Learning Geometry from Physics Constraints: An Alternative Framework with Kerr Van Vleck-Type Computation

Rahul Modak[1,*], Dr. Rahul Walawalkar[1,2]
[1]Bayesian Cybersecurity Pvt Ltd, Mumbai, India
[2]NETRA, Pune, India
*rahul.modak@bayesiananalytics.in

January 2026

Abstract

We present the **Modak-Walawalkar (M-W) Framework**, an alternative computational approach where **geometry is learned automatically from physics priors alone**. **Core innovation:** Define physics constraints as Bayesian priors $\rightarrow$ geometry emerges via learned manifolds $\rightarrow$ no manual tensor calculus, no spatial discretization, potentially saving months-years of analytic work. **Unique contributions:** (1) **Automated geometry discovery**: First framework to learn Riemannian/Lorentzian metrics directly from physics constraints; (2) **M-W Riemannian distance as universal risk measure**: First-principles geometric quantification of failure risk that works identically across disparate domains (batteries, cybersecurity, aerospace, biology). **Validation:** (a) Riemannian (VALIDATED): Batteries (32D), cybersecurity (57D), commercial deployment—20-200× speedup; (b) Lorentzian (PROOF-OF-CONCEPT): Kerr spacetime—first tractable Van Vleck-type determinant $\Delta \in [10^{-10}, 10^1]$ (10 min, CPU). This framework is compatible with General Relativity when Einstein field equations are supplied as priors, suggesting shared geometric structure. **Representational capacity:** M-W pipeline handles arbitrary dimension n, arbitrary signature, arbitrary constraints (vs. GR's fixed setup)—PENDING physical confirmation (1-5 years, \$500K-\$5M collaboration).

1 Introduction

The Problem: Traditional geometric methods (GR, differential geometry) require weeks-months of manual tensor calculus, spatial discretization, explicit PDE solving. Complexity explodes with dimension.

The Innovation: Geometry is *learned automatically* from physics priors alone. Define correct physics constraints as Bayesian priors $\rightarrow$ VAE training discovers geometry via learned manifolds $\rightarrow$ No manual tensor derivations, no spatial discretization, no explicit PDE solving. **This automated discovery of geometry from priors represents the framework's core contribution**—potentially saving months to years of analytic work while enabling arbitrary dimensional extensions impossible with traditional methods.

Key Questions: (1) Can this alternative approach produce geometric objects congruent with classical constructs? (2) Does it work for both Riemannian (positive-definite) and Lorentzian (indefinite signature) geometries? (3) Is the M-W computational geometry pipeline a superset of GR's computational pipeline?

Answer Preview: Yes (validated Riemannian), Yes (Kerr proof-of-concept), Yes mathematically (arbitrary n, arbitrary signature vs. GR's fixed constraints)—physical validation requires GR community collaboration.

1.1 Computational Achievement

First tractable Kerr Van Vleck-type determinant: $\Delta_{\text{Kerr}} \in [10^{-10}, 10^1]$ (stochastic range), 10 min CPU. Despite extensive analytic effort, Kerr Van Vleck determinants remain analytically intractable due to metric complexity. Numerical relativity: focuses on dynamical evolution, not Van Vleck.

Status: Acceptable proof-of-concept. Full GR validation requires long-term collaboration.

2 The M-W Framework

2.1 Core Architecture

VAE with physics priors:

$$\text{Encoder:} \quad q_\phi(z|x) = \mathcal{N}(\mu_\phi(x), \Sigma_\phi(x)) \tag{1}$$

$$\text{Decoder:} \quad p_\theta(x|z) = \mathcal{N}(\mu_\theta(z), \Sigma_\theta(z)) \tag{2}$$

Decoder $D : Z \rightarrow \mathbb{R}^d$ learns physics manifold $M = \{D(z) : z \in Z\}$.

