1) Using K maps, find the minimal Boolean expressions of the following SOP and POS representations

(a) $f(w,x,y,z) = \sum(7, 13, 14, 15)$

(b) $f(w,x,y,z) = \sum(1, 3, 4, 6, 9, 11, 14, 15)$

(c) $f(w,x,y,z) = \prod(1, 4, 5, 6, 11, 12, 13, 14, 15)$

(d) $f(w,x,y,z) = \sum(1, 3, 4, 5, 7, 8, 9, 11, 15)$

(e) $f(w,x,y,z) = \prod(0, 4, 5, 7, 8, 9, 13, 15)$

Soln:

$f(w,x,y,z) = \sum(7, 13, 14, 15)$

wx \ yz	00	01	11	10
00	[0]	[1]	[3]	[2]
01	[4]	[5]	1 [7]	[6]
11	[12]	1 [13]	1 [15]	1 [14]
10	[8]	[9]	[11]	[10]

Pair 1	Pair 2	Pair 3
$wx\bar{y}z$	$\bar{w}xyz$	$wxyz$
$wxyz$	$wxyz$	$wxy\bar{z}$
wxz	xyz	wxy

$= wxz + xyz + wxy$

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(e) $f(w,x,y,z) = \prod(0, 4, 5, 7, 8, 9, 13, 15)$

Soln:

$f(w,x,y,z) = \sum(7, 13, 14, 15)$

wx \ yz	00	01	11	10
00				
01			1	
11		1	1	1
10				

Pair 1	Pair 2	Pair 3
$wx\bar{y}z$	$\bar{w}xyz$	$wxyz$
$wxyz$	$wxyz$	$wxy\bar{z}$
wxz	xyz	wxy

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Soln:

$f(w,x,y,z) = \sum(7, 13, 14, 15)$

wx \ yz	00	01	11	10
00				
01			1	
11		1	1	1
10				

Pair 1	Pair 2	Pair 3
$wx\bar{y}z$	$\bar{w}xyz$	$wxyz$
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Soln:

$f(w, x, y, z) = \sum(7, 13, 14, 15)$

wx \ yz	00	01	11	10
00				
01			1	
11		1	1	1
10				

Pair 1	Pair 2	Pair 3
$wx\bar{y}z$	$\bar{w}xyz$	$wxyz$
$wxyz$	$wxyz$	$wxy\bar{z}$
wxz	xyz	wxy

