EECE 5644 Homework 1

Show your work.

Name: ______________________________

1. Let $\underline{u} = [2,7,1]^T$, $\underline{v} = [3\ 0\ 4]^T$, $\underline{w} = [1,5,8]^T$

   Find: (a) $3\underline{u} - 4\underline{v}$

   (b) $2\underline{u} + 4\underline{v} - 5\underline{w}$

2. Find the coefficients c_1, c_2, c_3 such that $\sum_{i=1}^{3} c_i \underline{u}_i = [0,3,5]^T$. Here, $\underline{u}_1 = [1,-1,1]^T$, $\underline{u}_2 = [1,3,2]^T$, and $\underline{u}_3 = [2,0,1]^T$.

3. Find $\underline{u}\cdot\underline{v} = \underline{u}^T\underline{v}$, where $\underline{u} = [2, 3, 5, 7]^T$ and $\underline{v} = [1, -1, 1, -1]^T$.

4. Let $\underline{u} = [5, 4, 1]^T$, $\underline{v} = [3, -4, 1]^T$, $\underline{w} = [1, -2, 3]^T$.
   (a.) Compute the generalized cosine between all possible pairs.
   (b.) Which pair of distinct vectors is most colinear?
   (c.) Which pair are most orthogonal?
   (d) Are $\underline{u}$, $\underline{v}$, and $\underline{w}$ linearly independent?

5. Let $\underline{u} = [1, 2, 1]^T$ and $\underline{v} = [3, 5, 8]^T$. Find

Find:

(a) the generalized cosine between $\underline{u}$ and $\underline{v}$,

(b) the projection of $\underline{u}$ onto $\underline{v}$,

(c) the L_2 distance between $\underline{u}$ & $\underline{v}$,

(d) the L_1 distance between $\underline{u}$ & $\underline{v}$.

6. A hyperplane can be described by two vectors $\underline{p}$ and $\underline{w}$, and is the set of vectors $\underline{x}$ such that $(\underline{x} - \underline{p})^T \underline{w} = \underline{0}$.

In $\mathbb{R}^2$, a hyperplane consists of all vectors $[x_1\ x_2]^T$ such that $x_2 = m x_1 + b$, for fixed m and b. Find $\underline{p}$ and $\underline{w}$ in terms of m and b. Let $\underline{p} = [0, ?]^T$ and $\underline{w} = [?\ 1]^-$ for unique answers.

7. Write the vector $\underline{z} = [1\ 0\ 5]^T$ as a linear combination of $\underline{p}_1 = [1\ 0\ 1]^T$, $\underline{p}_2 = [1\ 1\ 3]^T$, and $\underline{p}_3 = [2\ 0\ -1]^T$.

8. Find all t, K so that $\underline{u} = [t \;\; K]^T$ and $\underline{v} = [3 \;\; 5]^T$ are orthogonal and $\underline{u}$ has unit length.

9. Consider a hyperplane which is normal to $\underline{u}_1 = [1 \; -1 \;\; 1]^T$, and passes through $\underline{W} = [2 \;\; 2 \;\; 2]^T$. Prove or disprove that $\underline{x} = [2 \;\; 3 \;\; 3]$ is in the hyperplane.

10 Let L be a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$, with $L([0\ 1]^T) = [-1\ 0]^T$, and $L([1\ 0]^T) = [3\ 5]^T$.

(a) Find a matrix representation for L in the standard ordered basis.

(b) Find $L([2\ 2]^T)$ and $L([1\ -1]^T)$

(c) Prove or disprove: $[2\ 3]^T$ is an eigenvector of L.

11. Prove or disprove: $\underline{v} = [2\ 3]^T$ is an eigenvector of $M = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}$

12. Consider the linear operator from $\mathbb{R}^2$ to $\mathbb{R}^2$ defined by matrix $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $V = \{[1\ 1]^T, [1\ -1]^T\}$, both in the standard ordered basis.

(a) Show that V is a basis for $\mathbb{R}^2$

(b) Find matrix K to express the linear operator in the basis V.

13. Let $\underline{X}$ denote a Gaussian column vector with mean vector $\bar{m}_X = [2\ 3]^T$, and covariance matrix $C_X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. The random vector $\underline{Y} = A\underline{X}$, where

$$A = \begin{bmatrix} -1 & -2 \\ 2 & 3 \end{bmatrix}.$$

(a.) Determine $E[\underline{Y}]$.

(b.) Find and simplify the covariance matrix of $\underline{Y}$.

(c.) Completely specify the probability density function of vector $\underline{Y}$.

14. Let X and Y be random variables with joint pdf

$$f_{XY}(x,y) = \begin{cases} \frac{K}{16}, & \text{if } -4 < x < 4 \text{ and } 2 < y < 4, \\ 0, & \text{otherwise} \end{cases}$$

(a) Find constant K.

(b.) Prove or disprove: X and Y are orthogonal.

(c) Prove or disprove: X and Y are independent.

(d.) Find $P[Y \leq 3 \mid X \geq 0]$.

15. Coin C_1 is selected with probability $1/3$; otherwise coin C_2 is chosen for a single flip. C_1 produces heads with probability $1/7$ and C_2 produces heads with probability $7/8$.

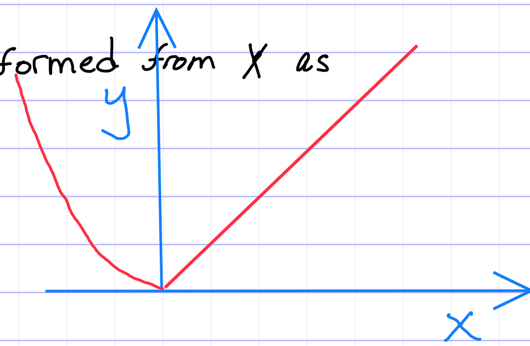
(a) A head is produced. Find the chance that C_2 was selected.

(b) Prove or disprove: the chance of heads is $1/2$.

16. Let X have probability density function $f_X(x) = \begin{cases} 2x, & 0 < x < \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$.
Find the characteristic function of X.

17. Let Y be a random variable formed from X as

$$Y = \begin{cases} X, & X > 0 \\ 2X^2, & X \leq 0 \end{cases}$$



(a.) Compute the cdf of Y, $F_Y(y)$, in terms of the cdf of X, $F_X(x)$.

(b) Find the pdf of Y, $f_Y(y)$, when $f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$.