Formulas

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Trigonometry Formulae

Length of arc of a circle (s)

$s = r\theta$

Area of sector of a circle

$A = \frac{1}{2}r^2\theta$

Relation between Trigonometric functions

$sin^2\theta + cos^2\theta = 1$

$1 + tan^2\theta = sec^2\theta$

$1 + cot^2\theta = cosec^2\theta$

Opposite real number identities

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

$\tan(-x) = -\tan x$

$\cot(-x) = -\cot x$

$\sec(-x) = \sec x$

$\text{cosec}(-x) = -\text{cosec}\, x$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Sin x	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
Cos x	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
Tan x	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	~	0	~	0
Cot x	~	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	~	0	~
Sec x	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	~	-1	~	1

Cosec x	~	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	~	-1	~

Quadrant	I	II	III	IV
+ve functions	All	Sin Cosec	Tan Cot	Cos Sec
	After	School	To	Cinema

Function	Domain	Range
Sin cos	R	$[-1, 1]$
Tan sec	$R - \left\{(2n+1)\frac{\pi}{2}\right\}$	R
Cot cosec	$R - n\pi$	$\mathrm{R} - (-1,1)$

Trigonometric Functions of Sum and Difference

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\sin(x-y) = \sin x \cos y - \cos x \sin y$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\cot(x+y) = \frac{\cot x \cot y - 1}{\cot y + \cot x}$$

$$\cot(x-y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}$$

$$\tan(x+y+z) = \frac{\tan x + \tan y + \tan z - \tan x \tan y \tan z}{1 - \tan x \tan y - \tan y \tan z - \tan z \tan x}$$

$\sin(x+y)\sin(x-y) = sin^2 x - sin^2 y$

$\cos(x+y)\cos(x-y) = cos^2 x - sin^2 y$

	$-x$	$\frac{\pi}{2}-x$	$\frac{\pi}{2}+x$	$\pi-x$	$\pi+x$	$\frac{3\pi}{2}-x$	$\frac{3\pi}{2}+x$	$2n\pi-x$	$2n\pi+x$
S	-	+	+	+	-	-	-	-	+
C	+	+	-	-	-	-	+	+	+
T	-	+	-	-	+	+	-	-	+

Product ←→ Sum/Difference

$$2\sin x\cos y = \sin(x+y)+\sin(x-y)$$
$$2\cos x\sin y = \sin(x+y)-\sin(x-y)$$
$$2\cos x\cos y = \cos(x+y)+\cos(x-y)$$
$$2\sin x\sin y = \cos(x-y)-\cos(x+y)$$
$$\sin x+\sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$
$$\sin x-\sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$$
$$\cos x+\cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$
$$\cos x-\cos y = -2\sin\frac{x+y}{2}\cos\frac{x-y}{2} = 2\sin\frac{x+y}{2}\sin\frac{y-x}{2}$$

Trigonometric Functions of 2x

$$\sin 2x = 2\sin x\cos x = \frac{2\tan x}{1+tan^2 x}$$
$$\cos 2x = cos^2 x - sin^2 x = 2\ cos^2 x - 1 = 1-2\ sin^2 x = \frac{1-tan^2 x}{1+\ tan^2 x}$$
$$\tan 2x = \frac{2\tan x}{1-tan^2 x}$$

Trigonometric Functions of 3x

$$\sin 3x = 3\sin x - 4\ sin^3 x$$
$$\cos 3x = 4\ cos^3 x - 3\cos x$$

$$\tan 3x = \frac{3\tan x - \ tan^3 x}{1 - 3\ tan^2 x}$$

Trigonometric Functions of Square

$$sin^2 x = \frac{1 - \cos 2x}{2}$$

$$cos^2 x = \frac{1 + \cos 2x}{2}$$

Trigonometric Functions of Cube

$$sin^3 x = \frac{3\sin x - \sin 3x}{4}$$

$$cos^3 x = \frac{3\cos x + \cos 3x}{4}$$

Sin and Cos to TAN!

$$\sin x = \frac{2\ tan^2\ \frac{x}{2}}{1 + tan^2\ \frac{x}{2}}$$

$$\cos x = \frac{1 - tan^2\ \frac{x}{2}}{1 + tan^2\ \frac{x}{2}}$$

Trigonometric Functions of some special angles

$$\sin 18 = \frac{\sqrt{5} - 1}{4}$$

$$\cos 36 = \frac{\sqrt{5} + 1}{4}$$

$$\cos 18 = \frac{\sqrt{10 + 2\sqrt{5}}}{4}$$

$$\sin 36 = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$$

Trigonometrical Equations

$\sin x = \sin \alpha \ \leftrightarrow x = n\pi + (-1)^n \alpha$

$\cos x = \cos \alpha \ \leftrightarrow x = 2n\pi \pm \alpha$

$\tan x = \tan \alpha \ \leftrightarrow x = n\pi + \alpha$

Properties of Triangle

Law of sines/sine formula

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of cosines/cosine formula

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ca}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Projection Formulae

