Laplace Transform-3

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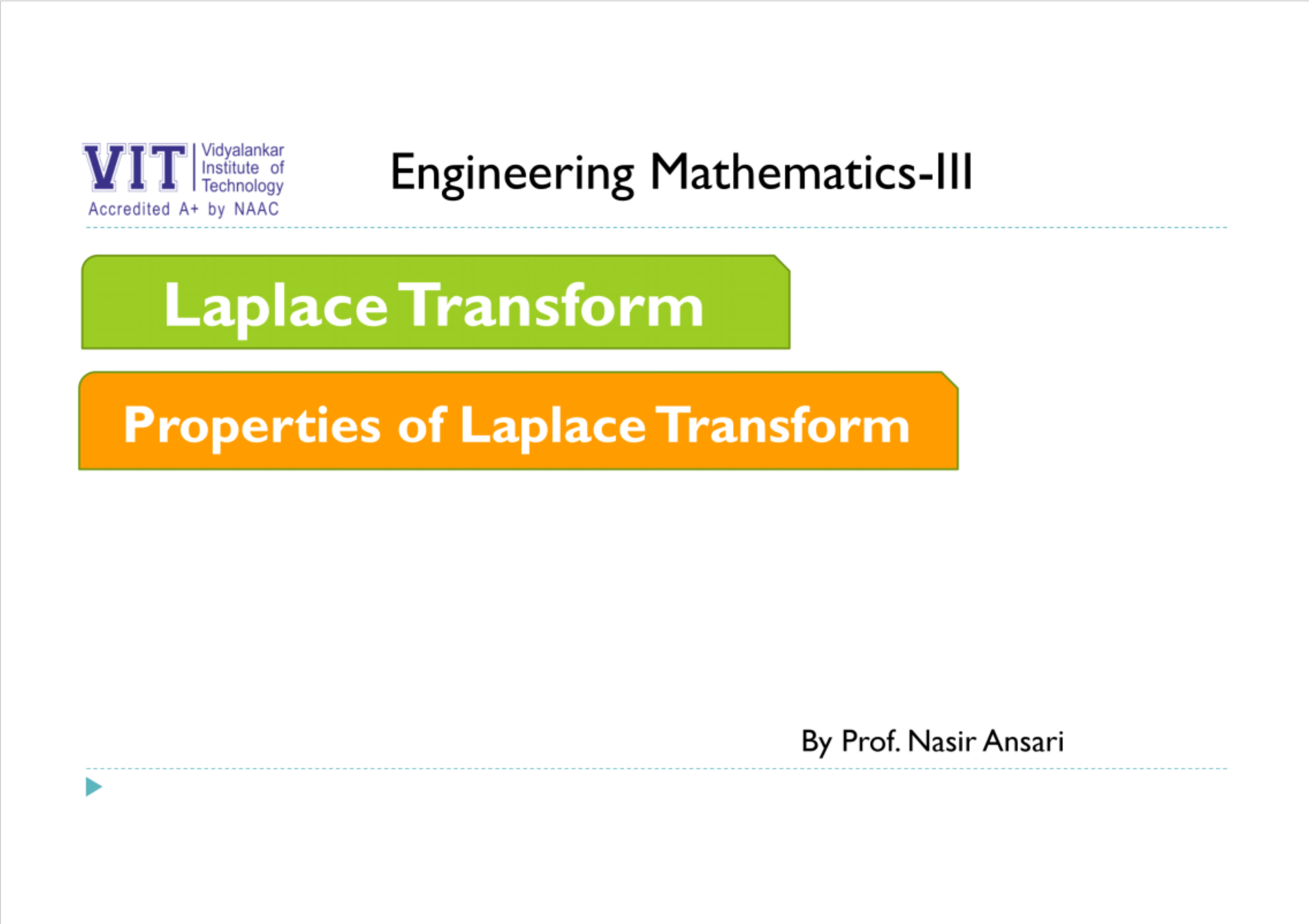
Laplace Transform-3



