Quantization

Quantization is the process of mapping continuous or high-resolution discrete values (such as floating-point numbers or 32-bit samples) into a finite set of discrete levels. It is a key operation in Digital Signal Processing (DSP), Analog-to-Digital Conversion (ADC), and compression algorithms.

(Figure: Analog waveform being mapped to discrete quantized levels)

Levels, L = 2^M → Example: 8 = 2^3 → 3 bits can uniquely represent 8 levels.

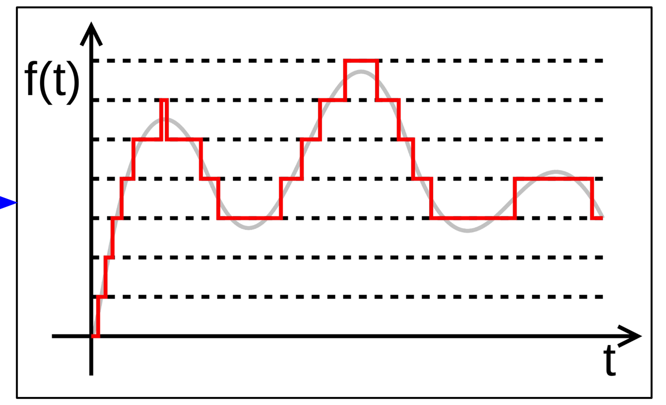
Level-to-Bit Mapping: ---→

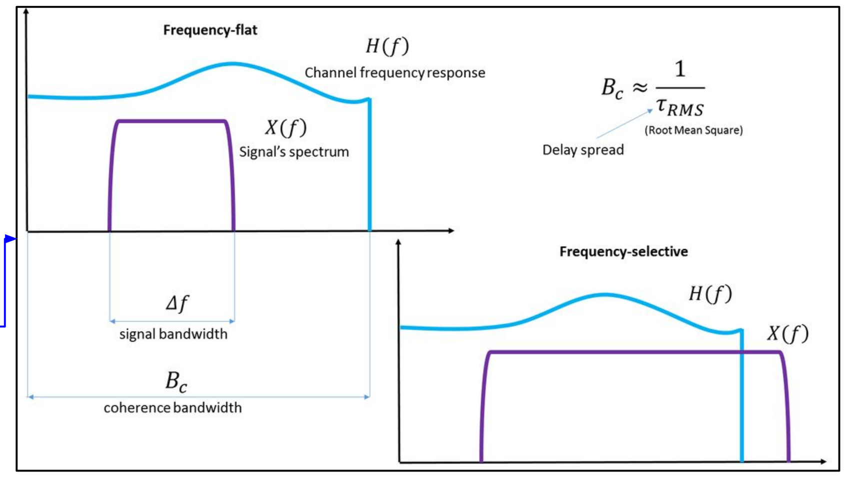
000 (0)	100 (4)
001 (1)	101 (5)
010 (2)	110 (6)
011 (3)	111 (7)

Types of Quantization

① **Uniform Quantization**
Equal step size across the input range. For example, an 8-bit ADC quantizes a 0–5 V signal into Δ = 5 V / 255 ≈ 19.6 mV steps (since 2^8 = 256 levels).

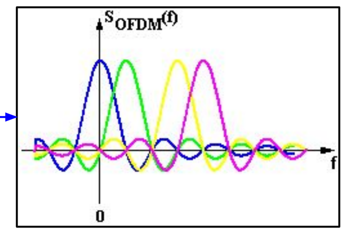
② **Non-Uniform Quantization**
Uses variable step sizes — smaller steps for more sensitive signal regions. For example, Log-Likelihood Ratio (LLR) quantization in 5G systems.





Why is OFDM robust to Multipath fading?

- **Multipath fading**: A Tx'd signal often travel through multiple path b4 reaching the Rx. Each path will suffer different delay, attenuation, and phase shift. (Freq. selective) This causes ISI due to delayed versions of previous symbols interfering with the current one.
- OFDM divides the wideband channel to multiple narrowband sub-channels. When a subcarrier bandwidth is narrow enough, the channel appears flat and experience only flat fading (Constant gain & phase).
- Each sub-carrier sees only mild distortion, which is much easier to equalize.



