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G2

Acyclic graph:-A graph is acyclic if it has no cycles.

Tree:-A tree is a connected acyclic graph.

Forest

In graph theory, a **forest** is **an undirected, disconnected, acyclic graph**. In other words, a disjoint collection of trees is known as forest. Each component of a forest is tree. **Forest:**-Any graph without cycles is a forest, thus the components of a forest are trees.

The tree with 2 points, 3 points and 4-points

Binary Trees:

If the outdegree of every node is less than or equal to 2, in a directed tree than the tree is called a binary tree. A tree consisting of the nodes (empty tree) is also a binary tree. A binary tree is shown in fig:

Basic Terminology:

Root: *A binary tree has a unique node called the root of the tree.*

Left Child: *The node to the left of the root is called its left child.*

Right Child: *The node to the right of the root is called its right child.*

Parent: *A node having a left child or right child or both are called the parent of the nodes.*

Siblings: *Two nodes having the same parent are called siblings.*

Leaf: *A node with no children is called a leaf. The number of leaves in a binary tree can vary from one (minimum) to half the number of vertices (maximum) in a tree.*

Descendant: A node is called descendant of another node if it is the child of the node or child of some other descendant of that node. All the nodes in the tree are descendants of the root.

Left Subtree: The subtree whose root is the left child of some node is called the left subtree of that node

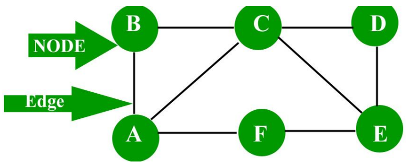
Pendant Vertices

Let **G** be a graph, A vertex **v** of **G** is called a **pendant vertex** if and only if **v** has **degree 1**. In other words, pendant vertices are the vertices that have **degree 1**, also called **pendant vertex**.

Graph Measurements: There are few graph measurement methods available:

1. Length –

Length of the graph is defined as the number of edges contained in the graph.

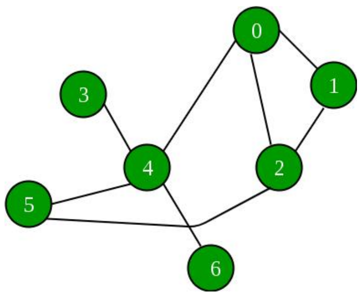


Length of the graph: 8

AB, BC, CD, DE, EF, FA, AC, CE

2. The distance between two Vertices –

The distance between two vertices in a graph is the number of edges in a shortest or minimal path. It gives the available minimum distance between two edges. There can exist more than one shortest path between two vertices.

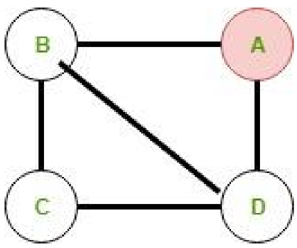


Shortest Distance between 1 – 5 is ***2***

1 → 2 → 5

3. Eccentricity of graph –

It is defined as the maximum distance of one vertex from other vertex. The maximum distance between a vertex to all other vertices is considered as the eccentricity of the vertex. It is denoted by e(V).



Eccentricity from:

(A, A) = 0

(A, B) = 1

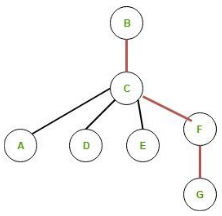
(A, C) = 2

(A, D) = 1

Maximum value is 2, So Eccentricity is ***2***

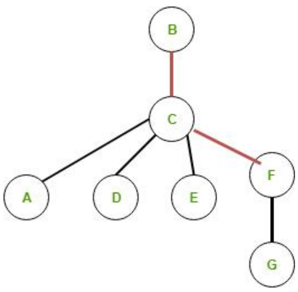
4. Diameter of graph –

The diameter of graph is the maximum distance between the pair of vertices. It can also be defined as the maximal distance between the pair of vertices. Way to solve it is to find all the paths and then find the maximum of all. It can also be found by finding the maximum value of eccentricity from all the vertices.



*Diameter: **3***
BC → CF → FG
Here the eccentricity of the vertex B is 3 since (B,G) = 3. (Maximum Eccentricity of Graph)

5. Radius of graph –A radius of the graph exists only if it has the diameter. The minimum among all the maximum distances between a vertex to all other vertices is considered as the radius of the Graph G. It is denoted as r(G). It can also b ity from all the vertices.

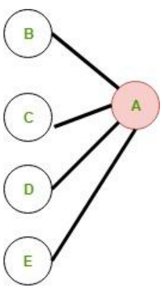


*Radius: **2***
All available minimum radius:
BC → CF,
BC → CE,
BC → CD,
BC → CA

6. Centre of graph –

It consists of all the vertices whose eccentricity is minimum. Here the eccentricity is equal to the radius. For example, if the school is at the center of town it will reduce the distance buses has to travel. If eccentricity of two vertex is same and minimum among all other both of them can be the center of the graph.

