Time Series Analysis final Project - Competitive

MATH 1318 Time Series Analysis Final Project

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5 After the absolute value transformation, we boserve many signficicant lags in **29**

6 both ACF and PACF. Also, EACF do not suggest an ARMA(0,0) model. **29**

7 From the EACF, we can identify ARMA(1,0), ARMA(1,1), and ARMA(2,1) models for absolute **29**

8 value series. **29**

9 These models correspond to parameter settings of [max(1,1),1], [max(1,2),1] and [max(2,2),2]. **29**

10 So the corresponding tentative GARCH models are GARCH(0,1), GARCH(1,1), GARCH(1,2). **29**

11 After the square transformation, we boserve many signficicant lags in both ACF and PACF. Also, EACF do not suggest an ARMA(0,0) model. **30**

12 From the EACF, we can identify ARMA(1,1), ARMA(1,2), and ARMA(2,2) models for squared series. **30**

13 These models correspond to parameter settings of [max(1,1),1], [max(1,2),1], [max(1,2),2], and [max(2,2),2]. So the corresponding **30**

14 tentative GARCH models are GARCH(1,1), GARCH(2,1), GARCH(2,2). **30**

1 Introduction

Bitcoin is a type of cryptocurrency, i.e. it is a digital currency which uses encryption techniques to generate units of the currency and verify the transfer of funds. Bitcoin is a decentralised currency, which operates independently of a central bank. An estimated 2.9 to 5.8 million unique users have a *cryptocurrency wallet*, of which most use bitcoin. The price of bitcoin has gone through various cycles of appreciation and depreciation, known as bubbles and bursts, with price fluctuations up to a magnitude of a few thousand USD in the space of a day, so that the currency has become renown for its volatility. The bitcoin historical price data gathered from the CoinMarketCap. This time series will be modelled using regression, ARIMA and GARCH methods. The report details;

- Description of the time series
- Model specification
- Model fitting and selection
- Diagnostic checking
- Predict the value of bitcoin for the following 10 days

2 Initial Diagnosis

```r
# Import Libraries
library(TSA)
library(fUnitRoots)
library(forecast)
library(CombMSC)
library(lmtest)
library(fGarch)
library(rugarch)
library(zoo)
library(ggplot2)
require(readr)
require(FitAR)

Bitcoin <- read.csv("../data/Bitcoin_Historical_Price.csv", header=TRUE)
Bitcoin$Date = as.Date(Bitcoin$Date,'%Y-%m-%d')
Bitcoin.zoo <- zoo(Bitcoin$Close, Bitcoin$Date)
Bitcoin.raw = Bitcoin.zoo
```

Data is converted to time series object using zoo library. Figure 1 shows the daily closing price of bitcoin from the 27th Apr 2013 to the 3rd Mar 2018, given in USD.

```r
autoplot.zoo(Bitcoin.zoo) +
  ylab('Closing Price (USD)') +
  xlab('Time (days)') +
  ggtitle("Time Series Plot for Daily Bitcoin Prices")
```

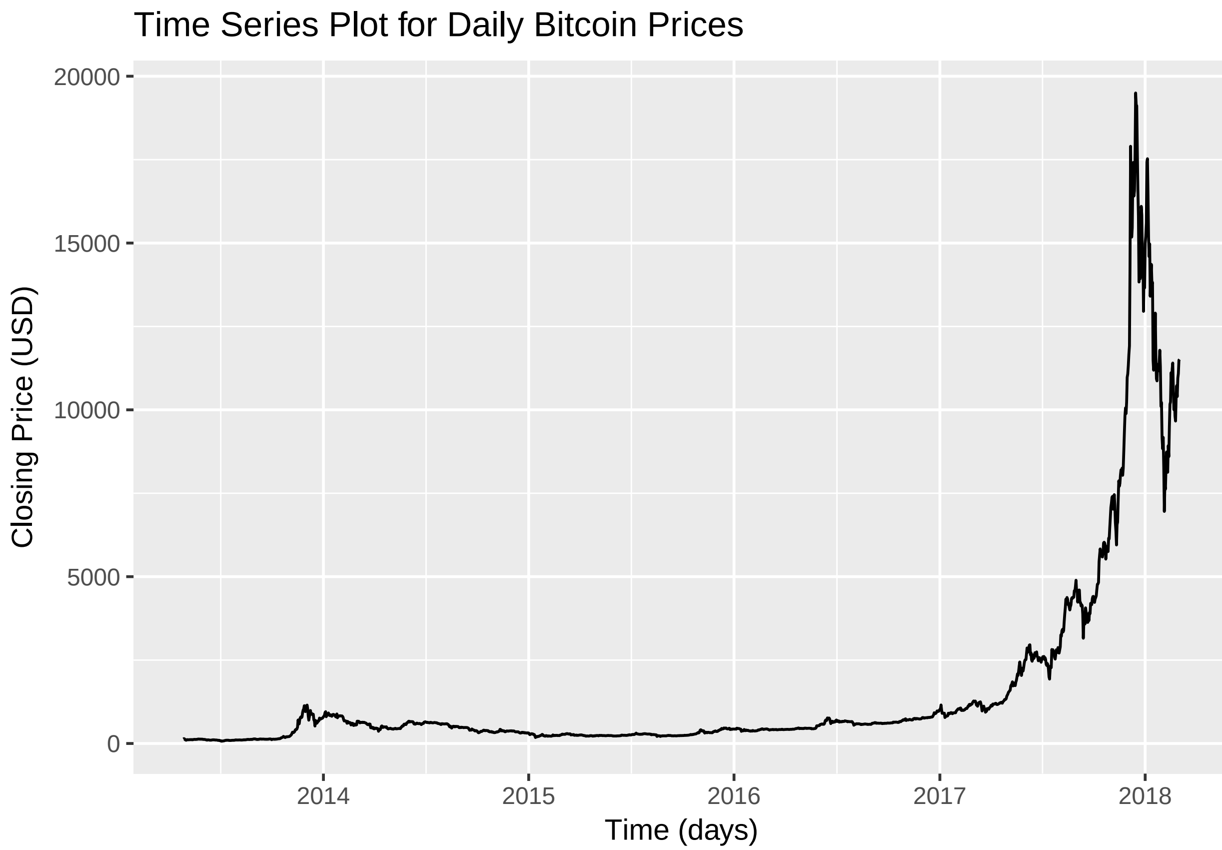


Figure 1: Time Series of Daily Bitcoin Prices

Figure 2 shows time series of last one year

```
Bitcoin.2017 = Bitcoin[Bitcoin$Date > as.Date("2017-04-01"),]
Bitcoin.2017.zoo = zoo(Bitcoin.2017$Close, Bitcoin.2017$Date)
autoplot(Bitcoin.2017.zoo) +
  geom_point(size=.5) +
  ylab('Closing Price (USD)') +
  xlab('Time (days)') +
  ggtitle("Time Series Plot for Daily Bitcoin Prices (2017-2018)")
```

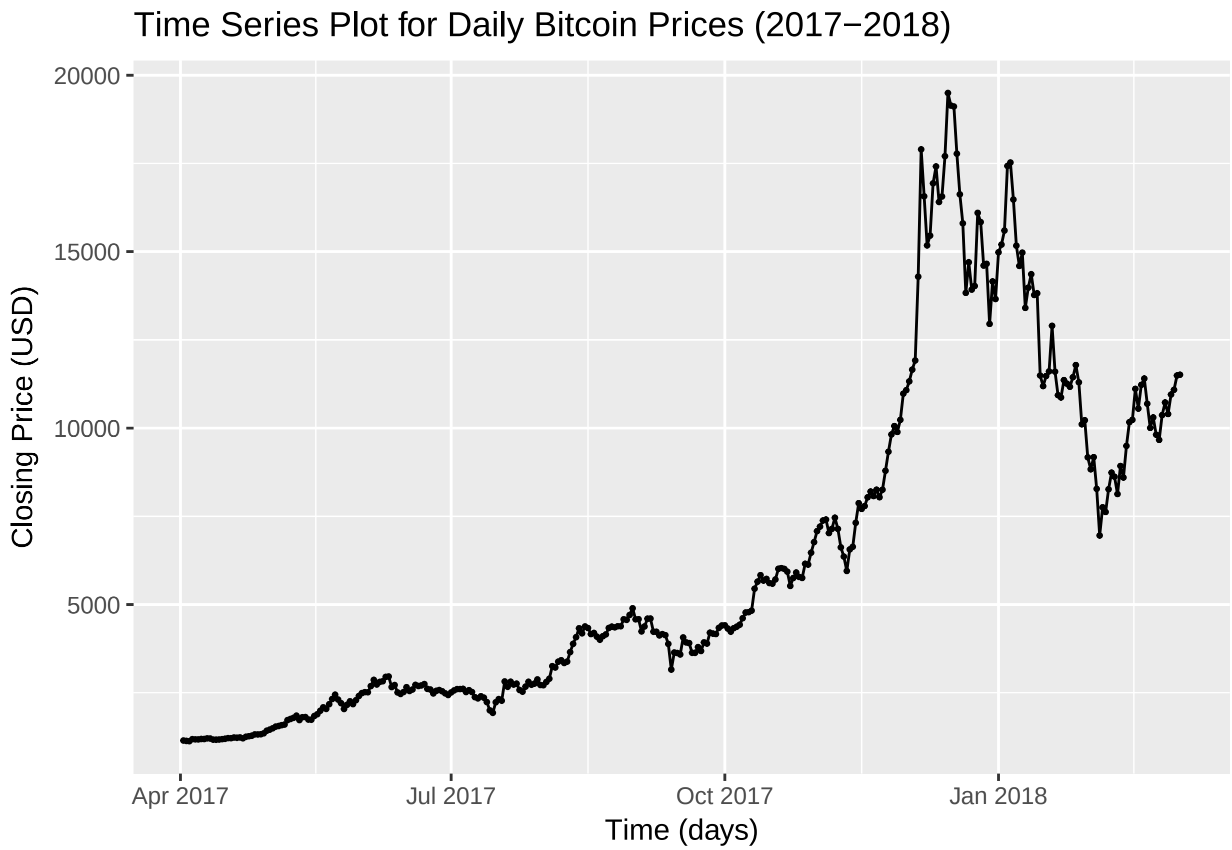
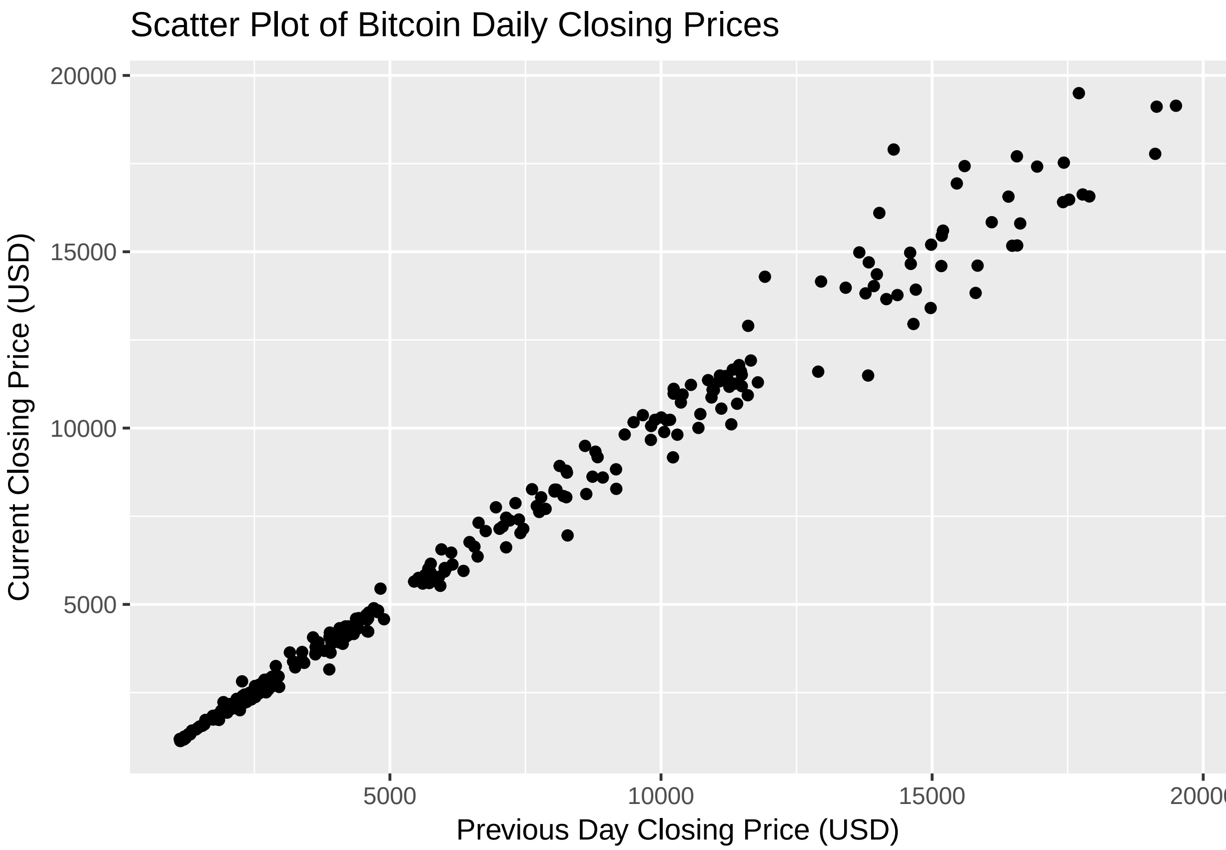


Figure 2: Subset Time Series of Daily Bitcoin Prices

2.1 Scatter Plot and correlation

```
ggplot(Bitcoin.2017,aes(zlag(Close), Close)) + geom_point() +
  ylab('Current Closing Price (USD)') +
  xlab('Previous Day Closing Price (USD)') +
  ggtitle("Scatter Plot of Bitcoin Daily Closing Prices")
```




```r
y = as.vector(Bitcoin.2017.zoo)
x = zlag(Bitcoin.2017.zoo)
index = 2:length(x)
cor(y[index],x[index])
```

```
## [1] 0.9935557
```