$= wxz + xyz + wxy$

(b) $f(w,x,y,z) = \sum(1,3,6,9,11,14,15)$

$wx \backslash yz$	00	01	11	10
00		1	1	
01	1			1
11			1	1
10		1	1	

Pair 1

$wxyz$

$wxy\bar z$

wxy

Pair 2

$\bar w x \bar y \bar z$

$\bar w x y \bar z$

$\bar w x \bar z$

Quad 1

$\bar w \bar x \bar y z$

$\bar w \bar x y z$

$w \bar x \bar y z$

$w \bar x y z$

$\bar x z$

$= wxy + \bar w x \bar z + \bar x z$

(d) $f(w,x,y,z$

Quad 1

$\bar w \bar x \bar y z$

$\bar w x \bar y z$

$\bar w \bar x y z$

$\bar w x y z$

$\bar w z$

Quad 2

$\bar w \bar x y z$

$\bar w x y z$

$wxyz$

$w\bar x y$

yz

(c) $f(w,x,y,z) = \prod(1,4,5,6,11,12,13,14,15)$

$wx \backslash yz$	00	01	11	10
00		0		
01	0	0		0
11	0	0	0	0
10			0	

Pair 1

$w+x+y+\bar z$

$w+\bar x+y+\bar z$

$w+y+\bar z$

Pair 2

$\bar w+\bar x+\bar y+\bar z$

$\bar w+x+\bar y+\bar z$

$\bar w+\bar y+\bar z$

Quad 1

$w+\bar x+y+z$

$w+\bar x+y+\bar z$

$\bar w+\bar x+y+z$

$\bar w+\bar x+y+\bar z$

$\bar x+y$

Quad 2

$w+\bar x+y+z$

$w+\bar x+\bar y+\bar z$

$w+\bar x+\bar y+z$

$\bar w+\bar x+\bar y+z$

$\bar x+z$

$= (w+y+\bar z)\cdot(\bar w+\bar y+\bar z)\cdot(\bar x+y)\cdot(\bar x+z)$

(e) $f(w,x,y,$

Pair 1

$w+x+y+z$

$w+\bar x+y+z$

$w+y+z$

Pair

$\bar w+x$

$\bar w+x$

$\bar w+$

(d) $f(w,x,y,z) = \sum(1,3,4,5,7,8,9,11,15)$

$wx \backslash yz$	00	01	11	10
00		1	1	
01	1	1	1	
11			1	
10	1	1	1	

Quad 1
$\bar{w}\bar{x}\bar{y}z$
$\bar{w}x\bar{y}z$
$\bar{w}\bar{x}yz$
$\bar{w}xyz$
$\bar{w}z$

Quad 2
$\bar{w}\bar{x}yz$
$\bar{w}xyz$
$wxyz$
$w\bar{x}yz$
yz

Pair 1
$\bar{w}x\bar{y}\bar{z}$
$\bar{w}x\bar{y}z$
$\bar{w}\bar{y}\bar{z}$

Pair 2
$w\bar{x}\bar{y}\bar{z}$
$w\bar{x}\bar{y}z$
$\bar{w}\bar{x}\bar{y}$

$= \bar{w}z + yz + \bar{w}\bar{y}\bar{z} + \bar{w}\bar{x}\bar{y}$

(e) $f(w,x,y,z) = \prod(0,4,5,7,8,9,13,15)$

$wx \backslash yz$	00	01	11	10
00	0			
01	0	0	0	
11		0	0	
10	0	0		

Pair 1
$w+x+y+z$
$w+\bar{x}+y+z$
$w+y+z$

Pair 2
$\bar{w}+x+y+z$
$\bar{w}+x+y+\bar{z}$
$\bar{w}+x+z$

Quad 1
$w+\bar{x}+y+\bar{z}$
$\bar{w}+\bar{x}+y+\bar{z}$
$w+\bar{x}+\bar{y}+\bar{z}$
$\bar{w}+\bar{x}+\bar{y}+\bar{z}$
$\bar{x}+\bar{z}$

uad2
$\bar{x}+y+\bar{z}$
$\bar{x}+\bar{y}+\bar{z}$
$\bar{x}+\bar{y}+z$
$\bar{x}+\bar{y}+z$
$+ z$

$= (w+y+z)\cdot(\bar{w}+x+z)\cdot(\bar{x}+\bar{z})$

2> Simplify the given Boolean expression using K map and implement the simplified expression using NOR gates

$y = \pi M(1, 3, 5, 8, 10) \cdot dc(12, 13, 14, 15)$.

Soln:

ab \ cd	00	01	11	10
00		0	0	
01		0		
11	X	X	X	X
10	0			0

3> A digital system is to be designed in which the months of the year is given as input in 4-bit form. The month January is represented as '0000', February as '0001' and so on. The output of the system is '0' corresponding to the input of the month the month containing 30 days or less otherwise it is '1'. Consider the excess number in the input beyond '1011' as don't care conditions. For this the systems of 4 variables (A, B, C, D) find the

→ Truth Table

→ SOP expression Σm

→ POS expression ΠM

→ Simplified SOP expression using K-Map

→ Basic gate implementation of simplified SOP expression

→ Simplified POS expression using K-map

→ Implementation of simplified POS expressions using NAND gate

Soln: Truth Table

Truth Table	I/P				O/P
	A	B	C	D	
0 → January	0	0	0	0	1
1 → February	0	0	0	1	0
2 → March	0	0	1	0	1
3 → April	0	0	1	1	0
4 → May	0	1	0	0	1
5 → June	0	1	0	1	0
6 → July	0	1	1	0	1
7 → August	0	1	1	1	1
8 → September	1	0	0	0	0
9 → October	1	0	0	1	1
10 → November	1	0	1	0	0
11 → December	1	0	1	1	1
12 →	1	1	0	0	x
13 →	1	1	0	1	x
14 →	1	1	1	0	x
15 →	1	1	1	1	x

