University of Engineering & Management, Kolkata

2nd Term Examination, January 2021

Course: B.Tech(All) **Semester: 1st**

Paper Name: Mathematics and Statistics - I

Paper Code: BSC103

Full Marks: 70 **Time: 2 hours**

Answer all the questions. Each question is of 10 marks.

1. **A.** **i)** In the Mean value theorem $f(b) - f(a) = (b - a)f'(c)$, determine c lying between a and b,if $f(x) = x(x-1)(x-2), a = 0$ and $b = \frac{1}{2}$.

ii) Expand $\log_e x$ in powers of $(x - 1)$.

OR

B. If $(x) = \log(1 + x),\ x > 0$, using Maclaurin's theorem , show that for $0 < \theta < 1$, $\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3(1+\theta x)^3}$. Deduce that $\log(1 + x) < x - \frac{x^2}{2} + \frac{x^3}{3}$ for $x > 0$

2. **A.** **i)** Find the values of a and b such that $\lim_{x\to 0} \frac{x(a+b\cos x)-c\sin x}{x^5} = 1$

ii) A wire of length 20 meters is bent so as to form a circular sector of maximum area. Find the radius of the circular sector.

OR

B. Find the maximum value of x^3y^2 subject to the constraint $x + y = 1$, using the method of Lagrange's multiplier.

3. **A.** **i)** Show that $\int_a^b (x-a)^3(b-x)^2\, dx = \frac{(b-a)^6}{60}$.

ii) Evaluate $\int_{-\infty}^{\infty} 2021^{-x^2} dx$.

OR

B. **i)** Find the area of the region bounded by the curve $y = x(x-a)(x-b)$ and the x axis, where a and b are positive numbers with $a < b$.

ii) Find the volume of the solid obtained by revolving the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about its axis of symmetry.

4. **A.** **i)** Find $\int_0^\infty e^{-x^2}x^2dx \times \int_0^\infty x^4e^{-x^4}dx$ (if exists).

ii) Show that : $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\, dx$

OR

B. **i)** Find the area bounded by $y^2 = x$ and its latus rectum.

ii) Find the volume of the solid obtained by revolving the cycloid $x = a(\theta + sin\theta), y = a(1 + cos\theta)$ about its base.

5. **A.** **i)** If $f(x,y) = x^2 tan^{-1}\frac{y}{x} - y^2 tan^{-1}\frac{x}{y}$ prove that $f_{xy} = \frac{x^2-y^2}{x^2+y^2}$

ii) If $v = f(x^2 + 2yz, y^2 + 2zx)$, then prove that

$$(y^2 - zx)\frac{\partial v}{\partial x} + (x^2 - yz)\frac{\partial v}{\partial y} + (z^2 - xy)\frac{\partial v}{\partial z} = 0.$$

OR

B. **i)** If $f(x,y) = x$ and $g(x,y) = xy$ then find $\frac{\partial(f,g)}{\partial(x,y)}$.

ii) If $\theta = t^n e^{-\frac{r^2}{4t}}$, find what value of n will make $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right) = \frac{\partial\theta}{\partial t}$

6. **A.** **i)** Given $\vec{A} = (6xy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (3xz^2 - y)\,\hat{k}$ is irrotational. Find the scalar function ϕ such that $\vec{A} = \mathbf{grad}(\phi)$.

ii) If θ be the angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point (1,-2,1) , then find the value of θ.

OR

B. **i)** An incomplete frequency distribution is given below:

Height (inches)	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99	Total
No. of plants	2	12	15	?	18	?	9	4	90

It is known that the median height of the plant is 57.5 inches. Calculate the missing frequency.

ii) Find the standard deviation from the following distribution.

Weight (in gm)	0-5	5-10	10-15	15-20	20-25	25-30
No. of articles	10	7	5	6	3	11

7. **A.** **i)** Examine the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \cdots$.

ii) Show that the series is conditionally convergent $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$.

OR

B. **i)** Three cards are drawn at random from a pack of 52 cards. Find the probability that **(a)** They are King, Queen, and Knave **(b)** All of them are aces.

ii) The chance that a doctor will diagnose a certain disease correctly is 60%.The chance that a patient will die under his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of the doctor who had the disease died. What is the chance that his disease was diagnosed correctly?

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