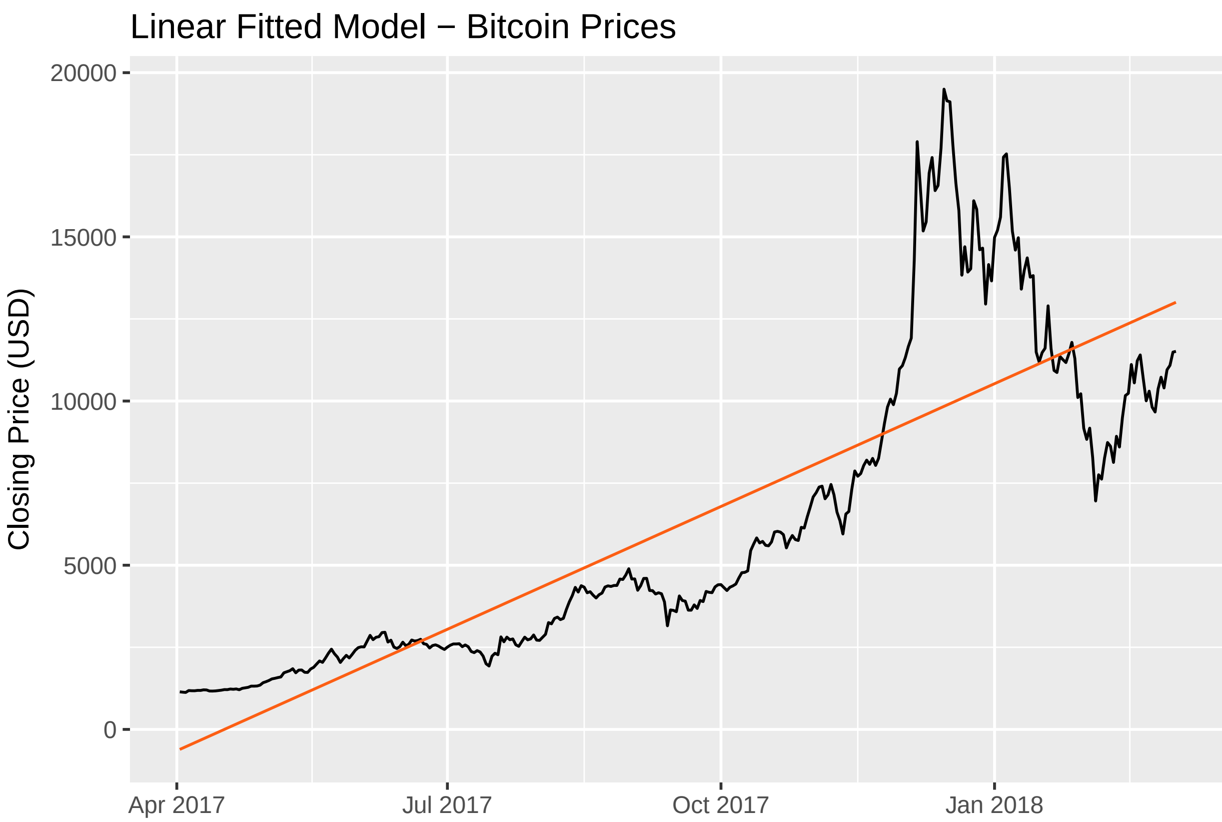
2.2 Linear Model

```r
model.ln = lm(Bitcoin.2017.zoo~time(Bitcoin.2017.zoo)) # label the linear trend model as model.ln
summary(model.ln)
```

```
##
## Call:
## lm(formula = Bitcoin.2017.zoo ~ time(Bitcoin.2017.zoo))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4954.5 -1579.6  -668.9   881.2  9660.6
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)            -7.021e+05  2.461e+04  -28.53   <2e-16 ***
## time(Bitcoin.2017.zoo)  4.065e+01  1.412e+00   28.79   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 2511 on 334 degrees of freedom
## Multiple R-squared:  0.7127, Adjusted R-squared:  0.7119
## F-statistic: 828.6 on 1 and 334 DF,  p-value: < 2.2e-16
```

```r
ggplot(Bitcoin.2017,aes(Date,Close))+
  geom_line()  +
  ylab('Closing Price (USD)') +
  xlab('') +
  ggtitle('Linear Fitted Model - Bitcoin Prices') +
  geom_line(aes(y=fitted(model.ln)),color='#fc5e13')
```



2.3 Residual Analysis - Linear Model

Below are the findings of residuals from linear model

```r
residual_analysis_qq <- function(myresiduals, title = 'QQ Plot of Residuals') {
data=as.data.frame(qqnorm( myresiduals , plot=F))
ggplot(data,aes(x,y)) +
  geom_point() +
  geom_smooth(method="lm", se=FALSE, color='#e36209', size=.4)+
  xlab('Theoretical') +
  ylab('Sample') +
  ggtitle(title)
}

checkresiduals(model.ln)
```

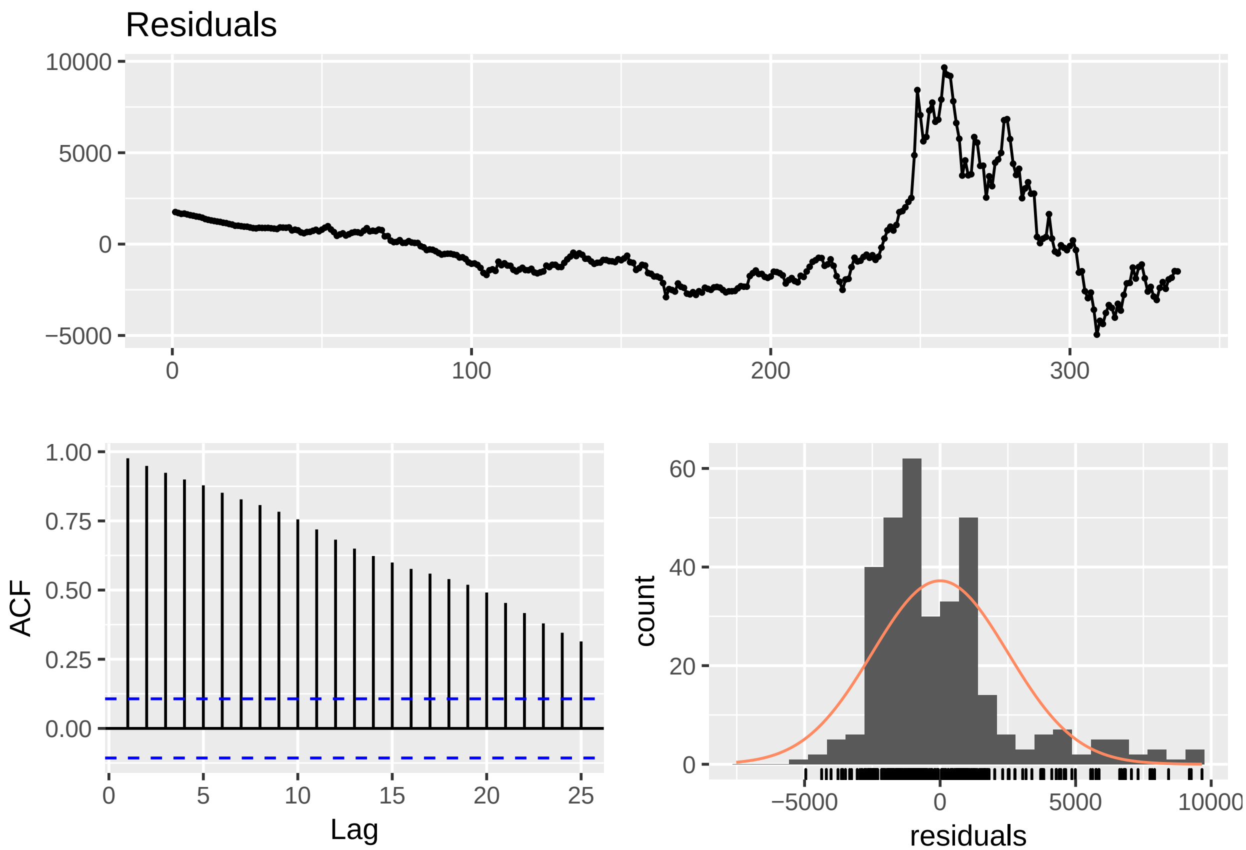


Figure 3: Residual Analysis Linear fitted Model

```
##
##  Breusch-Godfrey test for serial correlation of order up to 10
##
## data:  Residuals
## LM test = 321.71, df = 10, p-value < 2.2e-16
```

```
residual_analysis_qq(residuals(model.ln))
```

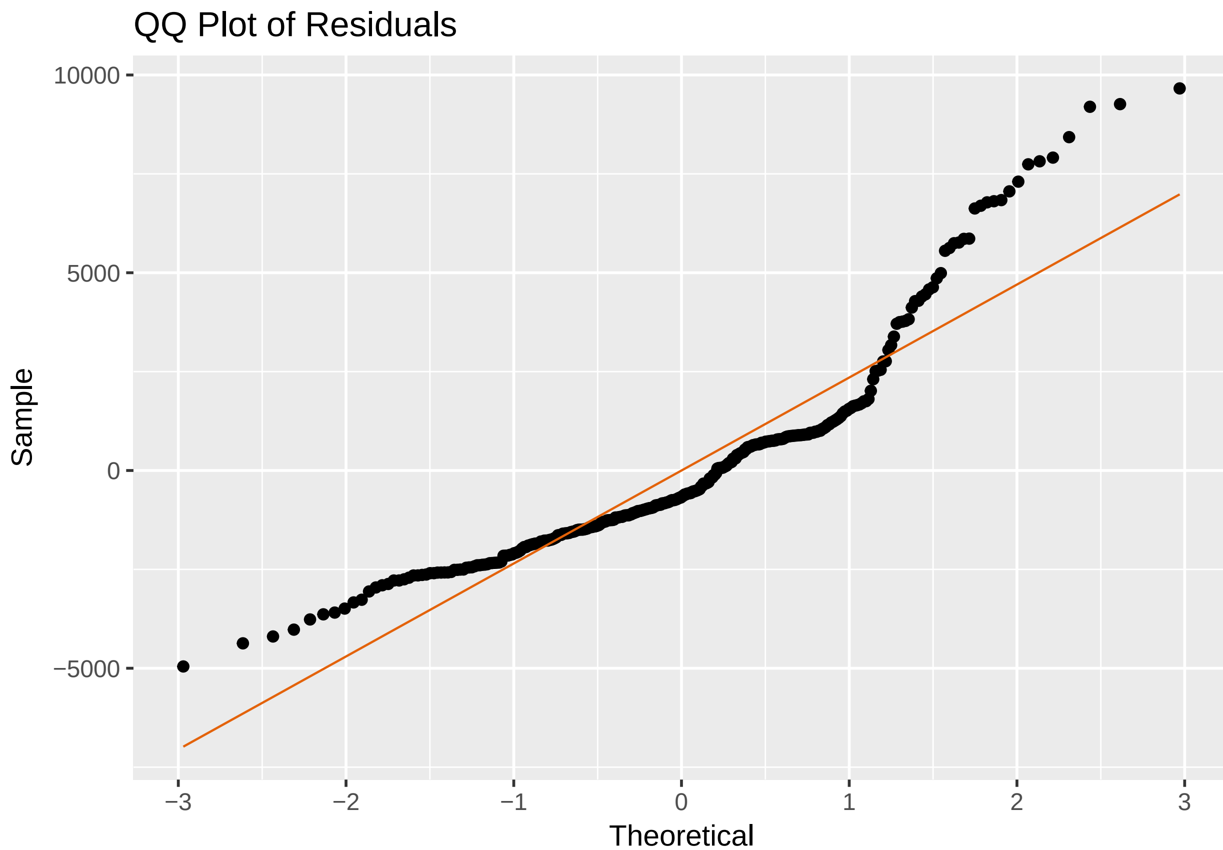


Figure 4: Residual Analysis Linear fitted Model

```r
shapiro.test(as.vector(residuals(model.ln)))
```

```
##
##  Shapiro-Wilk normality test
##
## data:  as.vector(residuals(model.ln))
## W = 0.87841, p-value = 1.204e-15
```