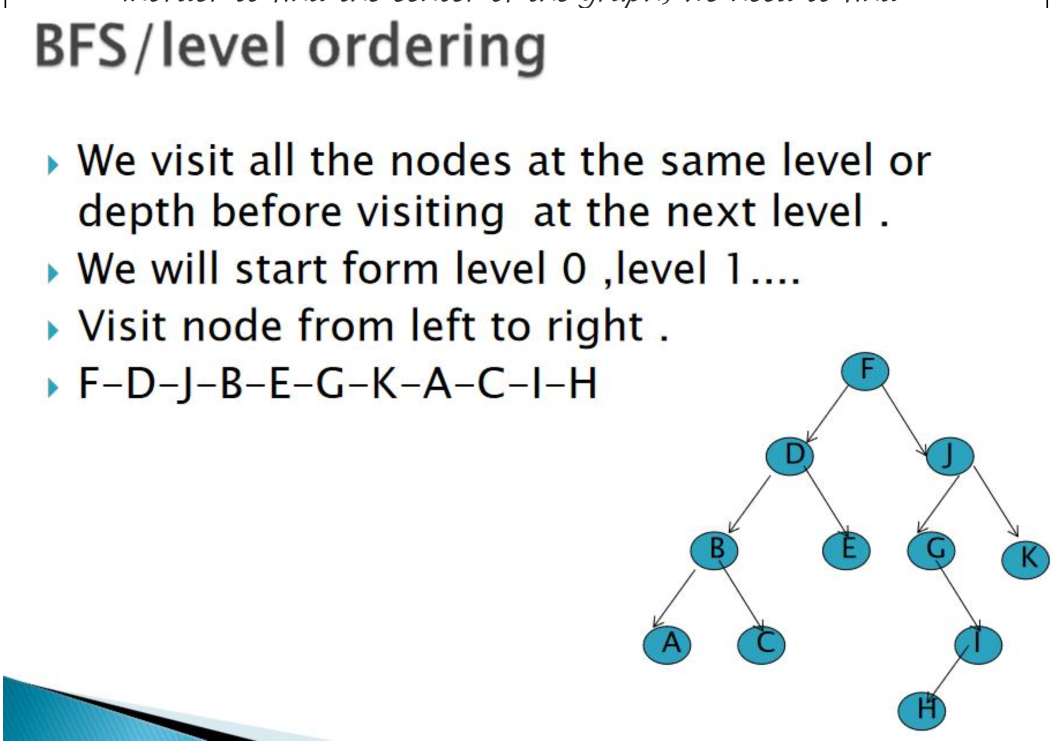
*Centre: **A***
Inorder to find the center of the graph, we need to find



Binary tree traversal

Processing of visiting each node in the tree exactly once in some order.

It can be two types



BFS(breath fist search)

level ordering

DFS(depth first search)

pre order

in order

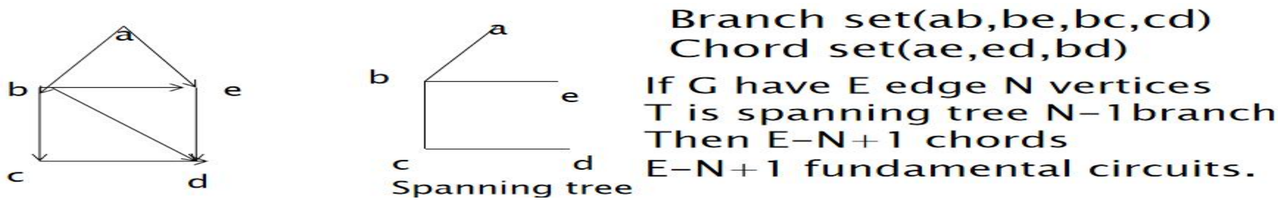
post order

Chord- an edge of G that is not given in spanning tree.

Fundamental circuits

G be a connected graph T be its spanning tree .

A circuits formed by adding a chord to spanning tree T that tree is called as fundamental circuits.



Cut Vertex

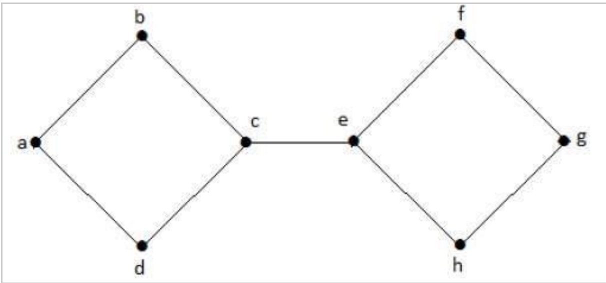
Let 'G' be a connected graph. A vertex V ∈ G is called a cut vertex of 'G', if 'G-V' (Delete 'V' from 'G') results in a disconnected graph. Removing a cut vertex from a graph breaks it in to two or more graphs.

