University of Engineering & Management (UEM), Jaipur | Kolkata

Institute of Engineering & Management (IEM), Jaipur | Kolkata

श्रद्धावान लभते ज्ञानम्
Good Education, Good Jobs

# Topic: Differential Equation


Instructor: Doyel Sarkar

Email: doyel.sarkar@uem.edu.in

# ❑Topics of Discussion:

❑ ODE: 1st order & 1st degree

✔ Solution techniques of 1st order & 1st degree ODE

H.W (1) :- $y^2 dx + (x^2 - xy - y^2) dy = 0$

H.W (2) :- $x^2 y dx - (x^3 + y^3) dy = 0.$

R(5):- If D.E. (1) can be expressed in the form:

$$y\,f(xy)\,dx + x\,g(xy)\,dy = 0\text{; then}$$

$\dfrac{1}{-Ny+Mx}$, $(Mx - Ny \neq 0)$ is an I.F. of D.E (1).

Problem :- $(1+xy)\,y\,dx + (1-xy)\,x\,dy = 0$

**Problem** :- $(1+xy)y\,dx + (1-xy)x\,dy = 0$

**Problem** :- $(1+xy)y\,dx + (1-xy)x\,dy = 0$

H.W (1) : $(x^2y^2 + xy + 1)y\,dx + (x^2y^2 - xy + 1)x\,dy = 0$

H.W (2) : $\{xy\sin(xy) + \cos(xy)\}y\,dx + \{xy\sin(xy) - \cos(xy)\}x\,dy = 0$

**R(6):-** If $M\,dx + N\,dy = 0$ is of the form
$x^a y^b (my\,dx + nx\,dy) + x^{a_1} y^{b_1} (m_1 y\,dx + n_1 x\,dy) = 0$, where
$a, b, a_1, b_1, m, n, m_1, n_1$ are constants & $mn_1 - m_1 n \neq 0$,
then $x^h y^k$ is an I.F. of $M\,dx + N\,dy = 0$, where

$$\frac{a+h+1}{m} = \frac{b+k+1}{n} \quad \& \quad \frac{a_1+h+1}{m_1} = \frac{b_1+k+1}{n_1}.$$

**Problem:-** Solve : $(x^7y^2 + 3y)\,dx + (3x^8y - x)\,dy = 0$

**Problem** :- Solve : $(x^7y^2+3y)\,dx+(3x^8y-x)\,dy=0$

Problem :- Solve : $(x^7y^2+3y)\,dx + (3x^8y - x)\,dy = 0$

**Problem:-** Solve: $(x^7y^2+3y)\,dx+(3x^8y-x)\,dy=0$

H.W (1) - $(3x + 2y^2) y \, dx + 2x (2x + 3y^2) dy = 0$

Solve : $3ydx - 2xdy + x^2 y^{-1} (10ydx - 6xdy) = 0.$

* Linear Equation :- A D.E. is of the form

$$\frac{dy}{dx} + Py = Q \quad \text{where} \quad \left.\begin{array}{l} P \equiv P(x) \\ Q \equiv Q(x) \end{array}\right]$$

is called a linear D.E. of first order.

Solution of the given D.E is

$$y \times (I.F.) = \int Q \times (I.F.)\, dx + c \;; \quad I.F. = e^{\int P\, dx}.$$

** $$\frac{dx}{dy} + Px = Q \quad ; \quad \text{where} \quad \left.\begin{array}{l} P \equiv P(y) \\ Q \equiv Q(y) \end{array}\right\}$$

Solution :- $$x \times (\text{I.F.}) = \int \{Q \times \text{I.F.}\}\, dy + c.$$

Problem : Solve : $x \cos x \dfrac{dy}{dx} + y(x \sin x + \cos x) = 1$

Problem : Solve : $x \cos x \frac{dy}{dx} + y(x \sin x + \cos x) = 1$

Problem : Solve : $x \cos x \frac{dy}{dx} + y(x \sin x + \cos x) = 1$

H.W(1) :- $\frac{dy}{dx} + y = e^{-x}$.

H.W(2) :- $\left(x - \frac{1}{y}\right)\frac{dy}{dx} + y^2 = 0$

H.W(3) :- $(x - \log y)\frac{dy}{dx} + y \log y = 0$

H.W(4) :- $(\sec x \tan x \tan y - e^x)\,dx + \sec x \sec^2 y\,dy = 0$.

* Bernoulli's Equation : The general form of B.E. is

$$\frac{dy}{dx} + Py = Q \cdot y^n \qquad \text{——— (1')}$$

Where P & Q are functions of x.

It is clear that this D.E. is non-linear, but it can be reduced to linear form as follows:

Multiplying both sides of (1) by $y^{-n}$, we get

$$y^{-n}\cdot\frac{dy}{dx} + P\cdot y^{1-n} = Q \quad \text{——— (2)}$$

Let us put, $z = y^{1-n}$. $\Rightarrow \frac{dz}{dx} = (1-n)\,y^{-n}\,\frac{dy}{dx}$.

Then (2) becomes: $\frac{dz}{dx} + (1-n)Pz = (1-n)Q$ ——— (3)

which is a linear D.E. in $z$.

**Problem:** Solve: $xy - \frac{dy}{dx} = y^3 \cdot e^{-x^2}$

Problem: Solve: $xy - \frac{dy}{dx} = y^3 \cdot e^{-x^2}$

Problem : Solve : $xy - \frac{dy}{dx} = y^3 \cdot e^{-x^2}$

H.W (1) :- Solve : $y' + y = y^3(\cos x - \sin x)$ [WBUT - 07, 10]

4. $(y^2 + 2x^2y)dx + (2x^3 - xy)dy = 0.$

5. $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0.$

6. $ye^{xy}dx + (xe^{xy} + 2y)dy = 0.$

7. $(\sin x \cos y + e^{2x})dx + (\cos x \sin y + \tan y)dy = 0.$

8. $(1+xy)ydx+(1-xy)xdy=0.$

9. $(xy^2+2x^2y^3)dx+(x^2y-x^3y^2)dy=0.$

10. $(3xy-2ay^2)dx+(x^2-2axy)dy=0.$

11. $(2x^2y-3y^4)dx+(3x^3+2xy^3)dy=0.$

12. $(e^x\sin y+e^{-y})dx+(e^x\cos y-xe^{-y})dy=0.$

13. $(2x^2y^2+y)dx-(x^3y-3x)dy=0.$

14. $(\cos x-x\cos y)dy-(\sin y+y\sin x)dx=0$

18. $\left(3x^2y^4 + 2xy\right)dx + \left(2x^3y^3 - x^2\right)dy = 0$

19. $y\left(y^2 - 2x^2\right)dx + x\left(2y^2 - x^2\right)dy = 0$

20. $\left(x^2y^2 + xy + 1\right)ydx + \left(x^2y^2 - xy + 1\right)xdy = 0$

21. $x^3y^3(2ydx + xdy) - (5ydx + 7xdy) = 0$

22. $3\left(x^2 + y^2\right)dx + x\left(x^2 + 3y^2 + 6y\right)dy = 0$

23. $x(4ydx + 2xdy) + y^3(3ydx + 5xdy) = 0$

24. Solve $(5x^2 + xy - 1)dx + (\frac{1}{2}x^2 - y + 2y^2)dy = 0$; given $y = 1$ when $x = 0$.