I. Change of Scale Property

Scaling

If $L\{f(t)\}=\phi(s)$ then $L\{f(at)\}=\frac{1}{a}\phi\left(\frac{s}{a}\right)$

$L\{f(t)\}=\int_0^\infty e^{-st} f(t)\,dt$

$=\phi(s)$

$L\{☺\}=\int_0^\infty e^{-st}(☺)\,dt$

$L\{f(at)\}=\int_0^\infty e^{-st} f(at)\,dt$

put $at=u$

$a\,dt=du$

t	0	∞
u	0	∞

$=\int_0^\infty e^{-s\left(\frac{u}{a}\right)} f(u)\,\frac{du}{a}$

$=\frac{1}{a}\int_0^\infty e^{-\left(\frac{s}{a}\right)u} f(u)\,du$

Laplace Tr of u

$s=s/a$

$=\frac{1}{a}\phi\left(\frac{s}{a}\right)$

If $L\{f(t)\}=\phi(s)$ then $L\{f(at)\}=\frac{1}{a}\phi\left(\frac{s}{a}\right)$

Q1. If $L\{f(t)\}=\frac{2s}{s^2+4}=\phi(s)$

then $L\{f(2t)\}=\,?$

Sol: Here $a=2$

$L\{f(at)\}=\frac{1}{a}\phi\left(\frac{s}{a}\right)$

$=\frac{1}{2}\phi\left(\frac{s}{2}\right)$

$=\frac{1}{2}\left[\frac{2(s/2)}{\left(\frac{s}{2}\right)^2+4}\right]$

$=\frac{1}{2}\left[\frac{s}{s^2+16}\right.$

$$= \frac{1}{2}\left[\frac{s}{\frac{s^2+16}{4}}\right]$$

$$= \frac{1}{2}\left[\frac{4s}{s^2+16}\right] = \frac{2s}{s^2+16}$$

② $L\{f(t)\} = \dfrac{s^2-s}{s^2+3s+5}$ $\quad L\{f(3t)\} = ?$

Ans $= \dfrac{1}{3}\left[\dfrac{s^2-3s}{s^2+9s+45}\right]$

Sol: Here $a=3$, $\phi(s) = \dfrac{s^2-s}{s^2+3s+5}$

$$L\{f(3t)\} = \frac{1}{3}\phi\left(\frac{s}{3}\right)$$

$$= \frac{1}{3}\left[\frac{\left(\frac{s}{3}\right)^2 - \left(\frac{s}{3}\right)}{\left(\frac{s}{3}\right)^2 + 3\left(\frac{s}{3}\right) + 5}\right]$$

$$= \frac{1}{3}\left[\frac{\frac{s^2-3s}{9}}{\frac{s^2+9s+45}{9}}\right]$$

$$= \frac{1}{3}\left[\frac{s^2-3s}{s^2+9s+45}\right]$$

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③ $L\{\sin\sqrt{t}\} = \dfrac{\sqrt{\pi}}{2s\sqrt{s}} e^{-\left(\frac{1}{4s}\right)}$

find $L\{\sin 2\sqrt{t}\} = ?$

Sol: Here $a=4$

$f(t) = \sin\sqrt{t}$

$f(4t) = \sin\sqrt{4t}$

$= \sin 2\sqrt{t}$

$a=4$

Ans $= \dfrac{1}{4}\phi\left(\dfrac{s}{4}\right)$

$$= \frac{1}{4}\left[\frac{\sqrt{\pi}}{2\frac{s}{4}\sqrt{\frac{s}{4}}} e^{-\left(\frac{1}{4\cdot\frac{s}{4}}\right)}\right]$$

$$= \left[\frac{\sqrt{\pi}}{s\sqrt{s}} e^{-1/s}\right]$$

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2. First Shifting Theorem

If $L\{f(t)\}=\phi(s)$ then $L\{e^{-at}f(t)\}=\phi(s+a)$

$$L\{e^{-at}f(t)\} = \phi(s+a)$$

$$L\{e^{at}f(t)\} = \phi(s-a)$$

① $L\{\sin at\} = \dfrac{a}{s^2+a^2}$ $\quad L\{e^{bt}\sin at\} = \dfrac{a}{(s-b)^2+a^2}$

