

Day- 4

* Except one element, All elements appears twice. Find that element

① BF $\rightarrow O(n^2)$ T.C

$O(1)$ S.C

even?

$2^k + 1 \Rightarrow \text{odd}$

Input: $ar[] = \{7, 3, 5, 4, 5, 3, 4\}$ $n=7$

② sorting $\rightarrow O(n \log n)$ T.C

$O(1)$ S.C

Output: 7

③ K+V $\rightarrow O(n)$ T.C

$O(n)$ S.C

K	V	
7	1	1
3	2	2
5	2	2
4	2	2

$$2 + 3 - 3 - 3 + 5 - 1 = 0$$

Note:

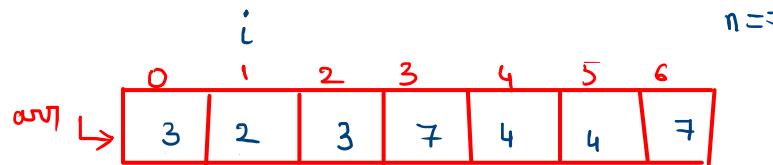
$$\textcircled{1} \quad x \oplus x = 0$$

$$\textcircled{2} \quad 0 \oplus x = x$$

$$\textcircled{3} \quad x \oplus y = y \oplus x$$

$\Rightarrow O(n) \quad T.C$

$x, y \in \omega$
 $\hookrightarrow \{0, 1, 2, \dots, y\}$ $O(1) \quad S.C$



$\Rightarrow O/P \quad \textcircled{2}$

$$res = arr[0]$$

$\text{for } (i=1; i < n; i++)$

{

$$res = res \wedge arr[i]$$

}

return res

$$res = 3 \wedge 2 \wedge 3 \wedge 7 \wedge 4 \wedge 4 \wedge 7$$

$$= 0 \wedge 2$$

= $\textcircled{2}$

② even | odd [without using +, -, %, /]

msb ↓ → lsb

0 → 0 0 0 0
⇒ 1 → 0 0 0 1 1
2 → 0 0 1 0 ✓
⇒ 3 → 0 0 1 1 1
4 → 0 1 0 0 ✓
⇒ 5 → 0 1 0 1 1
6 → 0 1 1 0 ✓
⇒ 7 → 0 1 1 1 1

even → lsb ↓

10 → 1 0 1 0
8 0 0 0 1
—————
0 0 0 0

odd →

9 → 1 0 0 1
8 0 0 0 1
—————
0 0 0 1

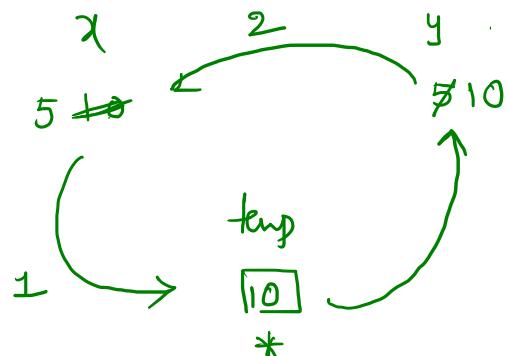
12 → 1 1 0 0
8 0 0 0 1
—————
0 0 0 0

13 → 1 1 0 1
8 0 0 0 1
—————
0 0 0 1

```
if( n & 1 == 1)  
    print(" odd")  
  
else  
    print(" even")
```

③ Swap 2 numbers, without using 3rd variable.

$x = 10$ code $x = 5$
 $y = 5$ \Rightarrow $y = 10$



+1 -1 *1 1

A₁-1 $a = a + b \checkmark$

$b = a - b \checkmark$

N₁ +1 *

$a = a - b \checkmark$

a	b
5	10
+5	5
10	

A₁2 [^]

1) $a = a \wedge b$

2) $b = a \wedge b$

3) $a = a \wedge b$

11:45

a	b
5	10
5 \wedge 10	
5 \wedge 10 \wedge 5	
10	

a	b
10	5

D → B

$$12 \rightarrow (\quad)_2$$

$$\begin{array}{r} 2 | 12 \\ 2 | 6 - 0 \\ 2 | 3 - 0 \\ 1 - 1 \end{array}$$

$(1100)_2$
↳ Binary rep

37 ⇒

$$\begin{array}{r} 2 | 37 \\ 2 | 18 - 1 \\ 2 | 9 - 0 \\ 2 | 4 - 1 \\ 2 | 2 - 0 \\ 1 - 0 \end{array}$$

