# Pipes and Cisterns

1. A pipe can fill a tank in 6 hours. Another pipe can empty the tank in 12 hours. If both pipes are opened simultaneously, the part of tank filled by both pipes in 1 hour is?

   a) $\frac{1}{9}$ th part

   b) $\frac{1}{6}$ th part

   **c) $\frac{1}{12}$th part**

   d) $\frac{1}{3}$ rd part

   **Solution** -

   Tank filled or work done by Pipe 1 in 1 hour $= \frac{1}{6}$

   Tank emptied or work done by Pipe 2 in 1 hour $= \frac{1}{12}$

   **Tank filled** or work done **by both pipes in 1 hour** $= \frac{1}{6} - \frac{1}{12} = \mathbf{\frac{1}{12}}$

   > **Tip:**
   > **We need to subtract because Pipe 2 is emptying the tank.**

2. Two pipes M and N can fill a tank in 22 hours and 33 hours respectively. In how much time will the tank be full, if both the pipes are opened simultaneously?

   a) 18 hours

   b) $\frac{19}{22}$ hours

   c) $1\frac{1}{11}$ hours

   d) **$13\frac{1}{5}$ hours**

   **Solution** -

   Tank filled or work done by M in 1 hour $= \frac{1}{22}$

   Tank filled or work done by N in 1 hour $= \frac{1}{33}$

**Tank filled** or work done **by both pipes in 1 hour** $= \frac{1}{22} + \frac{1}{33} = \frac{5}{66}$

**So the entire tank is full in** $\frac{66}{5} = 13\frac{1}{5}$ **hours**

> **Tip:**
> **if A completes** $\frac{1}{n}$ **amount of work in 1 day, he completes entire work in 'n' days.**

3. A tap can fill a tub in 24 hours. Due to a leak at the bottom of the tub, the tap fills the tub in 36 hours. If the tub is full, how much time will the leak take to empty it?

**a) 72 hours**
b) 48 hours
c) 60 hours
d) 96 hours

Solution -

Normally, tap can fill the tub in 24 hours. So in 1 hour it fills $= \frac{1}{24}$ part of tub

Due to leak, tap can fill the tub in 36 hours. So in 1 hour it actually fills $= \frac{1}{36}$ part of tub

**∴ Water removed by the leak in 1 hour** $= \frac{1}{24} - \frac{1}{36} = \frac{1}{72}$

**∴ Leak empties the tank in 72 hours.**

4. Pipe P can fill a tank in 38 hours. Pipe Q alone can fill it in 19 hours. Pipe R can empty the full tank in 133 hours. If all the pipes are opened together, how much time will be needed to make the tank full?

a) 28 hours
b) 95 hours
c) 19 hours

**d) 14 hours**

**Solution** -

Tank filled by P in 1 hour $= \frac{1}{38}$ ; Tank filled by Q in 1 hour $= \frac{1}{19}$

Tank emptied by R in 1 hour $= \frac{1}{133}$

**Tank filled by all 3 pipes together in 1 hour $= \frac{1}{38} + \frac{1}{19} - \frac{1}{133}$**

Making denominators common,

**Tank filled by all 3 pipes together in 1 hour** $= \frac{3.5}{133} + \frac{7}{133} - \frac{1}{133} = \frac{9.5}{133} = \frac{1}{14}$

**So the entire tank is full in $\frac{14}{1}$ = 14 hours** ⟶ Inverse or reciprocal of $\frac{1}{14}$

5. Two pipes M and N are opened together to fill a tank. Both the pipes fill the tank in time X minutes. When M alone is filling the tank, it takes 9 minutes more time than X to fill the tank. In a similar manner N takes 16 minutes more time than X to fill the tank. What is the value of X?

**a) 12 minutes**
b) 7 minutes
c) 25 minutes
d) 144 minutes

**Solution** -

This is similar to what we do in Time and Work.

> **Tip:**
> **In such cases, use the following trick -**
> $$\mathbf{X = \sqrt{Extra\ time\ of\ M \times Extra\ time\ of\ N}}$$

**$\therefore X = \sqrt{9 \times 16} = 12$ minutes = Time in which M and N together fill the tank**

6. Two pipes M and N can fill a tank in 22.5 and 15 minutes, respectively. If both the pipes are opened simultaneously, after how much time should N be closed so that the tank is full in 18 minutes?

a) 4 minutes
**b) 3 minutes**
c) 4.5 minutes
d) 2.5 minutes

Solution -

M fills tank in 22.5 minutes and M remains open for 18 minutes ⟶ **Given**
So total tank filled by M = Tank filled in 1 min x 18 minutes = $\frac{1}{22.5}$ x 18 = $\frac{4}{5}$
**This is the entire work done by M.**
**So whatever is the remaining work, it is done by only N**
Let N be open for T minutes.
**Total tank filled by N** = 1 - $\frac{4}{5}$ = $\mathbf{\frac{1}{5}}$ = Tank filled in 1 min x T minutes = $\frac{1}{15}$ x T
∴ $\frac{1}{5}$ = $\frac{1}{15}$ x T
∴ T = **3 minutes = N should be closed after this much time**

7. Pipe A can fill a tank 5 times faster than Pipe B and takes 32 minutes less than Pipe B to fill the tank. If both the pipes are opened together, then in how much time the tank would be full?

a) $\frac{32}{3}$ minutes
b) $\frac{32}{5}$ minutes
**c) $\mathbf{\frac{20}{3}}$ minutes**
d) $\frac{5}{32}$ minutes

Solution -

Let Pipe A take T minutes to fill the tank alone.
Since Pipe A is 5 times faster than Pipe B, Pipe B takes 5 times more time.
So time taken by Pipe B = 5T minutes

Also, 5T-T = 32 ⟶ **Given**

∴ T = **8 minutes = Time taken by A**

**Time taken by B** = 5 x 8 = **40 minutes**.

In 1 min, A + B fills $= \frac{1}{8} + \frac{1}{40} = \frac{3}{20}$ part of tank

**So entire tank is filled in $\frac{20}{3}$ minutes** ⟶ Inverse or reciprocal of $\frac{3}{20}$

8. A tap having diameter 'd' can empty a tank in 84 min. How long another tap having diameter '2d' take to empty the same tank?

a) **21 minutes**
b) 42 minutes
c) 168 minutes
d) 28 minutes

<u>Solution</u> -

**Area ∝ Water flowed or Work done**

Area = $\pi r^2$

Since, it is square of radius, when radius or diameter doubles, area becomes 4 times.