② $L\{t^n\} = \dfrac{\lceil n+1}{s^{n+1}}$, $\quad L\{e^{-bt}t^n\} = \dfrac{\lceil n+1}{(s+b)^{n+1}}$

③ $L\{\cos at\} = \dfrac{s}{s^2+a^2}$, $\quad L\{e^{-bt}\cos at\} = \dfrac{s+b}{(s+b)^2+a^2}$

④ $L\{\sinh at\} = \dfrac{a}{s^2-a^2}$, $\quad L\{e^{bt}\sinh at\} = \dfrac{a}{(s-b)^2-a^2}$

Property 1: Scaling
If $L\{f(t)\}=\phi(s)$, $L\{f(at)\} = \frac{1}{a}\phi\left(\frac{s}{a}\right)$

Property 2: If $L\{f(t)\}=\phi(s)$, $L\{e^{at}f(t)\} = \phi(s-a)$

If $L\{f(t)\}=\phi(s)$, $L\{e^{-at}f(t)\} = \phi(s+a)$

✓ ① Scaling

✓ ② First shift

2. First shift
3. multipli by t
4. Division by t
5. Derivatives
6. Integrals

step1 ① Scaling

step2 ② Deri / Integr
$\boxed{\text{OR}}$

step3 ③ Mult / Division

step4 ④ First shifting

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Q. If $L\{f(t)\} = \dfrac{s}{s^2+s+4}$ then Find $L\{e^{-3t} f(2t)\}$

First shifting

scaling

Sol:

$$\boxed{L\{f(t)\} = \frac{s}{s^2+s+4} = \phi(s)}$$

$a = 2,$

$$L\{f(2t)\} = \frac{1}{2}\phi\left(\frac{s}{2}\right) \quad \text{(Scaling)}$$

$$= \frac{1}{2}\left[\frac{s/2}{\left(\frac{s}{2}\right)^2 + \left(\frac{s}{2}\right) + 4}\right]$$

$$= \frac{1}{2}\left[\frac{s}{2} \times \frac{4}{s^2+2s+16}\right]$$

$$= \frac{s}{s^2+2s+16}$$

$$L\{e^{-3t} f(2t)\} = \frac{s+3}{(s+3)^2 + 2(s+3) + 16}$$

$$= \frac{s+3}{s^2+6s+9+2s+6+16}$$

$$= \frac{s+3}{s^2+8s+31}$$

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Q1. Find the Laplace Transform of $\frac{\cos 2t \sin t}{e^t}$ $= e^{-t}\cos 2t \sin t$

Sol:

$$\cos 2t \sin t = \frac{1}{2}(2\cos 2t \sin t)$$

$$= \frac{1}{2}[\sin 3t - \sin t]$$

$$L\{\cos 2t \sin t\} = \frac{1}{2}[L\{\sin 3t\} - L\{\sin t\}]$$

$$= \frac{1}{2}\left[\frac{3}{s^2+9} - \frac{1}{s^2+1}\right]$$

$$L\{e^{-t}\cos 2t \sin t\} = \frac{1}{2}\left[\frac{3}{(s+1)^2+9} - \frac{1}{(s+1)^2+1}\right]$$

$$= \frac{1}{2}\left[\frac{3}{s^2+2s+10} - \frac{1}{s^2+2s+2}\right]$$

Evaluate $\int_0^\infty \frac{\cos 2t \sin t}{e^t}\,dt$

$$\int_0^\infty e^{-t}\cos 2t \sin t\,dt$$

$$L\{\cos 2t \sin t\} = \frac{1}{2}\left[\frac{3}{s^2+9} - \frac{1}{s^2+1}\right]$$