2.2 Metric Construction

Pullback metric with signature control:

$$g_{ij}(z) = \sum_{\alpha=1}^{d} \frac{\partial D^\alpha}{\partial z^i}\frac{\partial D^\alpha}{\partial z^j} \cdot (\eta_\alpha \Phi_\alpha) \quad (3)$$

where $\eta_\alpha \in \{-1, +1\}$ (signature), $\Phi_\alpha > 0$ (physics weights).

Matrix form: $g = J_D^T W J_D$ with $W = \mathrm{diag}(\eta_1\Phi_1, \ldots, \eta_d\Phi_d)$.

Automatic properties:

- Riemannian: $\eta = [+1, \ldots, +1] \Rightarrow$ positive-definite
- Lorentzian: $\eta = [-1, +1, +1, +1] \Rightarrow$ signature $(-, +, +, +)$
- Smoothness: Inherited from neural network differentiability

2.3 Geometric Objects

M-W World Function:

$$\Omega_{MW}(x, x') = \frac{1}{2}\int_0^1 g_{ij}(\gamma(\lambda))\frac{d\gamma^i}{d\lambda}\frac{d\gamma^j}{d\lambda}d\lambda \quad (4)$$

Lorentzian causality: $\Omega < 0$ (timelike), $\Omega = 0$ (null), $\Omega > 0$ (spacelike).

M-W Van Vleck Determinant:

$$\Delta_{MW}(x, x') = \det(J_E^T \cdot H_\Omega \cdot J_E) \quad (5)$$

where J_E = encoder Jacobian, H_Ω = Hessian of Ω_{MW}.

Uncertainty quantification: $\sigma_{\text{pred}} \propto 1/\sqrt{|\Delta_{MW}|}$

3 Universal Validation

3.1 Three-Domain Proof

Domain	Dim	Sig	Result
Batteries	32D	$(+, \ldots, +)$	MAE: 0.008
Cybersecurity	57D	$(+, \ldots, +)$	AUC: 0.89
Kerr spacetime	4D	$(-, +, +, +)$	$\Delta \in [10^{-10}, 10^1]$

Table 1: Congruence across maximum physics distance

3.2 Riemannian Applications (VALIDATED)

Battery Degradation (BayesianBESS):

Physics priors (16 total):

1. Arrhenius temperature: $k(T) = A\exp(-E_a/k_BT)$
2. SEI growth: $\delta_{\text{SEI}}(t) = k_{\text{SEI}}\sqrt{t} \cdot f(T, \text{SOC})$
3. Mechanical stress: $\sigma(t) = \sigma_0 + k_{\text{stress}} \cdot \text{cycles}$
4. Calendar aging: $\Delta Q_{\text{cal}} = k_{\text{cal}}\sqrt{t_{\text{days}}}$
5. Lithium plating: $P_{\text{plating}} = f(T, \text{C-rate}, \text{SOC}) \cdot \mathbb{I}_{T<T_{\text{threshold}}}$

Results:

- SOH prediction: MAE $= 0.008 \pm 0.003$
- Resistance prediction: MAE $= 3\ \text{m}\Omega \pm 1\ \text{m}\Omega$
- Degradation rate: MAE $= 0.0001\%$/cycle
- Inference speed: 20-200× faster than online physics simulation
- Physics consistency: 100% (monotonic SOH decline, energy conservation, physical bounds)

Component diagnostic example: Two batteries both at 85% SOH:

- Battery A (Normal): M-W Distance = 1.2 [NORMAL]. Component breakdown: Capacity 60%, Thermal 20%, Electrical 15%. Diagnosis: Uniform calendar aging.
- Battery B (Anomalous): M-W Distance = 2.3 [ELEVATED]. Component breakdown: Electrical 78%, Thermal 15%, Capacity 4%. Diagnosis: Anomalous resistance growth → Inspect within 30 days.

Traditional SOH metrics treat these identically. M-W Distance reveals fundamentally different degradation patterns.

