Probability Assignment-1

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Problem statement

An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?

Solution

Let $X \in 0, 1$ where 0 represents black. Let X_1 represent the event representing drawing the first ball. X_2 represent the event of drawing the second ball. Then probability of the second ball being red is

$$\begin{aligned}\Pr\left(X_2 = 1\right) &\\ &= \Pr\left(X_2 = 1, X_1 = 1\right) + \Pr\left(X_2 = 1, X_1 = 0\right)\\ &= \Pr\left(X_2 = 1 | X_1 = 1\right)\Pr\left(X_1 = 1\right)\\ &\quad + \Pr\left(X_2 = 1 | X_1 = 0\right)\Pr\left(X_1 = 0\right) \quad (3.1)\end{aligned}$$

From the given information,

$$\Pr\left(X_1 = 0\right) = \Pr\left(X_1 = 1\right) = \frac{5}{10} = \frac{1}{2}. \quad (3.2)$$

Also,

$$\Pr\left(X_2 = 1 | X_1 = 0\right) = \frac{5}{5+2+7} = \frac{5}{12} \quad (3.3)$$

$$\Pr\left(X_2 = 1 | X_1 = 1\right) = \frac{5+2}{5+2+5} = \frac{7}{12} \quad (3.4)$$

Thus,

$$\Pr\left(X_2 = 1\right) = \frac{7}{12} \times \frac{1}{2} + \frac{5}{12} \times \frac{1}{2} = \frac{1}{2} \quad (3.5)$$

https://github.com/Rahulraj00/probability-assignments/tree/main/Q3/codes/Q3.py