$$\Downarrow$$

$$\int_0^\infty e^{-st}\cos 2t \sin t\,dt = \frac{1}{2}\left[\frac{3}{s^2+9} - \frac{1}{s^2+1}\right]$$

put $s=1$

$$\int_0^\infty e^{-t}\cos 2t \sin t\,dt = \frac{1}{2}\left[\frac{3}{10} - \frac{1}{2}\right] =$$

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Q2. Find the Laplace Transform of $e^t \sin 2t \sin 3t$

Sol:

$$\sin 2t \sin 3t = \frac{1}{2}(2\sin 3t \sin 2t)$$
$$= \frac{1}{2}(\cos t - \cos 5t)$$
$$L\{\sin 2t \sin 3t\} = \frac{1}{2}\left[L\{\cos t\} - L\{\cos 5t\}\right]$$
$$= \frac{1}{2}\left[\frac{s}{s^2+1} - \frac{s}{s^2+25}\right]$$
$$L\{e^t \sin 2t \sin 3t\} = \frac{1}{2}\left[\frac{(s-1)}{(s-1)^2+1} - \frac{(s-1)}{(s-1)^2+25}\right]$$
$$= \frac{1}{2}\left[\frac{(s-1)}{s^2-2s+2} - \frac{(s-1)}{s^2-2s+26}\right]$$

Scaling Property

If $L\{f(t)\} = \phi(s)$

$L\{f(at)\} = \frac{1}{a}\phi\left(\frac{s}{a}\right)$

First shifting

If $L\{f(t)\} = \phi(s)$

then

$L\{e^{at} f(t)\} = \phi(s-a)$

$L\{e^{-at} f(t)\} = \phi(s+a)$

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Q3. Find the Laplace Transform of $e^{-4t} \sin ht \sin t$

$$\sinh t \sin t = \left(\frac{e^{t} - e^{-t}}{2}\right) \sin t$$

$$= \frac{1}{2}\left[e^{t} \sin t - e^{-t} \sin t\right]$$

$$L\{\sinh t \sin t\} = \frac{1}{2}\left[L\{e^{t} \sin t\} - L\{e^{-t} \sin t\}\right]$$

$$L\{\sin t\} = \frac{1}{s^2+1} , \quad L\{e^{t} \sin t\} = \frac{1}{(s-1)^2+1}$$

$$L\{e^{-t} \sin t\} = \frac{1}{(s+1)^2+1}$$

$$= \frac{1}{2}\left[\frac{1}{(s-1)^2+1} - \frac{1}{(s+1)^2+1}\right]$$

$$L\{e^{-4t} \sinh t \sin t\} = \frac{1}{2}\left[\frac{1}{(s+4-1)^2+1} - \frac{1}{(s+4+1)^2+1}\right]$$

$$= \frac{1}{2}\left[\frac{1}{s^2+6s+10} - \frac{1}{s^2+10s+26}\right]$$

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Q4. Find the Laplace Transform of $e^{2t}(1+t)^2$

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Q5. Evaluate $\int_0^\infty e^{-t} \sinh 2t \sin 3t \, dt$

