ECON 390 Assignment 3

SID2

10/14/2021

Probability Estimation

```
nsim = 10000
set.seed(123)
```

1. (Points 3) Suppose that X approx Exp(1/2). Calculate the probability that X <= 3 using simulation i.e. Pr(X <= 3).

```
lambda = 1/2
X = rexp(nsim, rate = lambda)
sim_prob = mean((X <= 3) )
sim_prob
```

```
## [1] 0.7765
```

2. (Points 3) Now, again, suppose that X approx Exp(1/2). Calculate the probability that X2 <= 3 using simulation i.e. Pr(X2 <= 3).

```
sim_prob = mean((X^2 <= 3) & (X^2 >= -3))
sim_prob
```

```
## [1] 0.5775
```

3. (Points 30) Suppose that a firm can release one of three products or nothing at all. A firm will release a product that makes the most profit. The revenue of each product j is simply rj = j. So the revenue of the first product is 1, the second product is 2, the third product is 3, and not releasing anything (the zero product) is 0. However, suppose that the cost of production is cj Exp(1/2). Note, each product has it's own cost cj . The cost of not releasing anything is 0. Profits are then pij = rj - cj . From an econometric perspective, we can usually observe revenues but costs are trickier, so we are assuming they are random. Estimate the probability that the firm will release each product (including nothing at all). Again, a product is released if it results in the most amount of profit. Hint: Form a profit matrix that is Nsim $\times$ 4 where each row is a simulation and each column is the profit of a product. 1 Then, for each row, see which column as the highest amount of profit. Lastly, use these results to estimate the probability

```r
j = 1:4
rj = 1:4
lambda = 1/2
cj = rexp(4*nsim, rate = lambda)
cj = matrix(cj,ncol = 4)
rj = matrix(rep(1:4, each = nsim), ncol = 4)
numer = exp(rj - cj)
denom = apply(numer, 1, sum) + 1
cprob = numer/denom
cprobj = apply(cprob, 2, mean)
cprobj
```

```
## [1] 0.04864453 0.11970532 0.26239587 0.51037178
```

Expectation Estimation

1. (Points 3) Suppose that X aprox Exp(1/2). Calculate E[f(X)] where f(x) = x2.

```r
mean(X^2)
```

```
## [1] 8.028965
```

2. (Points 3) Now, again, suppose that X aprox Exp(1/2). Calculate E[f(X)] where f(x) = sin(x).

```r
mean(sin(X))
```

```
## [1] 0.4073608
```

3. (Points 3) Calculate E[f(vi))] where f(vi) = e2+vi 1+e2+vi and vi ~ N(0, 1).

```r
norm_draws = rnorm(nsim, mean = 0, sd = 1)
fvi = ((exp(2 +norm_draws))/(1 +(exp(2+norm_draws))))
mean(fvi)
```

```
## [1] 0.8467375
```

4. (Points 15) The CRRA utility function programmed in the second problem set is used a lot in econ to model risk averse agents whose preferences for risk change as their initial amount of wealth changes. Suppose that an agent has some initial wealth w0 = 100 and faces some unknown shock epsi tomorrow where epsi ~ N(0, 4). epsii is added to the initial wealth w0. Calculate the expected utility for agents with risk aversion parameters of n = 0.5, 1, 2, 5, 10. Note that expected utility is E[CRRA(w0 + epsi; n].

```r
CRRA = function(c, eta) {
  if(eta < 0 | !(length(eta) == 1)) {
    stop("invalid input")
  }
  count = 1;
  output = rep(0, length(c))
```

```
  for(i in c) {
    if(eta != 1) {
      output[count] = (i^(1-eta) - 1)/(1 - eta)
    } else {
      output[count] = log(i)
    }
    count = count + 1
  }
  return(output)
}

norm_draws = rnorm(nsim, mean = 0, sd = 4)
riskAverrsion = c(0.5, 1, 2, 5, 10)
count  = 1;
for(i in riskAverrsion) {
  expectedUtility = mean(CRRA(100 + norm_draws, i))
  print(paste("N: ", toString(i), " E[CRRA(w0 + epsi; n]: ", toString(expectedUtility)))
  count = count + 1
}
```

```
## [1] "N:  0.5  E[CRRA(w0 + epsi; n]:  17.9958996941065"
## [1] "N:  1  E[CRRA(w0 + epsi; n]:  4.60435319055448"
## [1] "N:  2  E[CRRA(w0 + epsi; n]:  0.98998365787588"
## [1] "N:  5  E[CRRA(w0 + epsi; n]:  0.249999997458694"
## [1] "N:  10  E[CRRA(w0 + epsi; n]:  0.111111111111111"
```

5. (Points 15) Repeat the last question, but with w0 = 1,000.

```
count  = 1;
for(i in riskAverrsion) {
  expectedUtility = mean(CRRA(1000 + norm_draws, riskAverrsion[count]))
  print(paste("N: ", toString(riskAverrsion[count]), " E[CRRA(w0 + epsi; n]: ", toString(expectedUtilit
  count = count + 1
}
```

```
## [1] "N:  0.5  E[CRRA(w0 + epsi; n]:  61.2454128477102"
## [1] "N:  1  E[CRRA(w0 + epsi; n]:  6.90774678481361"
## [1] "N:  2  E[CRRA(w0 + epsi; n]:  0.998999983393979"
## [1] "N:  5  E[CRRA(w0 + epsi; n]:  0.24999999999975"
## [1] "N:  10  E[CRRA(w0 + epsi; n]:  0.111111111111111"
```

The Wage Gap: Bad Controls

1. What is the data generating process and what are the data? We are generating the wage of men and women, and exploring the difference and correlation of regressors when looking at wage gap between men and women

2. Monte Carlo Simulation

```
low = 15
high = 30
```

```r
n = 10000

c1 = cbind(rep(0, n), rep(0, n))
c2 = cbind(rep(0, n), rep(0, n))
c3 = cbind(rep(0, n), rep(0, n), rep(0, n))

for(i in 1:n) {
  gender = sample(c("M", "F"), n, TRUE, c(0.5, 0.5))
  jobType = rep(0, n)
  jobType[which(gender == "M")] = sample(c("low", "high"), length(gender[gender == "M"]), TRUE, c(0.5, 
  jobType[which(gender == "F")] = sample(c("low", "high"), length(gender[gender == "F"]), TRUE, c(0.75,
  wage = rep(0, n)
  wage[which(jobType == "low")] = low
  wage[which(jobType == "high")] = high
  c1[i, ] = glm(wage ~ factor(gender))$coef
  c2[i, ] = glm(wage ~ factor(jobType))$coef
  c3[i, ] = glm(wage ~ factor(gender) + factor(jobType))$coef
}
```

3) Look at the mean of the coefficients in each regression across models. Compare the mean of the sex coefficient in the two models that control for sex. What does this tell us about controlling for occupation in this model?

```r
apply(c1, 2, mean)
```

```
## [1] 18.750996  3.748781
```

```r
apply(c2, 2, mean)
```

```
## [1]  30 -15
```

```r
apply(c3, 2, mean)
```

```
## [1]  3.000000e+01  5.142875e-15 -1.500000e+01
```

Due to a decrease in gender, controlling of occupation underestimates the true wage gpa