$a = b \cos C + c \cos B$

$b = c \cos A + a \cos C$

$c = a \cos B + b \cos A$

Area of Triangle

$$A = \frac{1}{2}\text{ab} \sin C = \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B$$

Inverse Trigonometry

f(x)

$\sin(sin^{-1} x) = x$

Similarly all functions. Check whether sin inverse of x exists or not…

$sin^{-1}(\sin x) = x$

Similarly all functions. Check whether sin of x exists or not…

(-x) functions

$sin^{-1}(-x) = -sin^{-1} x$

$tan^{-1}(-x) = -tan^{-1} x$

$cosec^{-1}(-x) = -cosec^{-1} x$

$cos^{-1}(-x) = \pi - cos^{-1} x$

$cot^{-1}(-x) = \pi - cot^{-1} x$

$sec^{-1}(-x) = \pi - sec^{-1} x$

$$tan^{-1}x + tan^{-1}y = \begin{cases} tan^{-1}\left(\frac{x+y}{1-xy}\right), & xy < 1 \\ \pi + tan^{-1}\left(\frac{x+y}{1-xy}\right), & x > 0, y > 0, xy > 1 \end{cases}$$

$$tan^{-1}x - tan^{-1}y = tan^{-1}\left(\frac{x-y}{1+xy}\right) \qquad xy > -1$$

$$sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\,tan^{-1}x$$

$$cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\,tan^{-1}x$$

$$tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\, tan^{-1}x$$

$$tan^{-1}x = sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

$$tan^{-1}x = cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$$

$$tan^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) = sin^{-1}x$$

$$tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = cos^{-1}x$$

$$sin^{-1}x = cos^{-1}\sqrt{1-x^2}$$

$$cos^{-1}x = sin^{-1}\sqrt{1-x^2}$$

$$cos(sin^{-1}x) = sin\,(cos^{-1}x) = \sqrt{1-x^2}$$

$$sin^{-1}x + sin^{-1}y = sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$$

$$sin^{-1}x - sin^{-1}y = sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$$

$$cos^{-1}x + cos^{-1}y = cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$$

$$cos^{-1}x - cos^{-1}y = cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right)$$

$$3\, sin^{-1}x = sin^{-1}(3x - 4x^3)$$

$$3\, cos^{-1}x = cos^{-1}(4x^3 - 3x)$$

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Differentiation [f(x)]

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = sec^2 x$$

$$\frac{d}{dx}(\cot x) = -cosec \quad x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\mathrm{cosec}\, x) = -\mathrm{cosec}\, x \tan x$$

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(a \pm b \pm c) = \frac{d}{dx}(a) \pm \frac{d}{dx}(b) \pm \frac{d}{dx}(c)$$

$$\frac{d}{dx}(ab) = a\frac{d}{dx}(b) + b\frac{d}{dx}(a)$$

$$\frac{d}{dx}\left(\frac{N}{D}\right) = \frac{D\frac{d}{dx}(N) - N\frac{d}{dx}(D)}{D^2}$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x.\log a$$

$$\frac{d}{dx}(\log x) = \frac{1}{x \log e} = \frac{1}{x}$$

ITF

$$\frac{d}{dx}(sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(tan^{-1}x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(cot^{-1}x) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}(sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(cosec^{-1}x) = \frac{-1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(|x|) = \frac{x}{|x|}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Differentiation of Determinant

$$F'(x) = \begin{vmatrix} f_1'(x) & f_2'(x) & f_3'(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1'(x) & g_2'(x) & g_3'(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix} + \begin{vmatrix} f_1'(x) & f_2'(x) & f_3'(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1'(x) & h_2'(x) & h_3'(x) \end{vmatrix}$$

Rolle's Theorum

If a function f is:

- Continuous in [a,b]
- Derivable in (a,b)
- f(a)=f(b)

Then, there exists atleast one real value c in (a,b) such that:

$$f'(c) = 0$$

Lagrange Mean Value Theorum (LMV)

If a function f is:

- Continuous in [a,b]
- Derivable in (a,b)

Then, there exists atleast one real value c in (a,b) such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Continuity

$$\lim_{x \to c} f(x) = f(c)$$

$$\lim_{x \to c^-} f(x) = f(c) = \lim_{x \to c^+} f(x)$$

NOTE: A polynomial/rational/trigonometric function is continuous everywhere!!!

