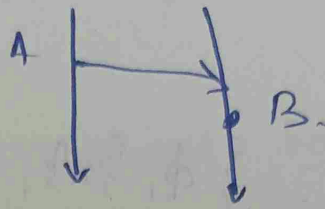
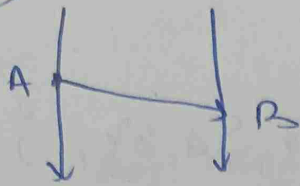
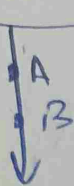
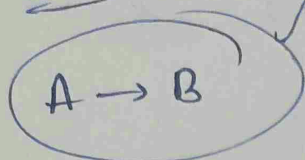


lec-03

→ irreflexive partial order



The set of all of the events in the system are ordered by this partial order,

but the "happen before" is a slightly weird partial order, as it doesn't have reflexivity,

A Standard Partial order has: (Set  $S$ , binary relation  $\leq$ )

- Reflexive  $\rightarrow \forall a \in S, a \leq a$ .

- Antisymmetric  $\rightarrow \forall a, b \in S, a \leq b \& b \leq a$   
then  $a = b$ .

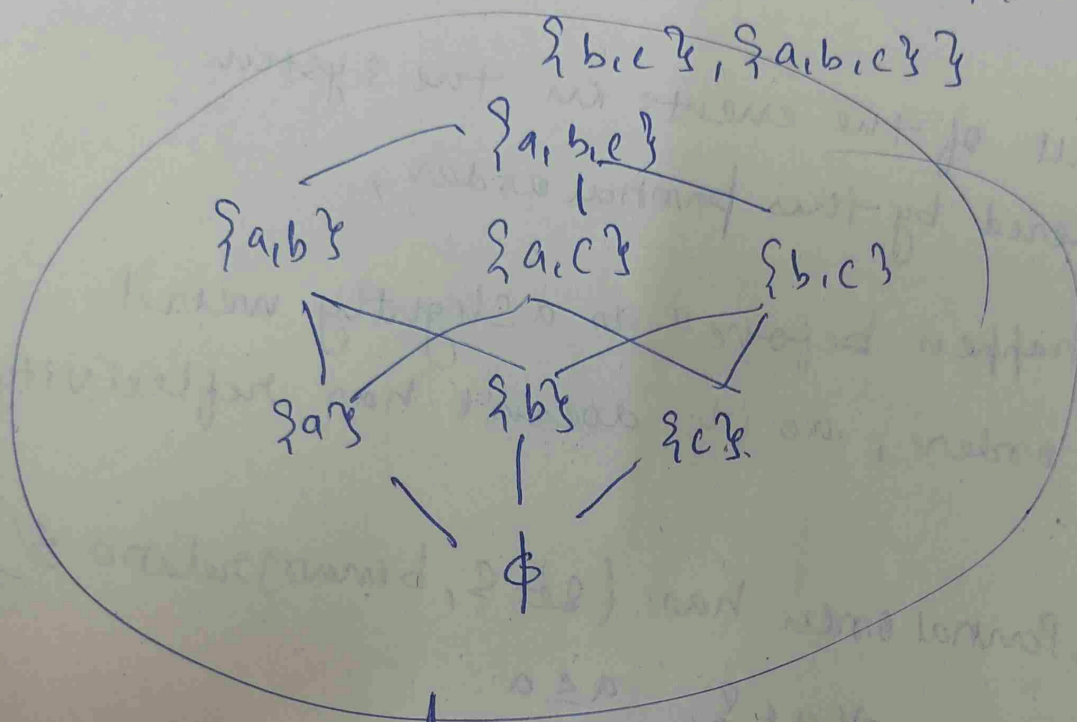
- Transitive:  $\forall a, b, c \in S$

$a \leq b \& b \leq c$  then  $a \leq c$ .

## Set Inclusion

$\{a, b, c\}$ .

Power Set  $= S = \{ \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\} \}$



So, this is an example of a partial order that is truly a partial order, as all three hold.

Reflexive  $\rightarrow \{a\} \leq \{a\}$   
subset

Antisymmetry ✓

Transitive ✓

~~Iska~~ lekin partial order mai kari bhi do  
elements kar paya ana jaroori ni h.

