(H0) :$\mu \geq value$
Alternative Hypothesis(H_A) : $\mu < value$

Null Hypothesis(H0) :$\mu \leq value$
Alternative Hypothesis(H_A) : $\mu > value$

Null Hypothesis(H0) :$\mu = value$
Alternative Hypothesis(H_A) : $\mu \neq value$



One Tailed Test

Two Tailed Test

Two sample Hypothesis Test

Comparing Two Populations

Independent samples t-test	Paired samples t-test
Independent random samples are collected	Samples collected are paired in some way
No of datapoints may be different in the 2 samples	No of datapoints are same
Examples: • Comparing performance of girls and boys • Comparing per capita income of 2 regions	Examples: • Testing efficacy of a medicine • Assessing the effectiveness of a marketing campaign by launching survey before and after the marketing campaign

Assumptions for **Independent samples t-test**

1. The data are continuous (not discrete)
2. Datapoints from the two groups follow a normal distribution.
3. The variances of the two populations are equal. If not, the Welch Unequal-Variance test is used

Comparing Two Populations

A study claimed that boys scored better or the same as girls in the Mathematics section of Competitive Tests.

A researcher wanted to check the validity of this claim.

The researcher collected a random sample for 80 female test takers and 120 male test takers to disprove the claim made in the study.

Construct a Hypothesis Test to help the researcher validate the hypothesis at a significance level of 0.05.

Analysis of Variance

Comparing Three or More Populations

An oil company wishes to develop a reasonably priced gasoline that will deliver improved mileage. The company prepares three different formulations for gasoline for use.

‘Analysis of variance’ is used to compare the effects of the 3 types of gasoline on the mileage in order to find the gasoline that delivers the highest average mileage.

ANOVA – Analysis of Variance

Hypothesis test of Analysis of variance:

H_0: $\mu 1 = \mu 2 = \mu 3 = \ldots \ldots$

H_a: Not all μ are equal

Assumptions:

1. Homogeneity of variance among the groups
2. Independent random sampling from the populations
3. Populations are normally distributed

F Statistic for ANOVA

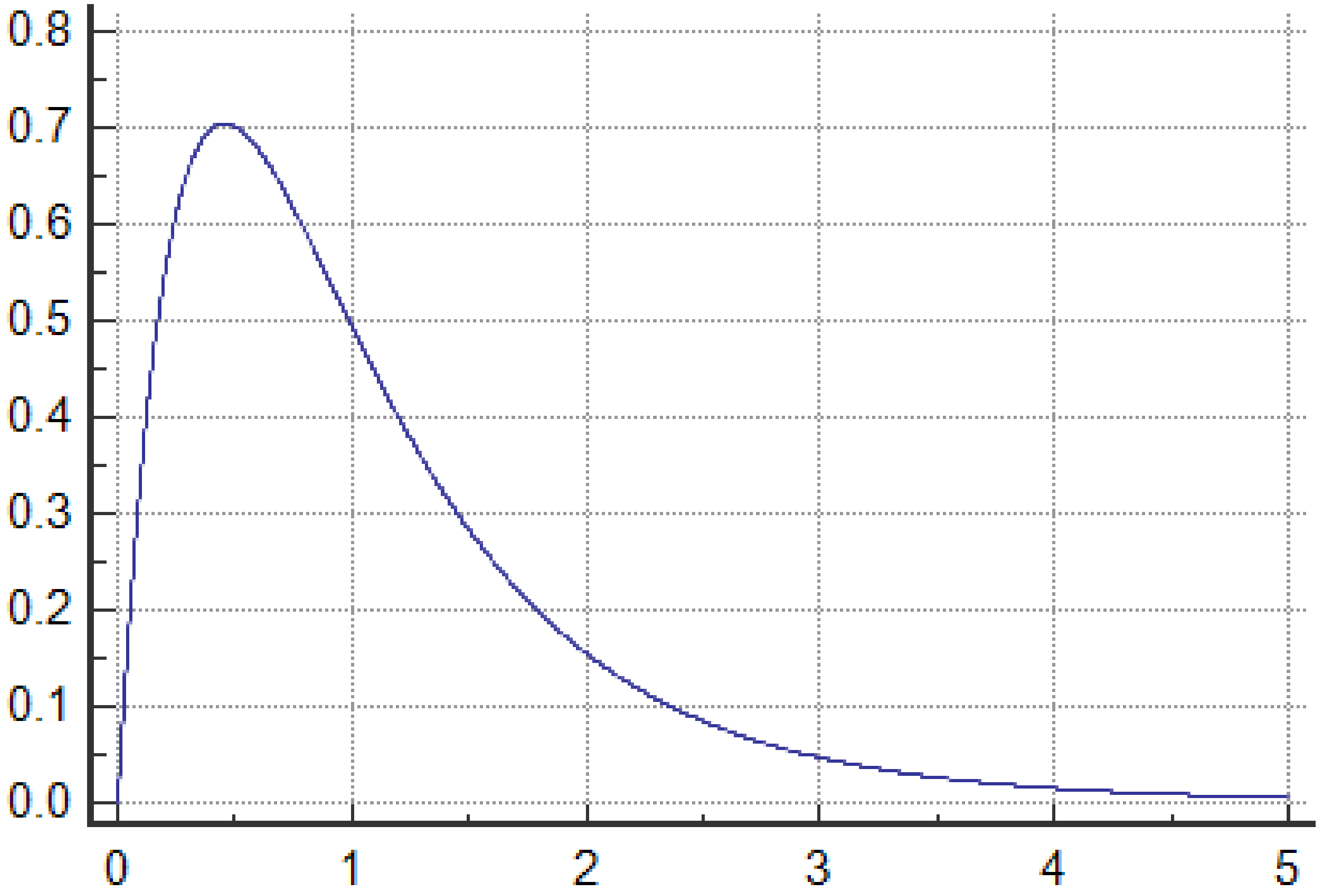
F-statistic is a ratio of two variances.