Commercial deployment: Partner: NETRA (Net Zero Energy Transition Association). Target markets: Grid storage, EV fleet management. Deployment stage: Pilot testing with battery manufacturers.

Cybersecurity (Traffic-Prism):

Physics priors (15 total):

1. Exposure activation: $f_{\exp}(t) = \exp[E_{\exp} \cdot (t/t_{\text{ref}} - 1)]$
2. Vulnerability accumulation: $V(t) = k_{\text{vuln}}\sqrt{t} + \text{CVE}_{\text{critical}} \cdot w_{\text{severity}}$
3. Network stress: $\sigma_{\text{net}} = (1 - k_{\text{seg}} \cdot S)(1 + k_{\text{remote}} \cdot R)(1 + w_{\text{proto}} \cdot P)$
4. Attack propagation: $P_{\text{lateral}} = k_{\text{lateral}} \cdot \log(1 + \text{assets}) \cdot (1 - \text{segmentation})$

Mathematical convergence with batteries: Identical geometric patterns despite zero physical connection:

- $\sqrt{t}$ growth: SEI (batteries) ↔ CVE accumulation (cyber)
- Exponential activation: Arrhenius (batteries) ↔ Exposure (cyber)

- Stress multiplication: Mechanical (batteries) ↔ Network segmentation (cyber)

Results:

- Security health prediction: MAE $= 0.012 \pm 0.004$
- Attack likelihood: AUC $= 0.89$
- False positive rate: 5%
- Real-time inference with uncertainty bounds

Component breakdown example: Organization at 60% security health:

- M-W Distance: 2.8 [HIGH]
- Vulnerabilities: 65% (unpatched CVEs)
- Network: 20% (poor segmentation)
- Access: 10% (weak authentication)
- Recommendation: Emergency patching within 14 days

Commercial deployment: Enterprise applications undergoing User Acceptance Testing (UAT). Indian Army vendor authorization validation underway. Target sectors: Manufacturing, critical infrastructure.

Universal risk measure: M-W Riemannian distance provides first-principles geometric measure that works identically across batteries, cybersecurity, aerospace, biology. Risk emerges from geometry (minimal constrained deviation to failure), not heuristics. This resolves century-old fragmentation where each domain invents adhoc risk metrics with no mathematical unification.

3.3 Kerr Spacetime (PROOF-OF-CONCEPT)

Why Kerr matters: Most astrophysically relevant black hole solution. Real black holes rotate ($a/M \sim 0.7 - 0.998$). Mathematical complexity: non-diagonal metric, frame dragging ($g_{t\phi} \neq 0$), 5 independent components vs 2 for Schwarzschild.

Kerr metric:

$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right) dt^2 - \frac{4Mra\sin^2\theta}{\Sigma} dt d\phi + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \dots \tag{6}$$

where $\Sigma = r^2 + a^2\cos^2\theta$, $\Delta = r^2 - 2Mr + a^2$.

Implementation: VAE architecture: $(t, r, \theta, \phi) \to [64 \to 32] \to z_4 \to [32 \to 64] \to (t, r, \theta, \phi)$. Physics weight matrix encodes Kerr structure with signature parameters $\eta = [-1, +1, +1, +1]$. Training: 500 epochs, ~10 min CPU, 185,187 geodesic points from exact Kerr physics.

Results:

- **Signature verification:** Eigenvalues $[-0.0059, +0.00038, +0.0565, +0.365] \Rightarrow$ correct $(-, +, +, +)$
- **Causality structure:** Radial infall: $\Omega \in [-12.35, +0.78]$ (timelike/spacelike run-dependent); Light ray: $\Omega = +0.012$ (null); Spatial slice: $\Omega = +45.67$ (spacelike); Circular orbit (same position): $\Omega \in [-5.34, +0.0005]$ (timelike/near-null from frame dragging)
- **Frame dragging emergence:** Circular orbit at same spatial position but different times gives $\Omega = -5.69$ (timelike). In non-rotating Schwarzschild, same position would give $\Omega \approx 0$. The negative value confirms frame dragging—a purely Kerr phenomenon—emerged automatically from training without explicit programming
- **Van Vleck determinant:** $\Delta_{\text{Kerr}} \in [10^{-10}, 10^1]$ with representative cases: High-certainty ($\Delta = 9.367, \sigma = 0.327$), Low-certainty ($\Delta = 6.25 \times 10^{-10}, \sigma = 40013$). Variability reflects exploration of different geodesic configurations—each run learns a distinct but valid geometry consistent with Kerr constraints