Orthogonality is achieved by,
Subcarrier Spacing $= \frac{1}{T} = \Delta f$ → OFDM symbol duration

$$\int_0^T e^{j2\pi f_k t} \cdot (e^{j2\pi f_m t})^* dt = \begin{cases} T & \text{if } k = m \\ 0 & \text{if } k \neq m \end{cases}$$

Orthogonality in OFDM:

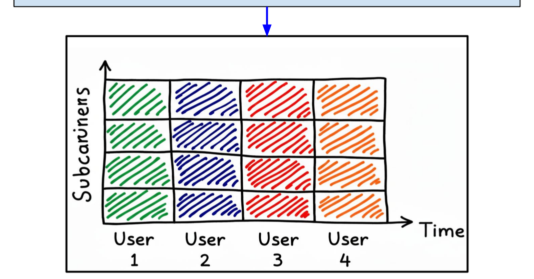
> In OFDM, the subcarriers (individual narrowband channels) are spaced such that they are mathematically orthogonal to each other, i.e., at the peak (maximum amplitude) of one subcarrier, all other subcarriers have zero amplitude, and so no interference.

What is OFDM & why is it used in modern wireless communication system?

> OFDM is a digital multi-carrier modulation technique where a high data-stream is split into multiple lower-rate streams and transmitted simultaneously ver several closely spaced orthogonal subcarriers.

1. **High spectral efficiency:** Subcarriers are orthogonal, i.e., they overlap in frequency without interfering, thus maximizing the data throughput.
2. **Robust to multipath fading:** Converts a frequency-selective fading channel into multiple flat-fading subchannels, reducing the inter symbol interference (ISI).
3. **Flexible resource allocation:** OFDM allows dynamic assignment of subcarriers to users ideal for systems like 4G/5G where the bandwidth varies.

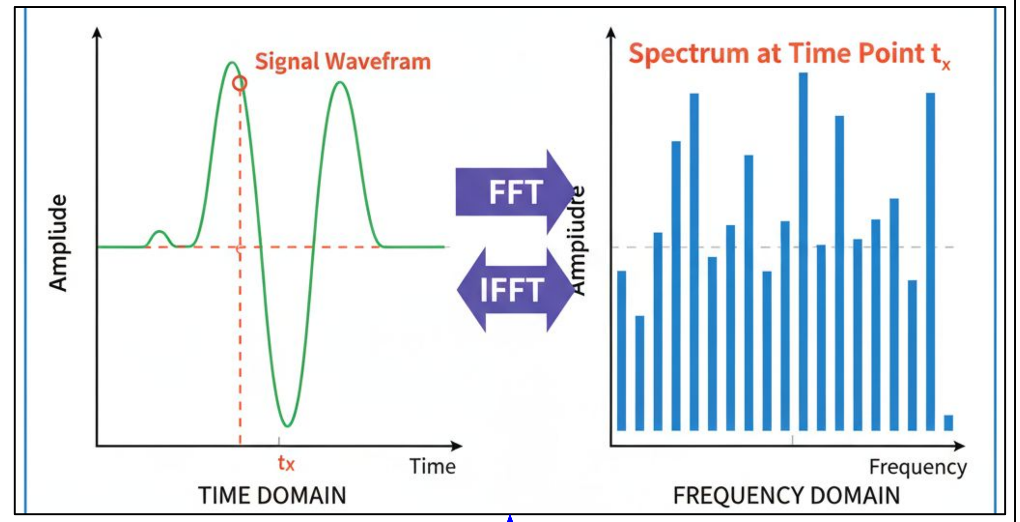
> In OFDM, all the subcarriers serve only 1 user at a time.
> Used in Wi-Fi 5 and prior versions.



