ELEC 6061: REAL TIME COMPUTER CONTROL SYSTEM

DISCRETE TIME CONTROLLER DESIGN FOR A TWO-DOF HELICOPTER

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We certify that this submission is the original work of members of the group and meets the Faculty's Expectations of Originality

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ABSTRACT

In our project, we make use of two degree of freedom helicopter system due to its highly non-linear nature to design a discrete time controller for pitch channel with the help of root locus method. The 2DOF helicopter system comprises of two identical propellers driven by DC motors where one propeller control the elevation of the helicopter (pitch angle) and other propeller is used to control the side to side motion (yaw angle) of the system. The root locus method helps in visualizing the poles and zeros as per system requirements which results in the stability, controllability, and steady state error analysis of the proposed controller. To ensure the validity of the proposed controller - Design specifications, Controller design, MATLAB (m-files) and simulation are provided. This system has its applications in various fields such as military, aerospace, industrial applications etc.

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1. INTRODUCTION

1.1. Purpose

As 2DOF helicopter is being used in various applications, it is inevitable to design an accurate controller to meet the design requirements. Root locus method acts a vital technique in improving stability and controllability of the system by convenience of placing poles and zeros to fulfill the specifications.

1.2. Background

Usually, the proposed controllers make use of its design methodology to examine their overall performance primarily based on time response specification for controlling Pitch and yaw movement. The 2DOF helicopter pitch and yaw control system is designed for Linear and Non-linear models and the effect of various nonlinearities is studied based on the overall performance of various controllers. The controllers are designed based on linearized model of helicopter so as to simplify the design process, with the idea of dynamic modeling of motor which is used to control the movement of elevators. In 2DOF helicopter system, pitch control movement is categorized under the longitudinal motion. Yaw controller which is on the back side of the 2DOF helicopter, which is used to control the side to side motions.

1.3. Problem statement

our objective is to design a discrete time controller for Pitch and Yaw channel. We study the design procedure of each controller, individually also while considering one controller design other controller effect is considered as disturbance and ignoring the effect of coupling. Moreover, design specifications involving various factors such as rise time, settling time, overshoot, sampling rate and steady state error with respect to reference input and step disturbance needs to be satisfied. We also need to obtain motor voltage to step reference input in the pitch channel. Further combined effects of pitch and yaw controllers are examined and observed in the report. Block diagram of the whole system with yaw and pitch channel coupled is as shown.

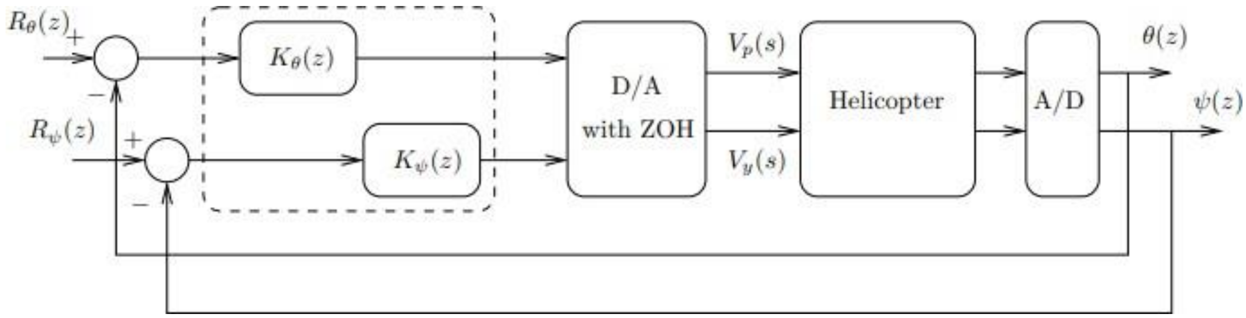

Figure -1 Block diagram of the 2DOF system with both of the controllers

2. DESIGN SPECIFICATIONS

To obtain the desired controller the specifications are provided as follows.