Isme aap kari bhi do element ko relate  
kar pad unse "total order" kehte hain

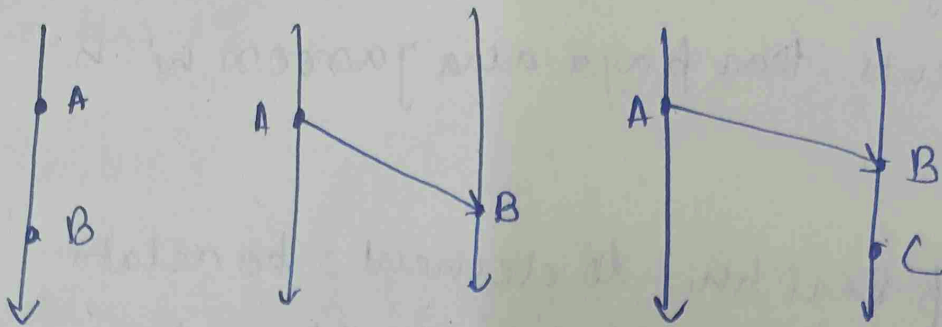
eg: - Integers.

$S = \mathbb{N}$  relation  $\leq \rightarrow$  less than or equal

$\mathbb{N}$

1  
2  
1  
1  
0

$\nexists! \forall e \in \mathbb{N}$  total order.



So, if you have got a bunch of events, you can use these defn to figure out to know what happened before what

Previously, we looked at a set of events and we sort of computed by hand what was in the happens before relation

but we wanna come up with an algorithm by which a computer can straightforwardly compute what's in that happens before relation.



Alternative to using physical clock

↳ logical clock

⇓  
Only tell about  
ordering of events  
and don't tell anything  
about elapsed time  
or time of day

So the simplest type of logical clock is:

Lamport clock

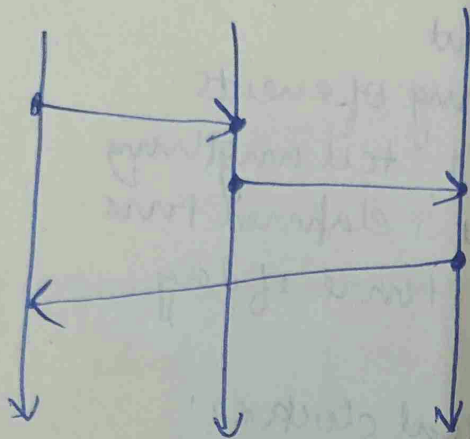
→ Just a way of assigning numbers to events

eg  $LC(A) = 3$   $\rightarrow$  ये 3 क्या ह?

$\boxed{\text{If } A \rightarrow B \text{ then } LC(A) < LC(B)}$

→ Lamport clocks are  
consistent with causality.

## Assigning Lcs to Events



① Every process has a counter initially 0.