Comparison: OFDM vs. FDM (Frequency Division Multiplexing)

Aspect	OFDM (Orthogonal Frequency Division Multiplexing)	FDM (Frequency Division Multiplexing)
Subcarrier Structure	Subcarriers **overlap without guard bands** due to orthogonality.	Subcarriers are **separated by guard bands** to prevent interference.
Spectral Efficiency	**High spectral efficiency**, as there are no guard bands — more data is transmitted in the same bandwidth.	**Low spectral efficiency**, since guard bands waste part of the spectrum.
Modulation Method	All subcarriers are **modulated jointly** using the **Inverse Fast Fourier Transform (IFFT)**.	Each subcarrier is **modulated independently**.
Multipath Handling	A **Cyclic Prefix (CP)** is added to combat inter-symbol interference caused by multipath delay.	**No Cyclic Prefix (CP)** is used; system is more sensitive to multipath distortion.

In OFDM, the available bandwidth is divided into multiple subcarriers, each carrying a portion of the data. These subcarriers are orthogonal, independently modulated, and transmitted in parallel.



Role of IFFT/FFT in OFDM

- **IFFT @ Tx**: freq. domain → time domain signals
- **FFT @ Rx**: time domain → freq. domain signals
- **IFFT operation**: combines all sub-carriers in a single time-domain signal. Freq. domain symbols $X_0, X_1, \ldots, X_{(N-1)}$ on N-subcarriers and QAM-modulated, a complex time-domain OFDM symbol.

$$S_n = \frac{1}{N} \cdot \sum_{k=0}^{N-1} X_k \cdot e^{\left(j2\frac{\pi kn}{N}\right)}, n = 0, 1, \ldots, N-1$$

($\frac{1}{N}$ ensures power consistency)

Calculation of Channel Delay Spread (τ_{max})

Definition

The **maximum delay spread** (τ_{max}) is defined as the time difference between the **first** and the **last significant multipath component** of a received signal.
It represents the **maximum excess delay** beyond which the signal power falls below a certain threshold — typically **30 dB** below the peak power level.

Measurement from Power Delay Profile (PDP)

Delay (μs)	Power (dB)
0.0	0.0 (Strongest)
0.5	−5.0
1.2	−15.0
2.1	−30.0 (Threshold)

From the above PDP data:

$$\tau_{max} = 2.1\ \mu s$$

Mean Excess Delay

The **mean excess delay** (τ_{mean}) is computed as the **power-weighted average** of the delay components:

$$\tau_{mean} = \frac{\sum_i P_i \tau_i}{\sum_i P_i}$$

where:

- P_i is the power of the i-th multipath component
- τ_i is the corresponding delay of that component

Why is CP added in OFDM?
- CP is a guard interval (kind of) added to each symbol to combat ISI and ICI.
- CP is a copy of the end portion of OFDM symbol prepended to its beginning.
- Eliminate ISI: Multipath propagation causes delayed copies of the signal to overlap with subsequent symbols. The CP acts as a buffer, absorbing this overlap so that the FFT window only captures the clean part of the symbol.
Preserves orthogonality (ICI): Without CP, multipath delays disrupt subcarrier orthogonality, causing interference between them. The CP ensures the delayed signal remains periodic within the FFT window, maintaining orthogonality.

CP Length calculation:
CP ≥ maximum channel delay spread (τ_max) + margin (10%-20% of τ_max)

OFDM: What is ICI and how is it mitigated

- In OFDM, ICI occurs when the orthogonality b/w subcarriers are lost, causing interference b/w them.
- **Causes**:
  - Doppler shift - due to high mobility, the freq. offsets b/w the Tx & Rx.
  - Phase Noise - from imperfect oscillators.
- **Mitigation Techniques**:
  - Freq. synchronization - use pilot tones
  - Guard interval - Cyclic prefix (CP)
  - Equalization: ZF and MMSE equalization can compensate for ICI.
  - Doppler compensation - In high mobility, use extended CP or adaptive OFDM parameters (i.e., shorter symbol duration).