- Percentage of overshoot for step reference input ≤ 20%
- Settling time of step response ≤ 16 sec
- . Rise time of step response ≤ 2 sec.
- Steady–state error for step reference input = 0
- Steady–state output in response to step disturbance = 0.
- The response (θ(t)) to step disturbance must settle within 16 s. For this item, we define the settling time as follows. Let θmax = max |θ(t)| (t ≥ 0). The settling time is the time after which |θ(t)| < 0.02 θmax

3. METHODOLOGY

- Sampling Rate (Ts):

  As per the given specification in terms of rise time (t_r), sampling period or rate is chosen between 6 to 10 samples within the rise time period. Hence, choosing sampling time Ts $\leq \frac{t_r}{10}$ (10 samples) would result in appropriate and smooth response of the system.

$$Ts \leq \frac{t_r}{10} \qquad \text{Where, } t_r = 2\text{sec}$$

$$\text{we choose, Ts} = 0.2 \text{ sec}$$

- Obtaining damping ratio(ξ)

$$M_p = e^{\frac{-\xi*\pi}{\sqrt{1-\xi^2}}}*100\%$$

  Where, M_p is percentage maximum overshoot and $M_p \leq 0.2$

  Hence, we obtained $\xi \geq 0.4559$

Choosing **ξ = 0.5** would have impact on settling time as well as overshoot as higher the zeta lower would be the settling time and overshoot.

- Natural frequency (w_n)

From the settling time $t_s = \frac{4.6}{\xi * w_n}$, w_n can be obtained.

It is given that $t_s \leq 16$ sec,

$$\frac{4.6}{\xi * w_n} \leq 16$$

$$\xi * w_n \geq 0.2875 \text{ where, } \xi = 0.5$$

$$\text{hence, } w_n \geq 0.575$$

Choosing w_n **= 0.575,** which will have effect on rise time of system as $t_r = \frac{1.8}{w_n}$ hence, higher the natural frequency lower would be the rise time.

4. ANALYSIS AND DESIGN OF PITCH CONTROLLER

Design and analysis of controller is carried out based on root locus technique.

- Transfer function G_P_theeta ($G_{p,\theta}(s)$) of the plant in time domain is as follows with its poles and zeros.

$$G_{p,\theta}(s) = \frac{37.2021}{S^2 - 0.2830S + 2.7452}$$

Poles = -0.1415$\pm$1.6508i

- Zero Order Hold(ZOH) discrete equivalent of the plant G_P_theeta_d ($G_{p,\theta}(z)$)can be obtained by MATLAB command c2d and it is as follows with its poles and zeros.

- $$G_{p,\theta}(z) = \frac{0.7236\ Z + 0.71}{Z^2 - 1.839Z + 0.945}$$

Poles = 0.9196$\pm$ 0.3152i

Zeros = -0.9812

After obtaining the ZOH Discrete equivalent of the plant, root locus of the plant G_P_theeta_d is obtained by MATLAB command rlocus.

Observing the root locus, it can be seen that root locus branches are not in the desired area of given specifications. In order to fall them under the desired area to meet the specifications, controller will be designed by adding poles and zeros to the root locus.

- Controller K_theeta_z ($K_{\theta}(z)$)

  Designing of controller is done by using matlab tool ControlSystemDesigner in which we added the poles and zeros based on the specifications and to fulfill those requirements.

$$K_{\theta}(z) = \frac{0.60575 * (Z^2 - 1.812Z + 0.8778)}{Z^2 - 0.7206Z - 0.2794}$$

Poles = 1, -0.2794

Zeros = 0.9060 $\pm$ 0.2387i

According to the specification states that Steady state (ss)error (e_{ss}) for step reference i/p to be zero the following conclusion is obtained.

$$e_{ss} = \frac{1}{1+\mathrm{K}_p}$$

Where, $\mathrm{K}_p = \lim_{Z\to 1} \mathrm{K}_\theta(z) * \mathrm{G}_{p,\theta}(z)$

So in order for the SS to be zero from the above equations its observed that $\mathrm{K}_\theta(z) * \mathrm{G}_{p,\theta}(z)$ has at least one pole Z = 1 and as plant has no poles at Z = 1 hence, Controller has to have at least one pole at Z= 1.