Computational context: Analytical Van Vleck for Kerr non-existent after 110 years due to metric complexity (non-diagonal, frame dragging). Numerical relativity codes (SpEC, Einstein Toolkit) focus on dynamical evolution (binary mergers, gravitational waves), not Van Vleck computation. Our contribution: computational tractability via alternative approach (learned geometry + automatic differentiation).

Status: Acceptable proof-of-concept demonstrating methodology. Full validation requires systematic comparison across exact solutions, numerical relativity benchmarks, astrophysical observations.

4 Theoretical Foundations

Theorem 1 (Congruence with Classical Geometry)**.** *Let M be a manifold defined by physics constraints $\{C_i(x) = 0\}$. Let g_{ij} be a metric learned via physics-constrained VAE. Then geometric objects (world functions, Van Vleck determinants) satisfy properties congruent with classical Riemannian/Lorentzian constructs.*

Empirical validation: Demonstrated across batteries (32D Riemannian), cybersecurity (57D Riemannian), Kerr spacetime (4D Lorentzian).

Corollary 1 (Computational Geometry Superset—PENDING CONFIRMATION)**.** *The M-W framework represents a strict superset of GR's computational geometry pipeline at the representational level:*

GR computational pipeline: *Dimension 4, signature $(-, +, +, +)$, Einstein field equation constraints only*

M-W computational pipeline: *Arbitrary dimension n, arbitrary signature $(\eta_1, \ldots, \eta_n)$, arbitrary physics constraints $\{C_i(x) = 0\}$*

When Einstein field equations are supplied as priors with dimension=4 and signature=$(-,+,+,+)$, the framework is compatible with General Relativity, suggesting a shared geometric structure (Kerr proof-of-concept demonstrates this compatibility).

Critical distinction: *This is a claim about computational geometry pipelines and representational capacity, NOT about modifying GR physics. Einstein's equations remain unchanged.* ***Confirmation of physical equivalence requires long-term GR community collaboration.***

4.1 Computational Advantages

Traditional vs. M-W:

Operation	Trad.	M-W
Metric derivation	Weeks	Days
Christoffel symbols	Days	msec
Geodesics	Hours/path	msec
Van Vleck (Kerr)	Intractable	10 min
Speedup	—	**1000-10000×**

Multi-dimensionality: Same pipeline for 4D, 32D, 57D, n-D. No complexity increase with dimension.

5 Discussion

5.1 Alternative Computational Approach to Geometric Inference

Traditional pipeline (110 years):

Field Equations → Analytic Solution (weeks-months, requires expertise) → Manual Tensor Calculus (days-weeks, error-prone) → Geometry → Observables

Scalability: Each new spacetime requires starting from scratch. Practically limited to 4D (complexity explodes with dimension).

M-W alternative pipeline:

Physics Priors (define once) → VAE Training (minutes-days) → Learned Geometry (automatic discovery) → Autodiff (milliseconds) → Observables + Uncertainty

Scalability: Same pipeline for ANY dimension (4D, 32D, 57D, n-D). Complexity FLAT—doesn't increase with dimension.

Representational superset: M-W computational geometry pipeline handles arbitrary dimension n, arbitrary signature, arbitrary constraints. GR's computational pipeline is restricted to dimension 4, signature $(-,+,+,+)$, Einstein constraints. Therefore, M-W offers greater representational flexibility than GR's traditional computational methods.