Note − Removing a cut vertex may render a graph disconnected.

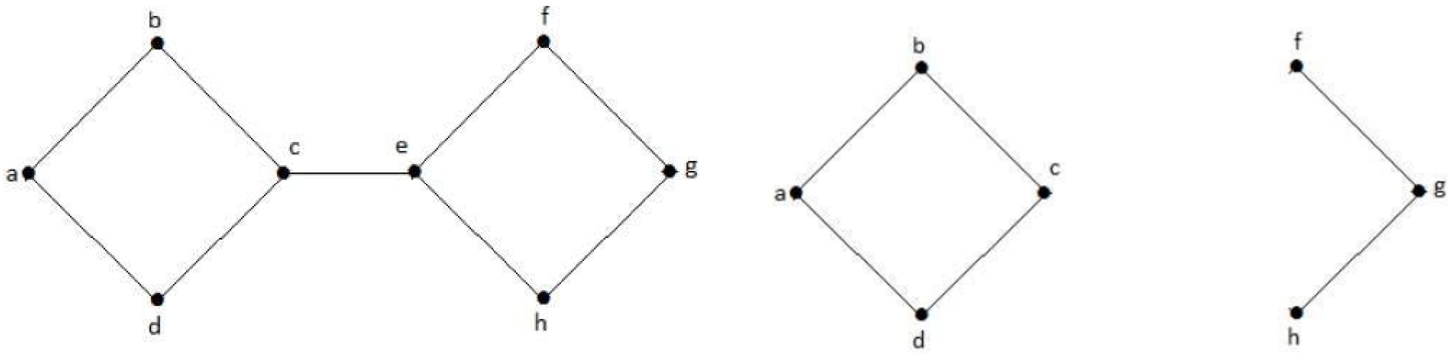
A connected graph 'G' may have at most (n−2) cut vertices.

Example

In the following graph, vertices 'e' and 'c' are the cut vertices.



By removing 'e' or 'c', the graph will become a disconnected graph.



Without 'g', there is no path between vertex 'c' and vertex 'h' and many other. Hence it is a disconnected graph with cut vertex as 'e'. Similarly, 'c' is also a cut vertex for the above graph.

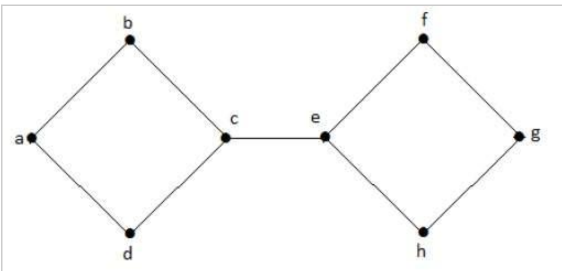
Cut Edge (Bridge)

Let 'G' be a connected graph. An edge 'e' ∈ G is called a cut edge if 'G-e' results in a disconnected graph.

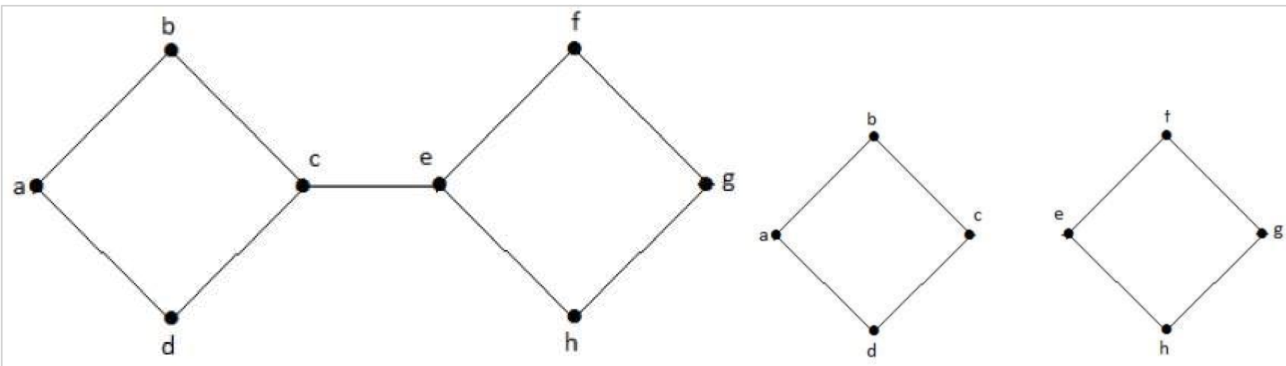
If removing an edge in a graph results in to two or more graphs, then that edge is called a Cut Edge.