$\int integration$

$$\int c\,dx = cx + C$$

$$\int c\,f(x)dx = c\int f(x)\,dx$$

$$\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

$$\int fg\,dx$$

- Convert * to +/-
- Substitution Method
- By part
- Special Case

$$\int \frac{f}{g}dx$$

- Try to remove Denominator
- Substitution Method
- It might be a question of partial fraction
- Special case

FORMULA:

Algebraic Functions:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{1}{x}dx = \log|x| + C$$

Trigonometry Functions:

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \tan x\,dx = -\log|\cos x| + C$$

$$\int \cot x\,dx = \log|\sin x| + C$$

$$\int \sec x\,dx = \log|\sec x + \tan x| + C$$

$$\int \operatorname{cosec} x\,dx = \log|cosec\ x - \cot x| + C$$

$$\int sec^2\,x\,dx = \tan x + C$$

$$\int cosec^2\,x\,dx = -\cot\ x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + C$$

Exponential Functions:

$$\int e^x\,dx = e^x$$

$$\int a^x\,dx = \frac{a^x}{\log_e a}$$

Formula based on two perfect squares:

- One is variable
- One is constant
- Coefficient of x must be 1

$\int \frac{1}{}\,dx$ – One is $tan^{-1}x$, rest two log

$$\int \frac{1}{x^2+a^2}dx = \frac{1}{a}tan^{-1}\frac{x}{a} + C$$
$$\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$$
$$\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$$

$\int \frac{1}{\sqrt{}}dx$ – One is $sin^{-1}x$, rest two log

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \log\left|x+\sqrt{x^2+a^2}\right| + C$$
$$\int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right| + C$$
$$\int \frac{1}{\sqrt{a^2-x^2}}dx = sin^{-1}\frac{x}{a} + C$$

$$\int \sqrt{}\,dx$$
$$= \frac{Variable}{2}(whose\ integration)$$
$$+ \frac{full\ constant}{2}(outcome\ of \int\ keeping\ \sqrt{}\ in\ denominator$$

$$\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\left(\sqrt{x^2+a^2}\right) + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + C$$
$$\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\left(\sqrt{x^2-a^2}\right) - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$$
$$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\left(\sqrt{a^2-x^2}\right) + \frac{a^2}{2}sin^{-1}\frac{x}{a} + C$$

Special:

$$\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} sec^{-1} \frac{x}{a} + C$$

Substitution and Mismatch:
If the mismatch is linear, perform integration and divide the result with differentiation of mismatch.

TRICKS:

1. When log appears in power, then log should be *single*, coefficient must be *one*. Eg, $5^{2\log_5 x} = 5^{\log_5 x^2} = x^{2^{\log_5 5}}$
2. Whenever there is $\int \frac{1}{\quad} dx, \int \frac{1}{\sqrt{\quad}} dx, \int \sqrt{\quad} dx$

   $1 + \cos x = 2cos^2 \frac{x}{2}$

   $1 - \cos x = 2sin^2 \frac{x}{2}$

   $1 + \sin x = 1 + \cos(\frac{\pi}{2} - x) = 2cos^2(\frac{\pi}{4} - \frac{x}{2})$

   $1 - \sin x = 1 - \cos(\frac{\pi}{2} - x) = 2sin^2(\frac{\pi}{4} - \frac{x}{2})$

3. Division:

   $\frac{P(x)}{Q(x)}$

   - Numerator >= Denominator – Divide =>

    - $Quotient + \frac{Remainder}{Divisor}$
- Numerator < Denominator =>
    - Substitution method
    - Partial method
    - Special trick

4. Integration:
    - $\int tan^2 x\, dx = tan^2 x - 1$
    - $\int cot^2 x\, dx = cosec^2 x - 1$
5. Denominator contains product of sin and cos:
    - Also create numerator in sin and cos
    - Power must be less than equal to 2
    - Distribute denominator over numerator
6. m and n are even:
    - $\int sin^m x\, dx$
    - $\int cos^n x\, dx$
    - $\int sin^m x\, cos^n x\, dx$
    - Power Reduce to 1
    - 2 Power:

$$sin^2 x = \frac{1 - \cos 2x}{2}$$
$$cos^2 x = \frac{1 + \cos 2x}{2}$$

    - 3 Power:

$$sin^3 x = \frac{3 \sin x - \sin 3x}{4}$$
$$cos^3 x = \frac{3 \cos x + \cos 3x}{4}$$

7. When sin and cos are multiplied in numerator and Denominator is constant, convert * to +/- using:
    - 2 sin A sin B
    - 2 sin A cos B

