Problem Statement:

Find the maximum isosceles triangle inscribable in a given ellipse, i.e,find the maximum value of xy, having given $\frac{x^2}{a^2}+\frac{y^2}{b^2}\!=\!1.$

Solution

Gradient Ascent

$$f(x) = absin\theta(1 - cos\theta) \tag{1}$$

$$f'(x) = ab\cos\theta - \cos^2\theta + \sin^2\theta \tag{2}$$

we have to attain the maximum value of area of rectangle. This can be seen in Figure. Using gradient ascent method we can find its maxima.

$$x_{n+1} = x_n + \alpha \nabla f(x_n) \tag{3}$$

$$\implies x_{n+1} = x_n + \alpha(abcos\theta - cos^2\theta + sin^2\theta))$$
 (4)

Taking $x_0 = 0.5$, $\alpha = 0.001$ and precision = 0.00000001, values obtained using python are:

$$Maxima = 1.9999 \tag{5}$$

$$|Maxima Point = 0.7853| (6)$$

Download the code Github link: Assignment 7.