K

AB \ CD	00
00	1
01	1
11	X
10	

SOP expressi

f(a, b, c, d

Σ

Simplified S

y =

POS express

K- map

AB \ CD	00	01	11	10
00	1 (0)	(1)	(3)	1 (2)
01	1 (4)	(5)	1 (7)	1 (6)
11	X (12)	X (13)	X (15)	X (14)
10	(8)	1 (9)	1 (11)	(10)

SOP expression $\sum m$

$$f(a,b,c,d) = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D} + AB\bar{C}\bar{D} + AB\bar{C}D + A\bar{B}\bar{C}D + \bar{A}BCD + ABCD + A\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}BC\bar{D} + ABC\bar{D}$$

$$= m_0 + m_4 + d_{12} + d_{13} + m_9 + m_7 + d_{15} + m_{11} + m_2 + m_6 + d_{14}$$

$$= m_0 + m_2 + m_4 + m_7 + m_6 + m_9 + m_{11} + d_{12} + d_{13} + d_{14} + d_{15}$$

$$\sum m = m_0 + m_2 + m_4 + m_6 + m_7 + m_9 + m_{11} + d_{12} + d_{13} + d_{14} + d_{15}$$

Simplified SOP expression using K-map

$$y = \sum m(0, 2, 4, 6, 7, 9, 11) + d(12, 13, 14, 15)$$

POS expression $\prod M$ =

$$\prod M = M_0 + M_2 + M_4 + M_6 + M_7 + M_9 + M_{11} + D_{12} + D_{13} + D_{14} + D_{15}$$

Simplified POS expression using K map

$y = \pi M(0, 2, 4, 6, 7, 9, 11)$

$D(12, 13, 14, 15)$

Basic gate implementation using simplified SOP expression

4> Using 4-
given Bu

Soln:

wx

4> Using 4-variable K map simplify the Boolean functions given by $F(w, x, y, z) = \sum m(1, 3, 4, 6, 8, 9, 11, 13, 15) + \sum d(0, 2, 14)$

Soln:

wx \ yz	00	01	11	10
00	X [0]	1 [1]	1 [3]	X [2]
01	1 [4]	[5]	[7]	1 [6]
11	[12]	1 [13]	1 [15]	X [14]
10	1 [8]	1 [9]	1 [11]	[10]

5> Using K-map simplify the product-of-sum from the function given by

$F = (A, B, C, D) = \prod M (0, 2, 3, 8, 9, 10, 11)$.

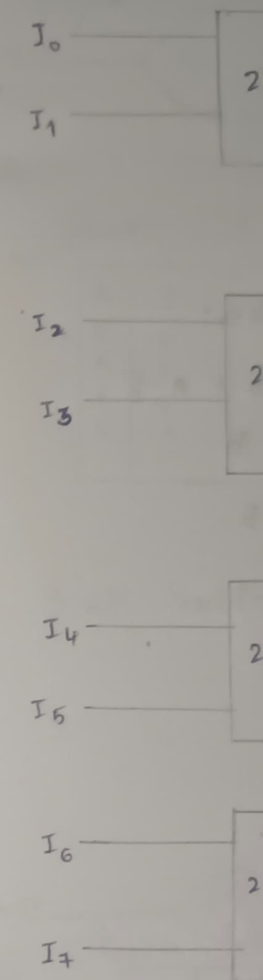
don't care (5, 13, 14, 15)

Soln:

AB \ CD	00	01	11	10
00	0		0	0
01		X		
11		X	X	X
10	0	0	0	0

6> Construct

Soln:

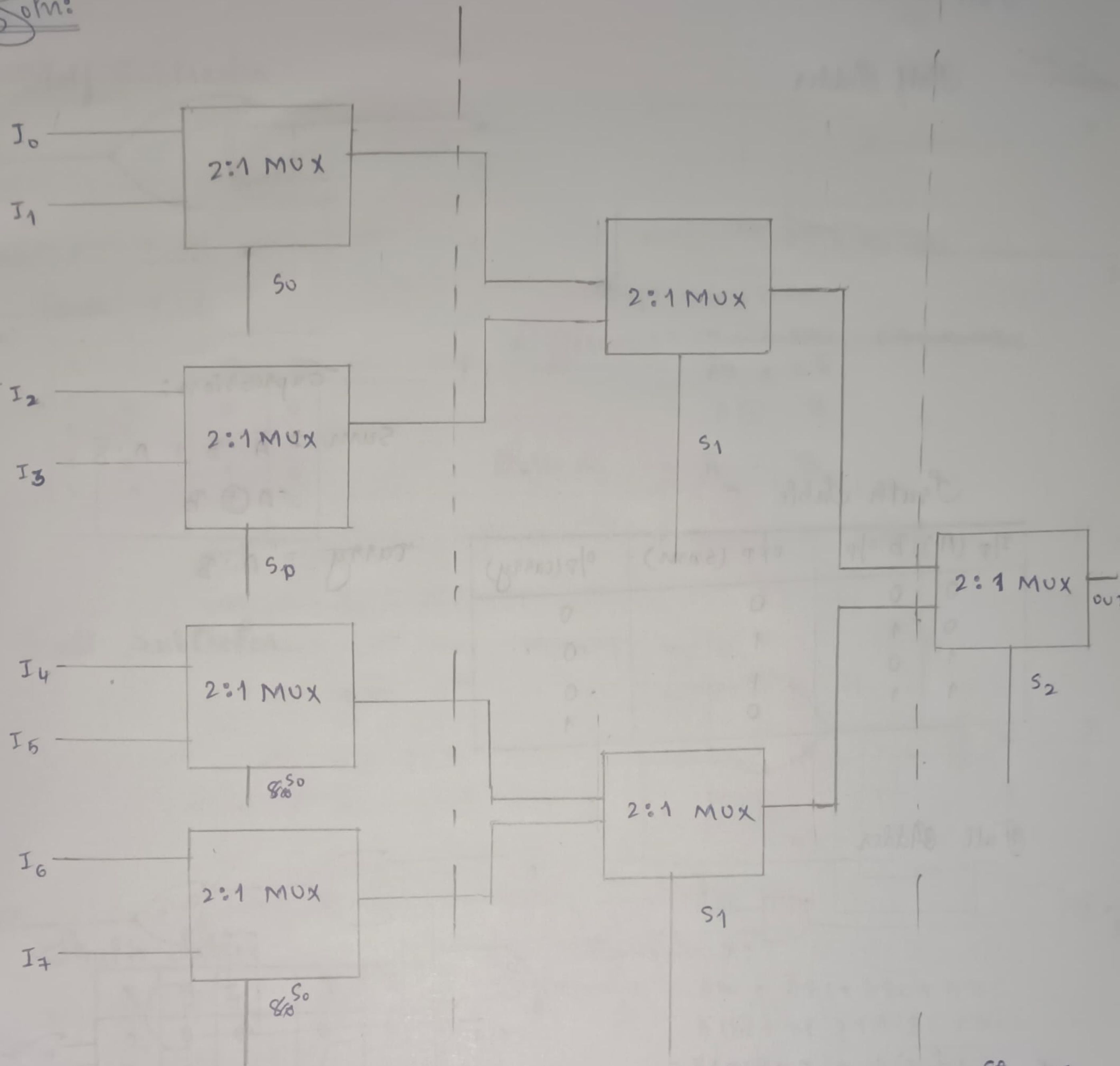


Stag

No. of

6> Construct 8:1 mux using only 2:1 mux

Soln:



Stage 1

No of ~~MUX~~ MUX = $8/2 = 4$

Stage 2

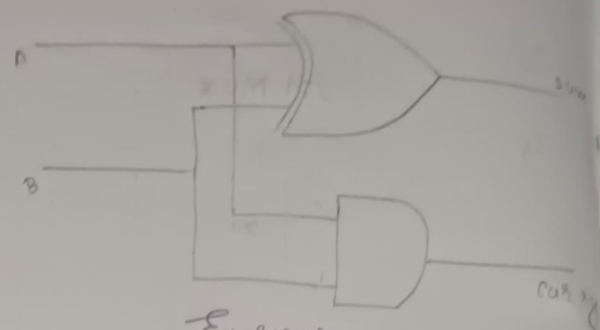
No of MUX = $4/2 = 2$

Stage 3

No of MUX = $2/2$ =

7> Design and explain working Full Adder / Half Adder and Full Subtractor

Soln: Half Adder



Expression:

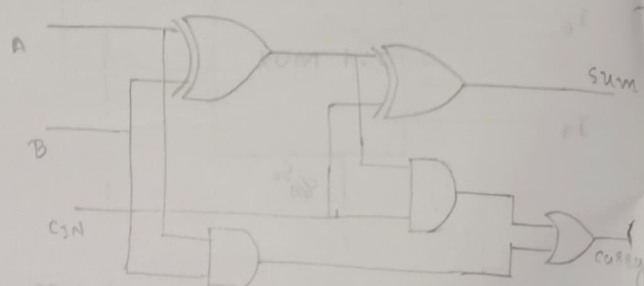
$$Sum = \bar{A}\cdot B + A\cdot\bar{B}$$
$$= A \oplus B$$

$$carry = A\cdot B$$

Truth Table

I/p (A)	(B) o/p	o/p (sum)	o/p (carry)
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Full Adder



Truth Table

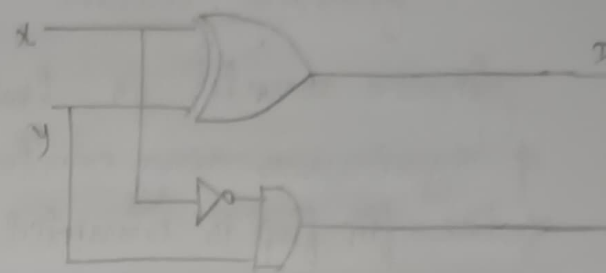
A	B	C_{IN}	Sum	CARRY
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Expression:

$$Sum = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$
$$= \bar{A}(\bar{B}C + B\bar{C}) + A(\bar{B}\bar{C} + BC)$$
$$= \bar{A}(B \oplus C) + A(\overline{B \oplus C})$$
$$= \bar{A}(x) + A(\bar{x}) \quad \text{let } B \oplus C = x$$
$$= A \oplus x$$
$$= A \oplus B \oplus C$$

$$\text{Carry} = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$
$$= C(\bar{A}B + A\bar{B}) + AB(\bar{C} + C)$$
$$= C(A \oplus B) + AB$$

Half Subtractor



Expression:

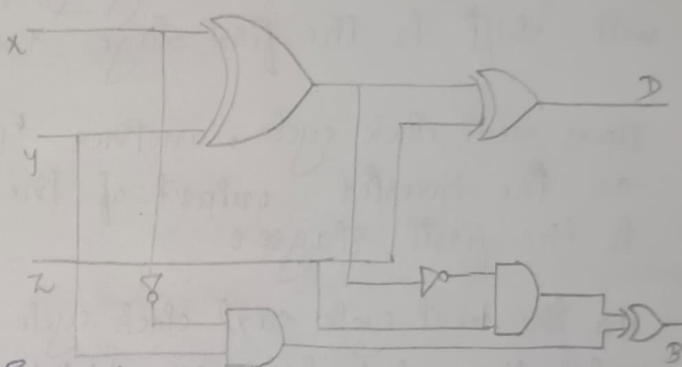
$$\text{Difference} = \cancel{\bar{A}BC + \bar{A}BC + ABC + ABC}$$
$$\bar{A}B + A\bar{B}$$
$$= A \oplus B$$

$$\text{Borrow} = \bar{A} \cdot B$$

Truth Table

x	y	D	B
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Full Subtractor



Truth Table

x	y	z	D	B
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

Expression:

$$\text{Difference} = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$
$$= \bar{A}(\bar{B}C + B\bar{C}) + A(\bar{B}\bar{C} + BC)$$
$$= \bar{A}(B \oplus C) + A(\overline{B \oplus C})$$
$$= \bar{A}(x) + A(\bar{x})$$

