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ex :- $T(n) = \begin{cases} 1 & n=1 \\ 2T(\frac{n}{2}) + n & n>1 \end{cases}$

$T(n) = 2T(n/2) + n$

$T(n/2) = 2T(\frac{n}{2^2}) + \frac{n}{2}$

$T(\frac{n}{2^2}) = 2T(\frac{n}{2^3}) + \frac{n}{2^2}$

$T(n) = 2\left(2T(\frac{n}{2^2}) + n/2\right) + n$

$= 2^2 T(\frac{n}{2^2}) + 2n$

$= 2^2\left(2T(\frac{n}{2^3}) + \frac{n}{2^2}\right) + 2n$

$= 2^3 T(\frac{n}{2^3}) + 3n = 2^k T(\frac{n}{2^k}) + k \cdot n$

now $\frac{n}{2^k} = 1 \Rightarrow n = 2^k \Rightarrow \log_2 n = k$

$= 2^{\log_2 n} + \log_2 n \cdot n$

$= 2^{\log_2 n} T(\frac{n}{2^{\log_2 n}}) + \log_2 n \cdot n$

$= n + n \log_2 n$

$= O(n \log n)$

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ex $T(n) \begin{cases} 1 & n \le 1 \\ 8T\left(\frac{n}{2}\right) + n^2 & n > 1 \end{cases}$

$T(n) = 8T\left(\frac{n}{2}\right) + n^2$

$T\left(\frac{n}{2}\right) = 8T\left(\frac{n}{2^2}\right) + \left(\frac{n}{2}\right)^2$

$T\left(\frac{n}{2^2}\right) = 8T\left(\frac{n}{2^3}\right) + \left(\frac{n}{2^2}\right)^2$

$T(n) = 8\left(8T\left(\frac{n}{2^2}\right) + \left(\frac{n}{2}\right)^2\right) + n^2$

$= 8^2 T\left(\frac{n}{2^2}\right) + 3n^2$

$= 8^2\left(8T\left(\frac{n}{2^3}\right) + \left(\frac{n}{2^2}\right)^2\right) + 3n^2$

$= 8^3 T\left(\frac{n}{2^3}\right) + 7n^2$

$= 8^k + T\left(\frac{n}{2^k}\right) + (2^k - 1)n^2$

$\frac{n}{2^k} = 1 \Rightarrow \log_2 n = k$

$= 8^{\log_2 n} + T\left(\frac{n}{2^{\log_2 n}}\right) + \left(2^{\log_2 n} - 1\right)n^2$

$= 2^{3\cdot\log_2 n} + (n-1)n^2 = n^3 + n^3 - n^2$

$= O(n^3)$