We designed the controller with two poles and two complex zeroes and that is achieved after certain trial and error of placing poles and zeros on the root locus additionally observing the step response of the closed loop system whether design requirements are met.

- Transfer function of closed loop system G_p_theeta_CL_d ($\mathrm{G}_{p,\theta,CL}(z)$) is as shown.

$$\mathrm{G}_{p,\theta,CL}(z) = \frac{0.4383Z^3 - 0.3641Z^2 - 0.3946Z + 0.3775}{Z^4 - 2.121Z^3 + 1.627Z^2 - 0.5616Z + 0.1135}$$

Poles = 0.8629 $\pm$ $0.2405i$ and 0.1979 $\pm 0.3198i$

Zeroes = -0.9812 and 0.9060$\pm 0.2387i$

5. ANALYSIS AND DESIGN OF YAW CONTROLLER

Design and analysis of controller is carried out based on root locus technique.

- Transfer function G_y_sai ($G_{y,\psi}(s)$) of the yaw channel in time domain is as follows with its poles and zeros.

$$G_{y,\psi}(s) = \frac{7.461}{S^2 + 0.2701S}$$

Poles = 0,-0.2701

- Zero Order Hold(ZOH) discrete equivalent of the yaw controller G_y_sai_d ($G_{y,\psi}(z)$) can be obtained by MATLAB command c2d and it is as follows with its poles and zeros.

- $G_{p,\theta}(z) = \frac{0.1466\ Z+0.144}{Z^2-1.947Z+0.9474}$

Poles = 1, 0.9474

Zeros = -0.9822

After obtaining the ZOH Discrete equivalent of the plant, root locus of the plant G_y_sai_d is obtained by MATLAB command rlocus.

Observing the root locus, it can be seen that root locus branches are not in the desired area of given specifications. In order to fall them under the desired area to meet the specifications, controller will be designed by adding poles and zeros to the root locus.

- Controller K_y_z ($K_{\psi}(z)$)

  Designing of controller is done by using matlab tool ControlSystemDesigner in which we added the poles and zeros based on the specifications and to fulfill those requirements.

$$K_{\psi}(z) = \frac{4.364 * (Z^2 - 1.872\ Z + 0.8774)}{Z^2 - 0.2633\ Z - 0.7367}$$

Poles = 1, -0.7367

Zeros = 0.9360 $\pm$ 0.0361i

According to the specification states that Steady state (ss)error (e_{ss}) for step reference i/p to be zero the following conclusion is obtained.

$$e_{ss} = \frac{1}{1+K_p}$$

Where, $K_p = \lim_{Z \to 1} K_\theta(z) * G_{y,\psi}(z)$

So in order for the SS to be zero from the above equations its observed that $K_\theta(z) * G_{y,\psi}(z)$ has at least one pole Z = 1, although plant has already one pole at Z = 1, Controller has to have at least one pole at Z= 1 inn order for the response to step disturbance to be zero according to the Internal Model Principle.

We designed the controller with two poles and two complex zeroes and that is achieved after certain trial and error of placing poles and zeros on the root locus additionally observing the step response of the closed loop system whether design requirements are met.

- Transfer function of open loop system Gol_y ($G_{y,\psi,oL}(z)$) is as shown.

$$G_{p,\theta,CL}(z) = \frac{0.6396Z^3 - 0.5692Z^2 - 0.6148Z + 0.5512}{Z^4 - 2.211Z^3 + 0.7235Z^2 1.185Z - 0.698}$$

Poles = - 0.7367,0.9474, 1, 1

Zeroes = -0.9822 and $0.9360 \pm 0.0361i$

- Transfer function of closed loop system G_y_sai_CL_d ($G_{y,\psi,CL}(z)$) is as shown.

