Probability Assignment-1

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Problem statement

An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?

Solution

Let $X \in 0, 1$ where 0 represents black. Let X_1 represent the event representing drawing the first ball. X_2 represent the event of drawing the second ball. Then probability of the second ball being red is

$$\begin{aligned}\Pr(X_2 = 1) \\ &= \Pr(X_2 = 1, X_1 = 1) + \Pr(X_2 = 1, X_1 = 0) \\ &= \Pr(X_2 = 1|X_1 = 1)\Pr(X_1 = 1) \\ &\quad + \Pr(X_2 = 1|X_1 = 0)\Pr(X_1 = 0) \end{aligned} \tag{3.1}$$

From the given information,

$$\Pr(X_1 = 0) = \Pr(X_1 = 1) = \frac{5}{10} = \frac{1}{2}. \tag{3.2}$$

Also,

$$\Pr(X_2 = 1|X_1 = 0) = \frac{5}{5+2+7} = \frac{5}{12} \tag{3.3}$$

$$\Pr(X_2 = 1|X_1 = 1) = \frac{5+2}{5+2+5} = \frac{7}{12} \tag{3.4}$$

Thus,

$$\Pr(X_2 = 1) = \frac{7}{12} \times \frac{1}{2} + \frac{5}{12} \times \frac{1}{2} = \frac{1}{2} \tag{3.5}$$

`https://github.com/Rahulraj00/probability-assignments/tree/main/Q3/codes/Q3.py`