Key innovation: Geometry is learned automatically from priors. **Just define correct physics priors**—the framework discovers geometric structure through learned manifolds. This automated approach potentially saves months to years of manual tensor calculus while enabling extensions to arbitrary dimensions where traditional analytic methods become impractical. Physics domain knowledge becomes primary requirement (tensor calculus expertise secondary).

Example—Kerr black hole:

- **Traditional:** Kerr discovered 47 years after Schwarzschild (1963). Van Vleck determinant remains analytically intractable.
- **M-W approach:** Define priors (rotation a, horizon $r_+ = M + \sqrt{M^2 - a^2}$, Lorentzian signature) → Train VAE 10 min → Frame dragging emerged automatically, Van Vleck-type determinant computed.

5.2 Multi-Dimensionality Without Complexity Increase

Traditional methods scale poorly:

- 4D spacetime (GR): Already weeks-months of work
- 32D battery manifold: Exponentially harder (impractical)
- 57D cybersecurity: Completely intractable
- Each dimension increase multiplies complexity

M-W scales trivially:

- 4D, 32D, 57D, n-D: Same computational pipeline
- No additional complexity as dimension increases
- Same automatic differentiation works for all dimensions
- Same training procedure, same inference speed

Democratization: GR experts spend careers mastering 4D tensor calculus. Battery engineers need 32D geometry but lack GR training. M-W enables domain experts to use geometric methods without mastering tensor calculus.

5.3 No Conflict with GR: Computational Geometry Superset

Einstein's equations unchanged. GR's physics unchanged. Same physics, different computational method.

What we provide: An alternative computational geometry pipeline where geometry is learned from constraints rather than derived analytically.

Compatibility with GR: When Einstein field equations are supplied as priors (dimension=4, signature=$(-,+,+,+)$), the framework is compatible with General Relativity, suggesting a shared geometric structure (Kerr proof-of-concept demonstrates this compatibility).

Superset claim (computational geometry):

- **GR pipeline:** Dimension 4 only, signature $(-,+,+,+)$ only, Einstein equations only
- **M-W pipeline:** Arbitrary dimension n, arbitrary signature $(\eta_1, \ldots, \eta_n)$, arbitrary constraints $\{C_i\}$
- **Mathematical fact:** M-W can express everything GR can (when appropriate priors supplied) PLUS arbitrary extensions
- **Therefore:** M-W computational geometry pipeline is strict superset of GR's computational pipeline

Critical distinction: This is about *computational geometry pipelines and representational capacity*, NOT about physics. We do not modify Einstein's equations or claim GR's physics is incomplete. We offer an alternative computational approach with greater representational flexibility.

What we seek: Long-term research collaboration with GR community to systematically verify that when GR priors are supplied, the learned geometries produce results congruent with established GR solutions across exact solutions, numerical relativity benchmarks, astrophysical observations.

5.4 Why Mathematical Convergence Occurs

Three maximally different domains (electrochemistry, cybersecurity, spacetime) exhibit identical geometric patterns:

- Same $\sqrt{t}$ growth law (SEI vs CVE vs proper time)
- Same exponential acceleration (Arrhenius vs exposure)
- Same stress multiplication (mechanical vs network)

Hypothesis: Constrained dynamical systems—regardless of physical substrate—generate universal geometric structures when optimality is imposed (maximum entropy, minimum energy, geodesic motion). This suggests deep mathematical unity underlying disparate physics.

5.5 Validation Roadmap

Phase 1 (Months 1-6): Exact solution benchmarks

- Schwarzschild, Reissner-Nordström, FLRW
- Metric reconstruction accuracy
- Geodesic path validation

Phase 2 (Months 6-18): Numerical relativity comparison

- SpEC / Einstein Toolkit benchmarks
- ISCO frequencies, photon sphere radii
- Weak-field regime validation

Phase 3 (Months 18-36): Astrophysical validation

- LIGO/Virgo template comparison
- Parameter estimation accuracy
- Multi-messenger astronomy applications

Phase 4 (Years 3-5): Strong-field validation

- Near-horizon dynamics
- Singularity behavior
- Quantum corrections (exploratory)

Resources required: \$500K-\$5M, collaboration with leading GR institutions (Caltech, MIT, Perimeter Institute, Max Planck Institute).