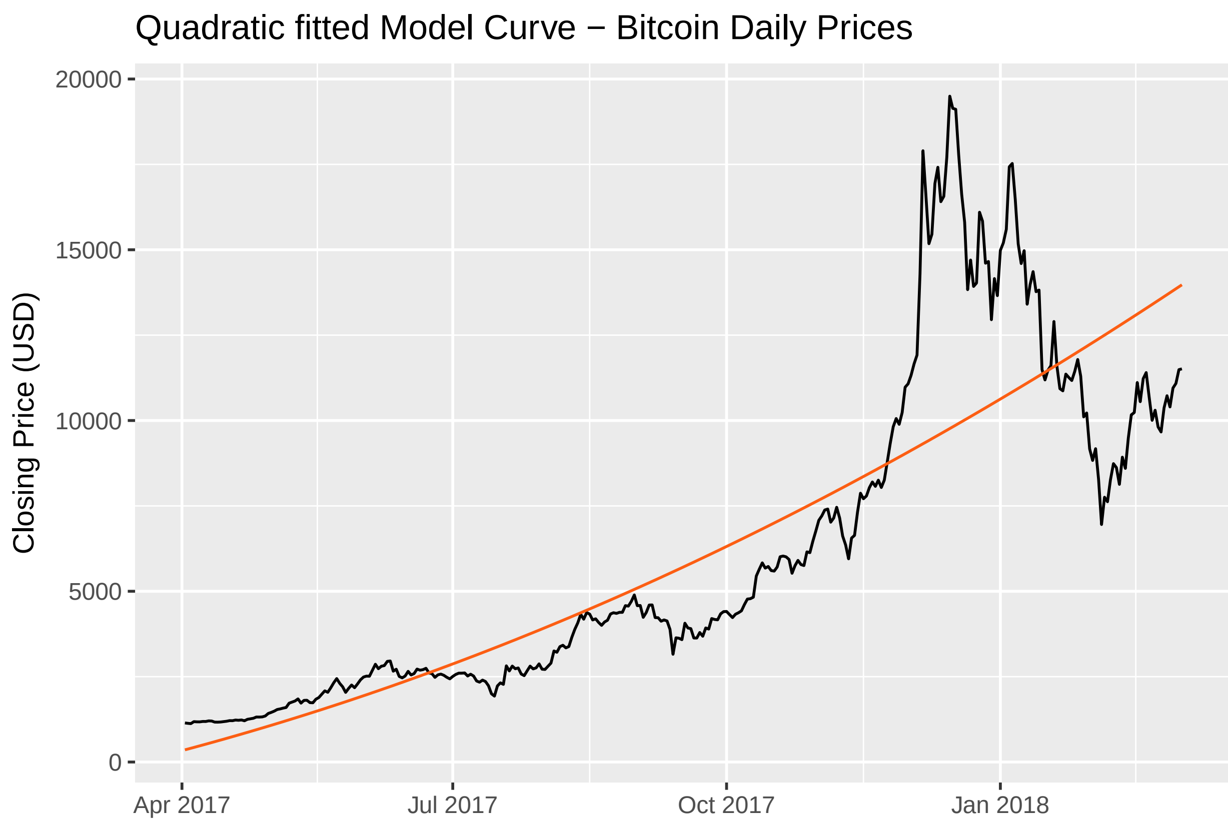
2.4 Quadratic Model

```r
t = as.vector(time(Bitcoin.2017.zoo))
t2 = t^2
model.qa = lm(Bitcoin.2017.zoo~ t + t2) # label the quadratic trend model as model.qa
summary(model.qa)
```

```
##
## Call:
## lm(formula = Bitcoin.2017.zoo ~ t + t2)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5490.1 -1286.7  -408.4   497.0  9733.1
```

```
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.504e+07  4.874e+06   3.085  0.00221 **
## t           -1.766e+03  5.594e+02  -3.156  0.00174 **
## t2           5.183e-02  1.605e-02   3.229  0.00137 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2476 on 333 degrees of freedom
## Multiple R-squared:  0.7214, Adjusted R-squared:  0.7198
## F-statistic: 431.2 on 2 and 333 DF,  p-value: < 2.2e-16
```

```r
ggplot(Bitcoin.2017,aes(Date,Close))+
  geom_line()  +
  ylab('Closing Price (USD)') +
  xlab('') +
  ggtitle('Quadratic fitted Model Curve - Bitcoin Daily Prices') +
  geom_line(aes(y=fitted(model.qa)),color='#fc5e13')
```



2.5 Residual Analysis - Linear Model

Below are the findings of residuals from linear model

```r
checkresiduals(model.qa)
```

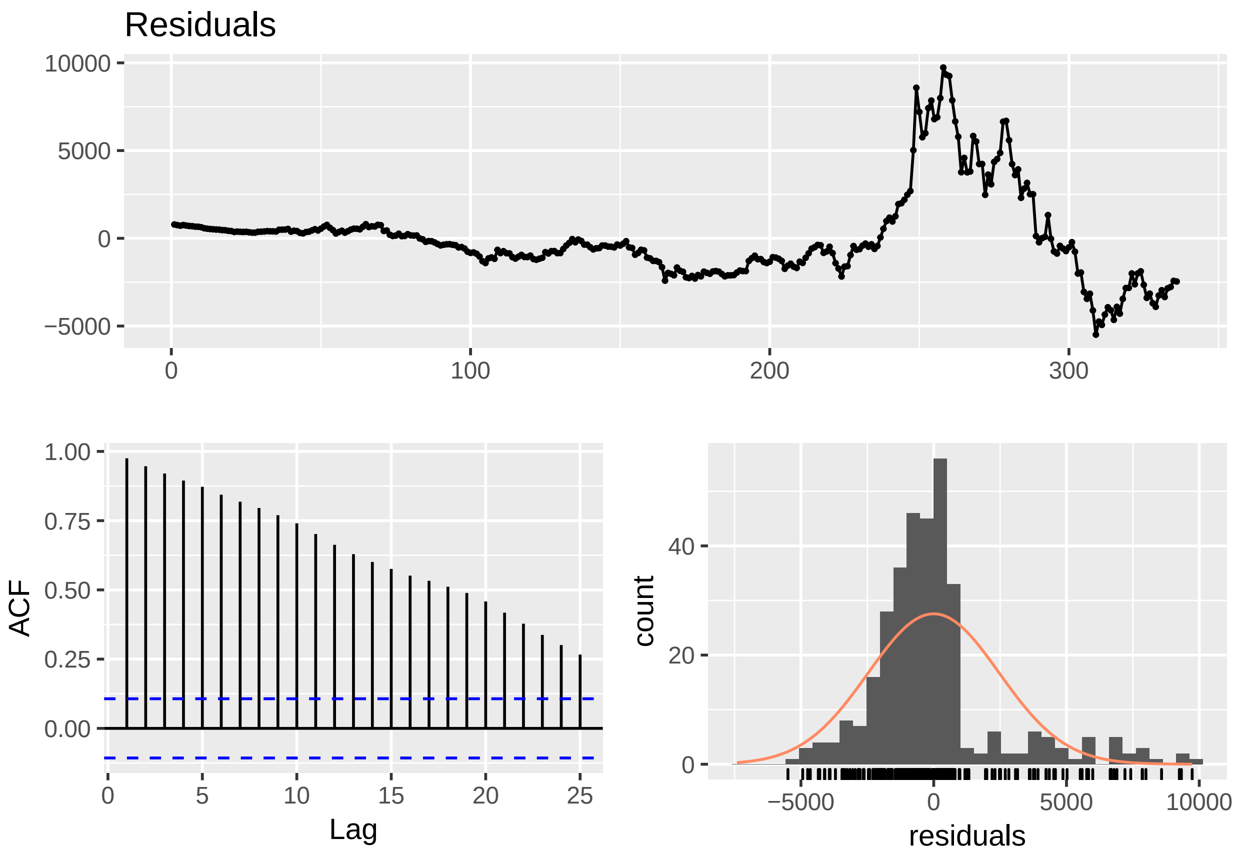


Figure 5: Residual Analysis Quadratic fitted Model

```
##
##  Breusch-Godfrey test for serial correlation of order up to 10
##
## data:  Residuals
## LM test = 321.7, df = 10, p-value < 2.2e-16
```

```
residual_analysis_qq(residuals(model.qa))
```

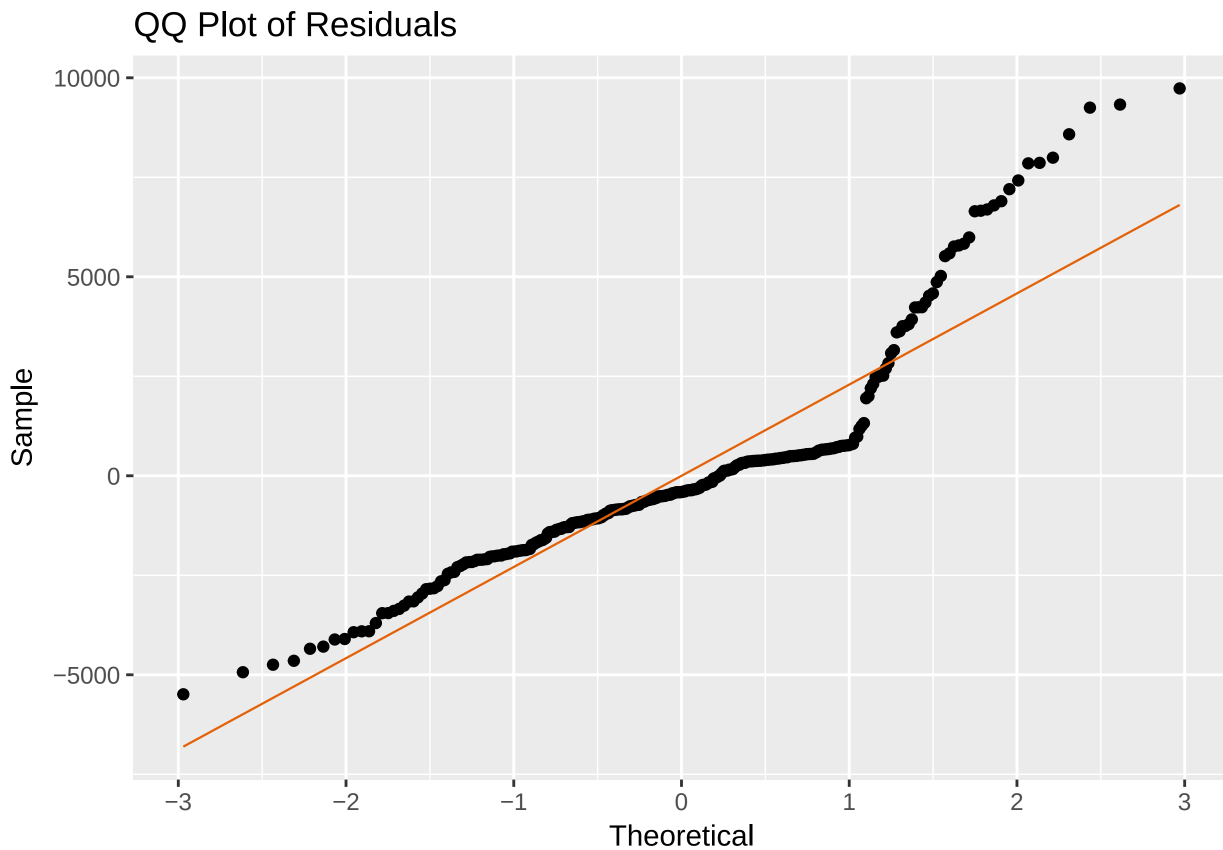


Figure 6: Residual Analysis Linear fitted Model

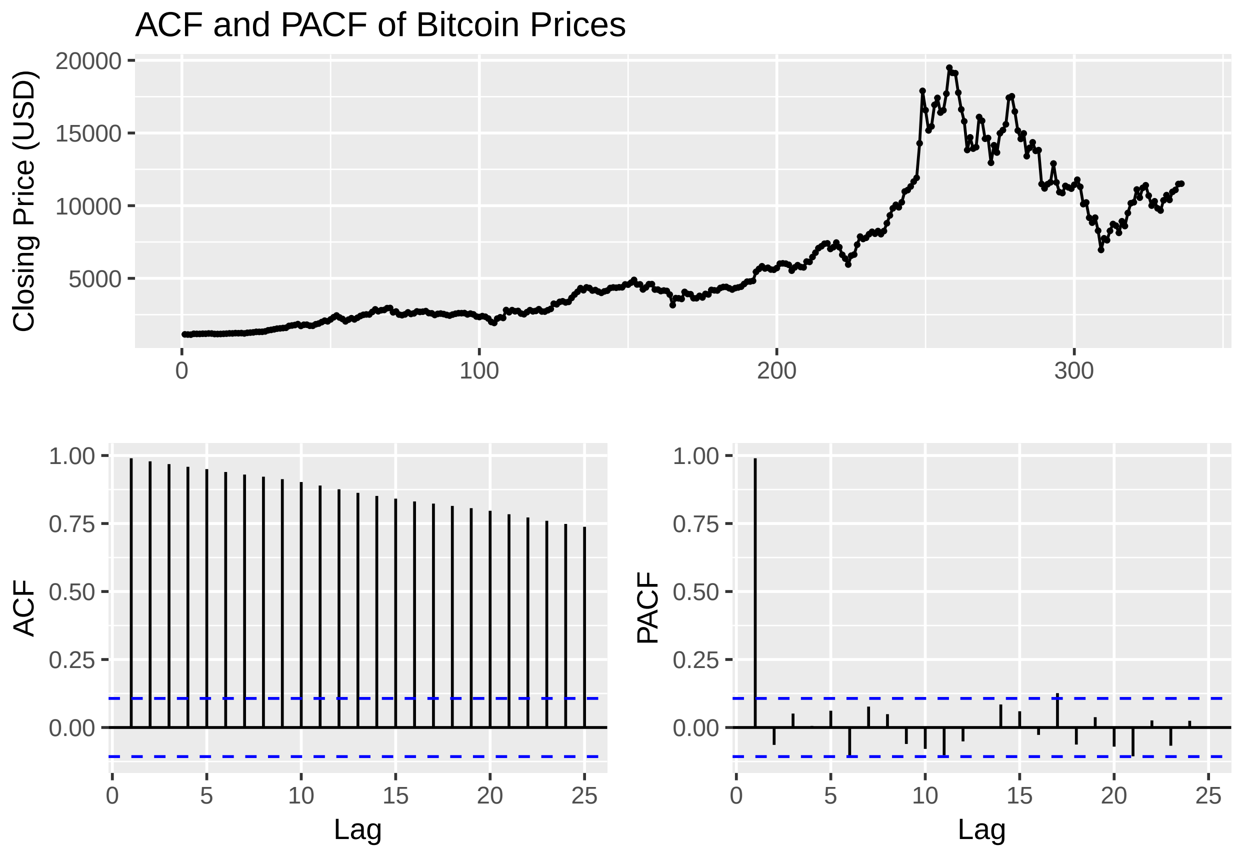
```
shapiro.test(as.vector(residuals(model.qa)))
```

```
##
##  Shapiro-Wilk normality test
##
## data:  as.vector(residuals(model.qa))
## W = 0.86085, p-value < 2.2e-16
```