Example

In the following graph, the cut edge is [(c, e)]



By removing the edge (c, e) from the graph, it becomes a disconnected graph.



In the above graph, removing the edge (c, e) breaks the graph into two which is nothing but a disconnected graph. Hence, the edge (c, e) is a cut edge of the graph.

Note − Let 'G' be a connected graph with 'n' vertices, then

- a cut edge e ∈ G if and only if the edge 'e' is not a part of any cycle in G.
- the maximum number of cut edges possible is 'n-1'.
- whenever cut edges exist, cut vertices also exist because at least one vertex of a cut edge is a cut vertex.
- if a cut vertex exists, then a cut edge may or may not exist.

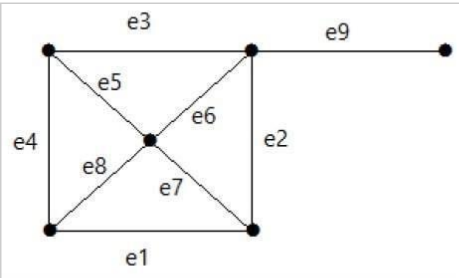
Cut Set of a Graph

Let 'G'= (V, E) be a connected graph. A subset E' of E is called a cut set of G if deletion of all the edges of E' from G makes G disconnect.

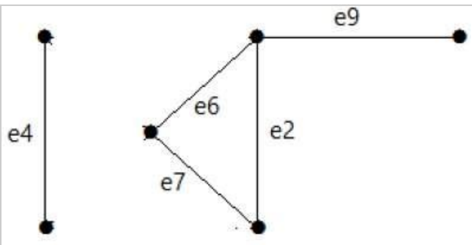
If deleting a certain number of edges from a graph makes it disconnected, then those deleted edges are called the cut set of the graph.

Example

Take a look at the following graph. Its cut set is E1 = {e1, e3, e5, e8}.



After removing the cut set E1 from the graph, it would appear as follows −



Similarly there are other cut sets that can disconnect the graph −

- E3 = {e9} – Smallest cut set of the graph.
- E4 = {e3, e4, e5}

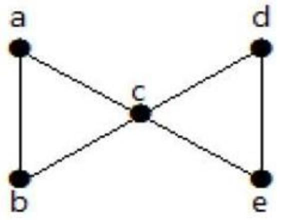
Edge Connectivity

The **edge connectivity** of a connected graph G is the minimum number of edges whose removal makes G disconnected. It is denoted by **λ(G)**.

When λ(G) ≥ k, then graph G is said to be **k-edge-connected**.

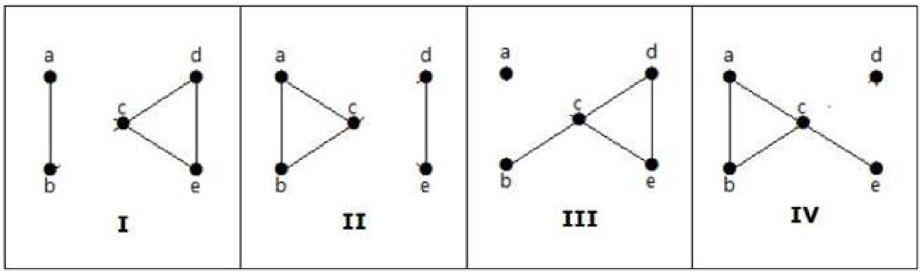
Example

Let's see an example,



From the above graph, by removing two minimum edges, the connected graph becomes disconnected graph. Hence, its edge connectivity is 2. Therefore the above graph is a **2-edge-connected graph**.

Here are the following four ways to disconnect the graph by removing two edges:



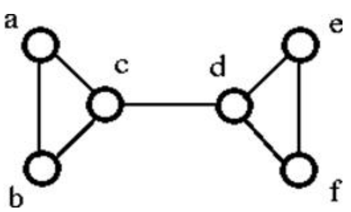
Vertex Connectivity

The connectivity (or vertex connectivity) of a connected graph G is the minimum number of vertices whose removal makes G disconnects or reduces to a trivial graph. It is denoted by K(G).

The graph is said to be k- connected or k-vertex connected when K(G) ≥ k. To remove a vertex we must also remove the edges incident to it.

Example

Let's see an example:



The above graph G can be disconnected by removal of the single vertex either 'c' or 'd'. Hence, its vertex connectivity is 1. Therefore, it is a 1-connected graph.

Separable Graph: A connected graph is said to be separable if its vertex connectivity is one. All other connected graphs are called nonseparable. In a separable graph a vertex whose removal disconnects the graph is called a cut-vertex, a cut-node, or an articulation point.