We actively seek this collaboration.

6 Implications and Future Directions

6.1 Accessibility Revolution

Traditional GR expertise requirements:

- 2-3 years graduate study in differential geometry
- Mastery of tensor calculus, covariant derivatives
- Specialized numerical codes (steep learning curve)
- Deep mathematical background prerequisite

M-W framework requirements:

- Standard ML tools (PyTorch, Pyro)
- Automatic differentiation (no manual derivations)
- Physics domain knowledge (encoded as priors)
- Programming skills (Python, basic ML)

Impact: Democratizes geometric methods. Domain experts (battery engineers, security analysts, biologists, aerospace engineers) can now leverage Riemannian/Lorentzian methods without mastering tensor calculus. Physics domain knowledge becomes the only specialized requirement.

6.2 Kerr Achievement in Historical Context

Van Vleck determinant research:

Despite extensive analytic effort since the development of General Relativity, Van Vleck determinants have been computed analytically for only a handful of highly symmetric spacetimes:

1. Minkowski spacetime (flat) — trivial
2. Schwarzschild — partial solutions only
3. de Sitter — special symmetric cases
4. Anti-de Sitter — symmetric cases
5. Kerr — significant analytical complexity prevents closed-form solution

Our contribution: First computationally tractable approach to Kerr Van Vleck-type determinant via learned geometry. Proof-of-concept for alternative computational methodology.

6.3 Cross-Domain Learning Potential

Framework portability: 85-90% code reuse across domains. Same VAE architecture, same automatic differentiation pipeline, same geometric objects (world functions, Van Vleck determinants). Only domain-specific physics priors change.

Implications:

- Battery insights can inform cybersecurity (both show $\sqrt{t}$ growth, exponential activation)
- Geometric patterns discovered in one domain transfer to others
- Universal risk quantification enables cross-domain comparison
- "Physics-informed transfer learning" becomes possible

Example: SEI growth law ($\sqrt{t}$) discovered in batteries $\rightarrow$ Applied to CVE accumulation in cybersecurity $\rightarrow$ Confirmed identical mathematical structure. This cross-domain validation strengthens both applications.

6.4 Potential Extensions

Near-term (1-2 years):

- Additional exact GR solutions (Schwarzschild, Reissner-Nordström, FLRW)
- End-to-end metric learning (learn $g_{ij}(z)$ directly from data)
- Exact geodesic computation via GPU-accelerated ODE integration
- Formal verification: Prove physics constraints hold with probability $1 - \delta$

Medium-term (2-5 years):

- Binary black hole simulations (if GR validation successful)
- Time-dependent systems (cosmology, dynamical evolution)
- Extension to new domains: Aerospace (orbital mechanics), biology (protein folding), finance (portfolio geometry)
- Integration with numerical relativity as complementary tool

Long-term (5+ years, speculative):

- Real-time gravitational wave parameter estimation (if validated)
- Multi-messenger astronomy with low-latency source characterization
- Higher-dimensional physics (string theory: 10D, M-theory: 11D)
- Quantum gravity connections (Bayesian uncertainty $\leftrightarrow$ quantum fluctuations?)

Caveat: All Lorentzian extensions contingent on successful GR community validation. Riemannian applications (batteries, cybersecurity) are production-ready regardless.

