**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans.  *Option B is correct.*

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans. Given That , *μ* = 38 and *σ* =6.

1. statement A is true
2. statement B is True .
3. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans.

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| As we know that if X ∼ N(µ1, σ1^2 ), and Y ∼ N(µ2, σ2^2 ) are two independent random variables then X + Y ∼ N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 ) . |
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| Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ). |
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| Therefore , |
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| 2X1~ N(2 u,4 σ^2) and |
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X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )

2X1 –(X1-X2)~N(2u-2u,4 σ^2 +2 σ^2)~N(0u,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans.

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| Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order. |
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| The Probability of getting value between a and b should be 0.99. |
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| So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99). |
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| The Probability towards left from a = -0.005 (ie. 0.01/2). |
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| The Probability towards right from b = +0.005 (ie. 0.01/2). |
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| So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities. |
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| By finding the Standard Normal Variable Z (Z Value), we can calculate the X values. |
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| Z=(X- μ) / σ |
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| For Probability 0.005 the Z Value is -2.57 (from Z Table). |
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| Z \* σ + μ = X |
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| Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4 |
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| Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6 |
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So, option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?