In ANOVA, F statistic is used the test statistics. F statistics helps to determine whether the variance between group means is larger that the variance within the groups.

$$F = \frac{Variance\ Between\ Sample\ Means}{Variance\ within\ the\ Samples}$$

if this F statistic is sufficiently large we can say that not all group means are equal.

F Distribution



Steps for One-way ANOVA

1. State Null Hypothesis H0 and alternate Hypothesis Ha

2. State significance level α.

3. Calculate the Grand Mean: $\bar{\bar{x}} = \frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} x_{ij}}{n_T}$ $n_T = n_1 + n_2 + n_3 + n_4 \ldots + n_k$ k is the number of groups and n represents number of data points

4. Calculate Group Means: $\bar{x_j} = \sum_{i=1}^{n_j} x_i$; n_j is number of samples in group j

Steps for One-way ANOVA

5

Calculate $SS_{between}$: Sum of Squares of Deviations Between Groups:

$$SS_{between} = \sum_{j=1}^{k} n_j\left(\bar{x}_j - \bar{\bar{x}}\right)^2$$

6

Calculate SS_{within} : Sum of Squares of Deviations Within Groups: $SS_{within} = \sum_{j=1}^{k}(n_j - 1){s_j}^2$

7

Calculate Degrees of Freedom for:

a. Between Group: k - 1

b. Within Group: n - k

8

Calculate

a. Mean Sum of Squares of deviation Between: $MS_{Between} = \frac{SS_{between}}{df_{between}}$

b. Mean Sum of Squares of deviation Within: $MS_{Within} = \frac{SS_{within}}{df_{within}}$