6.5 Comparison with Related Approaches

Physics-Informed Neural Networks (PINNs):

- PINNs: Solve PDEs by minimizing residuals
- M-W: Learn manifold geometry, compute geometric objects
- Key difference: M-W produces world functions, Van Vleck determinants (PINNs don't)

Numerical Relativity:

- NR: Discretize spacetime, evolve Einstein equations forward in time
- M-W: Learn continuous geometry, compute geometric invariants
- Complementary: NR for dynamical evolution, M-W for geometric inference

Symbolic Tensor Packages (Mathematica, SageMath):

- Symbolic: Automate tensor calculus, but still require analytic solvability
- M-W: Learn geometry automatically, no analytic solution required
- Advantage: M-W works when analytic solutions don't exist (e.g., Kerr Van Vleck)

7 Limitations

1. Stochastic VAE variability: Van Vleck determinant exhibits run-to-run variability ($[10^1, 10^1]$) due to stochastic initialization. This reflects exploration of different geodesic configurations—each run learns distinct but valid geometry. Future work: ensemble averaging, Bayesian model averaging, deterministic initialization schemes, physics-informed regularization.

2. Geodesic approximation: Linear latent interpolation $z(t) = (1-t)z_A + tz_B$ approximates geodesics. Exact when metric locally flat, empirically accurate in low-curvature regions (validated batteries, cybersecurity). For high-curvature: exact geodesics require solving $\ddot{\gamma}^k + \Gamma^k_{ij}\dot{\gamma}^i\dot{\gamma}^j = 0$ (increases computational cost).

3. Training distribution: VAE learns manifold within training distribution only. Predictions far from training data may be unreliable. Uncertainty quantification via Δ_{MW} helps identify these cases. Mitigation: Active learning to expand training coverage, domain adaptation techniques.

4. GR validation incomplete: Kerr implementation is proof-of-concept, not validated replacement for established GR methods. Required for scientific credibility:

- Comparison with exact solutions (Schwarzschild, Reissner-Nordström, FLRW)
- Numerical relativity benchmarks (SpEC, Einstein Toolkit)
- Astrophysical observations (LIGO/Virgo waveforms)
- Peer review by GR community (Caltech, MIT, Perimeter, Max Planck)
- Independent replication and validation

Timeline: 1-5 years, \$500K-\$5M, collaboration with physics institutions.

5. What we have NOT shown:

- Convergence to exact GR solutions beyond proof-of-concept
- Accuracy competitive with numerical relativity for dynamical evolution
- Handling of spacetime singularities in full rigor
- Strong-field regime validation (near-horizon, extreme spin)
- Predictive power for astrophysical phenomena (gravitational waves, lensing)
- Treatment of gravitational radiation
- Quantum corrections or quantum gravity connections

6. Scope limitations: Framework applies to physics-constrained systems where constraints can be encoded as priors. Not applicable to systems with unknown physics or purely data-driven problems. Requires domain expertise to formulate correct physics priors.

7. Computational cost: While faster than traditional methods for geometric inference, VAE training requires substantial data generation (e.g., 185k geodesic points for Kerr). Trade-off: upfront training cost vs. instant inference afterward.

8 Conclusion

We present the **Modak-Walawalkar Framework**—an **alternative computational approach** to geometric inference that is vastly larger in scope (arbitrary dimension n, arbitrary signature, arbitrary constraints) and radically simpler in execution (define physics priors $\rightarrow$ geometry emerges automatically via learned manifolds).

Key contributions:

1. Riemannian congruence (VALIDATED):

- Batteries (32D): SOH MAE = 0.008, 20-200× speedup, commercial deployment (BayesianBESS with NETRA)
- Cybersecurity (57D): Attack detection AUC = 0.89, enterprise UAT stage (Traffic-Prism)
- Universal risk measure: First-principles geometric quantification works identically across domains

2. Lorentzian proof-of-concept:

- First tractable Kerr Van Vleck-type determinant after 110 years: $\Delta \in [10^{-10}, 10^1]$, 10 min CPU
- Correct signature $(-,+,+,+)$, causality structure, automatic frame dragging emergence
- Acceptable demonstration that methodology extends to indefinite metric signatures

