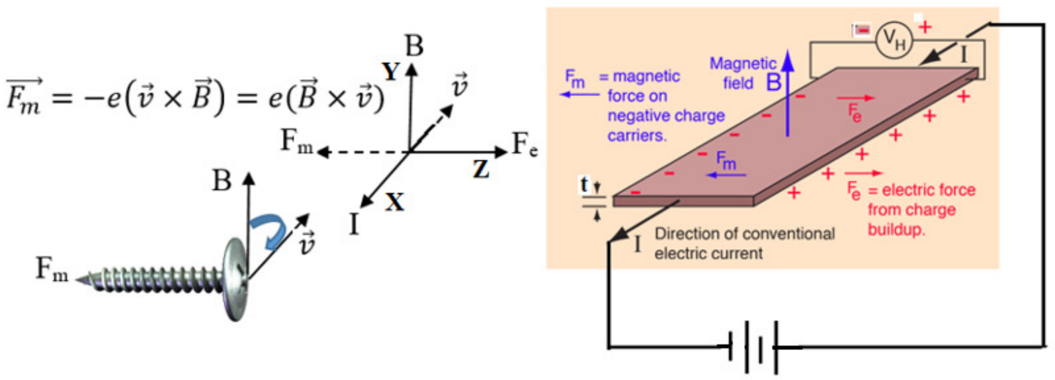
Hall-effect

When a current *I* flows along the X-direction (length *'l'*) of a rectangular sample of metallic conductor or a semiconductor placed in a magnetic field B acting along the Y-direction (thickness *'t'*) , an electric field is induced in the direction perpendicular to the direction of flow of the current and the direction of magnetic field (along the Z-direction - breadth *'b'*). The induced electric field is called the Hall-field (E_H). The electric potential difference developed along the breadth of the sample (Z-direction) due to the induced Hall-field is called Hall-voltage (V_H).

Hall-effect in a metallic conductor or an n-type semiconductor



Hall-effect in a metallic conductor or an n-type semiconductor

Dimension of the sample

Length - *l*, breadth - *b* and thickness - *t*

Lorentz force (F_m) and Coulombic attractive force (F_e) acting on free electrons

We consider that a current of I amperes flows along the positive X-direction due to free electrons in a metallic conductor or an n-type semiconductor. Then the velocity of these free electrons is along the negative X-axis. In the absence of magnetic field, the density of mobile electrons at any point inside the material is uniform. If a magnetic field is applied along the Y-direction, the magnetic Lorentz force $\vec{F}_m = -e(\vec{v}_d \times \vec{B}) = e(\vec{B} \times \vec{v}_d)$ acting on the drifting electrons deflects in the direction perpendicular to the magnetic field and the direction of the current as given by the screw rule. The mutually perpendicular vectors $\vec{B}$ and $\vec{v}_d$ are like lever arms. We imagine that these lever arms are fixed on the top head surface of the screw with groove. According to the screw rule, $(\vec{B} \times \vec{v}_d)$ means that the lever arm representing $\vec{B}$ must be rotated towards the lever arm representing $\vec{v}_d$; the direction of the movement of the tip of the screw represents the direction of the magnetic Lorentz force F_m. In this case, the magnetic Lorentz force F_m acts on the mobile electrons in the negative Z-axis direction. Therefore, they are deflected towards one side of the breadth and the density of free electrons is increased as shown in the above figure. Correspondingly, the density of immobile positive ions is increased on the other side of the breadth. Thus, Hall-field (E_H) is induced on the side of the breadth. The Hall-voltage developed across the breadth of the sample is $V_H = E_H \times b$. The negative terminal of a DC voltmeter is connected to the side of width where negatively charged free electrons are present relatively more and its positive terminal is connected to the side of width where the positively charged immobile ions are present. The polarity of the volt meter reveals the type of the given semiconductor. The immobile positive ions try to pull the deflected electrons by exerting the Coulombic attractive force $\vec{F_e} = (-e) \times E_H$ along the positive Z-axis direction to restore the normal condition. Thus the Coulombic attractive force (F_e) counterbalances the magnetic Lorentz force (F_m).

$$\text{At equilibrium, } |\vec{F_m}| = |\vec{F_e}|$$

$$Bev_d = eE_H - - - - - - - - - - - - - - - (1)$$

$$The\ current\ density\ (J) = n_n e v_d - - - - - (2)$$

In the case of an n-type semiconductor, n_n represents the concentration of free electrons in its conduction band.

In the case of a metallic conductor, n represents the concentration of free electrons in its conduction band.

Substituting for the drift velocity from the equation (2) in the equation (1),

$$E_H = \frac{V_H}{b} = \frac{BJ}{n_n e} = \frac{BI}{An_n e} = \frac{BI}{btn_n e} - - - - - - - - (3)$$

$$Hall - voltage\ (V_H) = \frac{1}{n_n e} \times \frac{BI}{t} = \frac{R_H BI}{t} - - - (4)$$

$$R_H = \frac{1}{n_n e}\ , the\ Hall - coefficient - - - - - - (5)$$

For an n-type semiconductor, $R_H = \frac{-1.18}{n_n e} = \frac{V_H t}{BI} - - - - - - - (6)$