Let $x = B \oplus C$

$$= A \oplus x$$
$$= A \oplus B \oplus C$$

$$\text{Borrow} = \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + ABC$$
$$= \bar{A}\bar{B}C + ABC + \bar{A}B\bar{C} + \bar{A}BC$$
$$= C(\bar{A}\bar{B} + AB) + \bar{A}B(\bar{C} + C)$$
$$= C(\overline{A \oplus B}) + \bar{A}B$$
$$= \bar{A}B + C(\overline{A \oplus B})$$

8> Describe the working of Johnson Counter along with timing ~~table~~ diagram and shift table

Soln:

Johnson Counter

Johnson counter or twisted ring counter is type of synchronous ring counter in which the complemented output of the flip flop is connected with the input of the first flip flop. The Johnson counter can be made with D flips or JK flip flops in the cascade setup

Working

The default state of Johnson counter is 0000 thus before starting the clock input we need to clear the counter using clear input.

Whenever the clock edge hits the counter the output of each flip flop will transfer to the next stage (flip flop) but the inverted flip flop will shift to the first stage making the stage 1000

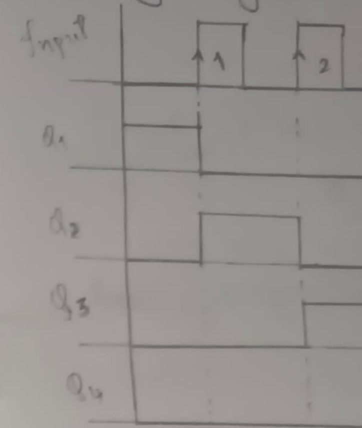
Upon next clock cycle, another '1' will stack in from the left side as the inverted output of the last stage will be shifted to the to the first stage.

On ~~the next cycle~~ next clock cycle, another '1' will add from left until the state becomes 1111

Now the last flip flop's output is '1', the next clock cycle will shift the invert of the last flip flop which is '0' into the the first flip flop. It will result in stacking '0' from the left side. This stacking of the first 0 will make the state the state 1111 into 0111

The next coming clock cycles will stack in 0's from the left making the states 0011, 0001 and 0000 with each clock cycle. Eventually it reaches its default state and it starts from the beginning again.