Combinatorial vs. Geometric

An abstract graph G can be defined as G = (V, E, Ψ) where the set V consists of the five objects named *a*, *b*, *c*, *d* and *e*, that is, V = {*a*, *b*, *c*, *d*, *e*} and the set E consists of seven objects (none of which is in set V) named 1, 2, 3, 4, 5, 6 and 7, that is,

$$E = \{1, 2, 3, 4, 5, 6, 7\}$$

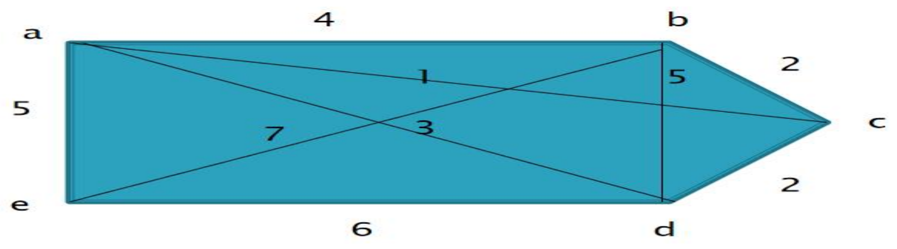
and the relationship between the two sets is defined by the mapping Ψ, which consists of combinatorial representation of the graph.

$$\Psi = \begin{bmatrix} 1 \longrightarrow (a, c) \\ 2 \longrightarrow (c, d) \\ 3 \longrightarrow (a, d) \\ 4 \longrightarrow (a, b) \\ 5 \longrightarrow (b, d) \\ 6 \longrightarrow (d, e) \\ 7 \longrightarrow (b, e) \end{bmatrix} \longrightarrow \text{Combinatorial representation of a graph}$$

Here, the symbol $1 \longrightarrow (a, c)$ says that object 1 from set E is mapped onto the (unordered) pair (a, c) of objects from set V.

Geometric graph **: -**

If we draw the graph with the help of Combinatorial relationship then the graph is called Geometric graph .



Network Flows

A network is represented by weighted graph in which

Vertices-stations

Edge- line

Through which the given assigned to each edge show the capacity.

Capacity is the max amount of flow possible per unit of time through that line.

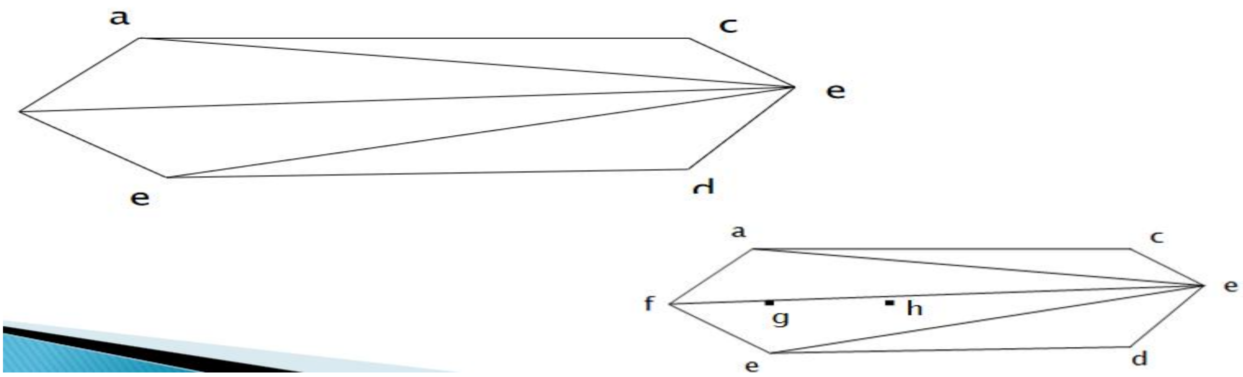
A networks has satisfy following **properties**

- It contain one and only one verities that have no incoming edge (is called source).

- It contain one and only one vertices that have no outgoing edge(is called destination).
- Weighted graph of edge is real positive number .

Homemorphic graph :

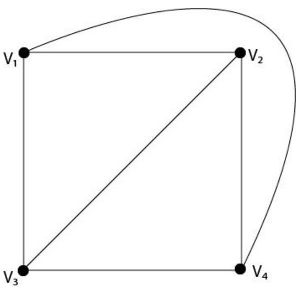
Two graph are said to be homemorphic if graph can be obtained from the other by creation of edges or merge of edge in series

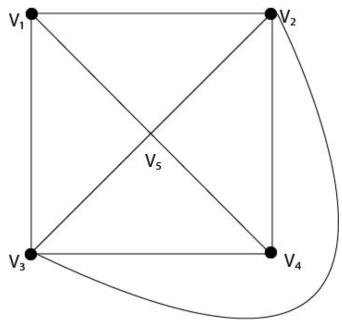


Planar Graph:

A graph is said to be planar if it can be drawn in a plane so that no edge cross.