Cross-check:-

$$(100101)_2$$



$$2^5 \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0 \\ 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1$$

$$\Rightarrow (1 \times 32) + 0 + 0 + (1 \times 4) + 0 + (1 \times 1) \\ = 37$$

$$\begin{array}{r}
 \underline{39} \rightarrow \\
 32 + \underline{7} \\
 \downarrow 4+2+1
 \end{array}
 \quad
 \begin{array}{ccccccccc}
 32 & 16 & 8 & 4 & 2 & 1 \\
 1 & 0 & 0 & 1 & 1 & 1
 \end{array}
 \Rightarrow (100111)_2$$

$$\begin{array}{r}
 \underline{97} \rightarrow \\
 \downarrow \\
 64 + \underline{33} \\
 \downarrow \\
 32 + 1
 \end{array}
 \quad
 \begin{array}{ccccccccc}
 64 & 32 & 16 & 8 & 4 & 2 & 1 \\
 1 & 1 & 0 & 0 & 0 & 0 & 1
 \end{array}
 \Rightarrow (11\ 00001)_2$$

*

If A, B are boolean variables.

A	B	$A + B$	$A \cdot B$	$A \oplus B$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

$+$: Bitwise OR (|)

\cdot : .. AND (&)

\oplus : n ex-OR (^)

↑ programmatically represent

$$\begin{array}{r} 13 \\ \oplus \\ 14 \\ \hline \end{array} = ?$$

Ex1: $13 \oplus 0$

$$\begin{array}{r} 1 \ 1 \ 0 \ 1 \\ + 0 \ 0 \ 0 \ 0 \\ \hline 1 \ 1 \ 0 \ 1 \rightarrow 13 \end{array}$$

Ex2: $13 \oplus 13 = 0$

$$\begin{array}{r} 1 \ 1 \ 0 \ 1 \\ + 1 \ 1 \ 0 \ 1 \\ \hline 0 \ 0 \ 0 \ 0 \rightarrow 0 \end{array}$$

13: 1 1 0 1

14: 1 1 1 0
 $\begin{array}{r} 1 \ 1 \ 1 \ 0 \\ + 0 \ 0 \ 1 \ 1 \\ \hline 0 \ 0 \ 1 \ 1 \end{array}$

$$\Rightarrow (0011)_2$$

↳ 3

Note: ① $x \oplus x = 0$
 $x \in N$

② $x \oplus 0 = x$

2) Rotate array by d elements [left], anti-clock wise direction

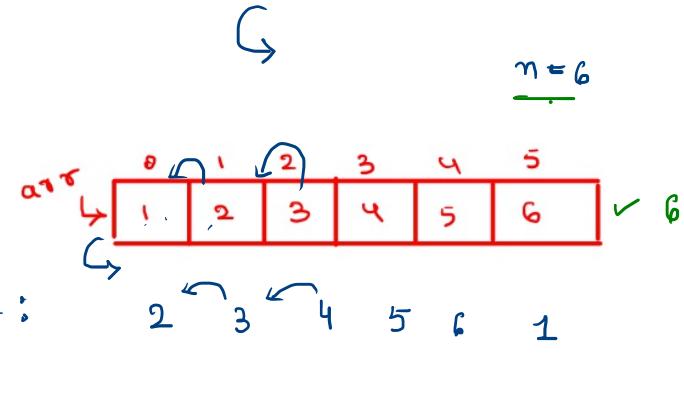
input:-

$\text{arr}[6] = \{ 1, 2, 3, 4, 5, 6 \}$ and $d = 2$

output :-

$d = 1 \Rightarrow 2, 3, 4, 5, 6, 1$

$d = 2 \Rightarrow 3, 4, 5, 6, 1, 2$ [final output]



$$d = \underline{9} \rightarrow 9$$
$$\hookrightarrow 6 + \frac{3}{\cancel{9}} \rightarrow 9 \% 6$$