Timing diagram

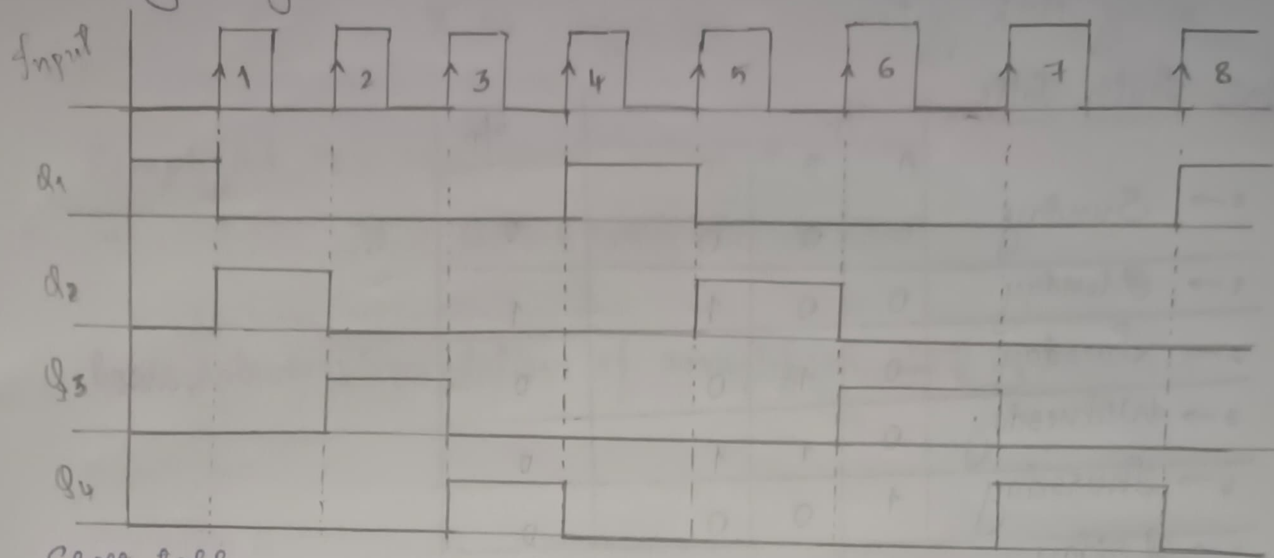


Shift table

CP	Q_1
0	0
1	1
2	1
3	1
4	1
5	0
6	0
7	0
8	0

a> The 3-bit in
is '000', '001'
having letters '
input condi
determine the
→ Truth table
→ SOP expression
→ POS expression
→ Simplified S
→ Basic gate
→ Simplified

Timing diagram



Shift table

CP	Q_1	Q_2	Q_3	Q_4
0	0	0	0	0
1	1	0	0	0
2	1	1	0	0
3	1	1	1	0
4	1	1	1	1
5	0	1	1	1
6	0	0	1	1
7	0	0	0	1
8	0	0	0	0

CP → clock pulse

a) The 3-bit inputs for the days of the week starting from Sunday is '000', '001' and so on, and the output for the days having letters 'E' and / or 'U' is '0' else a '1'. Treat the input condition '111' as don't care. For the above specifications determine the following

→ Truth Table
→ SOP expression Σm
→ POS expression ΠM
→ Simplified SOP expression using K-map
→ Basic gate implementation of simplified SOP expression
→ Simplified POS expression using K-map

→ Implementation of simplified POS expressions using NOR gates

Soln: Truth Table

	I/P A	B	C	O/P
0 → Sunday	0	0	0	0
1 → Monday	0	0	1	1
2 → Tuesday	0	1	0	0
3 → Wednesday	0	1	1	0
4 → Thursday	1	0	0	0
5 → Friday	1	0	1	1
6 → Saturday	1	1	0	0
7 →	1	1	1	x

ab \ cd	00	01	11	10
00	[0]	1 [1]	[3]	[2]
01	[4]	1 [5]	x [7]	[6]
11	[12]	[13]	[15]	[14]
10	[8]	[9]	[11]	[10]

SOP expression Σm

$$f(A \cdot B, C) = \bar{A}\bar{B}C + A\bar{B}C + ABC$$

$$= m_1 + m_5 + d_7$$

$$\Sigma m = m_1 + m_5 + d_7$$

Simplified SOP expression using K-map

$$y = \Sigma m(1, 5) + d(7)$$

POS ex

Simplifie

basic gat

POS expression $\prod M$

$$\prod M = M_1 + M_5 + D_7$$

$=$

Simplified POS expression using K-map

$$y = \prod M(1,5) \cdot D(7)$$

Basic gate implementation of simplified SOP expression

10> Illustrate T, JK flip flop with the help of logic diagram and the characteristic table