3 Models for Nonstationary Time Series

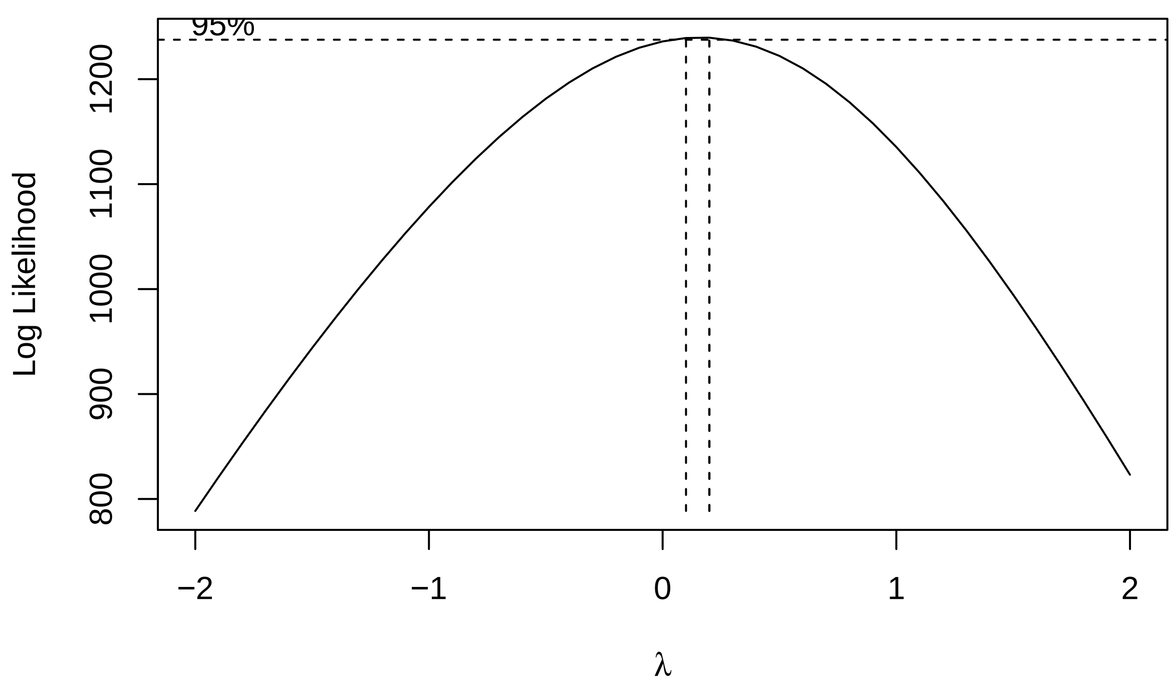
Auto regressive behaviour and non staionay Staionay is the first thing we need to check.

```
ggtsdisplay(Bitcoin.2017.zoo,
            main = 'ACF and PACF of Bitcoin Prices',
            ylab='Closing Price (USD)')
```



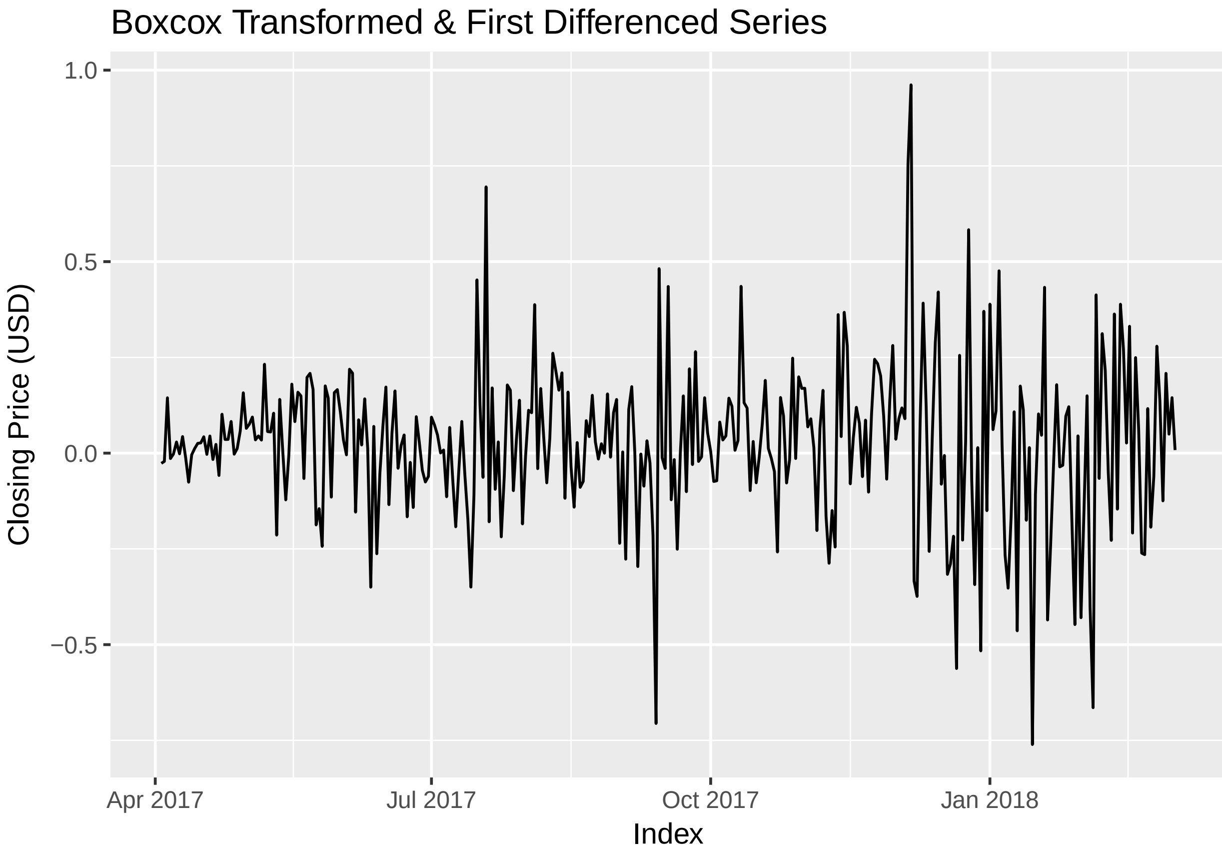
stategy to make stationay is transfromation.

```
Bitcoin.transform = BoxCox.ar(Bitcoin.2017.zoo, method = 'yule-walker')
```

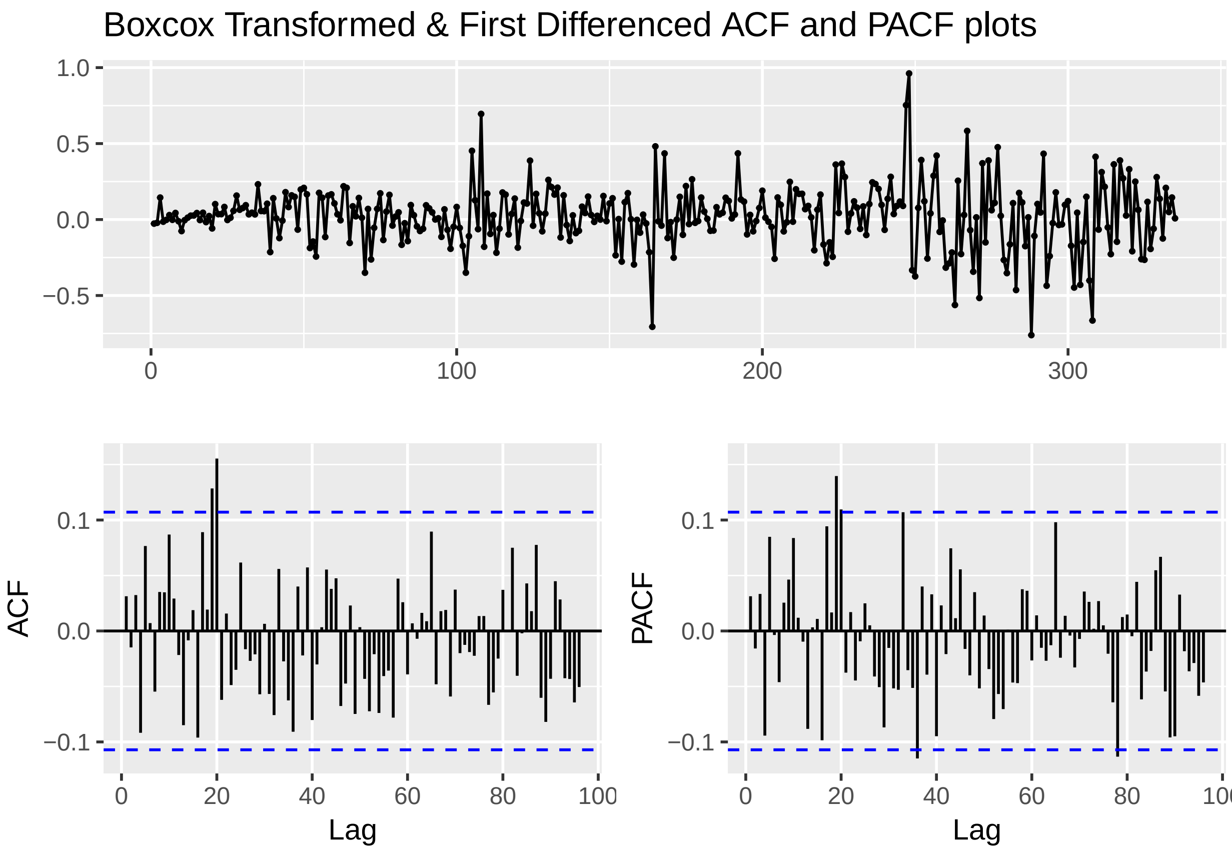



```
lambda = sum(Bitcoin.transform$ci)/length(Bitcoin.transform$ci)
Bitcoin.boxcox = (Bitcoin.2017.zoo^lambda - 1) / lambda
```

```
Bitcoin.diff = base::diff(Bitcoin.boxcox, differences = 1)
autoplot(Bitcoin.diff) +
  ylab('Closing Price (USD)') +
  ggtitle('Boxcox Transformed & First Differenced Series')
```




```
ggtsdisplay(Bitcoin.diff, lag.max = 96,ci.type='ma',
            main = 'Boxcox Transformed & First Differenced ACF and PACF plots',
            ylab='')
```




```r
adf.test(Bitcoin.diff)
```