$9 - 6 = 3$

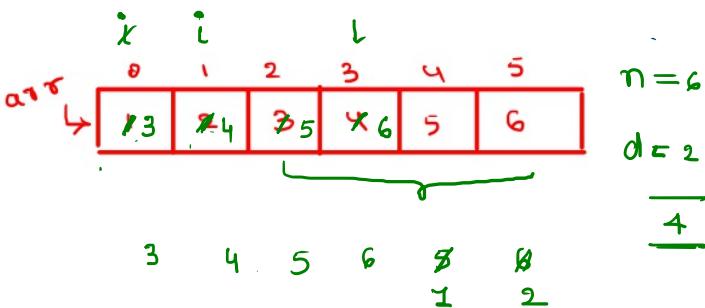
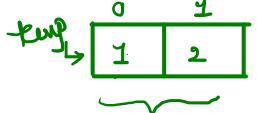
$$d = 17 \rightarrow$$
$$17 - 6 = 11$$

$$\Rightarrow d = n \% d$$

$a[6]$

0	1	2	3	4	5
1	2	3	4	5	6

Brute-Force



- * 1) take temp array of size d
- * 2) copy first d elements of given array into temp array \rightarrow loop (d)
- * 3) shift n-d elements to left, using a for loop [arr[i] = arr[i+d]] \rightarrow (n-d)
- * 4) put all elements of temp array to last d positions in given array \rightarrow (d)

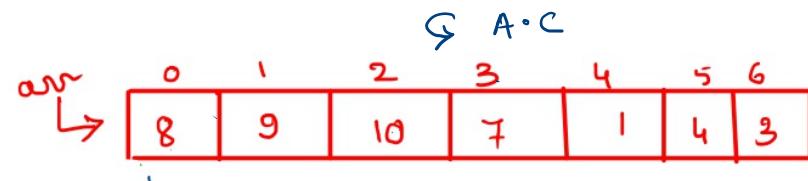
$$d = n \gamma d$$

$$d + n - d + d = n + d \quad d < n$$

$$\Rightarrow O(n) \text{ T.C}$$

$$\Rightarrow O(d) \text{ S.C}$$

Multiple-sub-arrays-reverse

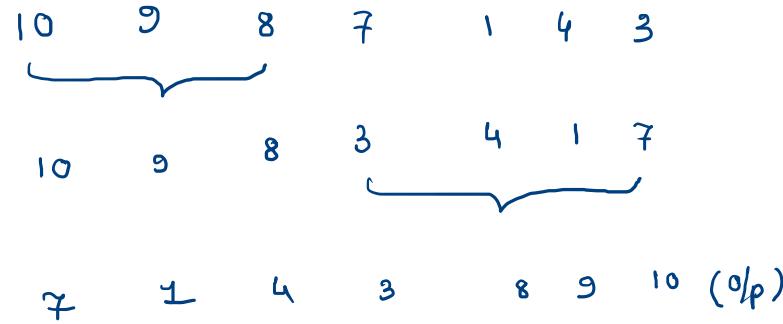


$n=7$
 $d=3$

⇒ 1) $\text{reverse}(\text{arr}, 0, d-1) : 0 \text{ to } 2 \Rightarrow o(d)$

⇒ 2) $\text{reverse}(\text{arr}, d, n-1) : 3 \text{ to } 6 \Rightarrow n-d$

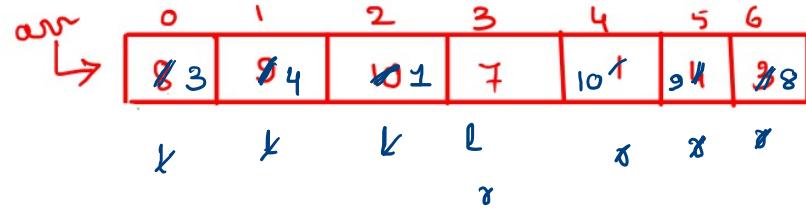
⇒ 3) $\text{reverse}(\text{arr}, 0, n-1) : 0 \text{ to } 6 \Rightarrow n$



$O(n) \cdot T.C$

$O(1) \cdot S.C$

* Reverse the array



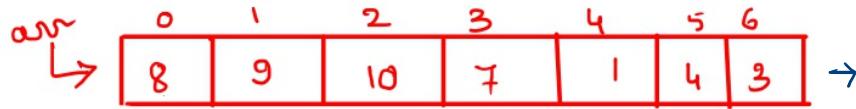
AP₁

rev[] =
 $i \leftarrow n-1$ to 0
rev.push(arr[i])

3 4 1 7 10 9 8

AP₂

2-ptr



arr

→

0	1	2	3	4	5	6
8	9	10	7	1	4	3

Home work : please do for same problem, clock wise

?

arr[6] = { 1, 2, 3, 4, 5, 6} and d=2

✓ d=1 ==> 6, 1, 2, 3, 4, 5

✓ d=2 ==> 5, 6, 1, 2, 3, 4 [final output]



- * Write an efficient program to find the sum of contiguous subarray within a one-dimensional array of numbers that has the largest sum.