$$G_{p,\theta,CL}(z) = \frac{0.6396Z^3 - 0.5692Z^2 - 0.6148Z + 0.5512}{Z^4 - 1.571Z^3 + 0.1543Z^2 + 0.5704Z - 0.1468}$$

Poles = - 0.5830, 0.2889 and $0.9326 \pm 0.0392i$

Zeroes = -0.9822 and $0.9360 \pm 0.0361i$

6. RESULTS FOR THE PITCH CONTROLLER

- **Root locus of the plant $G_{p,\theta}(z)$**

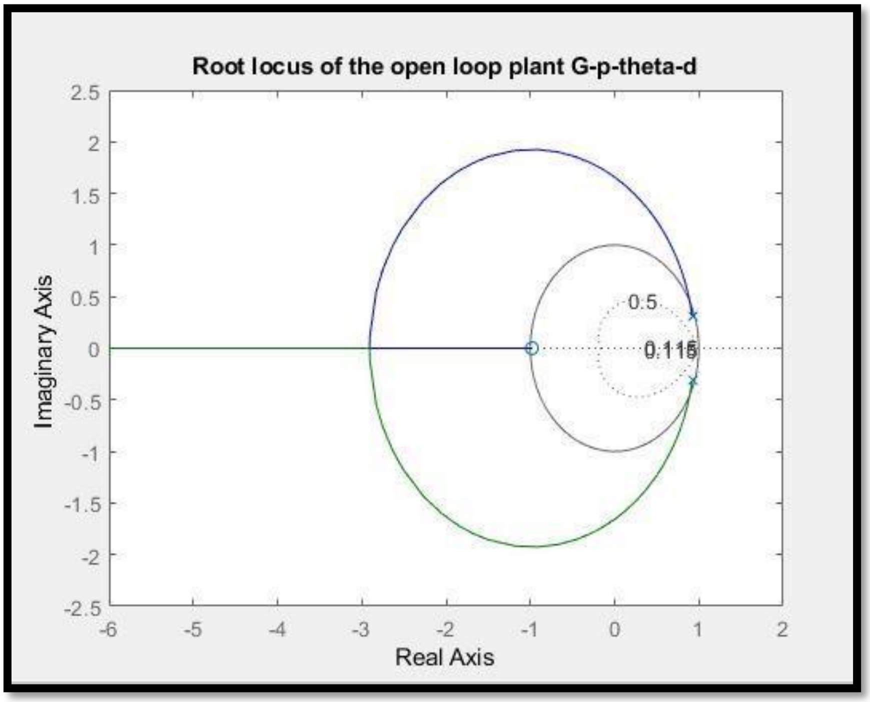

Figure- 2

From the curve it can be seen that root locus is not under the desired area (shown as dotted line), which can be made possible to enter into that region with the help of controller by adding the poles and zeros.

- **Root locus of the plant with controller $K_{\theta}(z)$**

Hence after designing Controller with additional two poles (one of them is at Z = 1) and complex zeroes it can be made possible to fall the root locus branches into the desired region.

