Q6 $1, 2, 3, \ldots, \sqrt{n}$

$$T(n) = O(\sqrt{n})$$

Q7 $\sum_{i=n/2}^{n} \sum_{j=i}^{n\,(j*2)} \sum_{\substack{k=1 \\ step = k*2}}^{n}$

$$\sum_{i=n/2}^{n} \sum_{\substack{j=1 \\ step\ j*2}}^{n} \log(n)$$

$$\sum_{i=n/2}^{n} (\log(n))^2$$

$$\left(\frac{n}{2}+1\right)(\log n)^2$$

$$T \cdot C = O(n \log n(\log n))$$

Q8 Recurrence relation :

$T(n) = T(n-3) + n^2 \qquad n^2 > 1$

$n <= 1$

using back substitution

$T(n) = T(n-6) + (n-3)^2$

$T(n-6) = T(n-9) + (n-6)^2$

=) Putting value

$T(n) = T(n-9) + (n-6)^2 + (n-3)^2 + n^2$

$T(n) = T(n-3k) + (n-3(k-1))^2 + (n-3(k-2))^2$

$+ \dots n^2$

$3k = n$

$k = \frac{n}{3}$

$T(n) = T(n - 3 \times \frac{n}{3}) + n^2 + (n-3)^2 + \dots + (n-3(\frac{n}{3}-1))^2$

$T(n) = T(1) + n^2 + (n-3)^2 + (n-6)^2 + \dots$
$+ (n - n + 3)^2$

=) taking only higher order terms

n^2 obtained k times

n^2 obtained $\frac{n}{3}$ times

$T(n) = 1 + (n^2 + n^2 + n^2 + \dots) + (Xn + 1n + 2n + \dots)$

$T(n) = 1 + kn^2 + \dots$

$T(n) = 1 + \frac{n^3}{3}$

$O(n^3)$ Ans

Q9

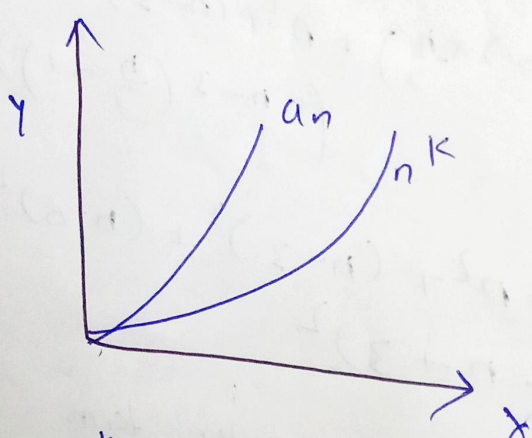
$n + \frac{n}{2} + \frac{n}{3} + \dots \frac{n}{n}$

$n \left(\underbrace{1 + \frac{1}{2} + \frac{1}{3} + \dots \frac{1}{n}}_{\log n}\right)$

$n \log n$

$O(n \log n)$

Q10



$n^k = O(a^n)$

$\therefore n^k \leq a^n \cdot c$

$\forall c > 0$ and $n \geq n_0$

suppose $n = n_0$

$\Rightarrow n_0^k \leq c \cdot a^{n_0}$

$= n_0 \le c \cdot B^{n_0}$

$k = a = 3$

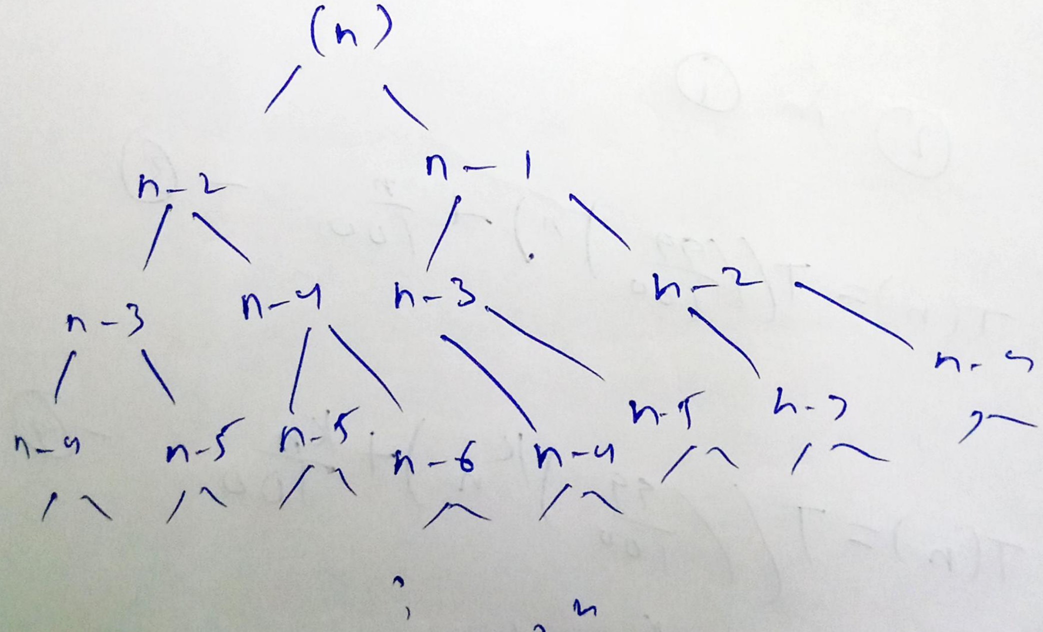
$\Rightarrow c \ge 1 \;\&\; n_0 \ge 1$

~~i goes as: 1, 3, 6, 10, 15, 21,~~ ...

Q11 $O(\sqrt{n})$

Q12 0, 1, 1, 2, 3, . . . Tn

$T(n) = T(n-2) + T(n-1) + 1$



$- \; 2^n$

$T = 1 + 2 + 4 + \dots + 2^n$

$a = 1$
$r = 2$

$$T = \frac{1(2^{n+1}-1)}{2-1}$$

$$= 2^{n+1} - 1$$

$$T(n) = O(2^n)$$

Q17 $T(n) = T\left(\frac{99}{100} n\right) + n/100$

$T(1) = 0$ — ①

put $n = \frac{99n}{100}$ in ①

$$T\left(\frac{99}{100} n\right) = T\left(\left(\frac{99}{100}\right)^2 n\right) + \frac{n}{100} \quad \text{— ②}$$

② in ①

$$T(n) = T\left(\left(\frac{99}{100}\right) n\right) + \frac{n}{100} \quad \text{— ③}$$

⋮

$$T(n) = T\left(\left(\frac{99}{100}\right)^k n\right) + \frac{kn}{100} \quad \text{— ④}$$

$$\left(\frac{99}{100}\right)^k n = 1$$

$$n = \left(\frac{100}{99}\right)^{k}$$

$$k = \log_{\frac{100}{99}} n \quad \text{— ⑤}$$

(i) in (c)

$$T(n) = \frac{n \log_{\frac{100}{99}} n}{100}$$

$$T(n) = O(n \log n)$$

Q18

a) $100 < \log(\log(n)) < \log(n) < \sqrt{n} < n <$
$\log(n!) < n\log n < n^2 < 2^n < 2^{2n} < 4^n <$
$n!$

b) $1 < n < 2n < 4n < \log\log n < \log\sqrt{n} <$
$\log n < \log 2n < 2\log n < \log(n!)$
$< n\log n < n^2 < (2^n)^2 < n!$

c) $96 < \log_8(n) < \log_2(n) < n\log_6 n <$
$n\log_2 n < \log(n!) < 5n < 8n^2 < 7n^3$
$< 8^{2n} < n!$