Example: The graph shown in fig is planar graph.



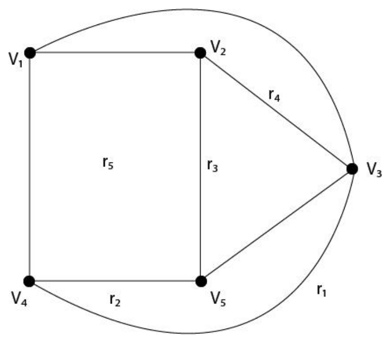


Region of a Graph: Consider a planar graph G=(V,E).A region is defined to be an area of the plane that is bounded by edges and cannot be further subdivided. A planar graph divides the plans into one or more regions. One of these regions will be infinite.

Finite Region: If the area of the region is finite, then that region is called a finite region.

Infinite Region: If the area of the region is infinite, that region is called a infinite region. A planar graph has only one infinite region.

Example: Consider the graph shown in Fig. Determine the number of regions, finite regions and an infinite region.



Solution: There are five regions in the above graph, i.e. r_1,r_2,r_3,r_4,r_5.

There are four finite regions in the graph, i.e., r_2,r_3,r_4,r_5.

There is only one finite region, i.e., r_1

Properties of Planar Graphs:

1. If a connected planar graph G has e edges and r regions, then $r \le \frac{2}{3} e$.
2. If a connected planar graph G has e edges, v vertices, and r regions, then $v-e+r=2$.
3. If a connected planar graph G has e edges and v vertices, then $3v-e\geq6$.
4. A complete graph K_n is a planar if and only if $n<5$.
5. A complete bipartite graph K_{mn} is planar if and only if $m<3$ or $n>3$.

Example: Prove that complete graph K_4 is planar.

Solution: The complete graph K_4 contains 4 vertices and 6 edges.

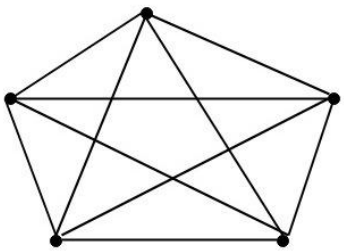
We know that for a connected planar graph $3v-e\geq6$.Hence for K_4, we have $3x4-6=6$ which satisfies the property (3).

Thus K_4 is a planar graph. Hence Proved.

Non-Planar Graph:

A graph is said to be non planar if it cannot be drawn in a plane so that no edge cross.

Example: The graphs shown in fig are non planar graphs.



K_5

These graphs cannot be drawn in a plane so that no edges cross hence they are non-planar graphs.

Properties of Non-Planar Graphs:

A graph is non-planar if and only if it contains a subgraph homeomorphic to K_5 or $K_{3,3}$

Example1: Show that K_5 is non-planar.

Solution: The complete graph K_5 contains 5 vertices and 10 edges.

Now, for a connected planar graph $3v-e\geq6$.

Hence, for K_5, we have $3 \times 5-10=5$ (which does not satisfy property 3 because it must be greater than or equal to 6).

Thus, K_5 is a non-planar graph.

Example2: Show that the graphs shown in fig are non-planar by finding a subgraph homeomorphic to K_5 or $K_{3,3}$.

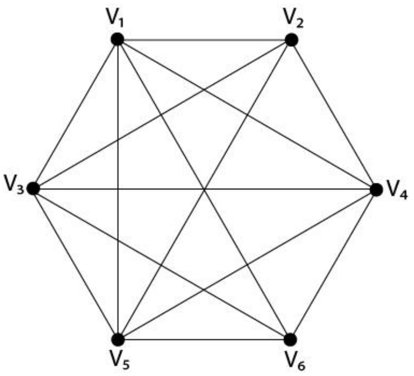


Fig: G_1

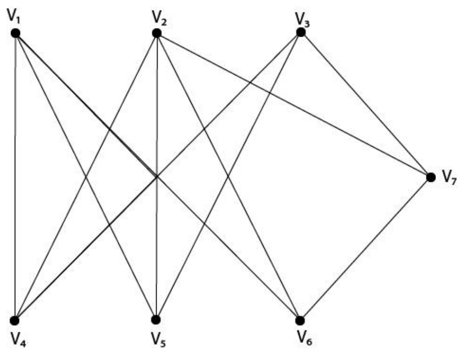


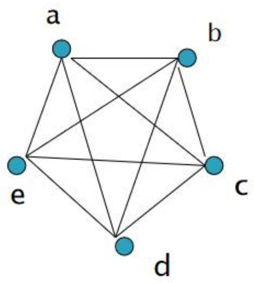
Fig: G_2

Solution: If we remove the edges (V_1,V_4),$(V_3,V_4)$and (V_5,V_4) the graph G_1,becomes homeomorphic to K_5.Hence it is non-planar.