Steps for One-way ANOVA

9. Calculate F statistics: $F = \frac{MS_{between}}{MS_{within}}$

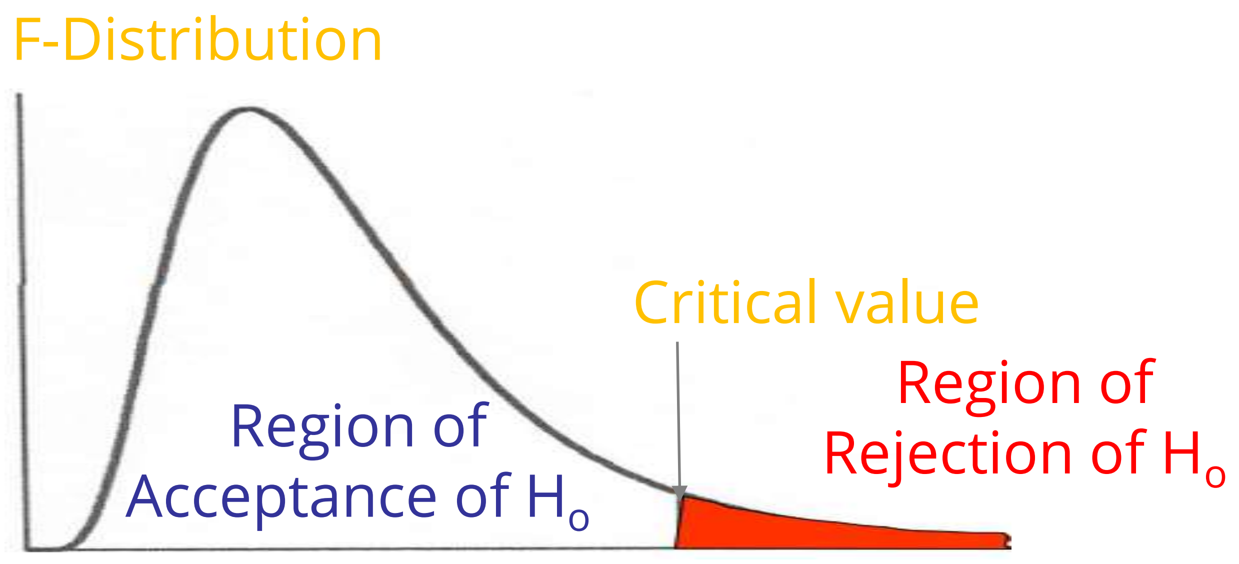
10. Get the p value using degrees of freedom k-1 and n-k.

11. Compare p value with significance level to conclude.

F Test : Equality of Two Variances

The F-Test for Equality of Variances between two groups is a statistical test used to compare the two sample variances to determine whether they are equal

The null hypothesis ($\mathbf{H_0}$) is that the variances of the two samples are equal, whereby the alternative hypothesis is that the variances of the two samples are not equal



F Test : Equality of Two Variances

Hypothesis

- $\mathbf{H_0}$: The variances are equal
- $\mathbf{H_1}$: The variances are not equal

Conditions

- Random sample: Sample is collected randomly from the populations
- Independent measurements: Each measurement on an individual is independent (from all other measurements)
- The population distributions is approximately a normal distribution

F Test: Equality of two variances

Step 1 & 2 : Formulate the null (H0) & alternative (H1), and select whether to use a One-Tailed or Two-Tailed Test.

$H_o : \sigma_1^2 = \sigma_2^2$

H_1: 3 Possible Cases

Two-tailed test	One-tailed test
$H_1 \quad : \sigma_1^2 \neq \sigma_2^2$	$H_1 \quad : \sigma_2^2 > \sigma_2^2$ $H_1 \quad : \sigma_1^2 < \sigma_2^2$

Population1 variance : σ_1^2
Population2 variance : σ_2^2

F Test: Equality of two variances

Step 3 : Calculate the appropriate test statistic

Test statistic:

F-statistic is the ratio of the larger sample variance ${S_1}^2$ to the smaller sample variance ${S_2}^2$.

- Determine whether the difference between the two sample variances is statistically significant.

The test statistic: $$F = \frac{{S_1}^2}{{S_2}^2}$$

F Test: Equality of two variances

Step 4 & 5 : Select a Level of Significance (α %) and determine the critical value(s) and Region of Acceptance & Rejection

10% , 5% , 1% are commonly used value for the Level of Significance (α %)

Based on the Level of Significant given and the Degree of Freedom, the critical value(s) can be determined and Region of Acceptance & Rejection setup.

- Degrees of freedom is the sample size minus 1. As there are two samples (variance 1 and variance 2), there would be two degrees of freedom: one for the numerator and one for the denominator.
  - $df1=n_1-1$
  - $df2=n_2-1$