- 2 cos A cos B
- 2 cos A sin B

8. If any one of m or n are odd:
   - $\int sin^m x\, dx$
   - $\int cos^n x\, dx$
   - $\int sin^m x\, cos^n x\, dx$
   - Substitution of integration of the entity multiplied to dx
   - Power of the entity will be ignored during third party substitution
   - The entity must be:
     - Of odd power
     - Of lowest power
9. m > 2, n > 2, n is even:
   - $\int tan^m x\, dx$
   - $\int cot^m x\, dx$
   - $\int sec^n x\, dx$
   - $\int cosec^n x\, dx$
   - Split into parts:
     - One Part: Power two => Put formula for two power
     - Another Part: remaining power
   - Open bracket:
     - 1. Substitution method
     - 2. Depend upon power
10. Purely Exponential Question? Try dividing numerator and denominator by e^x. Thank me later!
11. Some Integrations:

$$\int \frac{1}{\sin(x-\alpha)\sin(x-\beta)}dx$$

=> Multiply $\sin\big((x-\alpha)-(x-\beta)\big)$

=> Multiply outside Integration to counter:

$$\frac{1}{\sin(\beta-\alpha)}$$

$$\int \frac{1}{\cos(x-\alpha)\cos(x-\beta)}dx$$

=> Multiply $\sin\big((x-\alpha)-(x-\beta)\big)$

=> Multiply outside Integration to counter:

$$\frac{1}{\sin(\beta-\alpha)}$$

$$\int \frac{1}{\sin(x-\alpha)\cos(x-\beta)}dx$$

=> Multiply $\cos\big((x-\alpha)-(x-\beta)\big)$

=> Multiply outside Integration to counter:

$$\frac{1}{\cos(\beta-\alpha)}$$

12. If

$$\sqrt{\frac{Linear\ Numerator\ N}{Linear\ Denominator\ D}} = \sqrt{\frac{N}{D}} = \sqrt{\frac{N*N}{D*N}} = \frac{N}{\sqrt{Quad}}$$

Type of Quadratic:

- MIX: x^2+x+1 => Special Trick or Substitution
- PURE: : x^2+1 => Distribute denominator over numerator => Substitution or two perfect squares

13. If

$$\int \frac{Quad\ Numerator\ Q}{\sqrt{Quad\ Denominator\ D}}dx$$

Write numerator as follows:

$$Q = A(D) + B\frac{d}{dx}(D) + C$$

14. Integrations:

$$\int \frac{1}{Quad}dx$$

$$\int \frac{1}{\sqrt{Quad}}dx$$

$$\int \sqrt{Quad}\, dx$$

Convert Quadratic to perfect square

Use formula based on two perfect squares

How to make perfect square:

1. Make it separately
2. Coefficient of x^2 must be 1.
3. Add and subtract:

$$\left(\frac{middle\ term}{2x}\right)^2$$

15. Integrations:

$$\int \frac{linear}{Quad} dx$$

$$\int \frac{linear}{\sqrt{Quad}} dx$$

$$\int linear\sqrt{Quad}\, dx$$

Try for substitution:

1. If differentiation of quadratic is equal to linear: VOILA!!!
2. Else
   a. Rewrite linear of numerator as:

   $$Linear = A\frac{d}{dx}(Quad) + B$$

   b. Find value of A and B, by coefficient comparison of x^0 and x^1.
   c. Put A and B in Step(a) then put in integration. Distribute denominator over numerator.
   d. Apply substitution or perfect square.

16. If

$$\int \frac{1}{a\sin x + b\cos x + c} dx$$

- Only one from a, b or c can be 0. No more than that.

- Numerator must be purely constant.

$$\sin x = \frac{2\,tan^2\,\frac{x}{2}}{1 + tan^2\,\frac{x}{2}}$$

$$\cos x = \frac{1 - tan^2\,\frac{x}{2}}{1 + tan^2\,\frac{x}{2}}$$

Simplify:

$\tan x/2 = t$ //Substitution

It will become either quadratic or two perfect squares

Also:

- Divide by cos^2
- Substitute $\tan x$

17. If

$$\int \frac{\sin x\ \ (\&\&/||cos)}{\sin x + \cos x} dx$$

$$Numerator = A(Denominator) + B\left(\frac{d}{dx}(Denominator)\right)$$

18. If

$$\int \frac{x^2 \pm \sqrt{A}}{x^4 + kx^2 + A}$$

If: square root of x^4 is +/- with square root of A in the numerator:

1. YES: Divide N and D with x^2. Third party substitution.
2. NO: We have to write both in the numerator

$$\int \frac{x^2}{x^4 + 16} dx = \frac{1}{2}\int \frac{2x^2}{x^4 + 16} dx$$

$$= \frac{1}{2}\int \frac{(\boldsymbol{x^2} + \mathbf{4}) + (\boldsymbol{x^2} - \mathbf{4})}{x^4 + 16} dx$$

19. Substitution when question is purely exponential. (Take small power)

TIP: By part me substitution ke dauraan kabhi t^2 se substitute mat karna.

