Example – 1.1

Consider the most trivial array processor (beamformer) or (Spatial filter), for which $w_1 = w_2 = \cdots = w_M = 1$

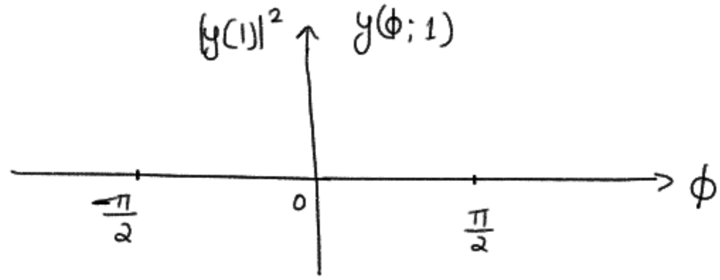
i.e $\bar{w} = 1_M$

Assume, that a constant value signal $\tilde{m}(t) = 1$ for every 't' arrives to our antenna array with DoA $= \phi$.

And let $d = \frac{\lambda_c}{2}$.

Do: In MATLAB, plot $|y(1)|^2$ in plain scale and dB as a function of ϕ.

$$\phi \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$



Discuss your findings?

Take $M = 2, 3, \ldots$.

Sol. Given: $m(t) = 1 \quad ; \quad \forall t$

$$\bar{w} = 1_M$$

Taking $M = 6 \rightarrow$ number of antenna elements

$$y(t) = \bar{w}^H . \bar{x}$$

and
$$\bar{x} = \tilde{m}(t) . \bar{s}(\phi)$$
$$= 1 . \bar{s}(\phi)$$

$$\therefore \quad y(t) = \bar{w}^H . \bar{s}(\phi)$$

Now; $\bar{s}(\phi) = \begin{bmatrix} 1 \\ e^{-j2\pi \cdot \frac{d}{\lambda_c} \cdot \sin(\phi)} \\ \vdots \\ e^{-j2\pi (M-1)\frac{d}{\lambda_c} \cdot \sin(\phi)} \end{bmatrix}$

Given; $d = \frac{\lambda_c}{2}$;

$$\bar{s}(\phi) = \begin{bmatrix} 1 \\ e^{-j\pi \cdot \sin(\phi)} \\ \vdots \\ e^{-j\pi (M-1) \sin(\phi)} \end{bmatrix}$$

$\therefore M = 6$; $\quad \bar{s}(\phi) = \begin{bmatrix} 1 \\ e^{-j\pi \sin\phi} \\ e^{-j2\pi \sin(\phi)} \\ e^{-j3\pi \sin\phi} \\ e^{-j4\pi \sin\phi} \\ e^{-j5\pi \sin\phi} \end{bmatrix}$

$$\therefore y(t) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ e^{-j\pi \sin\phi} \\ e^{-j2\pi \sin\phi} \\ e^{-j3\pi \sin\phi} \\ e^{-j4\pi \sin\phi} \\ e^{-j5\pi \sin\phi} \end{bmatrix}$$

$$\therefore \left[y(t) = 1 + e^{-j\pi \sin\phi} + e^{-j2\pi \sin\phi} + e^{-j3\pi \sin\phi} + e^{-j4\pi \sin\phi} + e^{-j5\pi \sin\phi} \right]$$

OBSERVATIONS :-

1> At $\phi = 0$, we have maximum energy , specifically equal to that of M^2.

$$\text{i.e} \quad M^2 = 6^2 = 36 .$$

2> As the number of antenna elements increases , the number of side lobes also increases . They are noticed to be $(M-2)$ side lobes .

3> For the given filter weights , i.e $\bar{w} = 1_M$, we observe that the received signal is able to differentiate at $\phi = 0$, better than at any other DoA / AoA .