The degree of freedom for larger sample variance is df1 with sample size n_1

and the smaller variance sample is df2 with sample size n_2

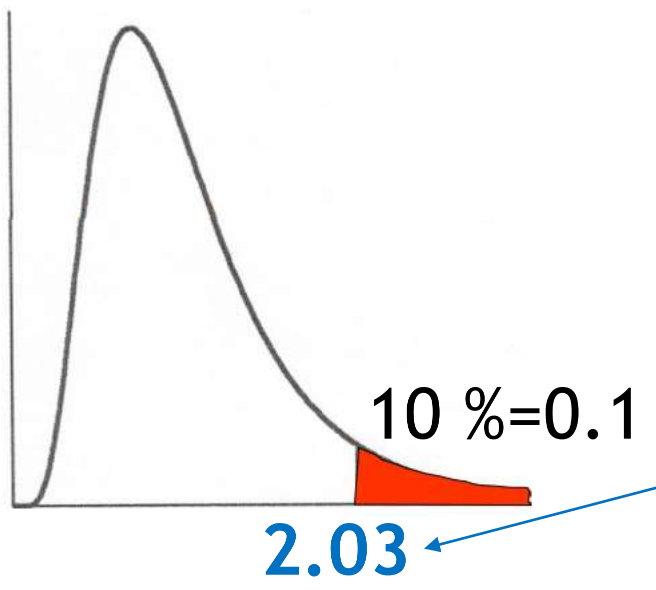
F Test: Equality of two variances

Step 4 & 5 : Select a Level of Significance (α %) and determine the critical value(s) and Region of Acceptance & Rejection

Example,

- Level of Significance = 10%
- One-Tailed Test
  - $n_1 = 18$
  - $n_2 = 10$

- Degree of Freedom
  - df1 =18-1 =17
  - df2 =10-1 =9



F-table of Critical Values for Significance Level = **0.10**

F-table of Critical Values of α = 0.10 for F(df1, df2)																			
	DF1=1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
DF2=1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19	60.71	61.22	61.74	62.00	62.26	62.53	62.79	63.06	63.33
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.48	9.49
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.20	5.18	5.18	5.17	5.16	5.15	5.14	5.13
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.79	3.78	3.76
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.14	3.12	3.11
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.90	2.87	2.84	2.82	2.80	2.78	2.76	2.74	2.72
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.67	2.63	2.59	2.58	2.56	2.54	2.51	2.49	2.47
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.34	2.32	2.29
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.38	2.34	2.30	2.28	2.25	2.23	2.21	2.18	2.16
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.28	2.24	2.20	2.18	2.16	2.13	2.11	2.08	2.06
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.03	2.00	1.97
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.15	2.10	2.06	2.04	2.01	1.99	1.96	1.93	1.90
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.90	1.88	1.85
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.05	2.01	1.96	1.94	1.91	1.89	1.86	1.83	1.80
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.02	1.97	1.92	1.90	1.87	1.85	1.82	1.79	1.76
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	[illegible]	1.99	1.94	1.89	1.87	1.84	1.81	1.78	1.75	1.72
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	[illegible]	2.03	[illegible]	1.96	1.91	1.86	1.84	1.81	1.78	1.75	1.72	1.69
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	[illegible]	2.00	1.98	1.93	1.89	1.84	1.81	1.78	1.75	1.72	1.69	1.66
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.91	1.86	1.81	1.79	1.76	1.73	1.70	1.67	1.63
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.89	1.84	1.79	1.77	1.74	1.71	1.68	1.64	1.61
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.87	1.83	1.78	1.75	1.72	1.69	1.66	1.62	1.59
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.64	1.60	1.57
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89	1.84	1.80	1.74	1.72	1.69	1.66	1.62	1.59	1.55
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.61	1.57	1.53
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.82	1.77	1.72	1.69	1.66	1.63	1.59	1.56	1.52
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.81	1.76	1.71	1.68	1.65	1.61	1.58	1.54	1.50
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85	1.80	1.75	1.70	1.67	1.64	1.60	1.57	1.53	1.49
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.79	1.74	1.69	1.66	1.63	1.59	1.56	1.52	1.48
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83	1.78	1.73	1.68	1.65	1.62	1.58	1.55	1.51	1.47
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.77	1.72	1.67	1.64	1.61	1.57	1.54	1.50	1.46
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.47	1.42	1.38
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.66	1.60	1.54	1.51	1.48	1.44	1.40	1.35	1.29
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.60	1.55	1.48	1.45	1.41	1.37	1.32	1.26	1.19
∞	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.55	1.49	1.42	1.38	1.34	1.30	1.24	1.17	1.00

F Test: Equality of two variances

Step 6 : Deciding whether to accept or reject Ho after checking whether the value the Test Statistic falls in the Region of Rejection.

F Test: Equality of two variances

Example:

- A bank has a long queue at Counter1 and a shorter queue at the Counter2. The manager of the bank wonders if there is variability in the number of customers at the two counters. The following samples are taken from the two counters. Carry out a one-tailed F-test with a level of significance of 5%, assuming a normal distribution.

	Counter1	Counter2
Sample Size, n	21	13
Variance, S^2	20	30

F Test: Equality of two variances

Step 1: Formulate the H0 & H1

- $\mathbf{H_0}$: The variances are equal .
- $\mathbf{H_1}$: The variances are not equal .