If we remove the edge $V_{2,V}$7) the graph G_2 becomes homeomorphic to $K_{3,3}$.Hence it is a non-planar.

Critical Planer:

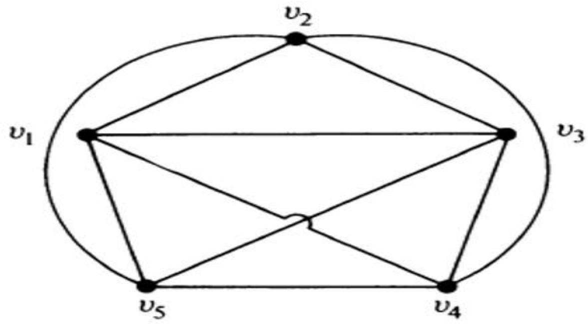
If a graph G is non planar but its sub graph is planar .this is called critical planar.



K_5

KURATOWSKI'S GRAPHS

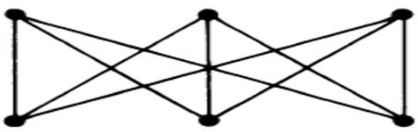
Two specify non planner graph are called kuratowski's graphs.The complete graphs with 5 vertices(k5).



1st KURATOWSKI'S GRAPHS

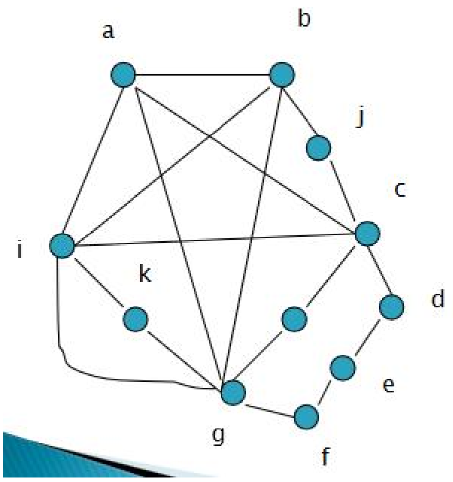
2nd KURATOWSKI'S GRAPHS :

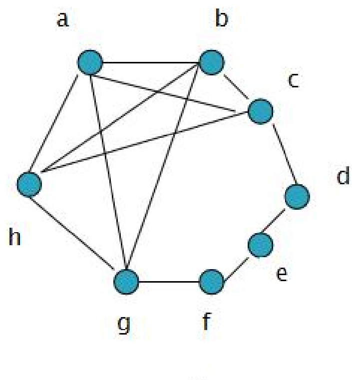
The regular graphs with six vertices and 9 edges .(k3,3)



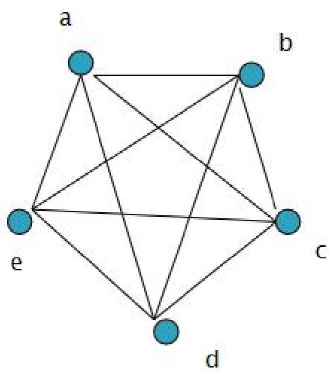
- **Kuwratoski's Theorem**: A graph is non-planar if and only if it contains a subgraph homeomorephic to $K_{3,3}$ or K_5

  Representation Example: G is Nonplanar





H



K_5

You may have notice some properties common to the two graphs of kuratowski …

- Both are regular graphs.
- Both are non planar.
- Remove of one edge or a vertex makes each a planar graph.
- Kuratowski's 1st graph is the nonplaner graphs with the smallest number of vertices.
- Kuratowski's 2nd graph is the nonplaner graph with the smallest number of edge.

DETECTION OF PLANARITY OF A GRAPH :

Elementary Reduction :

Step 1 : Since a disconnected graph is planar if and only if each of its components is planar, we need consider only one component at a time. Also, a separable graph is planar if and only if each of its blocks is planar. Therefore, for the given arbitrary graph G, determine the set.

$$G = \{G_1, G_2, \ldots\ldots G_k\}$$

where each G_i is a non separable block of G.

Then we have to test each G_i for planarity.

Step 2 : Since addition or removal of self-loops does not affect planarity, remove all self-loops.

Step 3 : Since parallel edges also do not affect planarity, eliminate edges in parallel by removing all but one edge between every pair of vertices.