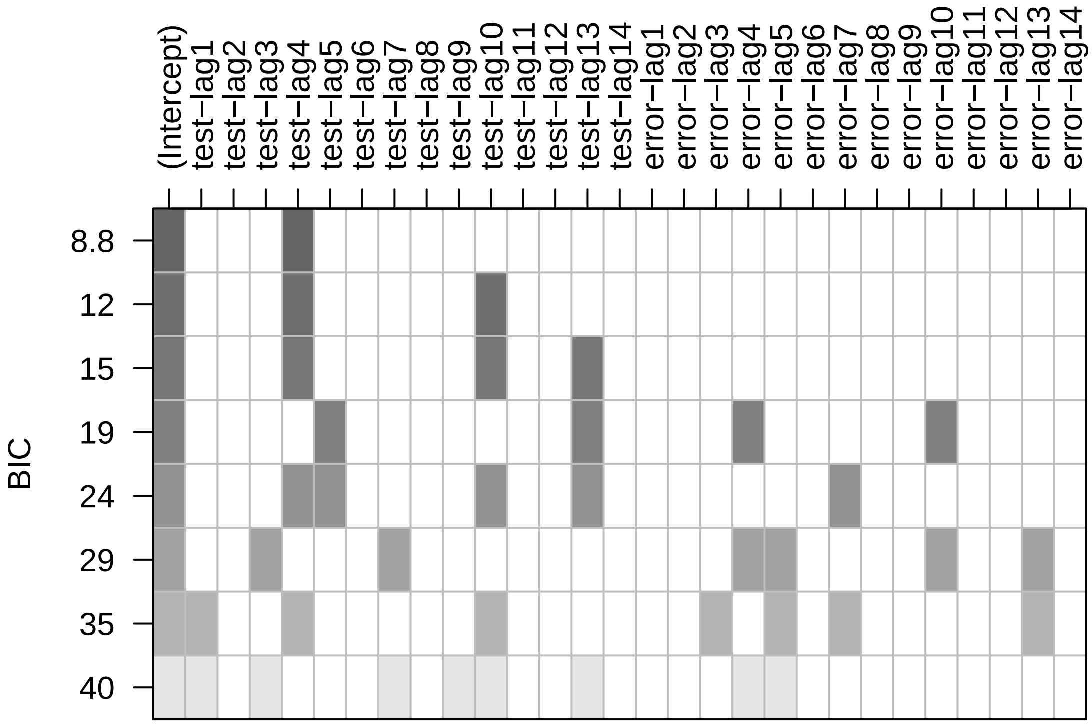
```
##
##  Augmented Dickey-Fuller Test
##
## data:  Bitcoin.diff
## Dickey-Fuller = -6.968, Lag order = 6, p-value = 0.01
## alternative hypothesis: stationary
```

```r
eacf(Bitcoin.diff)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 o o o o o o o o o o o  o  o  o
## 1 x o o o o o o o o o o  o  o  o
## 2 x o o o o o o o o o o  o  o  o
## 3 x o x o o o o o o o o  o  o  o
## 4 x x x o o o o o o o o  o  o  o
## 5 o x o x o o o o o o o  o  o  o
## 6 o x o o o o o o o o o  o  o  o
## 7 x x x o x x o o o o o  o  o  o
```

```r
# ARIMA(0,1,0),ARIMA(1,1,1),ARIMA(2,1,2),ARIMA(4,1,3)
```

```r
res1 = armasubsets(y=Bitcoin.diff,nar=14,nma=14,y.name='test',ar.method='mle')
plot(res1)
```




```r
#ARIMA(4,1,4),ARIMA(5,1,4)
```

```r
#The final set of possible models is
# ARIMA(0,1,0),ARIMA(1,1,1),ARIMA(2,1,2),ARIMA(4,1,3)
# ARIMA(4,1,4),ARIMA(5,1,4)

# ARIMA(1,1,1)
model_111_css = arima(Bitcoin.boxcox, order=c(1,1,1),method='CSS')
coeftest(model_111_css)
```

```
##
## z test of coefficients:
##
##     Estimate Std. Error z value Pr(>|z|)
## ar1 0.021830         NA      NA       NA
## ma1 0.022332         NA      NA       NA
```

```r
model_111_ml = arima(Bitcoin.boxcox, order=c(1,1,1),method='ML')
coeftest(model_111_ml)
```

```
##
## z test of coefficients:
##
##     Estimate Std. Error z value Pr(>|z|)
## ar1 0.020106         NA      NA       NA
## ma1 0.024617         NA      NA       NA
```

```r
# ARIMA(1,1,2)
model_112_css = arima(Bitcoin.boxcox,order=c(1,1,2),method='CSS')
coeftest(model_112_css)
```

```
##
## z test of coefficients:
```

```
##
##      Estimate Std. Error z value Pr(>|z|)
## ar1 -0.698636   0.258350 -2.7042 0.006846 **
## ma1  0.749424   0.260011  2.8823 0.003948 **
## ma2 -0.012098   0.059763 -0.2024 0.839579
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
model_112_ml = arima(Bitcoin.boxcox,order=c(1,1,2),method='ML')
coeftest(model_112_ml)
```

```
##
## z test of coefficients:
##
##      Estimate Std. Error z value Pr(>|z|)
## ar1 -0.713306   0.267548 -2.6661 0.007674 **
## ma1  0.764098   0.268822  2.8424 0.004478 **
## ma2 -0.010152   0.061030 -0.1663 0.867889
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# ARIMA(2,1,1)
model_211_css = arima(Bitcoin.boxcox,order=c(2,1,1),method='CSS')
coeftest(model_211_css)
```

```
##
## z test of coefficients:
##
##      Estimate Std. Error z value  Pr(>|z|)
## ar1 -0.729981   0.236531 -3.0862 0.0020274 **
## ar2 -0.010729   0.062640 -0.1713 0.8640056
## ma1  0.779953   0.230417  3.3850 0.0007119 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
model_211_ml = arima(Bitcoin.boxcox,order=c(2,1,1),method='ML')
coeftest(model_211_ml)
```

```
##
## z test of coefficients:
##
##       Estimate Std. Error z value Pr(>|z|)
## ar1  0.0091928         NA      NA       NA
## ar2 -0.0035792  0.0541984  -0.066   0.9473
## ma1  0.0335948         NA      NA       NA
```

```r
# ARIMA(2,1,2)
model_212_css = arima(Bitcoin.boxcox,order=c(2,1,2),method='CSS')
coeftest(model_212_css)
```

```
##
## z test of coefficients:
##
##      Estimate Std. Error  z value Pr(>|z|)
## ar1 -0.028530   0.079410  -0.3593   0.7194
## ar2  0.906936   0.075863  11.9549   <2e-16 ***
## ma1  0.085848   0.084591   1.0149   0.3102
```

```
## ma2 -0.913597   0.083838 -10.8972   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
model_212_ml = arima(Bitcoin.boxcox,order=c(2,1,2),method='ML')
coeftest(model_212_ml)
```

```
##
## z test of coefficients:
##
##       Estimate Std. Error  z value Pr(>|z|)
## ar1 -0.0021222  0.0653257  -0.0325   0.9741
## ar2  0.9248274  0.0619597  14.9263   <2e-16 ***
## ma1  0.0590080  0.0774408   0.7620   0.4461
## ma2 -0.9409861  0.0773613 -12.1635   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# ARIMA(3,1,2)
model_312_css = arima(Bitcoin.boxcox,order=c(3,1,2),method='CSS')
coeftest(model_312_css)
```

```
##
## z test of coefficients:
##
##     Estimate Std. Error z value Pr(>|z|)
## ar1 -0.96048    0.32229 -2.9802 0.002881 **
## ar2 -0.39084    0.21351 -1.8305 0.067170 .
## ar3  0.10504    0.06089  1.7250 0.084523 .
## ma1  1.01485    0.32371  3.1351 0.001718 **
## ma2  0.45021    0.21280  2.1157 0.034370 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
model_312_ml = arima(Bitcoin.boxcox,order=c(3,1,2),method='ML')
coeftest(model_312_ml)
```

```
##
## z test of coefficients:
##
##      Estimate Std. Error  z value  Pr(>|z|)
## ar1 -1.612748   0.083945 -19.2120 < 2.2e-16 ***
## ar2 -0.854073   0.108337  -7.8835 3.183e-15 ***
## ar3  0.048182   0.058346   0.8258    0.4089
## ma1  1.679110   0.064674  25.9626 < 2.2e-16 ***
## ma2  0.925998   0.065107  14.2228 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# ARIMA(4,1,2)
model_412_css = arima(Bitcoin.boxcox,order=c(4,1,2),method='CSS')
coeftest(model_412_css)
```

```
##
## z test of coefficients:
##
##      Estimate Std. Error z value Pr(>|z|)
```

```
## ar1 -0.737242   0.415004 -1.7765  0.07566 .
## ar2 -0.499844   0.577987 -0.8648  0.38715
## ar3  0.056956   0.071481  0.7968  0.42556
## ar4 -0.080260   0.070306 -1.1416  0.25363
## ma1  0.793473   0.414413  1.9147  0.05553 .
## ma2  0.540068   0.603325  0.8952  0.37070
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
model_412_ml = arima(Bitcoin.boxcox,order=c(4,1,2),method='ML')
coeftest(model_412_ml)
```

```
##
## z test of coefficients:
##
##      Estimate Std. Error  z value Pr(>|z|)
## ar1 -1.121373   0.055563 -20.1819   <2e-16 ***
## ar2 -0.911364   0.082083 -11.1030   <2e-16 ***
## ar3  0.053735   0.081959   0.6556   0.5121
## ar4 -0.022210   0.055153  -0.4027   0.6872
## ma1  1.175469   0.021048  55.8464   <2e-16 ***
## ma2  0.988325   0.027979  35.3241   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# ARIMA(4,1,4)
model_414_css = arima(Bitcoin.boxcox,order=c(4,1,4),method='CSS')
coeftest(model_414_css)
```

```
##
## z test of coefficients:
##
##      Estimate Std. Error  z value  Pr(>|z|)
## ar1 -0.551983         NA       NA        NA
## ar2  0.172921   0.038259   4.5198 6.190e-06 ***
## ar3  0.915619   0.060298  15.1850 < 2.2e-16 ***
## ar4  0.435558         NA       NA        NA
## ma1  0.580501         NA       NA        NA
## ma2 -0.196556   0.043484  -4.5202 6.178e-06 ***
## ma3 -0.920758   0.052864 -17.4176 < 2.2e-16 ***
## ma4 -0.551651         NA       NA        NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
model_414_ml = arima(Bitcoin.boxcox,order=c(4,1,4),method='ML')
coeftest(model_414_ml)
```

```
##
## z test of coefficients:
##
##      Estimate Std. Error z value  Pr(>|z|)
## ar1 -0.858500   0.281071 -3.0544 0.0022552 **
## ar2  0.075681   0.128007  0.5912 0.5543661
## ar3  1.122140   0.098181 11.4293 < 2.2e-16 ***
## ar4  0.639474   0.253747  2.5201 0.0117314 *
## ma1  0.923109   0.264539  3.4895 0.0004839 ***
```

```
## ma2 -0.032671   0.177524 -0.1840 0.8539858
## ma3 -1.117752   0.155372 -7.1940  6.29e-13 ***
## ma4 -0.719323   0.226069 -3.1819 0.0014633 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# ARIMA(5,1,4)
model_514_css = arima(Bitcoin.boxcox,order=c(5,1,4),method='CSS')
coeftest(model_514_css)
```

```
##
## z test of coefficients:
##
##        Estimate  Std. Error  z value  Pr(>|z|)
## ar1  9.9074e-02  1.9514e-04  507.703 < 2.2e-16 ***
## ar2  1.3895e-01  3.0111e-04  461.475 < 2.2e-16 ***
## ar3  5.0499e-01  7.2128e-05 7001.257 < 2.2e-16 ***
## ar4  1.6450e-01  1.5559e-04 1057.251 < 2.2e-16 ***
## ar5  9.0947e-02  3.6146e-04  251.613 < 2.2e-16 ***
## ma1 -8.1368e-02  5.7083e-03  -14.254 < 2.2e-16 ***
## ma2 -1.9260e-01  1.6938e-02  -11.371 < 2.2e-16 ***
## ma3 -5.1953e-01  1.7609e-02  -29.504 < 2.2e-16 ***
## ma4 -3.1371e-01  6.3856e-03  -49.127 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
model_514_ml = arima(Bitcoin.boxcox,order=c(5,1,4),method='ML')
coeftest(model_514_ml)
```

```
##
## z test of coefficients:
##
##      Estimate Std. Error z value Pr(>|z|)
## ar1  0.216613   0.581949  0.3722   0.7097
## ar2  0.125243   0.255393  0.4904   0.6239
## ar3  0.472679   0.519846  0.9093   0.3632
## ar4 -0.054074   0.426025 -0.1269   0.8990
## ar5  0.109269   0.078398  1.3938   0.1634
## ma1 -0.166104   0.585508 -0.2837   0.7766
## ma2 -0.144679   0.251819 -0.5745   0.5656
## ma3 -0.438182   0.523533 -0.8370   0.4026
## ma4 -0.056614   0.413995 -0.1368   0.8912
```

```r
source('sort.score.r')
sort.score(stats::AIC(model_111_ml,model_112_ml,model_211_ml,model_212_ml,model_312_ml,model_412_ml,mode
```

```
##              df        AIC
## model_312_ml  6 -102.82561
## model_212_ml  5 -102.65580
## model_412_ml  7 -102.28591
## model_414_ml  9 -101.78404
## model_111_ml  3 -100.97940
## model_112_ml  4 -100.65053
## model_211_ml  4  -98.98447
## model_514_ml 10  -95.11157
```

```r
sort.score(stats::BIC(model_111_ml,model_112_ml,model_211_ml,model_212_ml,model_312_ml,model_412_ml,mode
```

```
##              df       BIC
## model_111_ml  3 -89.53701
## model_112_ml  4 -85.39400
## model_211_ml  4 -83.72795
## model_212_ml  5 -83.58515
## model_312_ml  6 -79.94082
## model_412_ml  7 -75.58699
## model_414_ml  9 -67.45687
## model_514_ml 10 -56.97026
```

```r
fit <- Arima(Bitcoin.2017.zoo, order=c(3,1,2), lambda = lambda)
summary(fit)
```

```
## Series: Bitcoin.2017.zoo
## ARIMA(3,1,2)
## Box Cox transformation: lambda= 0.15
##
## Coefficients:
##           ar1      ar2     ar3     ma1     ma2
##       -1.6132  -0.8545  0.0479  1.6795  0.9262
## s.e.   0.0837   0.1081  0.0584  0.0643  0.0646
##
## sigma^2 estimated as 0.04215:  log likelihood=57.41
## AIC=-102.83   AICc=-102.57   BIC=-79.94
##
## Training set error measures:
##                    ME     RMSE      MAE       MPE     MAPE      MASE
## Training set 28.56341 517.2301 289.9583 0.5060474 3.975244 0.9891814
##                    ACF1
## Training set 0.06611109
```

3.1 Residual Analysis - ARIMA Model

Below are the findings of residuals from linear model

```r
checkresiduals(fit)
```

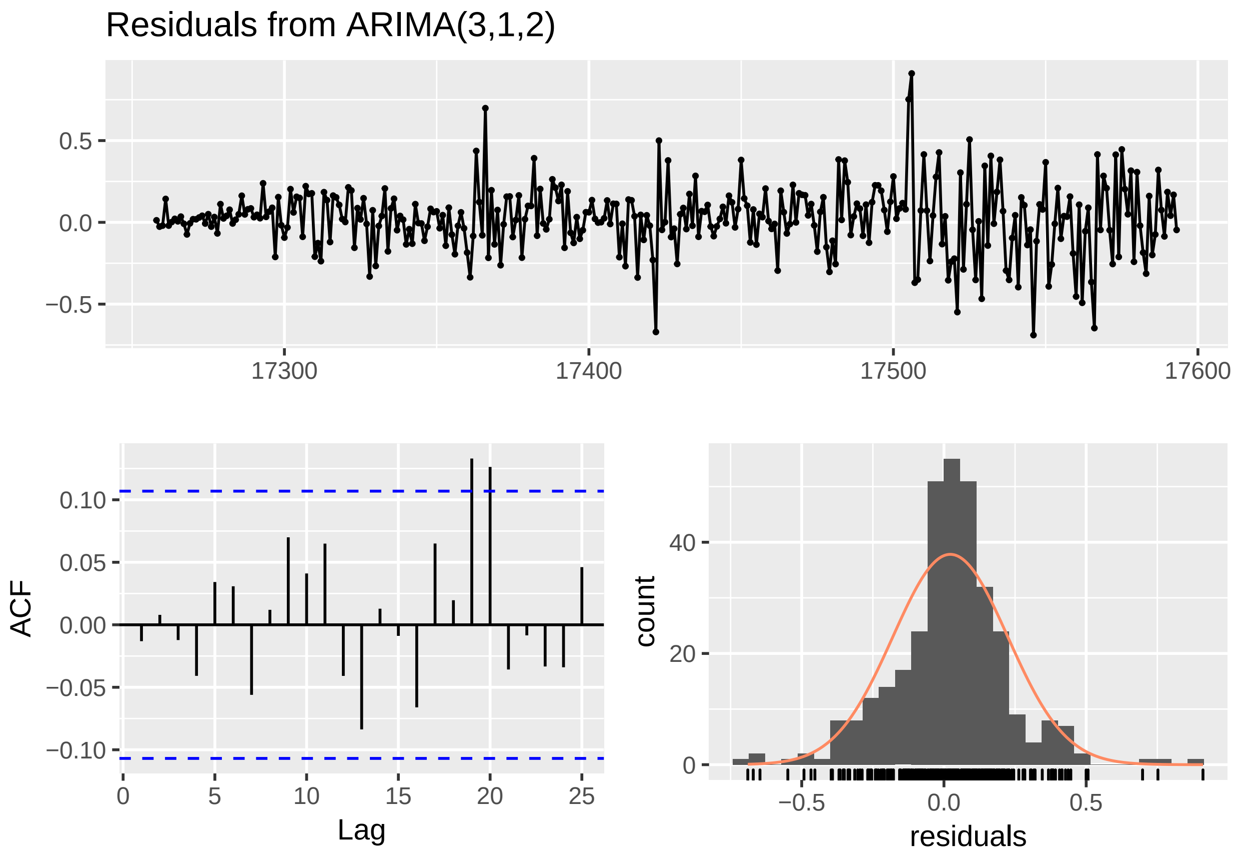


Figure 7: Residual Analysis Quadratic fitted Model

```
##
##  Ljung-Box test
##
## data:  Residuals from ARIMA(3,1,2)
## Q* = 4.8513, df = 5, p-value = 0.4343
##
## Model df: 5.   Total lags used: 10
```

```
residual_analysis_qq(residuals(fit))
```