Step 2: Use a One-Tailed or Two-Tailed Test

One-Tailed Test :

$$H_o : \sigma_1^2 = \sigma_2^2$$
$$H_1 : \sigma_1^2 > \sigma_2^2$$

F Test: Equality of two variances

Step 3 : Calculate the appropriate test statistic

$$F = \frac{{S_1}^2}{{S_2}^2}$$

$$= \frac{30}{20}$$

$$= \mathbf{1.5}$$

F Test: Equality of two variances

Step 4 & 5 : Select a Level of Significance (α %) and determine the critical value(s) and Region of Acceptance & Rejection

- Level of Significance = 5%=0.05.
- One-Tailed Test
  - $n_1 = 13$
  - $n_2 = 21$

- Degree of Freedom
  - $df1 = n_1 - 1 = 13 - 1 = 12$
  - $df2 = n_2 - 1 = 21 - 1 = 20$

From the F-Table with degrees of freedom as 12 and 20, the value is 2.28.

F-table of Critical Values for Significance Level = 0.05

F-table of Critical Values of α = 0.05 for F(df1, df2)

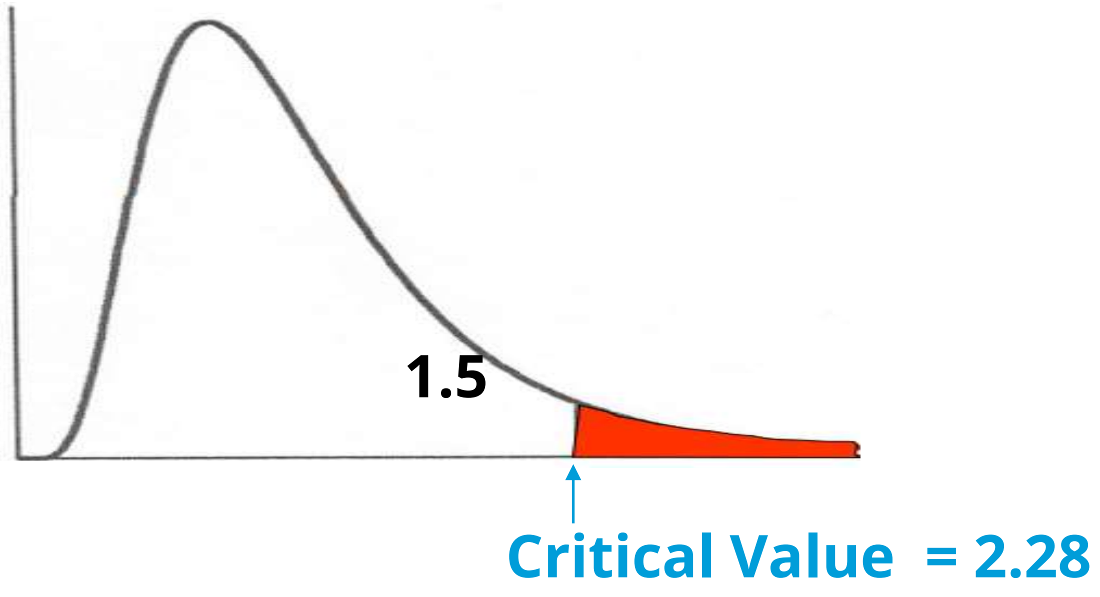
	DF1=1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
DF2=1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.95	248.01	249.05	250.10	251.14	252.20	253.25	254.31
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

2.28

F Test: Equality of two variances

Step 6 : Deciding whether to accept or reject Ho after checking whether the value the Test Statistic falls in the Region of Rejection.

- Since F=1.5 is lesser than the table value obtained (2.28), there is insufficient evidence at the 5% level to reject Ho .
- Hence, H_0 is accepted. i.e., there is no difference in the variance between customer at Counter1 and Counter2.