$s=1$

Sol:

$$\sinh 2t \sin 3t = \left(\frac{e^{2t} - e^{-2t}}{2}\right) \sin 3t$$

$$= \frac{1}{2}\left[e^{2t} \sin 3t - e^{-2t} \sin 3t\right]$$

$$L\{\sinh 2t \sin 3t\} = \frac{1}{2}\left[L\{e^{2t} \sin 3t\} - L\{e^{-2t} \sin 3t\}\right]$$

$$L\{\sin 3t\} = \frac{3}{s^2+9}, \quad L\{e^{2t} \sin 3t\} = \frac{3}{(s-2)^2+9}$$

$$L\{e^{-2t} \sin 3t\} = \frac{3}{(s+2)^2+9}$$

$$\int_0^\infty e^{-st} \sinh 2t \sin 3t \, dt = \frac{1}{2}\left[\frac{3}{(s-2)^2+9} - \frac{3}{(s+2)^2+9}\right]$$

put $s = 1$

$$\therefore \int_0^\infty e^{-t} \sinh 2t \sin 3t \, dt = \frac{1}{2}\left[\frac{3}{10} - \frac{3}{18}\right]$$

$$= \frac{3}{2}\left[\frac{18-10}{180}\right] = \frac{3}{2}\left[\frac{8}{180}\right]$$

$$= \frac{4}{60} = \frac{1}{15}$$

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3 . Multiplication by t^n , $n \in N$

If $L\{f(t)\} = \phi(s)$ then $L\{t^n f(t)\} = \frac{d^n}{ds^n}\phi(s)$ where $n \in N$

If $L\{f(t)\} = \phi(s)$, $L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}\phi(s)$

$L\{t\, f(t)\} = (-1)\frac{d}{ds}\phi(s)$

$L\{t^2 f(t)\} = (-1)^2 \frac{d^2}{ds^2}\phi(s)$

$L\{t^3 f(t)\} = (-1)^3 \frac{d^3}{ds^3}\phi(s)$

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Q1. Find the Laplace Transform of $t \sin at$

multi by t

std.

Sol:

$$L\{\sin at\} = \frac{a}{s^2+a^2}$$

$$L\{t \sin at\} = (-1)^1 \frac{d}{ds}\left(\frac{a}{s^2+a^2}\right)$$

$$\boxed{L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}\phi(s)}$$

$$= -1\left(\frac{-2as}{(s^2+a^2)^2}\right)$$

$$= \frac{2as}{(s^2+a^2)^2}$$

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Q2. Find the Laplace Transform of te^{at}

Mult. by t

std. func.

$L\{e^{at}\} = \frac{1}{s-a}$

$L\{t\,e^{at}\} = (-1)^1 \frac{d}{ds}\left(\frac{1}{s-a}\right)$

$= -\left[\frac{-1}{(s-a)^2}\right]$

$= \frac{1}{(s-a)^2}$

$\boxed{\text{OR}}$

$t\,e^{at}$

std. func

first shift

$L\{t\} = \frac{\Gamma 2}{s^2} = \frac{1}{s^2}$

$L\{e^{at}t\} = \frac{1}{(s-a)^2}$

(1) Scaling — (1)

(2) First shift — (4)

(3) multi by t^n — (3)

(4) division by t^n

(5) derivative — (2)

(6) Integrals

$\boxed{\text{ORDER}}$

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Q3. Find the Laplace Transform of $(1+te^{-t})^3$

Sol: $(1+te^{-t})^3 = 1^3 + 3(1)^2(te^{-t}) + 3(1)(te^{-t})^2 + (te^{-t})^3$

$= 1 + 3te^{-t} + 3t^2e^{-2t} + t^3e^{-3t}$

$L\{(1+te^{-t})^3\} = L\{1\} + 3L\{te^{-t}\} + 3L\{t^2e^{-2t}\} + L\{t^3e^{-3t}\}$

$L\{t\} = \frac{\lceil 2}{s^2} = \frac{1}{s^2}$, $L\{e^{-t}t\} = \frac{1}{(s+1)^2}$

$L\{t^2\} = \frac{\lceil 3}{s^3} = \frac{2}{s^3}$, $L\{e^{-2t}t^2\} = \frac{2}{(s+2)^3}$

$L\{t^3\} = \frac{\lceil 4}{s^4} = \frac{6}{s^4}$, $L\{e^{-3t}t^3\} = \frac{6}{(s+3)^4}$

