Optimization Assignment - 1

Thoutu Rahul Raj

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***Problem Statement* - Find the maximum isosceles triangle inscribable in a given ellipse,i.e,find the maximum value of xy,having given** $\frac{x^2}{a^2}+\frac{y^2}{b^2}$**=1.**

1 Solution

Given function is,

$$f(x) = absin\theta(1 - cos\theta) \tag{1}$$

1.1 Calculation using normal differentiation

Differentiating (1) yields,

$$\nabla f(x) = ab(cos\theta - cos2\theta) \tag{2}$$

Calculating the critical points: $\nabla f(x) = 0$

$$\implies \cos\theta = 0 \tag{3}$$

$$\implies cos2\theta = 0 \tag{4}$$

Therefore, the critical points are

$$\pi n, \quad \frac{\pi}{2} + \pi n \tag{5}$$

1.1.1 Finding absolute maximum Since given interval is $x \in [0, n\pi]$

f attains its maximum value on the interval [0,2] at x=1.

$$\implies \nabla f(1) = 0$$

$$\implies a = 1.999$$

1.2 Calculation of Maxima using gradient ascent algorithm

Maxima of the above equation (1), can be calculated from the following expression,

$$x_{n+1} = x_n + \alpha\nabla f(x_n) \tag{6}$$

1.3 Calculation of Maxima using gradient ascent algorithm

$$f(x) = absin\theta(1 - cos\theta) \tag{7}$$

$$f'(x) = abcos\theta - cos^2\theta + sin^2\theta \tag{8}$$

we have to attain the maximum value of area of rectangle. This can be seen in Figure.Using gradient ascent method we can find its maxima.

$$x_{n+1} = x_n + \alpha\nabla f(x_n) \tag{9}$$

$$\implies x_{n+1} = x_n + \alpha(abcos\theta - cos^2\theta + sin^2\theta)) \tag{10}$$

Taking $x_0 = 0.5, \alpha = 0.001$ and precision $= 0.00000001$, values obtained using python are:

$$\boxed{\text{Maxima} = 1.9999} \tag{11}$$

$$\boxed{\text{Maxima Point} = 0.7853} \tag{12}$$

2 Conclusion

1. At first, the given function has been differentiated and it is solved by setting f'(x) equal to zero. By using x values, f(x) values are calculated.

2. Later, the given function f(x) is solved by gradient ascent algorithm to find maxima and the point at which f(x) is maximum.

3. Then, the given function f(x) is solved by gradient descent algorithm to find minima and the point at which f(x) is is minimum.