Non-Parametric Tests

Parametric vs. Nonparametric Tests

Parametric	Nonparametric
Assumptions are made about the population distribution	Does not require any assumption about specific population distribution
Uses a mean value for central tendency	Does not deal with specific parameters like mean, variance or standard deviation. Median is used for central tendency.
Deals with interval and ratio type of data	Deals with enumerative data(frequency counts)

Nonparametric Test and Their Parametric Equivalent

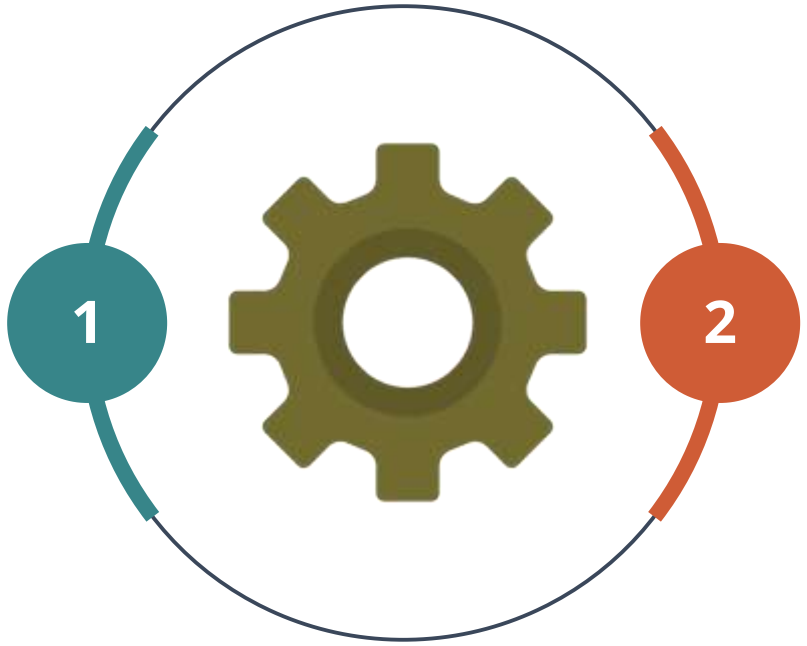
Nonparametric test	Parametric equivalent
Mann-Whitney U Test	Independent t-test
Wilcoxon Signed Rank Test	Paired t-test
Kruskal-Wallis H Test	One-way ANOVA
Spearman rank correlation coefficient	Pearson correlation coefficient

Note

The Chi-square test is the most used non-parametric test.
We will limit our scope to Chi-square test in this course.

Chi-square Tests

Chi-square test for goodness of fit



Chi-square test for independence of two variables/test of homogeneity

Chi-square test: Test of homogeneity or independence

Chi-Square Test of Homogeneity or Independence

- Attempt to determine if two categorical variables are associated
- Difference is the way the data for the study is collected
- Test of Homogeneity: Two or more distinct samples, independent samples are collected from the group. A categorical responses from each group is documented
- Test of Independence: One sample is collected from the group with two categorical responses documented

Chi-square test: Test of homogeneity or independence

Chi-Square Test of Homogeneity or Independence

- Examples of Test of Homogeneity
  - Survey which measured the citizens attitudes towards climate change first in 2016 and then in 2019
  - A study which randomly assigns a group of patients to take a medication A and another to take a medication B, and evaluate if the new medication help to improve the patients’ medical condition