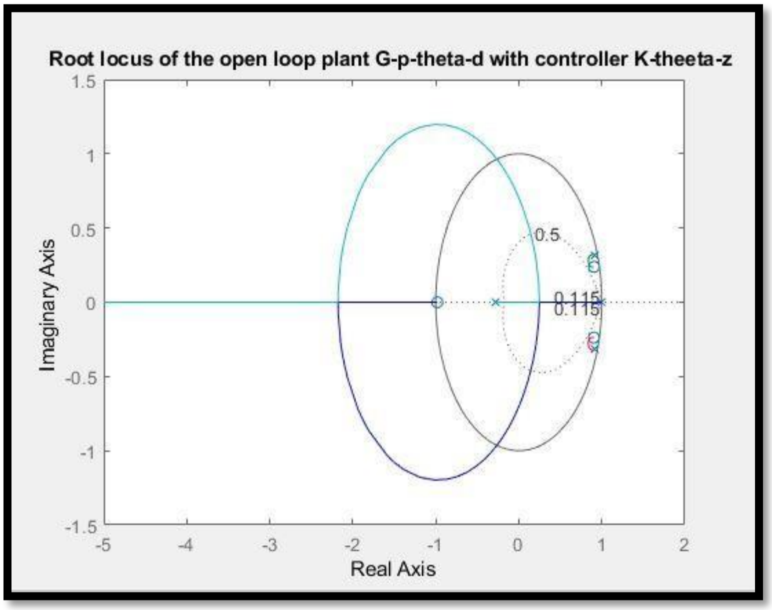

Figure- 3

- **Response of the closed loop system G_p_theeta_cl_d [$G_{p,\theta,CL}(z)$] to step reference input R[n] = 1[n]**

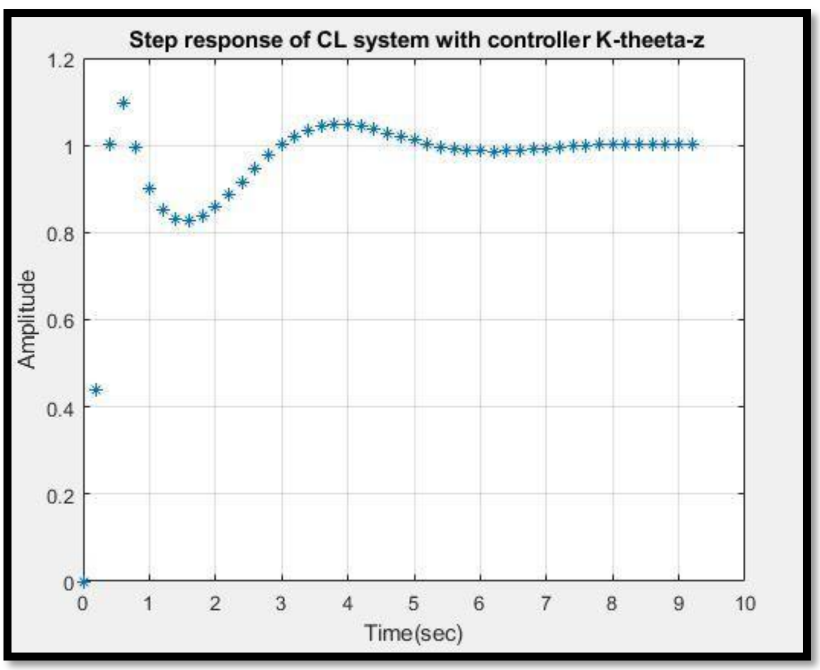

Figure- 4

Observing the step response its clear that desired specifications are met and also we can obtain transient specification about the closed loop system G_p_theeta_cl_d by the use of MATLAB command stepinfo().

Obtained specifications are as follows :-

RiseTime: 0.2000

SettlingTime: 4.8000

Overshoot: 9.6635

Peak: 1.0966

PeakTime: 0.6000

- **Response of G_p_theeta_CL_d [$G_{p,\theta,CL}(z)$] to step disturbance in d[n] = 1[n]**