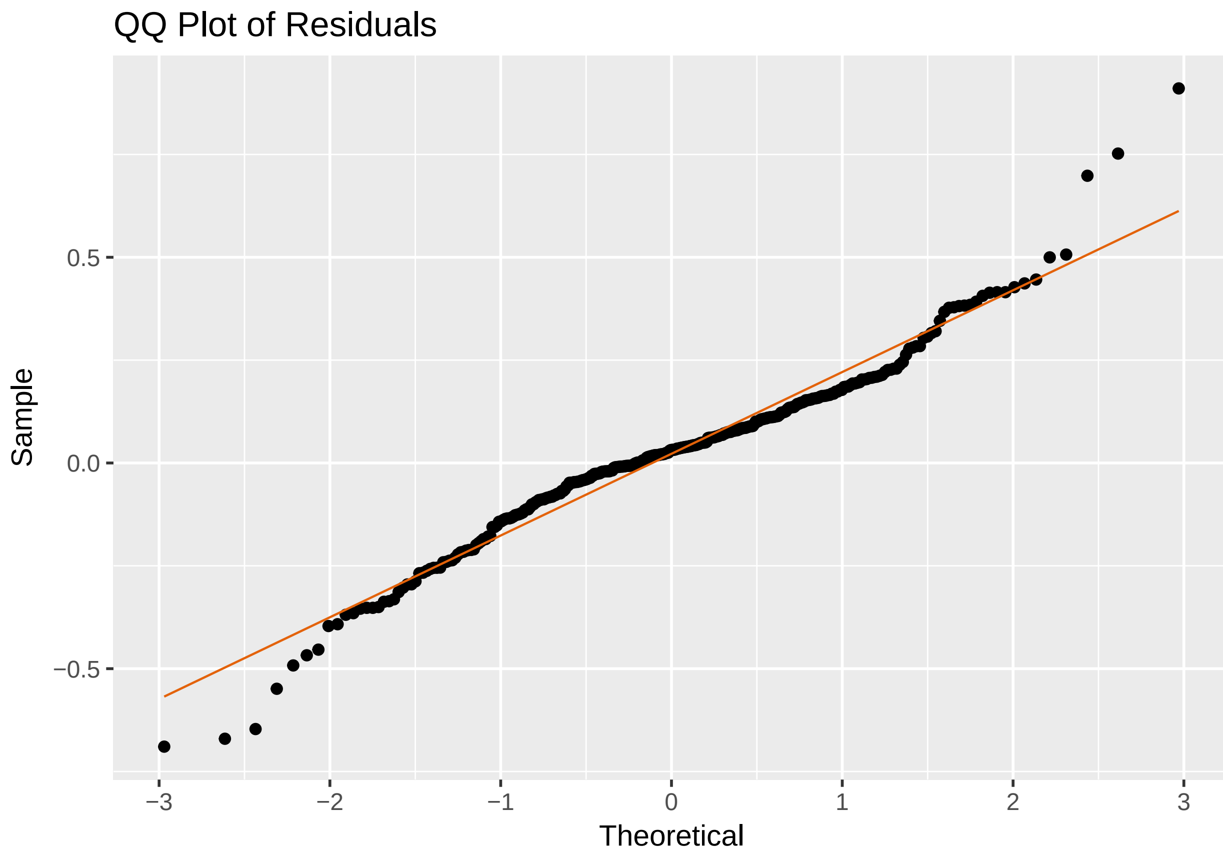
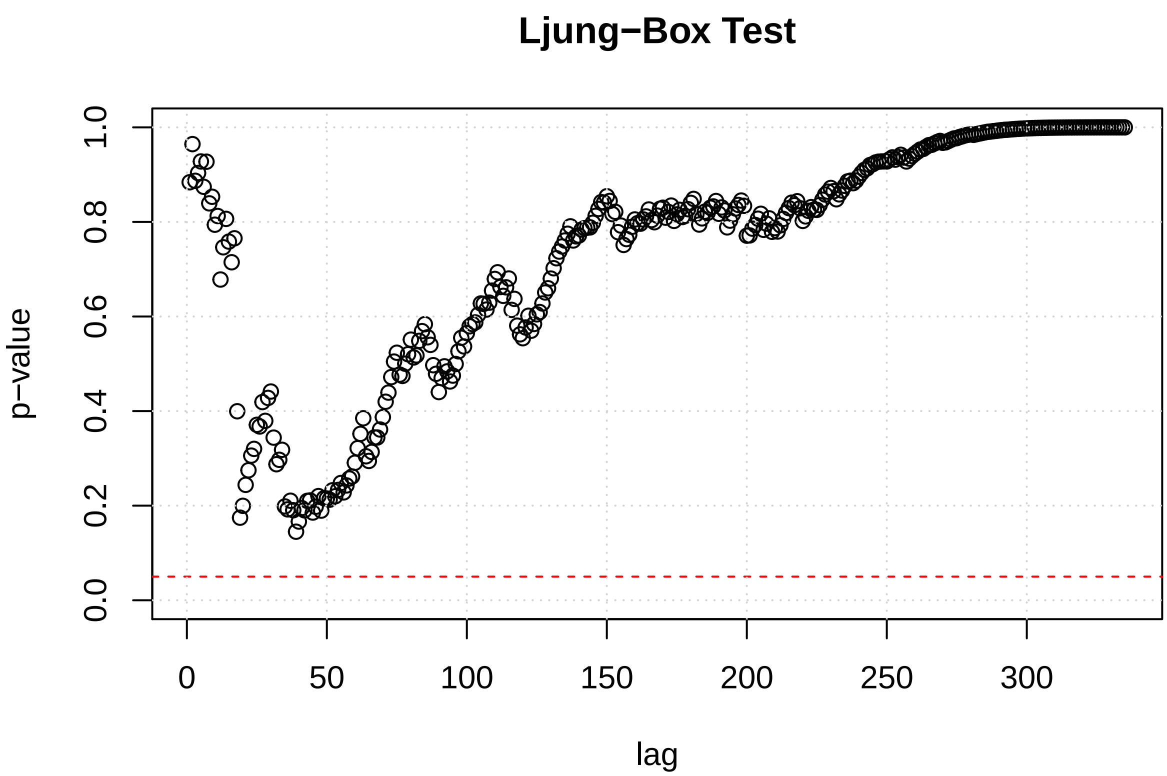


Figure 8: Residual Analysis Linear fitted Model

```r
shapiro.test(as.vector(residuals(fit)))
```

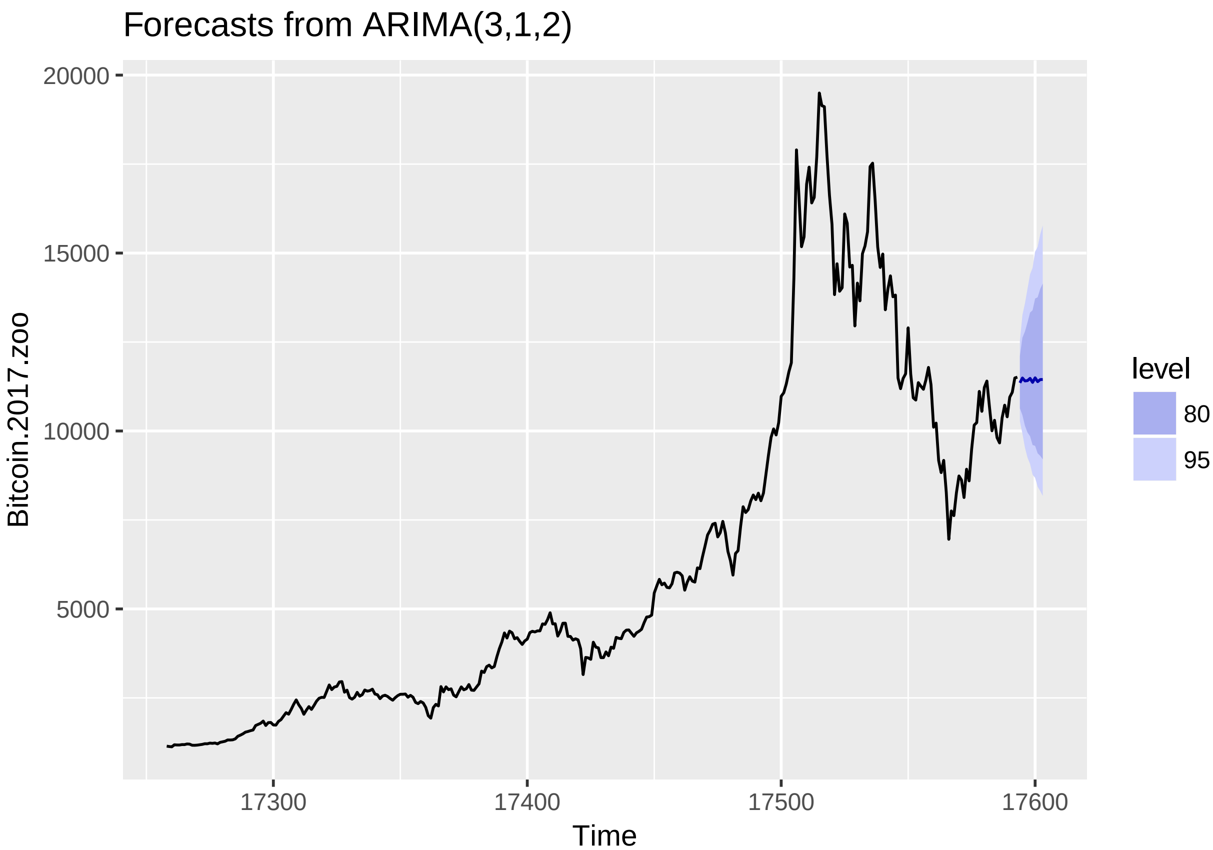
```
##
##  Shapiro-Wilk normality test
##
## data:  as.vector(residuals(fit))
## W = 0.96352, p-value = 1.918e-07
```

```r
x = residuals(fit)
k=0
LBQPlot(x, lag.max = length(x)-1 , StartLag = k + 1, k = 0, SquaredQ = FALSE)
grid()
```



3.2 Forecast

```
autoplot(forecast(fit,h=10))
```



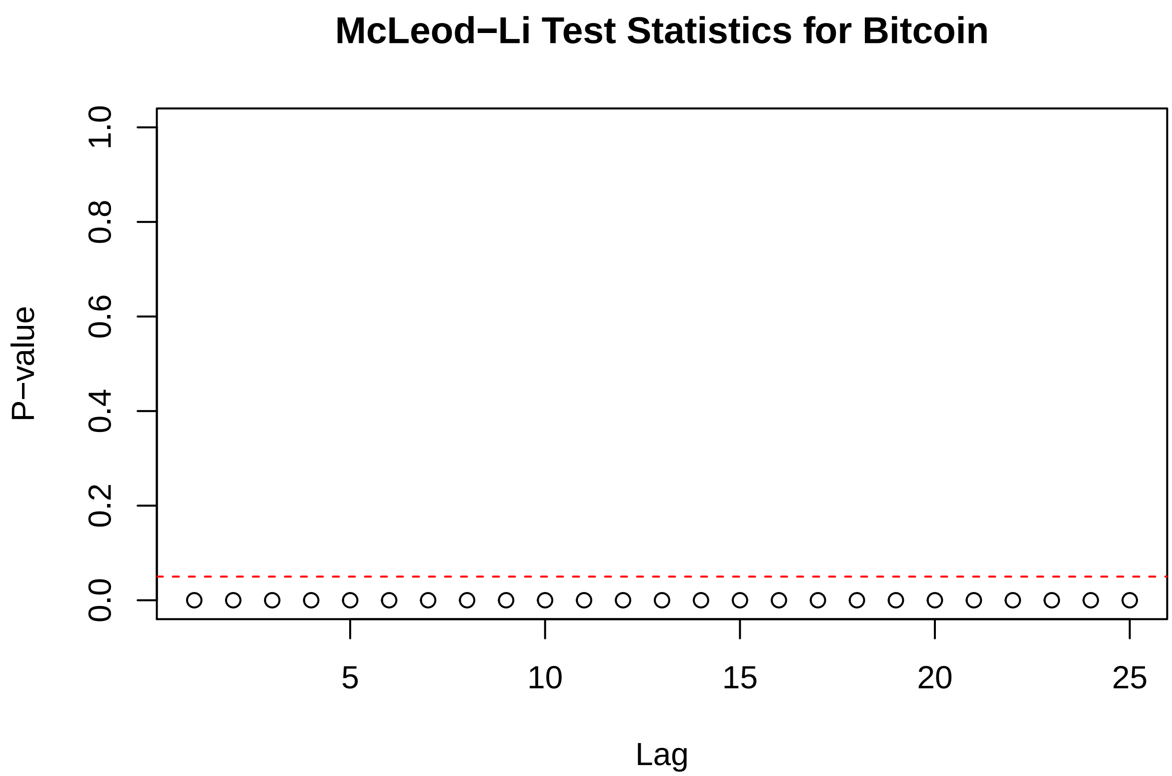

```r
Bitcoin.forecast <- read_csv("../data/Bitcoin_Prices_Forecasts.csv")
Bitcoin.forecast$Date = as.Date(Bitcoin.forecast$Date,'%d/%m/%y')
```