Channel Determination for Each Subcarrier

1. Channel Estimation

1. **Pilot-Based Estimation**
Known pilot symbols are inserted at specific subcarriers in the OFDM frame. These pilots, whose transmitted values are known at the receiver, are used to estimate the channel response at those subcarriers.
2. **Estimation Algorithms**
Two common estimation algorithms are used:
   - **Least Squares (LS)** estimation
   - **Minimum Mean Square Error (MMSE)** estimation

   The estimated channel frequency response for the k-th subcarrier is given by:

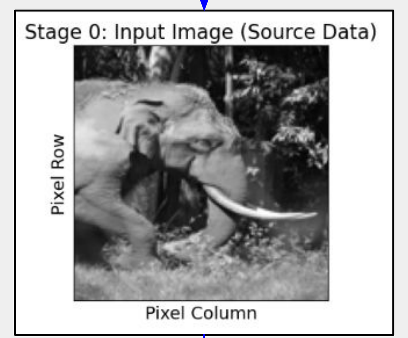
$$\hat{H}[k] = \frac{Y[k]}{X[k]}$$

   where $Y[k]$ is the received pilot symbol and $X[k]$ is the known transmitted pilot symbol.
3. **Interpolation**
Since pilots are not transmitted on every subcarrier, the channel must be interpolated for the subcarriers without pilots. Interpolation (linear, spline, or DFT-based) reconstructs the full channel response across all subcarriers.

OFDM SYSTEM MODEL

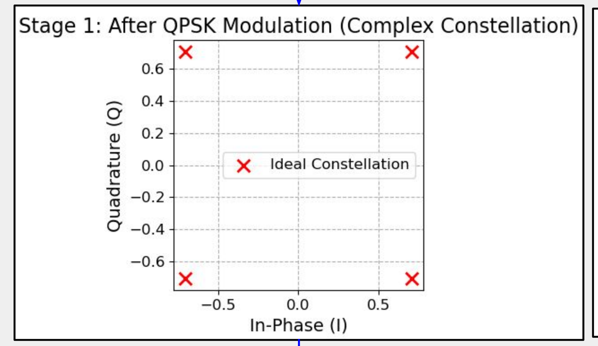
High Speed Data → Modulation → Serial-To-Parallel → N-Point Inverse FFT → Cyclic Prefix / Guard Insertion → Parallel-To-Serial → DAC → Channel h(t)

ADC → Parallel-To-Serial → Guard Removal → N-Point FFT → Parallel-To-Serial → Demodulation → Output Data



Input image = 128 x 128 = 16,384 pixels

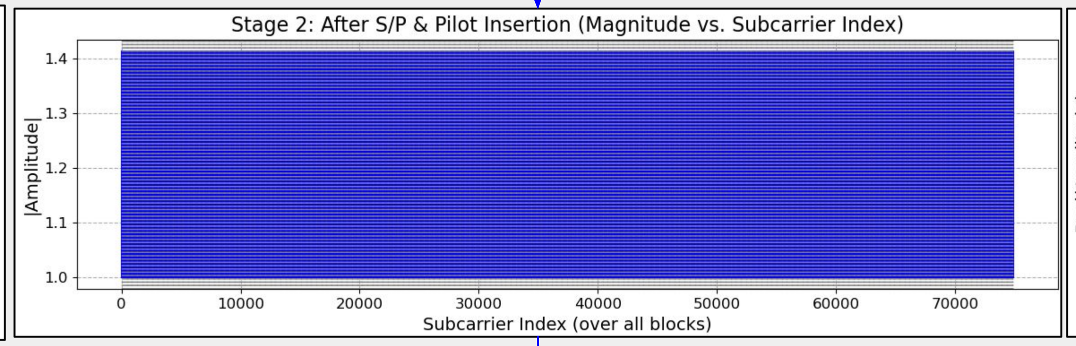
It's an 8-bit grayscale image, so total number of bits = 16384x8 = 131,072 bits



Total number of bits = 16384x8 = 131,072 bits

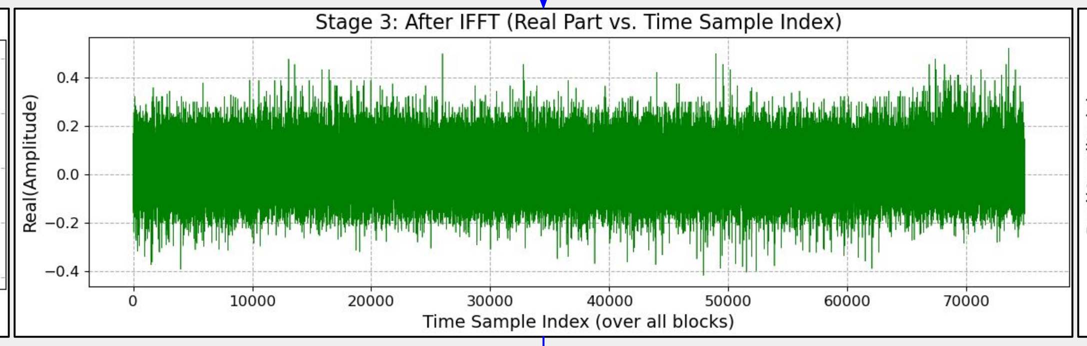
In QPSK, 2 bits are mapped to 1 complex symbol (i.e., bits_per_symbol = 2)
Therefore, symbols needed = 131,072 / 2 = 65,536 symbols.