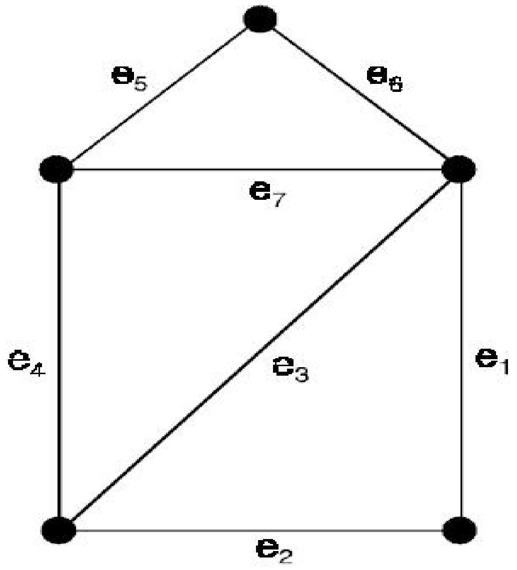
Step 4 : Elimination of a vertex of degree two by merging two edges in series does not affect planarity. Therefore, eliminate all edges in series.

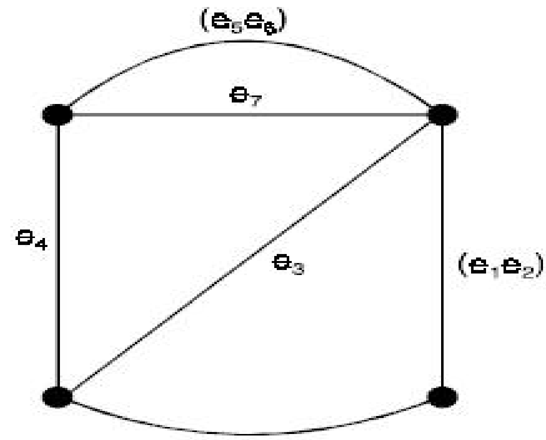
Graph H_i is

1. A single edge, or
2. A complete graph of four vertices, or
3. A non separable, simple graph with $n \geq 5$ and $e \geq 7$.

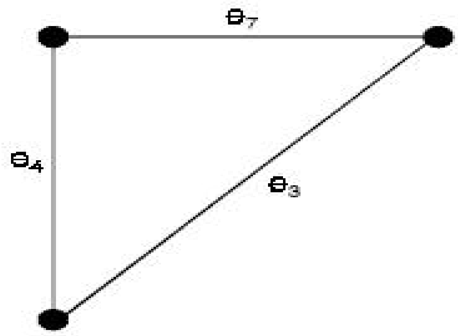
E<=3n-6

Carryout the elementary reduction process for the following graph graph is planer

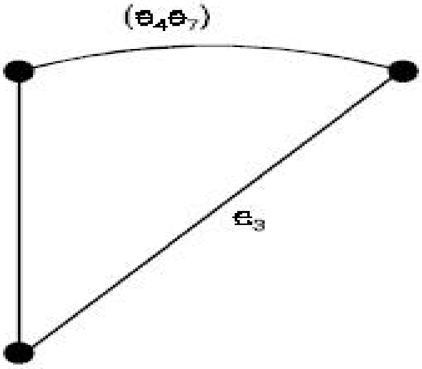




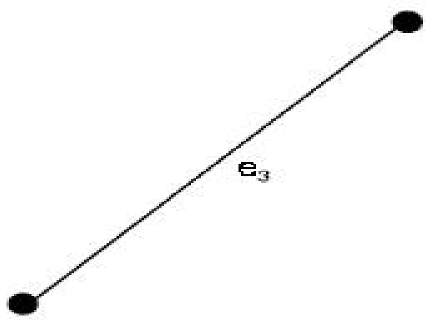
(a) Series Reduced



(b) Parallel Reduced

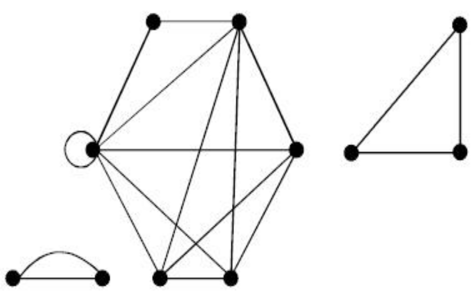


(c) Series Reduced



(d) Parallel Reduced

Another Example for better understanding



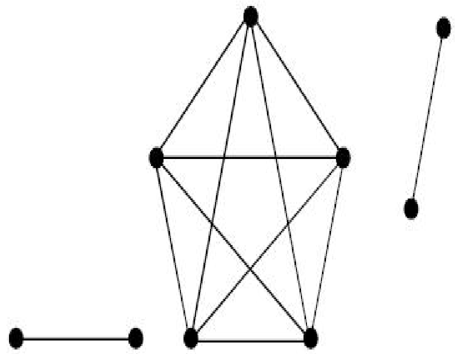


Fig. 2.54.

The reduction is now complete. The final reduced graph (shown in Figure above) has three blocks, of which the first and the third are obviously planar. The second one is evidently the complete graph K_5, which is non planar.