Definite Integrals

Properties:

1. If (-) is outside integration, you can:
   - Either send it inside or
   - Interchange the boundaries
2. $\int_a^b f(x)dx = \int_a^b f(t)dt$
3. $\int_a^b f(x)dx \int_a^c f(x)dx + \int_c^b f(x)dx$ Given: a < c < b
4. $\int_{-a}^a f(x)dx$ Check f(x), even or odd:
   - How to:
     - If $f(-x) = f(x)$, f is even
     - Else if $f(-x) = -f(x)$, f is odd
     - Else, neither even nor odd
   - If $f(x)$ is even:
     - $2\int_0^a f(x)dx$
   - If $f(x)$ is odd: 0
   - If $f(x)$ is neither even nor odd: Property or direct integration
5. Rambaadhh Property:
   - $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$
   - In certain questions, you gotta take them A and then add both equations to get: 2A = ….
6. $\int_0^a f(x)\,dx$ (base must be zero)
   - $f(a-x)$:
     - $+f(x)$ => $2\int_0^{\frac{a}{2}} f(x)\,dx$
     - $-f(x)$ => 0
     - Direct integration

Definite Integrals as the limit of sum:

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\,[\,f(a) + f(a+h) + f(a+2h) + \cdots + f(a+(n-1)h)\,]$$

Step 1: Write formula
Step 2: Write: $nh = b - a$
Step 3: Find values of all f(x)s in formula at step 1
Step 4: Put the above value
Step 5: Separate the series in formula
Step 6: Sum of series
Step 7: Multiply outside h inside the bracket
Step 8: Put value of nh
Step 9: Solve the limit

Always remember:

$$\int_a^{\frac{\pi}{2}} \log \sin x\,dx = -\frac{\pi}{2}\log 2$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

Partial Fraction

- Algebraic fraction
- Numerator < Denominator (in degrees)
- Denominator must be completely factorised

$$\frac{x+1}{(x+2)(x+3)(x+4)} = \frac{A}{x+2} + \frac{B}{x+3} + \frac{C}{x+4}$$

$$x = A(x+3)(x+4) + B(x+2)(x+4) + C(x+2)(x+3)$$

Then x ki aisi value chose karo ki ek term bache, baaki sab zero ho jae.

For eg,
x = -3, B nikal aaega
X = -2, A nikal aaega

$$\frac{x+1}{(x+2)^2(x+3)} = \frac{A}{x+3} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$
$$\frac{x+1}{(x^2+4)(x+2)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4}$$

By Part

$$\int uv \ dx = I \int II \ dx - \int \left(\frac{d}{dx}(I) \int II \ dx\right) dx$$

I – Selected such that differentiation is easy (function diminishes)
II – Selected such that integration is easy

I => I L A T E <= II
I => ITF LOG ALGEBRA TRIGONO EXPONENTIAL <= II

Matrix

Transpose: A = $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ A'= $\begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$

Properties of Transpose:

- (A')' = A
- (-A)' = -A'
- (A+B)' = A' + B'
- (AB)' = B'A'

A is a Symmetric Matrix when: A' = A

A is a Skew-Symmetric Matrix when: A' = -A

Each and every matrix can represented as a sum of a symmetric and skew-symmetric matrix:

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A') = P + Q$$

Symmetric: $P = \frac{1}{2}(A + A')$

Skew-Symmetric: $Q = \frac{1}{2}(A - A')$

Elementary Operations:

$C_i \leftrightarrow C_j$: Interchanging two columns (or rows)

$C_i \rightarrow kC_i$: Multiplying a constant to any column (or row)

$C_i \rightarrow C_i + kC_j$: Adding another column (or row) multiplied with a constant to any column (or row)

Inverse: If $AB = I_n$, then B is inverse of A

Inverse of a matrix by elementary operations:

ROW:

$A = I_n A$

Apply elementary row operations such that A becomes I_n

I_n = BA (I_n changes to B)

$I_n A^{-1} = (BA)A^{-1}$ (Multiplying with A^{-1})

$A^{-1} = BI_n = B$ (B is the inverse of A)

COLUMN:

$A = AI_n$

Apply elementary row operations such that A becomes I_n

$I_n = \text{AB}$ (I_n changes to B)

$A^{-1}I_n = A^{-1}(AB)$ (Multiplying with A^{-1})

$A^{-1} = I_nB = B$ (B is the inverse of A)

Determinants

Properties of Determinants: (Matrix = A)

- Every element is zero: $|A| = 0$
- Every element on *one* side of principal diagonal is zero: $|A|$ = product of elements of principal diagonal
- $|A|$ remains unchanged if its rows and columns are interchanged.
- $|A|$ changes by a minus sign only if any two rows or columns are interchanged.
- If any two parallel rows or columns are identical: $|A| = 0$
- If each element of a row or column of a determinant is multiplied by the same number k: $|A| = k|A|$

