Exercise 11.1. [2pts] Consider an elliptic curve \mathcal{E} defined by $y^2 = x^3 + x + 3$ over \mathbb{Z}_{13} . Is it singular?

Exercise 11.2. [10pts] Find all points on the elliptic curve \mathcal{E} defined by $y^2 = x^3 + 2x + 3$ over \mathbb{Z}_{13} . You can proceed like in class: for each value $x \in \mathbb{Z}_{13}$ find solutions of $y^2 = x^3 + 2x + 3$. (The table of square roots modulo 13 on page 10 of lecture 6 can be useful).

Exercise 11.3. [10pts] For the curve \mathcal{E} from the previous problem compute

- (a) (4,7) + (9,10),
- (b) (4,7) + (4,7).

Please, show computations (at least show the value of the slope λ).

Exercise 11.4. [10pts] Consider the curve \mathcal{E} defined on page 10 of lecture 11. Use the addition table on page 11 to compute the order and the cyclic subgroup generated by each of the following points:

- (a) (1,5),
- (b) (9,6),
- (c) (12, 2).