University of Engineering & Management, Kolkata

1st Term Examination, December 2020

Course: B.Tech(All) **Semester: 1st**

Paper Name: Mathematics and Statistics - I

Paper Code: BSC103

Full Marks: 70 **Time: 2 hours**

Answer all the questions. Each question is of 10 marks.

1. A. i) Show that $f(x,y)=\begin{cases}\frac{x^3+y^3}{x-y}\ , & (x,y)\neq(0,0)\\ 0\ , & (x,y)=(0,0)\end{cases}$ is not continuous at $(0,0)$.

ii) Let $f(x,y)=\begin{cases}\frac{xy}{x^2+y^2}\ , & (x,y)\neq(0,0)\\ 0\ , & (x,y)=(0,0)\end{cases}$. Evaluate $f_y(0,0)$.

OR

B. If $u=xf\left(\frac{y}{x}\right)+g\left(\frac{y}{x}\right)$, then show that

$$xu_x+yu_y=xf\left(\frac{y}{x}\right)\text{and}x^2u_{xx}+2xyu_{xy}+y^2u_{yy}=0.$$

2. A. i) Show that the following sequencesis convergent (a) $\left\{1+\frac{(-1)^n}{n}\right\}$, (b) $\left\{\frac{2n}{5n-2}\right\}$.

ii) State Leibnitz's test. Using that show that $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$ is convergent. Is this series absolutely convergent? Justify your answer.

OR

B. Test the convergence of the series $\frac{x}{1}+\frac{1}{2}\frac{x^3}{3}+\frac{1.3}{2.4}\frac{x^5}{5}+\frac{1.3.5}{2.4.6}\frac{x^7}{7}+\cdots$.

3. A. Find for which values of x the function $f(x)=x^3-6x^2+12x-3$ has maximum or minimum. Find the Global max and min in [-1, 3].

OR

B. Find the value of **i)** $\lim_{x\to 0}(1+x)^{1/x}$, **ii)** $\lim_{x\to 0}\frac{xe^x-\log(1+x)}{x^3}$.

4. A. i) Evaluate : $\lim_{n\to\infty}[(1+\frac{1}{n})+\left(1+\frac{2}{n}\right)+\left(1+\frac{3}{n}\right)+\cdots+\left(1+\frac{n}{n}\right)]^{\frac{1}{n}}$

ii) Evaluate :$\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}dx$

OR

B. i) Examine the convergence of the improper integral $\int_0^2 \frac{dx}{x(2-x)}$

ii) Evaluate $\int_{-\infty}^{\infty} 5^{-x^2} dx.$

5. A. i) Find the value of $\left(\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\, d\theta\right)\left(\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\sin\,\theta}} d\theta\right).$

ii) Prove that $\Gamma\left(\frac{1}{9}\right)\Gamma\left(\frac{2}{9}\right)\ldots\ldots\Gamma\left(\frac{8}{9}\right) = \frac{16}{3}\pi^4.$

OR

B. Show that $\beta(m,n) = \int_0^{\infty} \frac{x^{n-1}}{(1+x)^{m+n}} dx = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} dx.$

6. A. i) If θ be the angle between the surfaces $xy^2z = 3x + z^2$ *and* $3x^2 - y^2 + 2z = 1$ at the point (1,-2,1) , then find the value of θ.

ii) Find the direction from the point (2,1,-1) in which the directional derivative of $\phi = x^2yz^3$ is maximum? What is the magnitude of the maximum?

OR

B. Show that $r^n\vec{r}$ is an irrational vector for any value of n but it is solenoidal, if $n + 3 = 0$.

7. A. i) Verify Rolle's Theorem for the function $f(x) = \sin x\,, x\ \ in\ [0, \frac{\pi}{2}]$.

ii) Using mean value theorem prove that $0 < \frac{1}{\log(1+x)} - \frac{1}{x} < 1$.

OR

B. Expand the following functions in powers of x by the use of Maclaurin's theorem:

i) e^x , **ii)** $\log(1 + x)$.

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