$|kA| = k^n|A|$

$|A'| = -|A|$

$$\Delta = \begin{vmatrix} a+d & g & j \\ b+e & h & k \\ C+f & i & l \end{vmatrix} \; then \; \Delta = \begin{vmatrix} a & g & j \\ b & h & k \\ c & i & l \end{vmatrix} + \begin{vmatrix} d & g & j \\ e & h & k \\ f & i & l \end{vmatrix}$$

$$\begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix} = \begin{vmatrix} a+kd & d & g \\ b+ke & e & h \\ c+kf & f & i \end{vmatrix}$$

- The sum of the products of the elements of any row or column with the cofactors of the corresponding elements of some other R (or C) is zero.
- If A and B are matrices of same order: $|AB| = |A|.|B|$ and $|A^m| = |A|^m$

NOTE: If we apply $R_1 \rightarrow kR_1$, divide determinant with 'k' to counter.

Area of a triangle:

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Adjoint:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

adj A = $\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$

A(adj A) = $\begin{bmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{bmatrix}$

MARTIN RULE:

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$AX = B$

$A^{-1}AX = A^{-1}B$

$I_nX = A^{-1}B$

$X = A^{-1}B$

Relations and Functions

Types of relations:

- Reflexive: (a, a) ∈ R
- Symmetric: (a, b) ∈ R and (b, a) ∈ R
- Transitive: If aRb, bRc exists, then whether (a, c) ∈ R.

Types of functions:

- One-one: (*Injective*) if $f(x_1) = f(x_2)$ then $x_1 = x_2$
- Many-one: 2 or more elements of X have same image in Y
- Onto: (*Subjective*) each element of Y is image of atleast one element of X.
- Into: each element of Y is not image of any element of X.
- One-one correspondence: (*Bijective*) both one-one and onto

Odd and even f(x):

If f(-x) = f(x). f(x) is odd function

If f(-x) = -f(x), f(x) is even function

Applications of Derivatives

- $\frac{dy}{dx}$ of a curve $y = f(x)$ at any point P(x, y) represents the slope of the tangent.
- Equation of a tangent: $y - y_1 = \frac{dy}{dx}(x - x_1)$
- If derivative doesn't exists, tangent is parallel to y axis. Its equation: $x = x_1$
- $\tan\theta = \frac{m_1 - m_2}{1 - m_1 m_2}$
- If $m_1 = m_2$, curves are said to cut orthogonally
- If $m_1 = m_2$, curves touch each other
- $\Delta x = \frac{dy}{dx}\Delta y$
- 1st derivative test:
    - $f'(x) \geq 0$ for all $x \in (a, b)$: increasing
    - $f'(x) > 0$ for all $x \in (a, b)$: strictly increasing
    - $f'(x) < 0$ for all $x \in (a, b)$: strictly decreasing
    - $f'(x) \leq 0$ for all $x \in (a, b)$: decreasing
- Absolute Maxima and Minima:
    - Find f(x) at points where f'(x) = 0
    - Find f(x) at points where derivative fails to exist.
    - Find f(a) and f(b), where [a, b]: Range
    - Maximum value of all these is absolute maxima, while minimum is absolute minima.
- 2nd Derivative test:
    - f'(x) = 0, get all x (turning points)
    - f''(x), put x in f''(x) one by one
    - if f''(x) = +ve: Minima, minimum value f(x)
    - if f''(x) = -ve: Maxima, maximum value f(x)

- If derivative fails to exists,

c	c-h	c+h	
f'(c)	+ve	-ve	Maxima
f'(c)	-ve	+ve	Minima

- Word problems based on maxima/minima
  - Prepare function in only 2 variables, subject according to question
  - f'(x) = 0, get turning points
  - f''(x) – put turning points one by one
  - Theoretical Comment
  - Find the objective of the question with the help of previous step.

Applications of Calculus in Commerce and Economics

TC: Total Cost
TFC: Total Fixed Cost
TVC: Total Variable Cost
C: Cost function
AC: Average Cost
x: demand/number of units sold
p: price per unit
R: Revenue function
AR: Average Revenue
P(x) or π(x): Profit function
MC: Marginal Cost
MAC: Marginal Average Cost
MR: Marginal Revenue
K: Constant of integration
TC = TFC + TVC
AC = C/x
Demand function: relation between x and p (x = demand)
R = x.p (x = number of units sold)
AR = R/x = xp/x = p (AR is same as price per unit)
P(x) = R(x) – C(x)
Breakeven point:

1. R(x) = C(x)
2. R(x) – C(x) = 0
3. p(x) = 0

MC = $\frac{d}{dx}$(C)