$= \frac{1}{s} + 3\left(\frac{1}{(s+1)^2}\right) + 3\left(\frac{2}{(s+2)^3}\right) + \frac{6}{(s+3)^4}$

$= \frac{1}{s} + \frac{3}{(s+1)^2} + \frac{6}{(s+2)^3} + \frac{6}{(s+3)^4}$

$L\{t^2 e^{-2t}\}$

$L\{e^{-2t}\, t^2\}$

e^{-2t}: shifting

t^2: funct

① Function — t^2 — Expo

② property — multiply by t, shifting

eg. $e^{-2t}\, t^3 \cosh 2t$

① $L\{e^{-2t}\, t^3 \cosh 2t\}$ sint

shifting ③ mult ② function ①

t^3 — mult by t ③; e^{-2t} — shift ②; $\cosh 2t$ — funct ①

If $L\{f(t)\} = \log\left(\frac{s+3}{s-2}\right)$

then find $L\{t^3 e^{-3t} f(4t)\}$?

then find $L\{t^3 e^{-3t} f(4t)\}$

$L\left\{\frac{d}{dt}\left(t^3 e^{-3t} f(4t)\right)\right\}$

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Q4. Find the Laplace Transform of $te^{-4t}\sin 3t$

Sol:

$$L\{\sin 3t\} = \frac{3}{s^2+9}$$

$$L\{t\sin 3t\} = (-1)\frac{d}{ds}\left(\frac{3}{s^2+9}\right)$$

$$= (-1)\left(\frac{-6s}{(s^2+9)^2}\right)$$

$$= \frac{6s}{(s^2+9)^2}$$

$$L\{e^{-4t}t\sin 3t\} = \frac{6(s+4)}{((s+4)^2+9)^2}$$

$$L\{\sin 3t\} = \frac{3}{s^2+9}$$

$$L\{e^{-4t}\sin 3t\} = \frac{3}{(s+4)^2+9}$$

$$L\{te^{-4t}\sin 3t\} = (-1)\frac{d}{ds}\left(\frac{3}{(s+4)^2+9}\right)$$

$$=$$

$$=$$

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Q5. Find the Laplace Transform of $t^2 e^{-t} \sin 4t$

Sol: $L\{\sin 4t\} = \dfrac{4}{s^2+16}$

$$L\{t^2 \sin 4t\} = (-1)^2 \frac{d^2}{ds^2}\left(\frac{4}{s^2+16}\right)$$

$$= 4\frac{d^2}{ds^2}\left(\frac{1}{s^2+16}\right)$$

$$= 4\frac{d}{ds}\left(\frac{-2s}{(s^2+16)^2}\right)$$

$$= -8\frac{d}{ds}\left(\frac{s}{(s^2+16)^2}\right)$$

$$= -8\left[\frac{(s^2+16)^2(1) - (s)[2(s^2+16)(2s)]}{(s^2+16)^4}\right]$$

$$= -8\left[\frac{(s^2+16) - 4s^2}{(s^2+16)^3}\right] = -8\left[\frac{-3s^2+16}{(s^2+16)^3}\right]$$

$$L\{e^{-t}t^2 \sin 4t\} = -8\left[\frac{-3(s+1)^2+16}{((s+1)^2+16)^3}\right] = \frac{24(s+1)^2 - 128}{((s+1)^2+16)^3}$$

put $s = 0$

$$= \frac{24-128}{(17)^3}$$

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Q6. Find the Laplace Transform of $t^5 \cosh t$

$$\frac{e^t + e^{-t}}{2}$$

Sol: $t^5 \cosh t = t^5\left(\frac{e^t + e^{-t}}{2}\right)$

$$= \frac{1}{2}\left[e^t t^5 + e^{-t} t^5\right]$$

$$L\{t^5\} = \frac{\lceil 6}{s^6} = \frac{120}{s^6} \quad \therefore L\{e^t t^5\} = \frac{120}{(s-1)^6}$$

$$\therefore L\{e^{-t} t^5\} = \frac{120}{(s+1)^6}$$

$$\therefore L\{t^5 \cosh t\} = \frac{1}{2}\left[\frac{120}{(s-1)^6} + \frac{120}{(s+1)^6}\right] = 60\left[\frac{1}{(s-1)^6} + \frac{1}{(s+1)^6}\right]$$