3.3 MASE Error

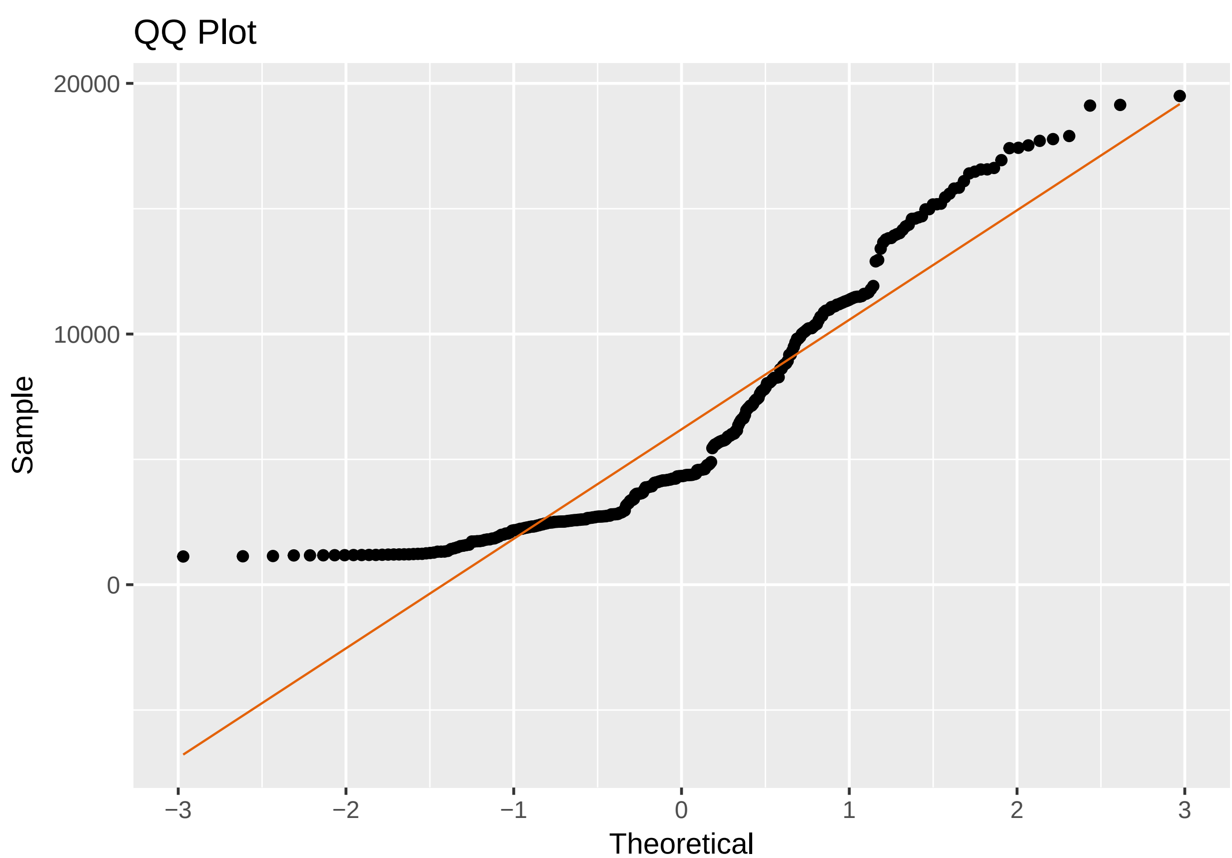
```r
source('MASE.r')
MASE(Bitcoin.forecast$`Closing price`,
     as.vector(tail(fitted(forecast(fit,h=10)),10)))
```

```
## $MASE
##       MASE
## 1 3.232775
```

```r
McLeod.Li.test(y=Bitcoin.2017.zoo,main="McLeod-Li Test Statistics for Bitcoin")
```

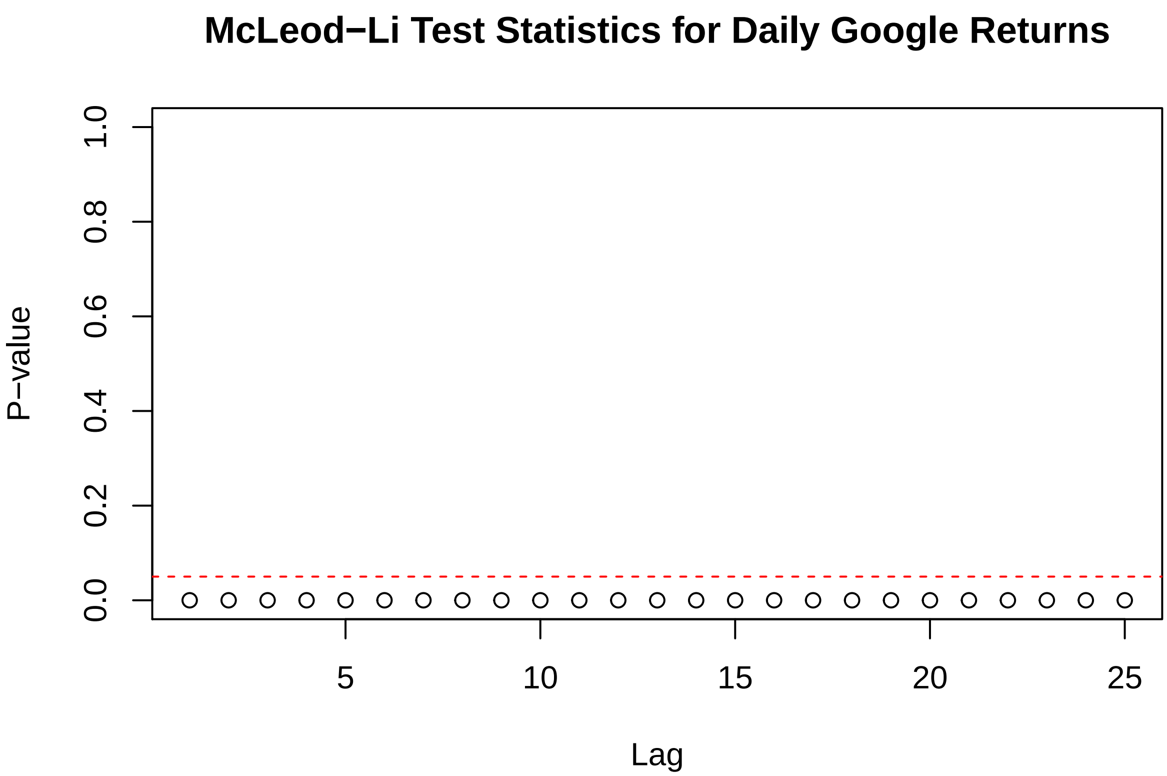



```
residual_analysis_qq(Bitcoin.2017.zoo, 'QQ Plot')
```



4 Heteroscedasticity Models

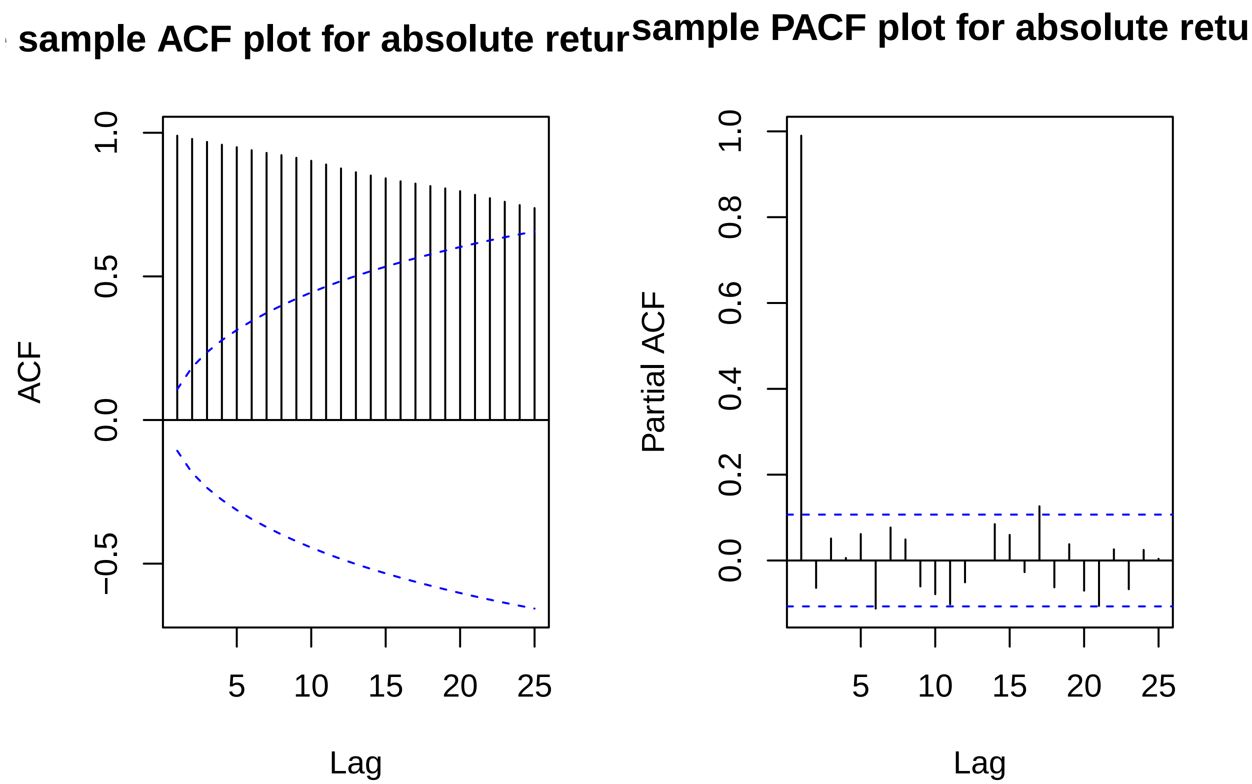
```
McLeod.Li.test(y=Bitcoin.2017.zoo,main="McLeod-Li Test Statistics for Daily Google Returns")
```



McLeod-Li test is significnat at 5% level of significance for all lags. This gives a strong idea about existence of volatiliy clustering.

```
#So we'll use absolute value and square transformations to figure out this ARCH effect.
abs.bitcoin = abs(Bitcoin.2017.zoo)
sq.bitcoin = Bitcoin.2017.zoo^2

par(mfrow=c(1,2))
acf(abs.bitcoin, ci.type="ma",main="The sample ACF plot for absolute return series")
pacf(abs.bitcoin, main="The sample PACF plot for absolute return series")
```




```r
eacf(abs.bitcoin)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x x x x x x x x x x x  x  x  x
## 1 o o o o x o o o o x o  o  x  o
## 2 x o o o o x o o o x o  o  o  o
## 3 o x o o o o o o o x o  o  o  o
## 4 o x x o o o o o o x o  o  o  o
## 5 x x x o o o o o o x o  o  o  o
## 6 x x x o o o o o o x o  o  o  o
## 7 x x x o o o o o o o o  o  o  o
```

5 After the absolute value transformation, we boserve many sign-ficicant lags in

6 both ACF and PACF. Also, EACF do not suggest an ARMA(0,0) model.

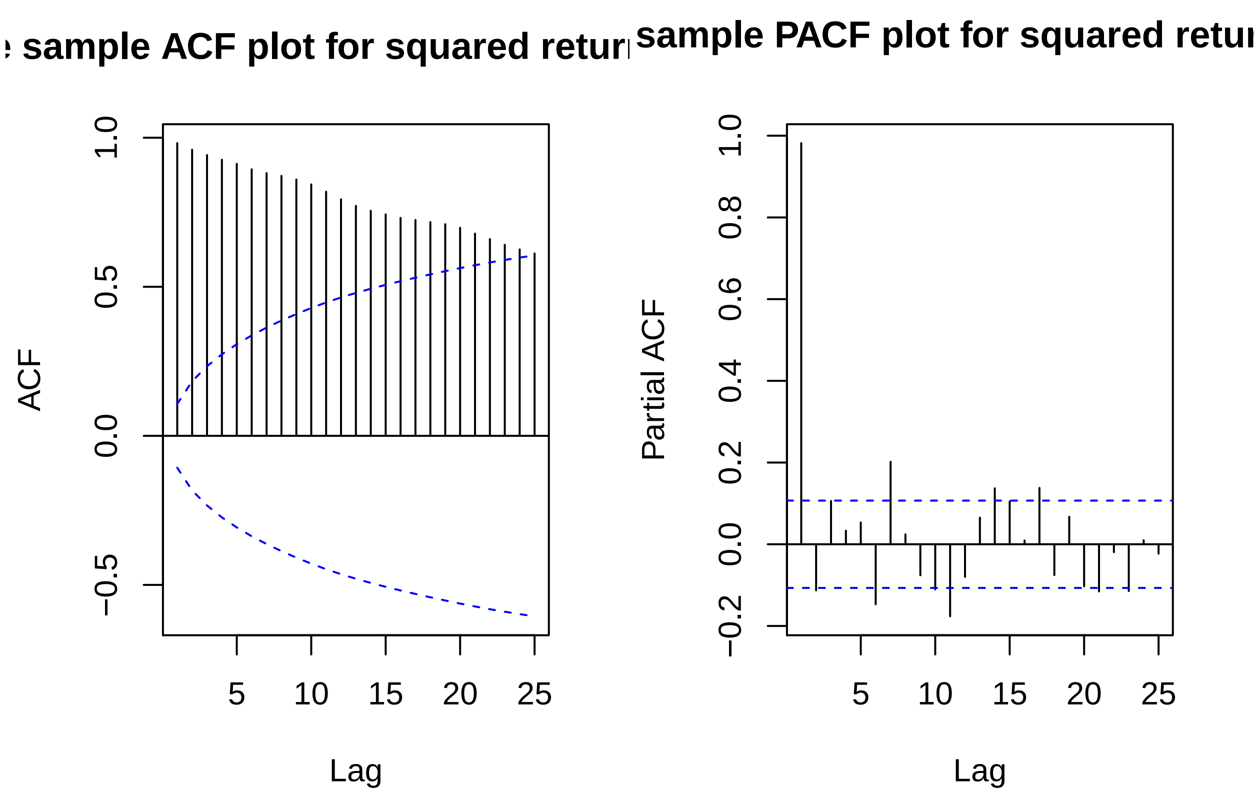
7 From the EACF, we can identify ARMA(1,0), ARMA(1,1), and ARMA(2,1) models for absolute

8 value series.

9 These models correspond to parameter settings of [max(1,1),1], [max(1,2),1] and [max(2,2),2].

10 So the corresponding tentative GARCH models are GARCH(0,1), GARCH(1,1), GARCH(1,2).

```
par(mfrow=c(1,2))
acf(sq.bitcoin, ci.type="ma",main="The sample ACF plot for squared return series")
pacf(sq.bitcoin, main="The sample PACF plot for squared return series")
```



```r
eacf(sq.bitcoin)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x x x x x x x x x x x  x  x  x
## 1 x o o o x x o o x x o  x  x  x
## 2 x x o o o x x o o x o  o  o  x
## 3 x x o o o x o o o x o  o  o  o
## 4 x o x o o x o x o x o  o  o  o
## 5 x x o x o o o o o x o  o  o  o
## 6 x x o x o o o x o x o  o  o  o
## 7 o x x x x x x o o o o  o  o  o
```

11 After the square transformation, we boserve many signficicant lags in both ACF and PACF. Also, EACF do not suggest an ARMA(0,0) model.