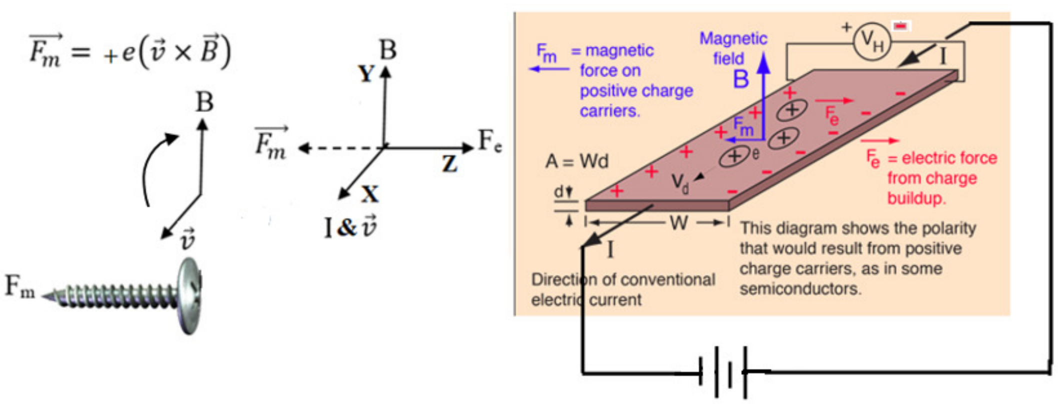
For a metallic conductor, $R_H = \frac{-1}{ne} = \frac{V_H t}{BI} - - - - - - - - - - (7)$

The negative sign indicates that the Hall-effect is due to electrons.

For the known values of *B, I* and *t*, the Hall-voltage can be measured and hence the Hall-coefficient can be found. If the Hall-coefficient is known, the concentration of electrons in the conduction band of the given semiconductor or a metallic conductor can be determined. The concentration of electrons in a metallic conductor is more than that in a semiconductor. Thus, we can distinguish between a conductor and a semiconductor.

Hall-effect in a p-type semiconductor

The Hall-effect experiment reveals the presence of holes in the valence band of a p-type semiconductor. We consider that a current of I amperes flows along the positive X-direction due to holes in a p-type semiconductor. Then the velocity of these holes is along the same positive X-axis.



Hall-effect in a p-type semiconductor

If a magnetic field is applied along the Y-direction, the magnetic Lorentz force $\vec{F}_m = e(\vec{v}_d \times \vec{B})$ acting on the drifting holes deflects in the direction perpendicular to the magnetic field and the direction of the current as given by the screw rule. The mutually perpendicular vectors $\vec{v}_d$ and $\vec{B}$ are like lever arms. We imagine that these lever arms are fixed on the top head surface of the screw with groove. According to the screw rule, $(\vec{v}_d \times \vec{B})$ means that the lever arm representing $\vec{v}_d$ must be rotated towards the lever arm representing $\vec{B}$; the direction of the movement of the tip of the screw represents the direction of the magnetic Lorentz force F_m. In this case, the magnetic Lorentz force F_m acts on the holes in the negative Z-axis direction. Therefore, they are deflected towards one side of the breadth and the density of holes is increased as shown in the above figure. Correspondingly, the density of immobile negative ions is increased on the other side of the breadth. Thus, Hall-field (E_H) is induced on the side of the breadth. The Hall-voltage developed across the breadth of the sample is $V_H = E_H \times b$.The positive terminal of a DC voltmeter is connected to the side of width where positively charged holes are present relatively more and its negative terminal is connected to the side of width where the negatively charged immobile ions are present. The polarity of the volt meter reveals the type of the given semiconductor. The immobile negative ions try to pull the deflected holes by exerting the Coulombic attractive force $\vec{F}_e = (+e) \times E_H$ along the positive Z-axis direction to restore the normal condition. Thus the Coulombic attractive force (F_e) counterbalances the magnetic Lorentz force (F_m).

$$\text{At equilibrium, } |\vec{F}_m| = |\vec{F}_e|$$

$$Bev_d = eE_H - - - - - - - - - - - - - - (1)$$

$$The\ current\ density\ (J) = p_p e v_d - - - - (2)$$

In the case of a p-type semiconductor, p_p represents the concentration of holes in its valence band.

Substituting for the drift velocity from the equation (2) in the equation (1),

$$E_H = \frac{V_H}{b} = \frac{BJ}{p_p e} = \frac{BI}{Ap_p e} = \frac{BI}{btp_p e} - - - - - - - - (3)$$

$$Hall - voltage\ (V_H) = \frac{1}{p_p e} \times \frac{BI}{t} = \frac{R_H BI}{t} - - - (4)$$

$$R_H = \frac{1}{p_p e}\ , the\ Hall - coefficient - - - - - - (5)$$

For a p-type semiconductor, $R_H = \frac{+1.18}{p_p e} = \frac{V_H t}{BI} - - - - - - (6)$

The positive sign indicates that the Hall-effect is due to holes.

For the known values of *B, I* and *t*, the Hall-voltage can be measured and hence the Hall-coefficient can be found. If the Hall-coefficient is known, the concentration of holes in the valence band of the given semiconductor can be determined.

Applications of the Hall-effect

1. The Hall-effect experiment is used to identify whether the given semiconductor is n-type or p-type.
2. By finding the Hall-coefficient of the given semiconductor, the concentration of charge carriers (electrons and holes) can be calculated.
3. By finding the conductivity of the given semiconductor and knowing the concentration of charge carriers (electrons and holes) at a particular temperature, the mobility of charge carriers can be calculated using the equations $|\mu_e| = \frac{\sigma_n}{n_n e}\ and\ |\mu_p| = \frac{\sigma_p}{p_p e}$.
4. By finding the mobility of charge carriers at a particular temperature, the relaxation time of charge carriers can be calculated using the equations

$$\tau_r(electrons) = \frac{m \times \mu_e}{e}\ and\ \tau_r(holes) = \frac{m \times \mu_h}{e}.$$

5. By knowing the current (I), Hall-coefficient (R_H), thickness of the sample (t) and measuring the Hall-voltage, the applied magnetic field can be calculated using the equation $B = \frac{V_H t}{R_H I}$. Gauss meter works based on this principle.