H.W

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Q7. Find the Laplace Transform of $t^3 \cos t$

mul by t   Real   funcn

OR

$$t^3 \cos t = t^3\left(\frac{e^{it}+e^{-it}}{2}\right)$$

$$= \frac{1}{2}\left(e^{it}t^3 + e^{-it}t^3\right)$$

$$L\{t^3\} = \frac{\lceil 4}{s^4}, \quad L\{e^{it}t^3\} = \frac{\lceil 4}{(s-i)^4}$$

$$= \frac{1}{2}\left[\frac{\lceil 4}{(s-i)^4} + \frac{\lceil 4}{(s+i)^4}\right]$$

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Q8. Evaluate $\int_0^\infty e^{-3t}\ t\ \cos t\ dt$

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4 . Division by t^n, $n \in N$, $n = 1, 2, 3, 4, \ldots$

If $L\{f(t)\} = \phi(s)$ then $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \phi(s)\,ds$ where $n \in N$

$\int_s^\infty$, $s \to 0$

$$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \phi(s)\,ds$$

$$L\left\{\frac{f(t)}{t^2}\right\} = \int_s^\infty \left(\int_s^\infty \phi(s)\,ds\right) ds$$

$$L\left\{\frac{f(t)}{t^3}\right\} = \boxed{\int_s^\infty \boxed{\int_s^\infty \boxed{\int_s^\infty \phi(s)\,ds}\,ds}\,ds}$$

1. $\log(ab) = \log a + \log b$
2. $\log a^k = k \log a$
3. $\log\left(\frac{a}{b}\right) = \log a - \log b$
4. $\log 1 = 0$
5. $\tan^{-1}\theta + \cot^{-1}\theta = \frac{\pi}{2}$
6. $\tan^{-1}a \pm \tan^{-1}b = \tan^{-1}\left(\frac{a \pm b}{1 \mp ab}\right)$
7. $\int \frac{1}{s}\,ds = \ln s$
8. $\int \frac{1}{s-a}\,ds = \ln(s-a)$
9. $\int \frac{1}{s^2+a^2}\,ds = \frac{1}{a}\tan^{-1}\left(\frac{s}{a}\right)$
10. $\int \frac{s}{s^2+a^2}\,ds = \frac{1}{2}\log|s^2+a^2|$
11. $\int \frac{1}{s^2-a^2}\,ds = \frac{1}{2a}\log\left|\frac{s-a}{s+a}\right|$

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Q1. Find Laplace Transform of $\frac{1-\cos t}{t}$

Sol: $L\{1-\cos t\} = L\{1\} - L\{\cos t\}$

$$= \frac{1}{s} - \frac{s}{s^2+1}$$

$$L\left\{\frac{1-\cos t}{t}\right\} = \int_s^\infty \left(\frac{1}{s} - \frac{s}{s^2+1}\right) ds$$

$$= \left[\ln s - \frac{1}{2}\ln(s^2+1)\right]_s^\infty$$

$$= \frac{1}{2}\left[2\ln s - \ln(s^2+1)\right]_s^\infty$$

$$= \frac{1}{2}\left[\ln s^2 - \ln(s^2+1)\right]_s^\infty$$

$$= \frac{1}{2}\left[\ln\left(\frac{s^2}{s^2+1}\right)\right]_s^\infty$$

$$= \frac{1}{2}\left[\{0\} - \left\{\ln\left(\frac{s^2}{s^2+1}\right)\right\}\right]$$

