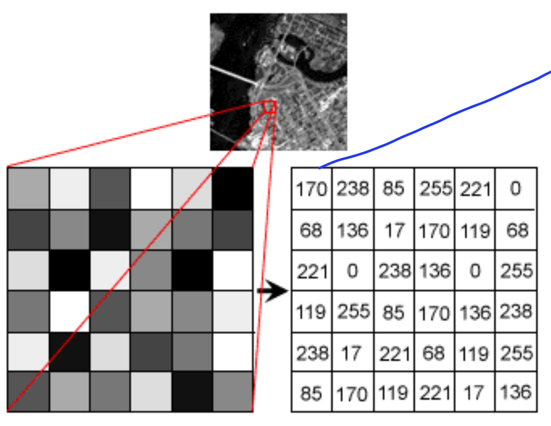
Comprehensive Explanation of OFDM Communication System with Digital Modulation

This code implements a complete OFDM (Orthogonal Frequency Division Multiplexing) communication system simulation with various digital modulation schemes. Let's break down all the mathematical concepts and technical components in detail.

1. Input
Grayscale image (128 x 128 pixels x 8 bits = 131,072 bits)

- First, the image is resized to 128 x 128 pixels.
- Each pixel value [0 to 255] is converted to 8-bit binary representation.
  ∴ if pixel value $= P$
  then, binary representation $b_7 b_6 \dots b_0$ is such that $P = \sum_{i=0}^{6} b_i \cdot 2^i$, where $b_i \in \{0, 1\}$



170 in binary

$$\begin{array}{r|l l} 2 & 170 & 0 \\ 2 & 85 & 1 \\ 2 & 42 & 0 \\ 2 & 21 & 1 \\ 2 & 10 & 0 \\ 2 & 5 & 1 \\ 2 & 2 & 0 \\ & 1 & \end{array}$$

$\therefore 170_{10} = 10101010_2$

2. BER Calculation loop (for each modulation scheme & SNR)

⇒ Modulation
The input binary stream is divided into groups of M-bits, where M = bits/symbol depending on the modulation scheme.

- Each M-bit group, $b = [b_{M-1}, b_{M-2}, \dots, b_0]$ is mapped to a complex symbol "S" from the constellation.
- o/p: Constellation points $\{S_0, S_1, S_2, \dots\}$

MODULATION	BITS PER SYMBOL
BPSK	1
QPSK	2
8PSK	3
16QAM	4
32QAM	5
64QAM	6

⇒ OFDM Symbol Generation
$N = \#$ of subcarriers
∴ A block of N complex symbols $(S_0, S_1, \dots, S_{N-1})$ is taken.

IFFT is applied to these symbols to generate time-domain OFDM symbols $\{x_0, x_1, \dots, x_{N-1}\}$

where $x_n = \frac{1}{\sqrt{N}} \sum_{K=0}^{N-1} S_K \cdot e^{\frac{j2\pi kn}{N}}$; for $n = 0, 1, 2, \dots, N-1$
↳ Ensures that the power of the time-domain signal is the same as the average power of the freq.-domain symbols.

⇒ Cyclic Prefix (CP) addition
The last CP samples of the time-domain OFDM symbol are prepended to the symbol.
symbols, $x_n = [x_0, x_1, \dots, x_{N-1}]_N$

Adding last CP samples, i.e
$x_{CP} = [x_{N-CP}, \dots, x_{N-1}, x_0, \dots, x_{N-1}]$.

Length of CP = N + CP

⇒ Channel Transmission
- AWGN: Noise $n(t)$ is added to the transmitted symbol $x_{cp}(t)$ to get the received signal,
  $y(t) = x_{cp}(t) + n(t)$
  ↳ Complex Gaussian Random Variable with zero mean & variance determined by SNR.

i.e., for a given SNR value → $SNR_{linear} = \frac{P_s}{\sigma_n^2}$

$\therefore \sigma_n^2 = \frac{P_s}{SNR_{Linear}}$

- Multipath Fading
  The received signal is convolved with the channel impulse response $h[n]$.

In this case, channel_taps $= [0.9, 0.2\, e^{-j\frac{\pi}{4}}]$
i.e., $h[n] = 0.9\, \delta[n] + 0.2 \cdot e^{j\pi/4} \cdot \delta[n-1]$
↳ LOS ↳ Delayed version

$\therefore r[i] = (y * h)[i] = \sum_K y[K] \cdot h[i-K]$
$= 0.9 \cdot y[i] + 0.2 \cdot e^{-j\frac{\pi}{4}} \cdot y[i-1]$

⇒ OFDM Demodulation
- CP removal:
  The first CP samples are discarded
  $y'[i] = r[i + CP]$, for $i = 0, 1, \dots, N-1$
- FFT:
  The FFT is applied to get the received frequency-domain symbols, Y_K.
  $Y_K = \sum_{n=0}^{N-1} y'[n] \cdot e^{-j\frac{2\pi nK}{N}}$, for $K = 0, 1, \dots, N-1$

⇒ Channel Estimation
- At the pilot subcarrier positions, $p \in$ pilot_positions, the received pilot symbol is related to the Tx'd pilot symbol X_p (which is known), and so the channel frequency response,
  $Y_p = X_p \cdot H_p + N_p$
  ↳ noise @ this subcarrier
- ∴ Channel estimate at the pilot subcarrier will be :-
  $\hat{H}_p = \frac{Y_p}{X_p} \approx H_p$ → ignoring noise for the ideal state.

⇒ Equalization
To compensate for channels effect, $\hat{S}_K = \frac{Y_K}{\hat{H}_K}$

⇒ Demodulation
- for each received and equalized symbol: $\hat{S}_K$, the euclidean distance to each point in the constellation is calculated:
  $d_i^2 = |\hat{S} - C_i|^2$, where C_i is the i^{th} constellation point.
- The constellation point with the minimum distance is chosen as the most likely transmitted symbol.
- The index of this closest constellation point is then mapped back to the corresponding M-bit sequence.

⇒ BER calculation
The demodulated bits are compared to the original transmitted bits, and the BER is calculated as the number of bit errors divided by the total number of transmitted bits.