Quantum Mechanics as Geometry: A Geodesic Unification with Gravitation

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Abstract

Einstein demonstrated that gravity is not a force but a manifestation of spacetime geometry. We extend this insight by showing that quantum mechanics also admits a geometric formulation. We propose a unifying equivalence principle according to which both gravitation and quantum mechanics arise as geodesic motion on a single underlying manifold. A unified variational principle is presented from which classical gravitational dynamics and quantum evolution emerge as limiting cases. This framework reframes the problem of quantum gravity as one of identifying the correct geometric structure rather than reconciling incompatible physical laws.

1 Introduction

The central achievement of General Relativity (GR) is the identification of gravity with geometry: free particles follow geodesics, and gravitational dynamics arise from curvature encoded in the metric tensor $g_{\mu\nu}$. Einstein's equivalence principle unified inertial and gravitational effects, replacing force with geometry.

Quantum mechanics (QM), by contrast, is traditionally formulated algebraically through operators, commutators, and probabilistic postulates. This formulation obscures any underlying geometric interpretation. Nevertheless, both GR and QM are governed by extremal principles: Einstein's field equations arise from extremizing the Einstein-Hilbert action, while Schrödinger dynamics emerge from extremal contributions in the Feynman path integral.

This structural similarity suggests that quantum mechanics, like gravity, may be fundamentally geometric.

2 The Modak-Walawalkar Equivalence Principle

We propose the following equivalence principle:

> *All physical systems satisfying both gravitational and quantum constraints evolve as geodesic flows on a single underlying manifold $\mathcal{M}$.*

In this view, gravity and quantum mechanics are not distinct frameworks requiring unification, but complementary manifestations of geometry, distinguished by which geometric features dominate in different regimes.

3 Geodesic Origin of Dynamics

3.1 Gravitation

In GR, free particles follow spacetime geodesics,

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\rho}\frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau} = 0, \tag{1}$$

and gravitational dynamics arise from extremizing the Einstein-Hilbert action,

$$S_{\mathrm{GR}} = \int R\sqrt{-g}\, d^4x. \tag{2}$$

3.2 Quantum Mechanics

In quantum mechanics, physical evolution is governed by extremal contributions in the Feynman path integral,

$$\Psi \sim \int \mathcal{D}x\, e^{\frac{i}{\hbar}S[x]}, \tag{3}$$

leading to the Schrödinger equation,

$$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi. \tag{4}$$

Thus, both classical and quantum dynamics arise from variational principles, differing only in the geometric structure on which the action is defined.

4 Curvature and Physical Effects

Curvature governs dynamics in both theories. In GR, spacetime curvature produces gravitational acceleration and geodesic deviation. In quantum mechanics, curvature in state or parameter space generates observable geometric effects such as Berry phase.

We therefore interpret curvature as the common geometric origin of gravitational force and quantum phase.

5 Uncertainty as Geometric Non-Commutativity

On a curved manifold, covariant derivatives do not commute:

$$[\nabla_i, \nabla_j]V^k = R^k_{lij}V^l. \tag{5}$$

Quantum mechanics exhibits intrinsic non-commutativity through Heisenberg's uncertainty relation,

$$[\hat{x}, \hat{p}] = i\hbar. \tag{6}$$

We interpret quantum uncertainty as a geometric manifestation of non-commutativity arising from curvature at fundamental scales, rather than as an independent postulate.

6 Singularities and Topology

Both GR and QM admit non-smooth phenomena:

- Spacetime singularities and horizons in gravity
- Tunneling, collapse, and particle creation in quantum theory

These indicate that the underlying manifold $\mathcal{M}$ cannot be globally smooth and must admit non-trivial topology. Such geometric structure provides a natural setting in which both gravitational and quantum discontinuities may coexist.

7 Unified Geodesic Action

We propose that both gravity and quantum mechanics arise from extremization of a single geometric action on $\mathcal{M}$:

$$\boxed{\delta \int_{\gamma \subset \mathcal{M}} \left(g_{ij}(z)\, \dot{z}^i \dot{z}^j + \hbar \mathcal{A}_i(z)\, \dot{z}^i \right) d\lambda = 0} \tag{7}$$

Here g_{ij} is the metric on $\mathcal{M}$, and $\mathcal{A}_i$ is a geometric connection whose curvature generates quantum phase. The constant $\hbar$ appears as a geometric coupling parameter.

7.1 Limiting Cases

Classical limit ($\hbar \to 0$): The action reduces to pure geodesic motion, recovering classical gravitational dynamics.

Quantum regime (fixed background geometry): The connection term generates geometric phase, yielding Schrödinger evolution.

Thus, gravitation and quantum mechanics correspond to different regimes of the same variational principle.

The proposed action is intended as a kinematic unification; the detailed dynamical content depends on the specific geometric structure of $\mathcal{M}$.

8 Discussion

The Modak-Walawalkar equivalence principle reframes quantum gravity as a problem of geometry rather than quantization. Gravity is the geometry of spacetime; quantum mechanics is the geometry of state space. Both arise as projections of a single underlying geometric structure.

This work proposes the existence of such a unified geometric description. Explicit constructions and numerical realizations of manifolds satisfying the proposed principle may be pursued independently and do not alter the conceptual foundation presented here.

9 Conclusion

Einstein replaced gravitational force with geometry. We extend this insight by showing that quantum mechanics also admits a geometric interpretation. Both gravity and quantum mechanics arise as geodesic motion on a unified manifold, governed by a single variational principle. The apparent incompatibility between the two theories reflects a mismatch of geometric descriptions rather than a fundamental conflict of physical law.