MAC = $\frac{d}{dx}$(AC) = $\frac{1}{x}$(MC – AC)

MR = $\frac{d}{dx}$(R) = p = AR (if p remains constant)
C = $\int MC\ dx + k$ (k can be determined if FC is given)
R = $\int MR\ dx + k$ (l can be determined if we are given the revenue on selling a specific number of units)

Linear Regression

Y on X:

$$b_{yx} = r\frac{\sigma_y}{\sigma_x} = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}$$

X on Y:

$$b_{xy} = r\frac{\sigma_x}{\sigma_y} = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\Sigma y^2 - \frac{(\Sigma y)^2}{n}}$$

Properties of linear regression:

- b_{yx}, b_{xy} and r or ρ(x, y) – same sign
- $|r|$ is the geometric mean of b_{yx} and b_{xy}
    - $b_{yx}.b_{xy} = r^2$ and $0 \le r^2 \le 1$
- Regression lines intersect at $(\bar{x}, \bar{y})$
- Regression lines coincide if ρ(x, y) = ± 1
- If r = 0, regression lines are parallel to coordinate axes
- Acute angle between regression lines:

$$\tan\theta = \left|\frac{1 - r^2}{b_{xy} + b_{yx}}\right|$$

$r = \sqrt{b_{xy}.b_{yx}}$ or $r = -\sqrt{b_{xy}.b_{yx}}$

r: Coefficient of correlation

$$b_{yx} = \frac{cov(x, y)}{{\sigma_x}^2}$$

Linear Programming

- Objective function (Variable x and y)
- Constraints (linear equations and inequations)
- Conditions
- Plot on graph
- Find optimal solution – select particular value of x and y that give desired value.

Differential Equations

Order of differential equation: Number of differentiations done to reach the equation

Degree of differential equation: Power of differentiation term used to find order

$$\frac{d^2y}{dx^2} = \frac{dy}{dx}$$

Order = 2

Degree = 1 (Power of $\frac{d^2y}{dx^2}$)

Solution of differential equation:

$$\frac{dy}{dx} = 3$$

$y = 3x$ – Particular Solution

$y = 3x + C$ – General Solution

Verification of solution of differential equations:

Verify $y = 3x + 10$ is solution of $\frac{dy}{dx} = 3$

- Check whether $y = 3x + 10$ satisfies $\frac{dy}{dx} = 3$

OR

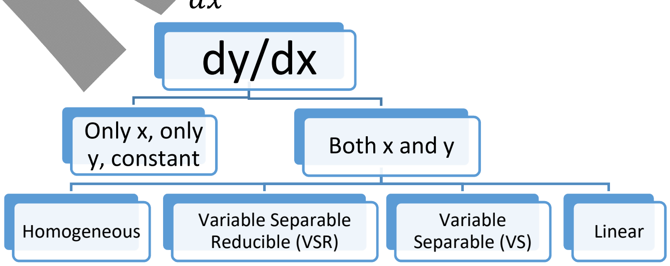
- Given $y = 3x + 10$, prove that $\frac{dy}{dx} = 3$

Formation of differential equation from solution:

Arbitrary constant (AC): Number of AC = Order of differential equation

Solving of differential equations:

- Make $\frac{dy}{dx}$ subject



1. Variable Separable

a. Identification:
    i. Make dy/dx subject
    ii. Dy/dx = x, y, constant, f(x, y)
    iii. Factorise f(x) if each factor contains only one variable
b. Working:
    i. All y left and all x right
    ii. Integrate both sides
    iii. Add integration constant to one side only.

2. Variable Separable Reducible

a. Identification
    i. Dy/dx – subject
    ii. Dy/dx = x and y both
        1. $\frac{dy}{dx} = \frac{ax+by+c}{dx+ey+f}$
            a. If ratio of x = ratio of y: VSR
            b. Else: Homogeneous Reducible
        2. There exists a linear relation in x and y which is used as mismatch: Homogeneous Reducible (Only one linear per question, if more than one then $R_x=R_y$.
b. Working
    i. Let linear = t
    ii. Differentiate both sides and prepare conversion: $\frac{dy}{dx} \to \frac{dt}{dx}$
    iii. Put dy/dx in term dt/dx in left and put linear as t in right
    iv. Make dt/dx subject, simplify RHS
    v. All t left, all x right
    vi. Integrate both sides
    vii. Put value of t back

3. Homogeneous

a. Identification
    i. Dy/dx = x and y
        1. x/y used as mismatch: invertendo

2. y/x is used as mismatch
3. When degree of each term of Numerator and Denominator is same.

b. Working

- **i.** Put dependent variable (y in dy/dx, i.e., variable on top) = v(independent variable (x in dy/dx, i.e., variable below)) OR y = vx in left as well as right
- **ii.** Use product rule in left, cancel x from right
- **iii.** Transfer v from left to right and simplify RHS
- **iv.** After simplifying RHS, all v left, all x right. Integrate both sides.
- **v.** Put value of v = y/x

4. Linear

a. Identification

- **i.** The variable on numerator (of dy/dx) must be present only once in the equation.
- **ii.** $\boldsymbol{\frac{dy}{dx} + P_y = Q}$
- **iii.** Coefficient of dy/dx must be 1
- **iv.** Integrating Factor (I.F.): $= e^{\int Pdx}$ (Do not use +C integration constant)
- v. PS if y is present more than one times, but x is present only once, apply invertendo.

