Apriori Analysis Examples : Assignment 2

① $f(n) = n - 10$, $g(n) = n + 10$; $f(n) = \Theta(g(n))$?

For Θ, it should satisfy (O) & (Ω).

Big O	Omega
$f(n) \leq c \cdot g(n)$	$f(n) \geq c \cdot g(n)$
$n - 10 \leq 1 \cdot (n+10)$	$n - 10 \geq c \cdot (n+10)$
TRUE $(c=1)$	For $c = \frac{1}{2}$; True

So $f(n) = \Theta(g(n))$ is TRUE.

② $f(n) = n$, $g(n) = n$; $f(n) = \Theta(g(n))$?

TRUE : No explanation needed.

For $c = 1$, it'll satisfy both O & Ω.

③ $64^{\log_2 n} \cdot 32^{\log_2 n} = O(n^5)$

$f(n) = 64^{\log_2 n} \cdot 32^{\log_2 n}$

$= n^{\log_2 2^6} \cdot n^{\log_2 2^5}$

$= n^{11}$

$f(n) \leq c \cdot g(n)$

$n^{11} \leq c \cdot (n^5)$

$\boxed{\text{FALSE}}$

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(4) $\frac{4^n}{2^n} = O(2^n)$

$f(n) = \frac{(2^2)^n}{2^n} = 2^{2n-n} = 2^n$ $\left\{ \begin{array}{l} f(n) \le c \cdot g(n) \\ 2^n \le c \cdot 2^n \\ \text{for } c \ge 2 \end{array} \right.$

$\boxed{\text{TRUE}}$

(5) $128^{\log_2 n} \cdot n^2 = \Theta(n^9)$

$f(n) = n^{\log_2 2^7} \cdot n^2$

$= n^7 \cdot n^2 = n^9$ $\Rightarrow$ TRUE for O & Ω

$\Rightarrow$ TRUE for Θ as well.