$$= -\frac{1}{2}\ln\left(\frac{s^2}{s^2+1}\right)$$

$$= \frac{1}{2}\ln\left(\frac{s^2+1}{s^2}\right)$$

$$\left[\ln\left(\frac{s^2}{s^2+1}\right)\right]_s^\infty$$

$$= \lim_{s\to\infty} \ln\left(\frac{s^2}{s^2+1}\right) - \lim_{s\to s} \ln\left(\frac{s^2}{s^2+1}\right)$$

$$= \lim_{s\to\infty} \ln\left(\frac{1}{1+\frac{1}{s^2}}\right) - \ln\left(\frac{s^2}{s^2+1}\right)$$

as it

as $s\to\infty$, $\frac{1}{s^2}\to 0$

$$= \log 1 - \ln\left(\frac{s^2}{s^2+1}\right)$$

$$= 0 - \ln\left(\frac{s^2}{s^2+1}\right)$$

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Q2. Find Laplace Transform of $\dfrac{e^{-t}\sin t}{t}$

Sol: $L\{\sin t\} = \dfrac{1}{s^2+1}$

$L\left\{\dfrac{\sin t}{t}\right\} = \int_s^\infty \dfrac{1}{s^2+1}\,ds$

$= \left[\tan^{-1} s\right]_s^\infty$

$= \tan^{-1}\infty - \tan^{-1} s$

$= \dfrac{\pi}{2} - \tan^{-1} s$

$= \cot^{-1} s$

$L\left\{e^{-t}\dfrac{\sin t}{t}\right\} = \cot^{-1}(s+1)$

$\because \tan^{-1}\theta + \cot^{-1}\theta = \dfrac{\pi}{2}$

$\cot^{-1}\theta = \dfrac{\pi}{2} - \tan^{-1}\theta$

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Q3. Find Laplace Transform of $\dfrac{e^{-2t}\sin 2t\cosh t}{t}$

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Two times integrati

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Q4. Find Laplace Transform of $\dfrac{1-\cos t}{t^2}$

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Q5. Show that $\int_0^\infty \frac{\sin at}{t}\, dt = \frac{\pi}{2}$

e^{-0t}

Sol: $L\{\sin at\} = \frac{a}{s^2+a^2}$

$L\left\{\frac{\sin at}{t}\right\} = \int_s^\infty \frac{a}{s^2+a^2}\, ds$

$= a\int_s^\infty \frac{1}{s^2+a^2}\, ds$

$= a \times \frac{1}{a}\left[\tan^{-1}\left(\frac{s}{a}\right)\right]_s^\infty$

$= \tan^{-1}\infty - \tan^{-1}\left(\frac{s}{a}\right)$

$= \frac{\pi}{2} - \tan^{-1}\left(\frac{s}{a}\right)$

$= \cot^{-1}\left(\frac{s}{a}\right)$

$L\left\{\frac{\sin at}{t}\right\} = \cot^{-1}\left(\frac{s}{a}\right)$

$\int_0^\infty e^{-st}\,\frac{\sin at}{t}\, dt = \cot^{-1}\left(\frac{s}{a}\right)$

put $s=0$

$\int_0^\infty \frac{\sin at}{t}\, dt = \cot^{-1}(0) = \frac{\pi}{2}$

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Q6. Show that $\int_0^{\infty} e^{-t} \frac{\sin^2 t}{t} dt = \frac{1}{4} \log 5$

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5 . Laplace Transform of Derivatives

If $L\{f(t)\}=\phi(s)$ then $L\{f'(t)\}= - f(0)+ s\,L\{f(t)\}$

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6 . Laplace Transform of Derivatives

$$\text{If} \quad L\{f(t)\} = \phi(s) \quad \text{then} \quad L\left\{\int_0^t f(u)\,du\right\} = \frac{1}{s}\phi(s)$$