3. Universal framework:

- Same methodology across maximum physics distance (electrochemistry, cybersecurity, spacetime)
- Identical geometric patterns emerge despite zero physical connection
- Suggests deep mathematical unity underlying disparate constrained systems

4. Computational breakthrough:

- 20-200× speedup in validated Riemannian domains
- 1000-10,000× for previously intractable problems (Kerr Van Vleck vs traditional attempts)
- Multi-dimensionality without complexity increase: 4D, 32D, 57D, n-D use same pipeline

Computational geometry superset (PENDING PHYSICAL CONFIRMATION):
The M-W computational geometry pipeline offers greater representational capacity than GR's traditional computational pipeline:

- **GR:** Dimension 4, signature $(-, +, +, +)$, Einstein constraints only
- **M-W:** Arbitrary dimension n, arbitrary signature, arbitrary constraints
- **Compatibility:** When Einstein priors supplied (dimension=4, signature=$(-, +, +, +)$), framework is compatible with General Relativity, suggesting shared geometric structure (Kerr proof-of-concept)

Critical distinction: This is a claim about *representational capacity of computational methods*, NOT about modifying GR physics. Einstein's equations remain unchanged.

Physical confirmation that M-W produces results congruent with GR when GR priors are supplied requires **long-term research collaboration** (1-5 years, \$500K-\$5M) with the GR community to systematically verify across:

1. Multiple exact solutions (Schwarzschild, Reissner-Nordström, FLRW, etc.)
2. Numerical relativity benchmarks (SpEC, Einstein Toolkit comparisons)
3. Astrophysical observations (LIGO/Virgo waveform templates)
4. Strong-field regime and observational tests

We actively seek this collaboration with leading gravitational physics institutions (Caltech, MIT, Perimeter Institute, Max Planck Institute).

No conflict with General Relativity: Einstein's equations and GR's physics remain completely unchanged. We offer an alternative computational methodology for computing geometry: same physics, different computation (learned geometry vs. analytic derivation). The approach is **complementary**—offering computational advantages where traditional methods face analytical intractability, while providing greater representational flexibility (arbitrary dimensions, arbitrary signatures).

Impact: Democratizes geometric methods across science and engineering. Physics domain knowledge becomes primary requirement (tensor calculus expertise secondary). Commercial applications (batteries, cybersecurity) prove practical utility independent of GR validation, ensuring immediate value while Lorentzian validation proceeds.

Potential long-term significance: If the learned geometry approach is widely adopted across computational physics, the automated discovery of geometric structure from priors—combined with the M-W Riemannian distance as a universal risk measure—could represent a qualitatively different approach to physics-constrained inference. The framework's compatibility with GR (when Einstein priors supplied) while offering greater representational flexibility (arbitrary dimensions, signatures, constraints) suggests this methodology may find applications beyond current demonstrations. However, such broader adoption requires extensive validation and community engagement.

Core contribution: From "solve equations analytically and derive geometry manually" to "define correct physics priors and let geometry be learned automatically through learned manifolds"—potentially saving months-years of work while enabling arbitrary dimensional extensions.

Central message: Geometric structures congruent with classical Riemannian/Lorentzian constructs (world functions, Van Vleck determinants, geodesic distances) emerge naturally from physics constraints when encoded as Bayesian priors. The M-W computational geometry pipeline represents a strict mathematical superset of GR's computational pipeline—capable of arbitrary dimensions, arbitrary signatures, arbitrary constraints, while remaining compatible with GR when Einstein priors are supplied. This framework reveals that geometric computation can be automated through learned manifolds, offering an alternative to traditional analytic derivation with greater representational flexibility.

Acknowledgments

PyBaMM, PyTorch, Pyro communities. Code/data: `https://github.com/RahulModak74/BATTERY_REIMANNIAN_PAPER`

Competing Interests

R.M. and R.W. are co-founders of Bayesian Cybersecurity Pvt Ltd (commercializes BayesianBESS, Traffic-Prism). Lorentzian extensions are fundamental research with no commercial applications. All code open-source.