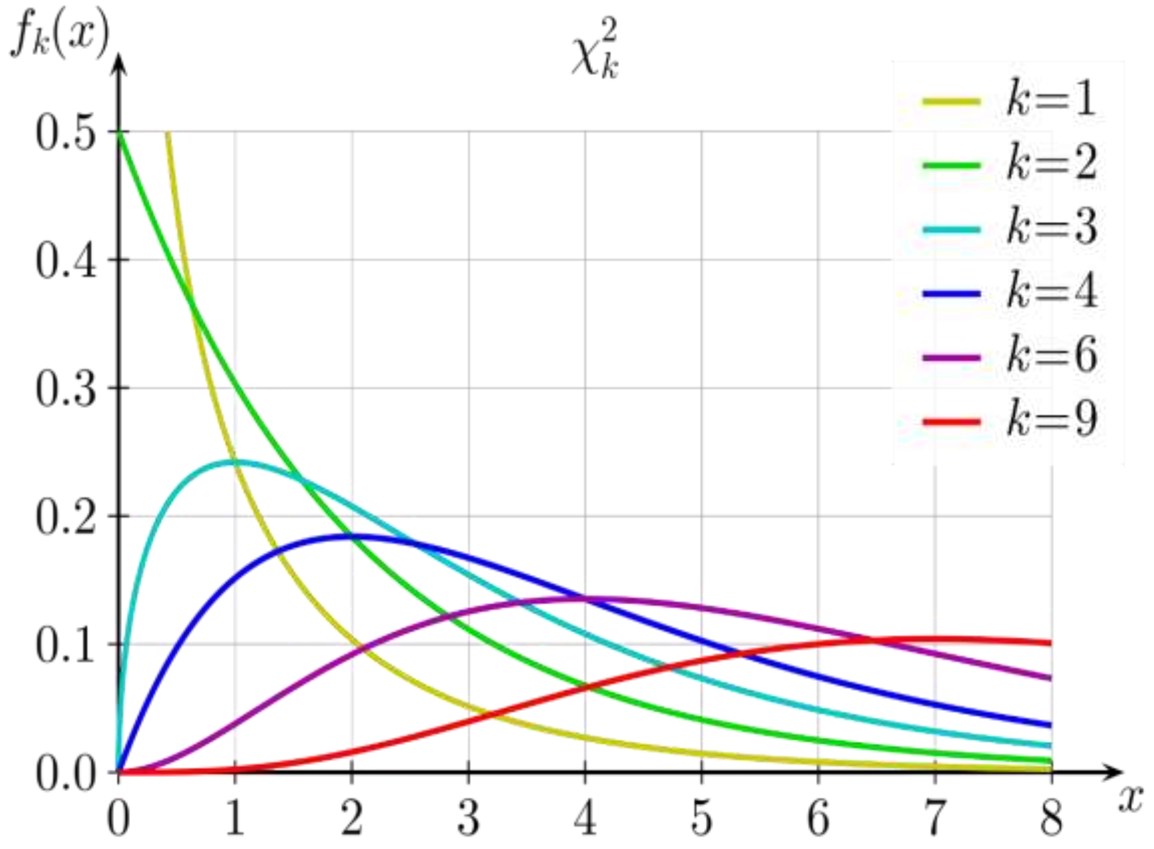
- Examples of Test of Independence
  - From government census data, we took a random sample of the housing and recorded the household types and region
  - An election poll asked 1000 people if they would vote for (a) woman and (b) Asian American president

Chi-square Test for Independence

- City has a newly opened nuclear plant, and there are families staying dangerously close to the plant. A health safety officer wants to take this case up to provide relocation for the families that live in the surrounding area.
- To make a strong case, he wants to prove with numbers that an exposure to radiation levels is leading to an increase in diseased population. He formulates a contingency table of exposure and disease. This table contains observed frequencies.
- Does the data suggest an association between the disease and exposure?

Contingency table	Disease		Total
Exposer	Yes	No	
Yes	37	13	50
No	17	53	70
Total	54	66	120

Chi-square Distribution



Contingency table	**Disease**		**Total**
Exposer	Yes	No	
Yes			50
No			70
Total	54	66	120

Chi Square test statistics :

$$\chi^2 = \sum \frac{(Observed\ Frequency\ - Expected\ Frequency)^2}{Expected\ Frequency}$$

Steps For Chi-square Test for Independence

1. State the null and alternative hypotheses.

   H_0: The 2 categorical variables are independent
   H_a: The 2 categorical variables are not independent

2. Select a random sample and record the observed frequencies for each cell of the contingency table.

3. Compute the expected frequency for each cell.

   Expected cell frequency = $\frac{(row\ total) * (column\ total)}{(grand\ total\ of\ all\ cells)}$

Steps For Chi-square Test for Independence

4

Compute the value of the test statistic.

$$\chi^2 = \sum \frac{(observed\ frequency - expected\ frequency)^2}{expected\ frequency}$$