Largest Subarray Sum Problem

-2	-3	4	-1	-2	1	5	-3
0	1	2	3	4	5	6	7

$$4 + (-1) + (-2) + 1 + 5 = 7$$

Maximum Contiguous Array Sum is 7

BF (n)

→ generate all sub-arrays

$$\underline{n=3} \quad \boxed{1 \ 2 \ 3}$$

$$n \leq 4$$

$$1+2+3+4$$

$$\text{len}(1) = 3 \quad \boxed{1} \quad \boxed{2} \quad \boxed{3}$$

$$\text{len}(2) = 2 \quad \boxed{1 \ 2} \quad \boxed{2 \ 3}$$

$$\text{len}(3) = 1 \quad \boxed{1 \ 2 \ 3} \quad \boxed{1 \ 3} \times$$

$$1+2+3 = 6$$

General:

$$1+2+3+\dots+n$$

$$= \frac{n(n+1)}{2} \checkmark$$

Brute-Force

↳ Generate All sub-arrays $\left[\frac{n(n+1)}{2} \right] \approx O(n^2)$

↳ Find out each & every sub-array sum $\Rightarrow O(n)$

$$n^2 * n \Rightarrow \frac{O(n^3)}{O(1)} \quad T.C \quad S.C$$

* Kadane's Algo

-2	-3	4	-1	-2	1	5	-3
0	1	2	3	4	5	6	7
i	i	i	i	i	i	i	i

$$\begin{aligned} -2 > -\infty &\checkmark \\ -3 > -2 &\times \end{aligned}$$

Sum = -2 + 4 + 1 + 5 + -3

Curr-Sum = 0 - 2 + 4 + 1 + 5 + -3 + -2

Alg

```
function fun(arr[], n)
{
    sum=-infinity
    curr_sum=0
    for(i=0;i<n;i++)
    {
        curr_sum=curr_sum+arr[i]
        if(curr_sum>sum)
            sum=curr_sum
        *if(curr_sum<0)
            curr_sum=0 ✓
    }
    return sum✓
}
```



T.C : O(n)

S.C : O(1)

* Search an element in row-wise, column-wise sorted

```
Input: mat[4][4] = { {10, 20, 30, 40},
                    {15, 25, 35, 45},
                    {27, 29, 37, 48},
                    {32, 33, 39, 50}};
x = 29
```

Output: Found at (2, 1)

Explanation: Element at (2,1) is 29

```
Input : mat[4][4] = { {10, 20, 30, 40},
                     {15, 25, 35, 45},
                     {27, 29, 37, 48},
                     {32, 33, 39, 50}};
x = 100
```

Output : Element not found

Explanation: Element 100 is not found

$n \times m$ $\Rightarrow O(n * m)$
 $m = n$ $\Rightarrow O(n^2)$

	0	1	2	3
0	10	20	30	40
1	15	25	35	45
2	27	29	37	48
3	32	33	39	50

$n \times n$

let $\underline{n \times m}$

i	j	
0	0	1
1	10	20
2	15	25
3	22	29
	30	35
	33	37
	39	45
	48	50

mat

$i \rightarrow 2$

4×4

key = 29

$i = 0, j = 3$

$mat[i][j] == key$ c_1

`print(i, j)`
return;

$mat[i][j] > key$ c_2
 $j--$

$mat[i][j] < key$ c_3
 $i++$

let Mat \rightarrow square

$O(n+m) [T.C]$

AP₁: $n \text{ rows}$ } $O(n) T.C$
 $n \text{ col}$

$O(1) [S.C]$

AP₂: $2n \Rightarrow O(n) T.C$

square_matrix

↓

```
⇒ search(int[][] mat, int n, int x)
{
    int i = 0, j = n - 1;
    while (i < n && j >= 0)
    {
        C1 if (mat[i][j] == x)
        {
            print("element found"); ✓
            return;
        }
        C2 if (mat[i][j] > x)
            j--;
        C3 else
            i++;
    }
    print("element is not found");
    return; ✓
}
```