If Number of subcarriers, N = 64 (56 data subcarriers + 8 pilots)
OFDM Truncation: To ensure the number of symbols is a perfect multiple of data subcarriers per OFDM block, the actual number of symbols are truncated to 65,520 symbols. I.e., 65520/56 = 1170 OFDM time-domain symbols.

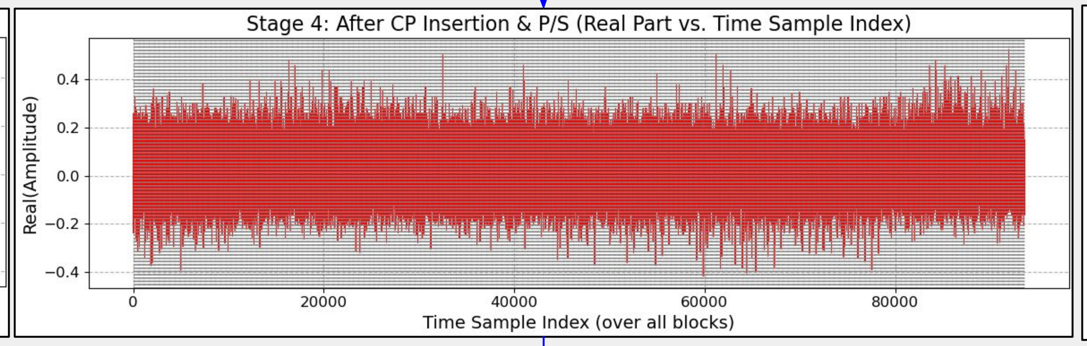


Total subcarriers = 64 (56 data + 8 pilot)
Therefore, total samples = 1170 OFDM symbols x 16 subcarriers = 74,880

The constant magnitude (around 1.414, the power of a QPSK symbol) confirms that all data subcarriers and pilot subcarriers have been set. But these are still in frequency domain.

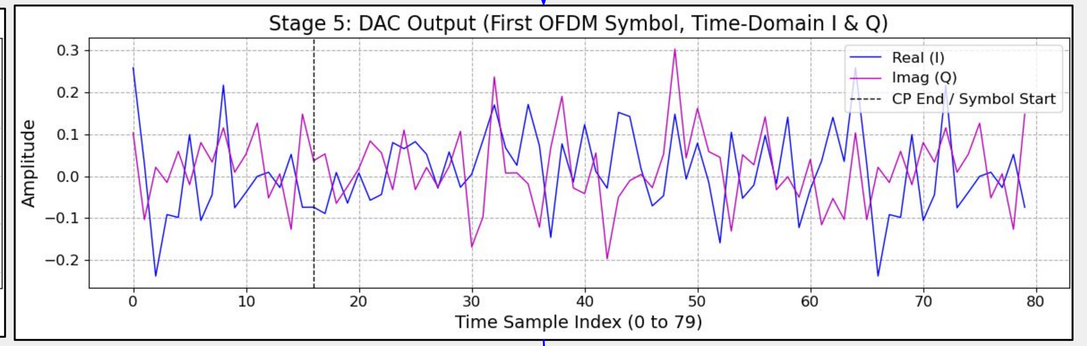


The IFFT combines the 64 orthogonal sine waves (subcarriers) into a composite time-domain signal. This signal is no longer discrete points like the constellation; it is a rapid sequence of complex numbers.