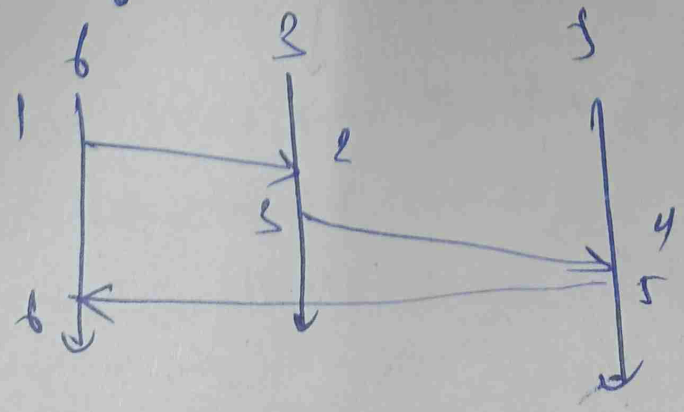
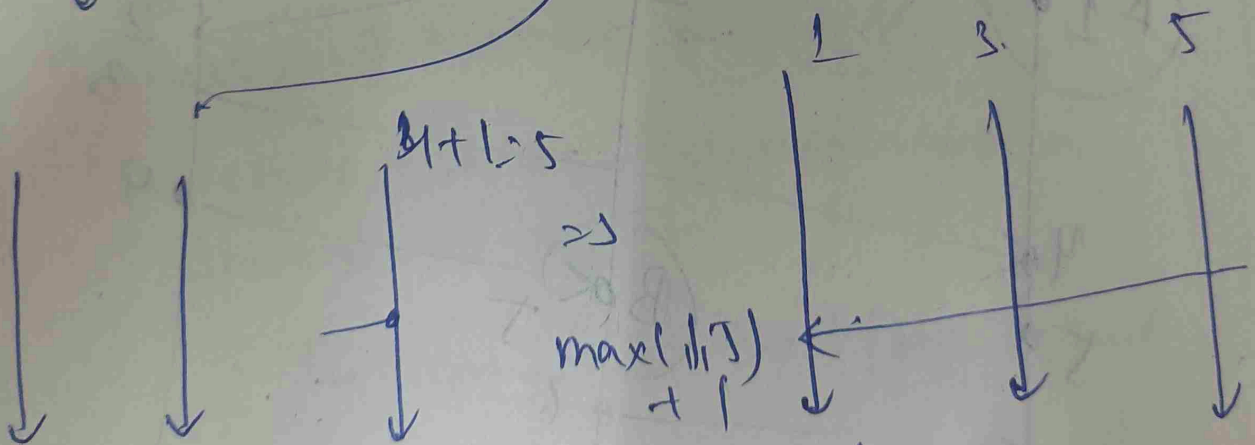
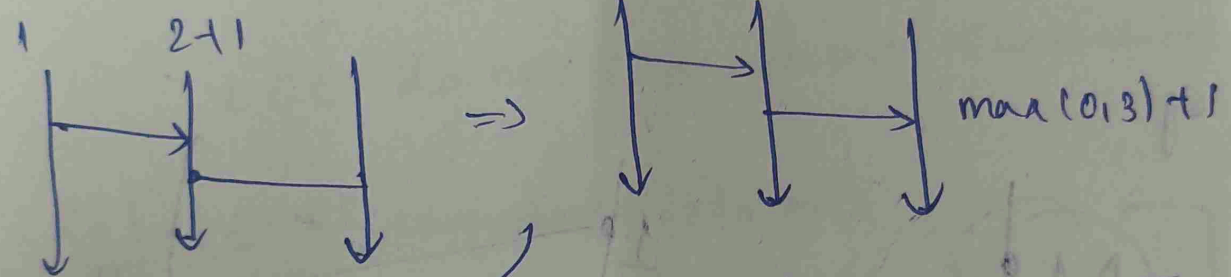
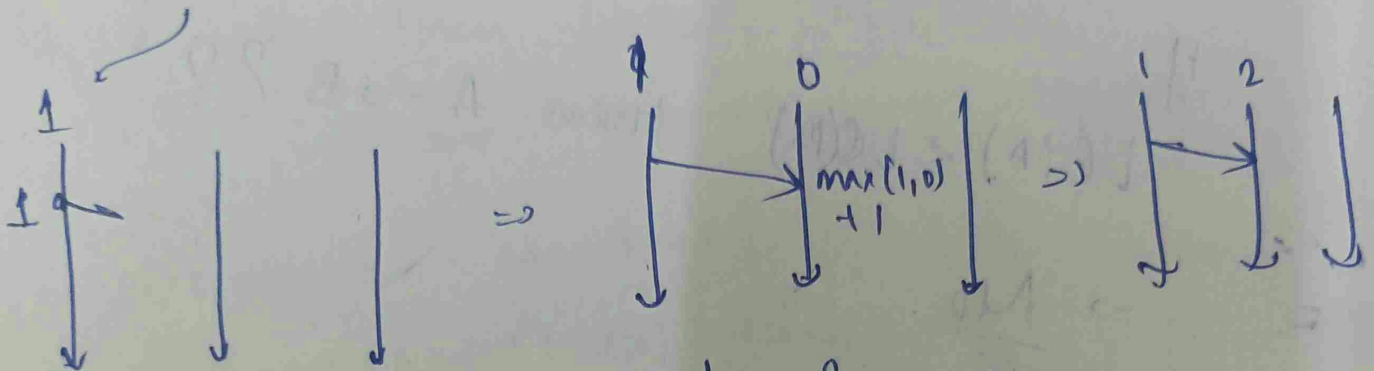
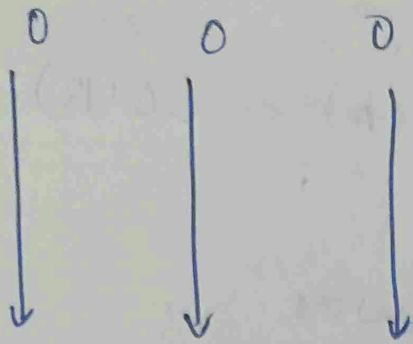
② on every ~~an~~ event, a process increments its counter.