5

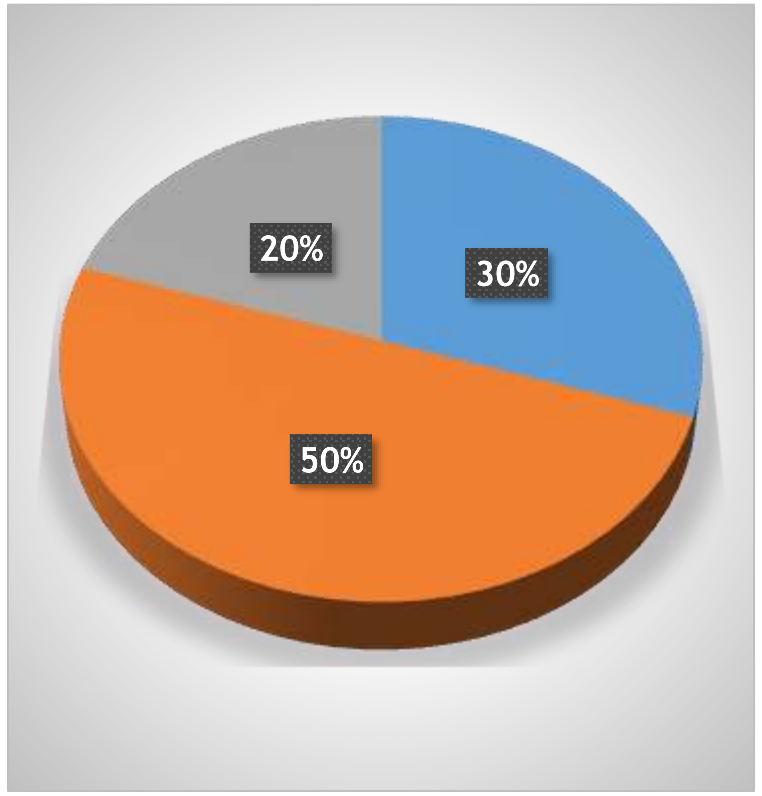
Obtain p value for degrees of freedom (# of rows – 1) * (# of columns – 1)

6

Reject H_0 if p-value is less than α

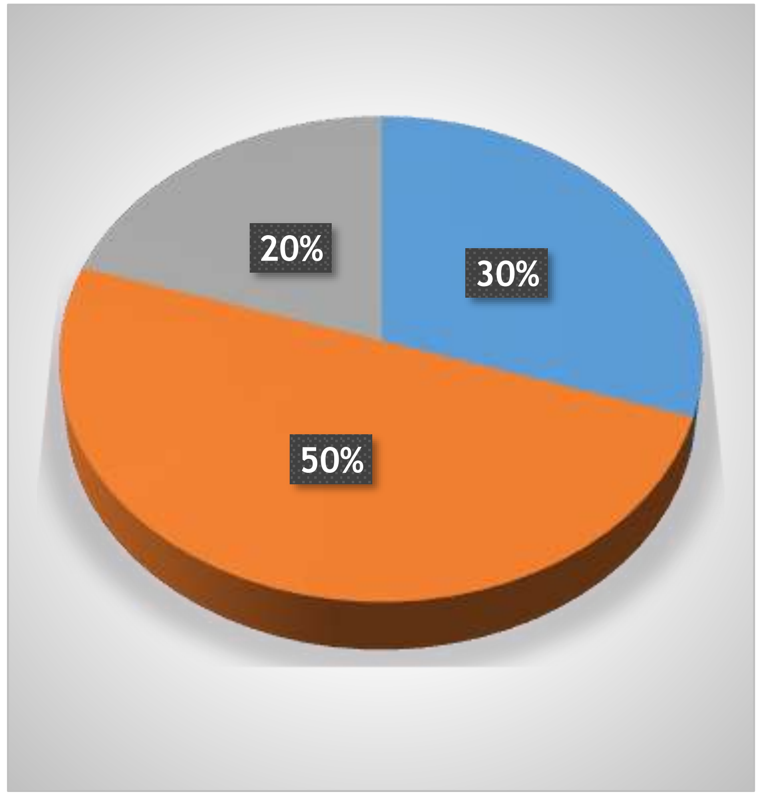
Chi-squared Test for Goodness of Fit

- A reach firm published the results of their market share study for a product.
- The study showed that for company A the market share stabilized at 50%, 30 % for company B and 20% for company C.
- To expand its market, company C developed an improved version of their product and conducted the study to see if that changes market shares.



Chi-squared Test for Goodness of Fit

- The population of interest here would be a multinomial population, for customers of each of the three company's product.
- The survey would provide proportion of population preferring each company's product.
- To check if the introduction of the new product brought a change in market share, a chi square test for goodness of fit has to be done.



Steps for Chi-square Test for Goodness of Fit

1

State the null and alternative hypotheses.
H_0: The population follows a multinomial distribution with specified probabilities for each of the k categories
H_a: The population does not follow a multinomial distribution with the specified probabilities for each of the k categories

2

Select a random sample and record the observed frequencies fi for each category.

3

Assume the null hypothesis is true and determine the expected frequency e_i in each category by multiplying the category probability by the sample size.

Steps for Chi-square Test for Goodness of Fit

4

Compute the value of test statistic $\chi 2 = \sum \frac{(fi-ei)2}{ei}$

5

Calculate p-value and compare with α to conclude