12 From the EACF, we can identify ARMA(1,1), ARMA(1,2), and ARMA(2,2) models for squared series.

13 These models correspond to parameter settings of [max(1,1),1], [max(1,2),1], [max(1,2),2], and [max(2,2),2]. So the corresponding

14 tentative GARCH models are GARCH(1,1), GARCH(2,1), GARCH(2,2).

```r
m.11 = garch(Bitcoin.2017.zoo,order=c(1,1),trace = FALSE)
summary(m.11) # All the coefficients are significant at 5% level of significance.
```

```
##
## Call:
## garch(x = Bitcoin.2017.zoo, order = c(1, 1), trace = FALSE)
##
## Model:
## GARCH(1,1)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## 0.2458 0.4968 0.7083 0.8982 1.2064
##
## Coefficient(s):
##     Estimate  Std. Error  t value Pr(>|t|)
## a0 1.969e+07          NA       NA       NA
## a1 9.815e-01          NA       NA       NA
## b1 4.590e-08          NA       NA       NA
```

```
##
## Diagnostic Tests:
##  Jarque Bera Test
##
## data:  Residuals
## X-squared = 18.774, df = 2, p-value = 8.381e-05
##
##
##  Box-Ljung test
##
## data:  Squared.Residuals
## X-squared = 302.55, df = 1, p-value < 2.2e-16
```

```
m.11_2 = garchFit(formula = ~garch(1,1), data =Bitcoin.2017.zoo )
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 1)
##  ARMA Order:                0 0
##  Max ARMA Order:            0
##  GARCH Order:               1 1
##  Max GARCH Order:           1
##  Maximum Order:             1
##  Conditional Dist:          norm
##  h.start:                   2
##  llh.start:                 1
##  Length of Series:          336
##  Recursion Init:            mci
##  Series Scale:              4677.035
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                        U          V   params includes
##     mu     -13.25374276  13.25374 1.325374     TRUE
##     omega    0.00000100 100.00000 0.100000     TRUE
##     alpha1   0.00000001   1.00000 0.100000     TRUE
##     gamma1  -0.99999999   1.00000 0.100000    FALSE
##     beta1    0.00000001   1.00000 0.800000     TRUE
##     delta    0.00000000   2.00000 2.000000    FALSE
##     skew     0.10000000  10.00000 1.000000    FALSE
##     shape    1.00000000  10.00000 4.000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1  beta1
##      1      2      3      5
##  Persistence:                  0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
```

```
##
## R coded nlminb Solver:
##
##   0:     414.12432:  1.32537 0.100000 0.100000 0.800000
##   1:     395.93293:  1.31047 0.0659485 0.101483 0.781411
##   2:     233.09559: 0.811091 1.00000e-06 0.386126 0.513406
##   3:     228.28802: 0.811068 0.00278674 0.386136 0.513414
##   4:     228.09216: 0.810228 0.00267505 0.387047 0.512580
##   5:     226.17230: 0.815568 0.00128314 0.384634 0.515234
##   6:     226.12595: 0.815611 0.000874644 0.384694 0.515272
##   7:     226.07325: 0.815975 0.00108646 0.384569 0.515464
##   8:     226.05615: 0.816396 0.00101887 0.384432 0.515684
##   9:     226.02974: 0.817252 0.00112365 0.384227 0.516156
##  10:     225.99259: 0.818917 0.00102218 0.383500 0.516962
##  11:     225.95230: 0.822303 0.00115447 0.382071 0.518538
##  12:     225.84830: 0.830744 0.000983166 0.380929 0.520082
##  13:     225.65941: 0.827567 0.00117393 0.385684 0.516095
##  14:     225.24016: 0.800259 0.000796139 0.424438 0.483201
##  15:     224.76096: 0.808031 0.00226237 0.467075 0.449230
##  16:     222.71183: 0.816061 0.000740948 0.509530 0.415067
##  17:     222.15583: 0.823177 0.00191793 0.551904 0.380675
##  18:     220.80218: 0.837996 0.00167042 0.639493 0.310151
##  19:     218.20747: 0.869164 0.00215705 0.724541 0.239552
##  20:     216.01827: 0.881098 0.00181985 0.754890 0.213999
##  21:     214.76116: 0.881180 0.000740494 0.754893 0.213994
##  22:     214.73188: 0.881593 0.00115532 0.755620 0.213385
##  23:     214.52256: 0.881788 0.000945344 0.755984 0.213079
##  24:     214.47259: 0.882076 0.000800222 0.756347 0.212776
##  25:     214.39931: 0.882293 0.000903503 0.756734 0.212451
##  26:     211.73937: 0.895468 0.000502912 0.789999 0.184468
##  27:     211.62683: 0.895493 0.000730736 0.790000 0.184468
##  28:     211.57356: 0.895550 0.000665235 0.790158 0.184322
##  29:     211.55884: 0.895622 0.000624245 0.790318 0.184175
##  30:     210.18553: 0.912225 0.000431817 0.848511 0.130412
##  31:     208.35499: 0.903062 0.000765140 0.983141 1.00000e-08
##  32:     208.35475: 0.909066 0.000371878  1.00000 1.00000e-08
##  33:     208.08405: 0.904282 0.000685178  1.00000 1.00000e-08
##  34:     207.99150: 0.905788 0.000610466  1.00000 1.00000e-08
##  35:     207.97768: 0.906003 0.000528316  1.00000 1.00000e-08
##  36:     207.97047: 0.906213 0.000558753  1.00000 1.00000e-08
##  37:     207.97029: 0.906127 0.000556062  1.00000 1.00000e-08
##  38:     207.97029: 0.906147 0.000555322  1.00000 1.00000e-08
##  39:     207.97029: 0.906143 0.000555482  1.00000 1.00000e-08
##  40:     207.97029: 0.906143 0.000555482  1.00000 1.00000e-08
##
## Final Estimate of the Negative LLH:
##  LLH:  3047.311    norm LLH:  9.069379
##           mu        omega       alpha1        beta1
## 4.238061e+03 1.215098e+04 1.000000e+00 1.000000e-08
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu         omega        alpha1         beta1
## mu     -0.0010267428 -2.306595e-04 -9.495482e-03 -1.263897e-01
## omega  -0.0002306595 -3.139387e-08 -1.167578e-02  5.037773e-03
```

```
## alpha1 -0.0094954818 -1.167578e-02 -1.651509e+02 -1.757162e+02
## beta1  -0.1263897442  5.037773e-03 -1.757162e+02 -4.305337e+02
## attr(,"time")
## Time difference of 0.005672932 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.03700995 secs
```

```
summary(m.11_2)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = Bitcoin.2017.zoo)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7fc09768c0c8>
##  [data = Bitcoin.2017.zoo]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1       beta1
## 4.2381e+03  1.2151e+04  1.0000e+00  1.0000e-08
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     4.238e+03   5.899e+00   718.42   <2e-16 ***
## omega  1.215e+04          NA       NA       NA
## alpha1 1.000e+00   9.908e-02    10.09   <2e-16 ***
## beta1  1.000e-08   6.125e-02     0.00        1
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  -3047.311    normalized:  -9.069379
##
## Description:
##  Fri May 25 16:57:47 2018 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  36.47852  1.19892e-08
##  Shapiro-Wilk Test  R    W      0.8008819 0
```

```
##  Ljung-Box Test     R    Q(10)  2450.448  0
##  Ljung-Box Test     R    Q(15)  3329.88   0
##  Ljung-Box Test     R    Q(20)  4010.308  0
##  Ljung-Box Test     R^2  Q(10)  68.1558   1.005253e-10
##  Ljung-Box Test     R^2  Q(15)  74.25125  7.728455e-10
##  Ljung-Box Test     R^2  Q(20)  83.69672  9.175625e-10
##  LM Arch Test       R    TR^2   60.79107  1.618925e-08
##
## Information Criterion Statistics:
##      AIC      BIC      SIC     HQIC
## 18.16257 18.20801 18.16229 18.18068
```

```
m.12 = garch(Bitcoin.2017.zoo,order=c(1,2),trace = FALSE)
summary(m.12)# All the coefficients but aplha_2 are significant at 5% level of significance.
```

```
##
## Call:
## garch(x = Bitcoin.2017.zoo, order = c(1, 2), trace = FALSE)
##
## Model:
## GARCH(1,2)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## 0.2490 0.4838 0.6508 0.7765 1.1059
##
## Coefficient(s):
##     Estimate  Std. Error  t value Pr(>|t|)
## a0 1.859e+07          NA       NA       NA
## a1 7.087e-01          NA       NA       NA
## a2 6.945e-01          NA       NA       NA
## b1 1.771e-07          NA       NA       NA
##
## Diagnostic Tests:
##  Jarque Bera Test
##
## data:  Residuals
## X-squared = 15.797, df = 2, p-value = 0.0003713
##
##
##  Box-Ljung test
##
## data:  Squared.Residuals
## X-squared = 299.33, df = 1, p-value < 2.2e-16
```

```
m.12_2 = garchFit(formula = ~garch(2,1), data =Bitcoin.2017.zoo, trace = FALSE )
summary(m.12_2)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(2, 1), data = Bitcoin.2017.zoo, trace = FALSE)
##
```

```
## Mean and Variance Equation:
##  data ~ garch(2, 1)
## <environment: 0x7fc09ab60678>
##  [data = Bitcoin.2017.zoo]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1      alpha2       beta1
## 2.5750e+03  5.3452e+03  1.0000e+00  3.1818e-02  1.2038e-03
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     2.575e+03   1.379e+01  186.761   <2e-16 ***
## omega  5.345e+03          NA       NA       NA
## alpha1 1.000e+00   1.202e-01    8.317   <2e-16 ***
## alpha2 3.182e-02   1.209e-01    0.263    0.792
## beta1  1.204e-03   7.531e-02    0.016    0.987
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  -2989.655    normalized:  -8.897782
##
## Description:
##  Fri May 25 16:57:47 2018 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  58.98889  1.550982e-13
##  Shapiro-Wilk Test  R    W      0.7675237 0
##  Ljung-Box Test     R    Q(10)  1945.51   0
##  Ljung-Box Test     R    Q(15)  2684.468  0
##  Ljung-Box Test     R    Q(20)  3201.47   0
##  Ljung-Box Test     R^2  Q(10)  4.552476  0.9190026
##  Ljung-Box Test     R^2  Q(15)  35.54104  0.002058026
##  Ljung-Box Test     R^2  Q(20)  48.24354  0.0003931735
##  LM Arch Test       R    TR^2   11.52186  0.4848018
##
## Information Criterion Statistics:
##      AIC      BIC      SIC     HQIC
## 17.82533 17.88213 17.82489 17.84797
```

```
m.22 = garch(Bitcoin.2017.zoo,order=c(2,2),trace = FALSE)
summary(m.22) # Higher order parameters are insignificant
```

```
##
## Call:
## garch(x = Bitcoin.2017.zoo, order = c(2, 2), trace = FALSE)
##
```

```
## Model:
## GARCH(2,2)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## 0.2564 0.4932 0.6589 0.7803 1.1086
##
## Coefficient(s):
##     Estimate  Std. Error  t value Pr(>|t|)
## a0 1.750e+07          NA       NA       NA
## a1 7.072e-01          NA       NA       NA
## a2 6.931e-01          NA       NA       NA
## b1 1.661e-03          NA       NA       NA
## b2 1.168e-07          NA       NA       NA
##
## Diagnostic Tests:
##  Jarque Bera Test
##
## data:  Residuals
## X-squared = 15.913, df = 2, p-value = 0.0003504
##
##
##  Box-Ljung test
##
## data:  Squared.Residuals
## X-squared = 298.35, df = 1, p-value < 2.2e-16
```

```
m.22_2 = garchFit(formula = ~garch(2,2), data =Bitcoin.2017.zoo, trace = FALSE, cond.dist = "QMLE" )
summary(m.22_2)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(2, 2), data = Bitcoin.2017.zoo, cond.dist = "QMLE",
##     trace = FALSE)
##
## Mean and Variance Equation:
##  data ~ garch(2, 2)
## <environment: 0x7fc0957aa720>
##  [data = Bitcoin.2017.zoo]
##
## Conditional Distribution:
##  QMLE
##
## Coefficient(s):
##         mu       omega      alpha1      alpha2       beta1       beta2
## 3.7323e+03  3.3504e+04  9.0427e-01  9.7561e-02  1.0000e-08  1.0000e-08
##
## Std. Errors:
##  robust
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
```

```
## mu     3.732e+03   1.991e+02   18.744   <2e-16 ***
## omega  3.350e+04   2.275e+03   14.728   <2e-16 ***
## alpha1 9.043e-01   3.782e-01    2.391   0.0168 *
## alpha2 9.756e-02   3.116e-01    0.313   0.7542
## beta1  1.000e-08   2.205e-01    0.000   1.0000
## beta2  1.000e-08   3.022e-01    0.000   1.0000
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  -3059.749    normalized:  -9.106397
##
## Description:
##  Fri May 25 16:57:47 2018 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  47.53842  4.755152e-11
##  Shapiro-Wilk Test  R    W      0.7674797 0
##  Ljung-Box Test     R    Q(10)  2714.587  0
##  Ljung-Box Test     R    Q(15)  3840.044  0
##  Ljung-Box Test     R    Q(20)  4839.897  0
##  Ljung-Box Test     R^2  Q(10)  11.28675  0.3356185
##  Ljung-Box Test     R^2  Q(15)  39.93851  0.0004632923
##  Ljung-Box Test     R^2  Q(20)  43.65357  0.00167418
##  LM Arch Test       R    TR^2   12.60656  0.3982738
##
## Information Criterion Statistics:
##      AIC      BIC      SIC     HQIC
## 18.24851 18.31667 18.24789 18.27568
```