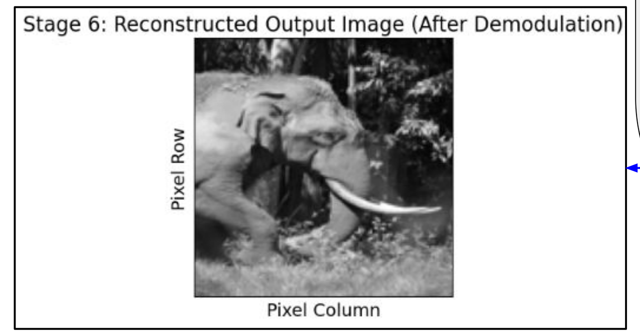
With CP = 16, it increases the length to 80 samples per block.

Therefore, total number of samples has increased from 74,880 (1,170×64) to 93,600 (1,170×80)

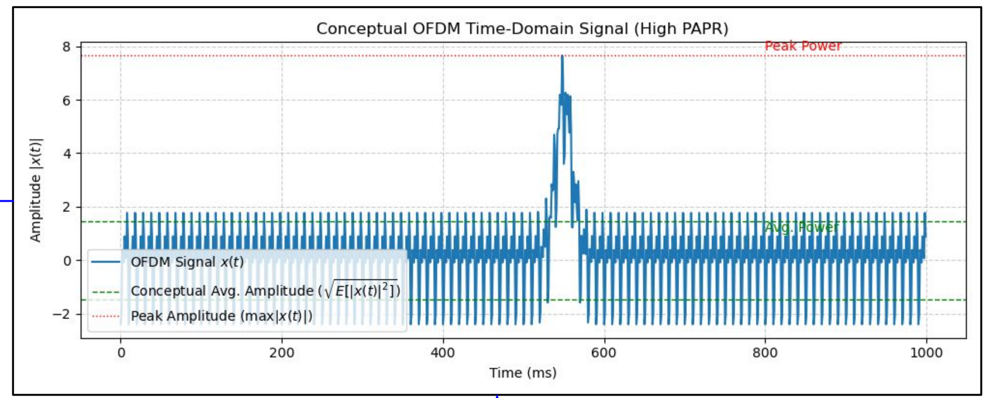


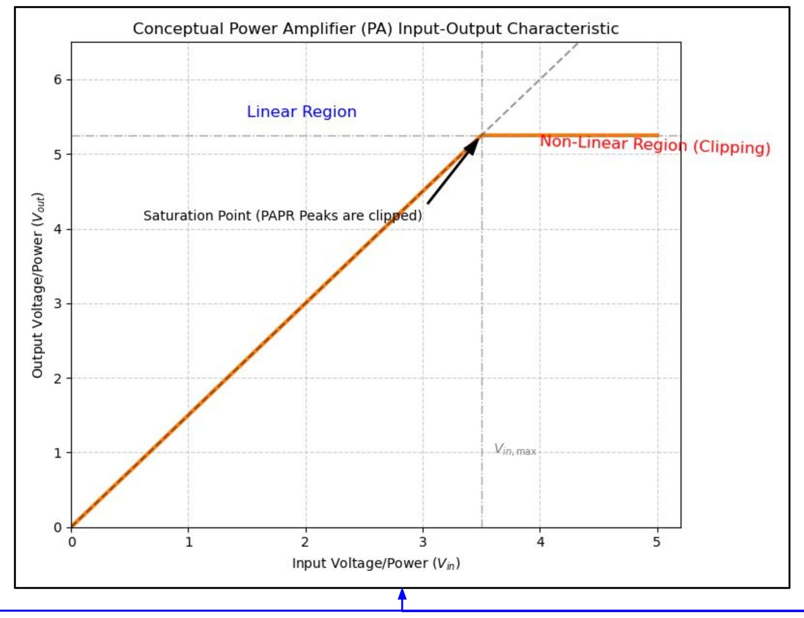
A **zoomed-in view** of the first single OFDM block (80 samples) from the signal in Stage 4, showing both the **In-Phase (I, real)** and **Quadrature (Q, imaginary)** components.

This shows the final digital signal that would be fed into the Digital-to-Analog Converter (DAC) and the RF upconverter. The dashed line at index **16** shows the boundary between the 16-sample CP and the 64-sample data portion of the OFDM symbol.