When area becomes 4 times, work done or water flowed also becomes 4 times.

If water flow increases by 4 times, time taken reduces by 4 times.

**So time taken by tap with diameter '2d' to empty the tank = $\frac{84}{4}$ = 21 minutes.**

9. A tank has three taps P, Q and R. Taps P and Q can fill the tank in 1.5 and 2 hours, respectively. Tap R can empty the completely filled tank in just half hour. Tap P is opened at 8 am, tap Q is opened at 9am and tap R is opened at 10 am. At what exact time will the tank be empty?

a) 12 pm
b) 1.25 pm
**c) 12.12 pm**
d) 12.28 pm

**Solution** -

Let the tank get empty in T hours counting from 8 am.

A is on for T hours and work done by A = Work in 1 hour x T hours $= \frac{T}{1.5} = \frac{2T}{3}$

Similarly, Tap B starts at 9 am i.e. it's on for (T-1) hours & work done is $= \frac{T-1}{2}$

Similarly, Tap C starts at 10 am i.e. for (T-2) hours & work done is $= \frac{T-2}{\frac{1}{2}} = 2(T-2)$

**Initially tank is empty and after T hours too, it is empty. So, total work done is 0.**

$\therefore \frac{2T}{3} + \frac{T-1}{2} - 2(T-2) = 0$

$\therefore$ **T =** $\frac{21}{5}$ hours **=** 4.2 hours **= 4 hours 12 minutes**

This time is needed for tank to get empty.

**Exact time will be 4 hours 12 min from 8 am = 12.12 pm**

10. Two pipes M and N can fill a pool in 40 minutes and 60 minutes, respectively. The entire pool can completely be emptied by another pipe P in 30 minutes. M, N and P are opened alternatively and each is kept open for just 1 minute. In how much time will the pool be filled?

a) 180 minutes
**b) 167 minutes**
c) 177 minutes
d) 165 minutes

**Solution** -

In 1st minute, only Pipe M is on. Water flowed in 1 minute $= \frac{1}{40}$

In 2nd minute, only Pipe N is on. Water flowed in 1 minute $= \frac{1}{60}$

In 3rd minute, only Pipe C is on. Water drained in 1 minute $= \frac{1}{30}$

So in 3 minutes, total parts of pool filled $= \frac{1}{40} + \frac{1}{60} - \frac{1}{30} = \frac{1}{60}$

**So, after 3 minutes, only 1 part gets filled.**

**Now, after initial 2 minutes i.e. before starting pipe C, how much of the pool is filled?**

It is $\frac{1}{40} + \frac{1}{60} = \frac{5}{60}$ = **that means 5 parts get filled in initial 2 min** → **Point 1**

Now, 60 parts -5 parts = 55 parts

In total 3 minutes 1 part gets filled

In total **? minutes** 55 parts get filled

∴ ? = 55 x 3 = **165 minutes**
Once the 55 parts are filled in 165 minutes,
In 166th and 167th minute Pipe M and Pipe N are turned on one after the other.
Part of pool filled in these two minutes = $\frac{5}{60}$ = Remaining 5 parts → **From Point 1**
**This fills all 60 parts of pool in 167 minutes**

11. 3 pipes have diameters 2 cms, 3 cms and 4 cms. The ratio of water flowing through them is equal to ratio of the square of their diameters. The biggest pipe when open alone, can fill the entire pool in just 126 minutes. When all the pipes are opened together, the pool would be filled in how much time?

**a)** $\mathbf{\frac{2016}{29}}$ **minutes**
b) $\frac{2111}{126}$ minutes
c) $\frac{215}{63}$ minutes
d) 155 minutes

**Solution** -

**Ratio of square of diameter of pipes = Ratio of Water flowed → Given**
But, Water flow $\propto \frac{1}{\text{Time taken}}$ ∴ $(\mathbf{d})^2 \propto \frac{\mathbf{1}}{\textbf{Time taken}}$

∴ If diameter becomes ½ i.e. 4 cms to 2 cms, time needed becomes $(2^2) = 4$ times.

For diameter 4 cms, time needed is 126 minutes.

**So for 2 cms diameter time needed** = 126 x 4 **= 504 minutes**

If diameter becomes 3 cms from 2 cms it means it increases by 1.5 times.

**Time taken by pipe of diameter 3 cm =** $\frac{504}{(1.5)^2}$ **= 224 minutes**

**When all 3 pipes are opened, in 1 min they fill how much of the pool?**

It is given by $= \frac{1}{126} + \frac{1}{504} + \frac{1}{224}$

Making denominators common, we get $= \frac{1 \text{ x } 16}{126 \text{ x } 16} + \frac{1 \text{ x } 4}{504 \text{ x } 4} + \frac{1 \text{ x } 9}{224 \text{ x } 9} = \frac{29}{2016}$

**So, the entire pool is filled in $\frac{2016}{29}$ minutes**