Soln: T flip flop

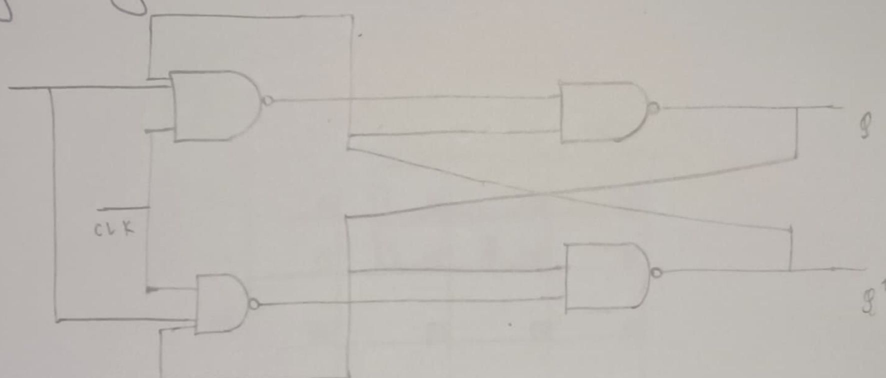
The T flip flop means Toggle flip flop. It changes the output on each clock edge and gives an output that is half the frequency of the signal to the input.

The most common flip flop used to make the T flip flop is MC74HC73A (Dual (JK flip flop)

The T flip flop can be derived from JK, SR, and D flip flop

The easiest way to construct T flip flop is from JK flip flop.

Logic diagram



Truth Table

T	Q_N	Q_{N+1}
0	0	0
0	1	1
1	0	1
1	1	0

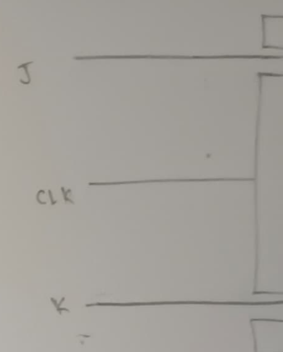
Characteristic equation for T flip flop –

$$Q_{N+1} = Q_N' T + Q_N T' = Q_N \text{ XOR } T$$

JK flip flop

The JK flip flop ...
is no change ...
The JK flip ...
it is conside...

Logic diagra...



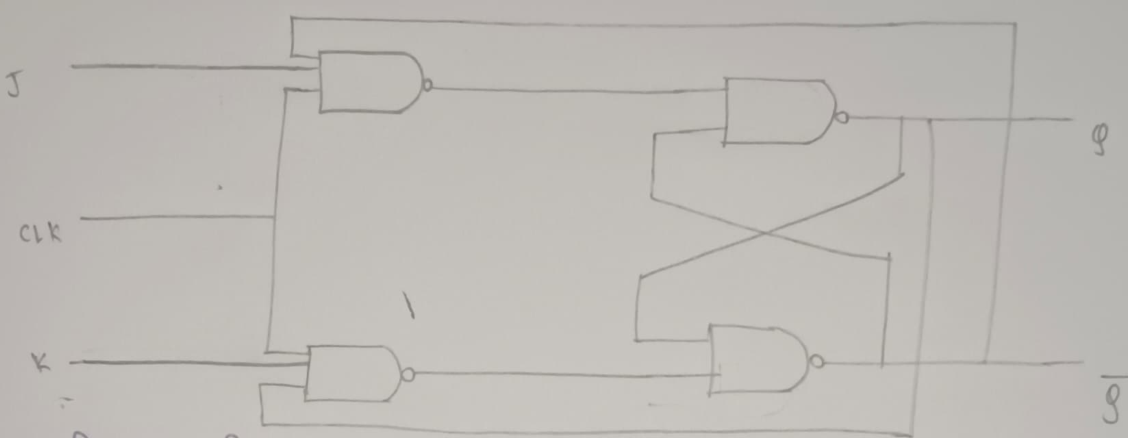
Truth Table

J	K
0	0
0	0
0	1
0	1
1	0
1	0
1	1
1	1

JK flip flop

The JK flip flop is similar to the SR flip flop but there is no change in state when the J and K inputs are low. The JK flip flop is the most widely used flip flop and it is consider to be the universal flip flop circuit

Logic diagram



Truth Table

J	K	Q_N	Q_{N+1}
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

Characteristic equ^n for JK flip flop

$$Q_{N+1} = JQ'_N + K'Q_N$$