③ When sending a message, a ~~no~~ process includes its current counter along with the message.

④ When receiving a message, set your counter to

$$\max(\text{local counter, message counter}) + 1$$

So, we have a way of assigning numbers to events now, let's try.



we know

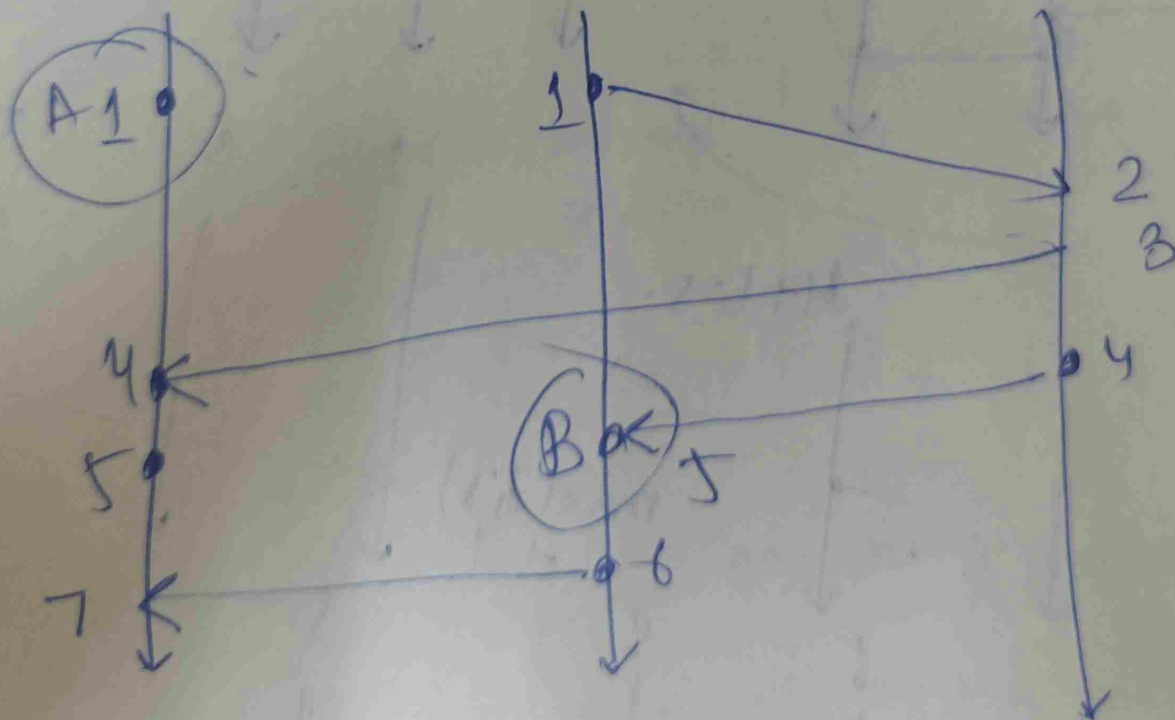
if  $A \rightarrow B$  then  $LC(A) < LC(B)$

but what about other way

if  $LC(A) < LC(B)$  then  $A \rightarrow B$  ??

$\rightarrow$  No

eg:-





Although ~~DA~~  $LC(B) > LC(A)$  but I can't say  
by three times rule that ~~A → B~~  $A \rightarrow B$ ,

if  $A \rightarrow B$  then  $LC(A) < LC(B)$

LCs are consistent with causality

it's not the case that

if  $LC(A) < LC(B)$  then  $A \rightarrow B$

LCs do not characterize causality.

Rs paper (Recommend to read)

Schwarz & Mattern (1994)

"Detecting Causal Relationships in  
Distributed Systems"

"In Search of the Holy Grail"

Mattern was one of the people

who developed another kind of  
logical clock that does characterize  
causality unlike Lamport clock

Even after the limitations of Lamport clock,  
we still use it cause.

if  $A \rightarrow B$ , then  $\neg Q \Rightarrow \neg P$

if  $\neg (LC(A) < LC(B))$ , then  $\neg (A \rightarrow B)$

if  $LC(A)$  is not less than  $LC(B)$

then  $A$  did not happen  
before  $B$

we can't say that  $A$  happened before  $B$