Probability

Sample Space: S

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)=P(A)+P(B-A)$$
$$=P(B)+P(A-B)$$

Mutually Exclusive Events: $P(A\cup B)=P(A)+P(B)$

Exhaustive Events: $P(A\cup B)=S$

Conditional Events:

Two events (A and B) of same experiment.

Probability of happening of A given that event B has already happened:

$$P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$$

Probability of happening of B given that event A has already happened:

$$P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}$$

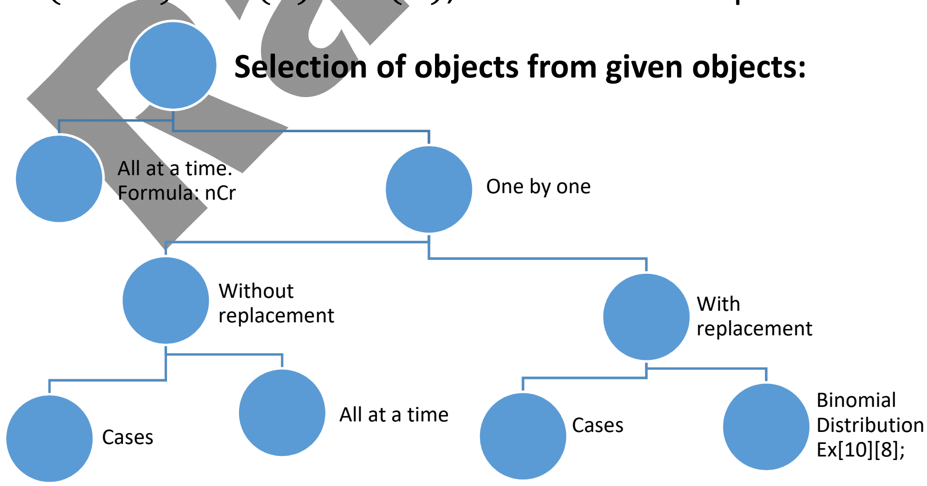
Property: An event F such that $P(F)\neq 0$ then

$$P\left(\frac{A\cup B}{F}\right)=P\left(\frac{A}{F}\right)+P\left(\frac{B}{F}\right)-P\left(\frac{A\cap B}{F}\right)$$

Property:

$$P\left(\frac{A'}{B}\right)=1-P\left(\frac{A}{B}\right)$$

If $P(A\cap B)=P(A)\times P(B)$, A and B are independent events



Law of total probability:

$$P(A) = P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + \cdots + P(E_n)P\left(\frac{A}{E_n}\right)$$

Sum of infinite terms of a GP:

$$\frac{a}{1-r}$$

Baye's Theorem:

A has *fuckin'* happened! What is the probability A is happened using E_n?

$$P\left(\frac{E_n}{A}\right) = \frac{P(E_n)P\left(\frac{A}{E_n}\right)}{P(A)}$$

// niche wala niche, upar wala upar, reciprocal upar

Probability Distribution (P.D.)

x	P
	$\sum P = 1$

Where x is:

- Either given in question OR
- Observations about which P.D. is required

Mean, Variance and Standard Deviation:

$$Mean = \sum Px$$

$$\sigma^2 = \sum Px^2 - \left(\sum Px\right)^2$$

$$\sigma = \sqrt{\sigma^2}$$

Binomial Distribution (B.D.):

Used when:

- When experiment has more than 1 trial, in each trial there must be two options (Happening or not happening)
    - n: Number of trials
    - p: Probability of success in one trial

  - Success:
    - Either explained in ques
    - Or observations about which probability or B.D. is required
- One by one with replacement

Q = 1 – p

B.D. = $(q+p)^n$

Parameter of B.D. = (n, p)

$P(r\ success) = {}^{n}C_{r}\ q^{n-r}p^{r}$

P(at most r success) = P(0)+P(1)+P(2)+…+P(r)

P(at least r success) = P(r)+P(r+1)+P(r+2)+…P(n)

$\mu = np\ (mean)$

$\sigma^2 = npq$

$\sigma = \sqrt{npq}$