For sudden high peaks, it is difficult to quantize the signal. And all the signals information is lost in the quantization process.





Peak-to-Average Power Ratio (PAPR) Problem in OFDM

Definition

The **Peak-to-Average Power Ratio (PAPR)** of an OFDM signal is defined as:

$$\text{PAPR} = \frac{\max |x(t)|^2}{E[|x(t)|^2]} = \frac{\text{Peak Power}}{\text{Average Power}}$$

Explanation

In OFDM, the transmitted signal is a **sum of multiple subcarriers**. When these subcarriers **constructively interfere**, they produce **large peaks** in the time domain signal.
These high peaks make the **quantization process difficult** and can result in **signal distortion**.

Hence, OFDM systems suffer from a **high PAPR problem**, since the instantaneous signal power can be much higher than its average power.

Consequences of High PAPR

1. **Power Amplifier (PA) Inefficiency**
   - High PAPR forces the power amplifier to operate in a **non-linear region**, leading to energy loss and distortion.
2. **Signal Distortion**
   - When peaks are clipped, it introduces **non-linear distortion**, increasing the **Bit Error Rate (BER)**.
3. **Hardware Complexity**
   - High PAPR requires **expensive and highly linear power amplifiers**, which increase system cost and power consumption.

Illustration

- When the input voltage V_{in} increases beyond the linear region of the amplifier, the output V_{out} enters the **non-linear zone**, causing distortion.
- High PAPR signals frequently reach this region, degrading signal quality.

Role of FFT in OFDM

- **FFT**: extracts each sub-carrier's data,

$$Y_k = \sum_{n=0}^{N-1} r_n \cdot e^{\left(-j2\frac{\pi kn}{N}\right)}, k = 0, \ldots, N-1$$

freq. domain symbols corrupted by channel effects.

- **Equalization**: Compensates for channel distortion,

$$\hat{X}_k = \frac{Y_k}{H_k}$$

Peak Power and Average Power of a Signal

For a sinusoidal signal:

- x(t) = A · sin(2πft)

where:

- A → amplitude
- f → frequency

θ (degrees)	sin(θ)
0°	0.0
30°	0.5
45°	0.707
60°	0.866
90°	1.0

1. Peak Power (P_{peak})

The peak power corresponds to the maximum instantaneous power of the signal. For a sinusoidal waveform, since $\sin^2(\theta)$ reaches its maximum value of 1 (i.e., sin(θ)=±1):

$P(t) = v(t)^2 / R$

Assuming R = 1Ω: $P(t) = x^2(t) = A^2 \cdot \sin^2(2\pi ft)$ → Hence, $P_{peak} = A^2$

2. Average Power (P_{avg})

The average power over one period T is given by: $P_{avg} = (1/T) \int_0^T A^2 \cdot \sin^2(2\pi ft)\, dt$

Using the trigonometric identity: $\sin^2(\theta) = (1 - \cos(2\theta))/2$ → $P_{avg} = (A^2/T) \int_0^T (1 - \cos(4\pi ft))/2\, dt$

Since the cosine term averages to zero over one complete period, $P_{avg} = A^2 / 2$

3. Peak-to-Average Power Ratio (PAPR)

For a pure sine wave:

$\text{PAPR} = P_{peak} / P_{avg} = A^2 / (A^2 / 2) = 2$

In decibels: $\text{PAPR(dB)} = 10 \cdot \log_{10}(2) \approx 3$ dB

2. Channel Equalization (Determination of Transmitted Bits)

1. **Zero-Forcing (ZF) Equalization**
In this approach, the received symbol is divided by the estimated channel coefficient for each subcarrier:

$$\hat{X}[k] = \frac{Y[k]}{\hat{H}[k]}$$

2. **Minimum Mean Square Error (MMSE) Equalization**
The MMSE equalizer minimizes the overall error by considering both channel distortion and noise power:

$$\hat{X}[k] = \frac{Y[k] \cdot \hat{H}^*[k]}{|\hat{H}[k]|^2 + \sigma^2}$$

where

$$\sigma^2 = \frac{N_0}{E_S}$$

represents the noise variance normalized to the symbol energy.