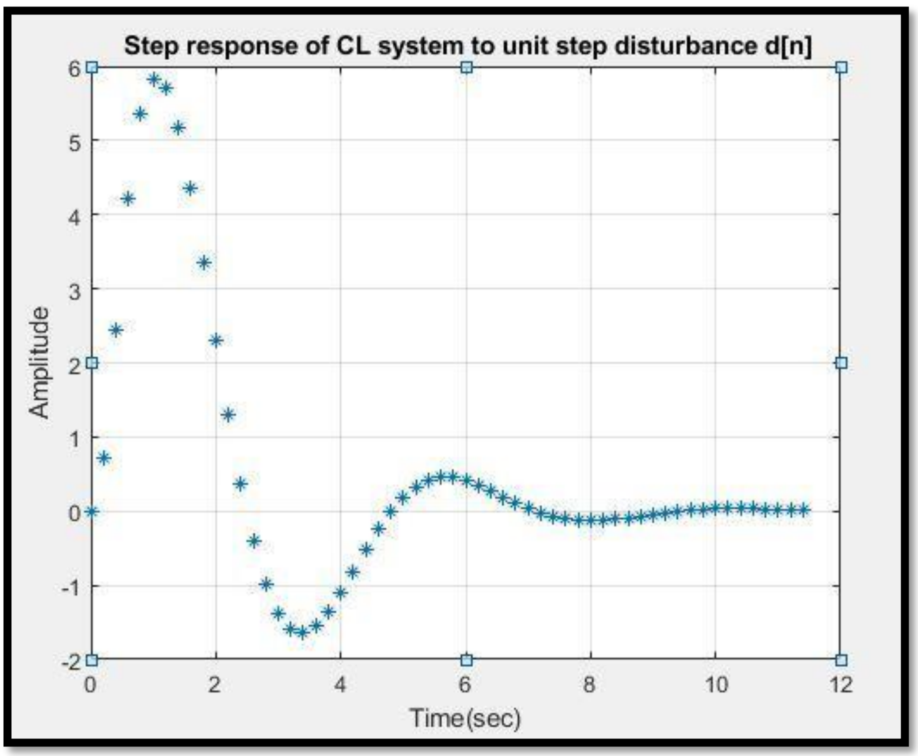

Figure- 5

Response of the system depicts that maximum value it approaches is approximately 6 and as per the given specification it should be settled within 16 seconds.

Now $|\theta(t)|$ = 0.02*θ_{max}, it is used to define settling time of response to step disturbance.

Hence from the obtained graph, $|\theta(t)|$ = 0.12 which means settling time can be observed around point when the amplitude of the graph becomes 0.12 and it is nothing but at nearly 7 seconds.

So, desired specification is met as response of closed loop system to disturbance settled within 16 sec.

- **Motor voltage Vp[n] in response to step reference input R[n] = 1[n]**

Motor voltage plot to step reference input is shown below as control signal which is nothing but the output of controller to step ref. input. Additionally, it can be seen that maximum value it can take is 0.6V for the step input size of 1.

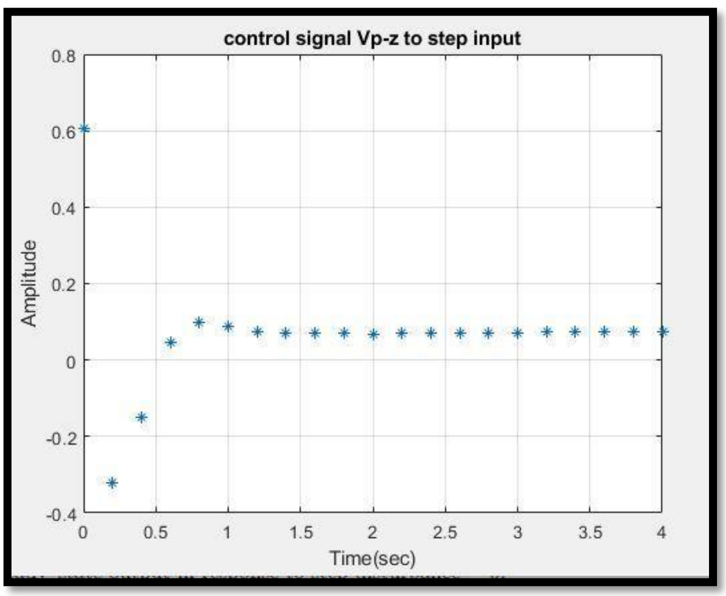

Figure- 6

Now as per the desired specifications, in order to find max size of step reference input so that motor voltage should not exceed 8V, by mathematical equation it is obtained that Max step size should be 13.

In order to verify this Simulink is used in which we provided step signal of R[n] = 13*1[n] to the controller and obtained the control signal which is nothing but input voltage Vp[n] to the motor. Block diagram and the response plot is shown as follows.

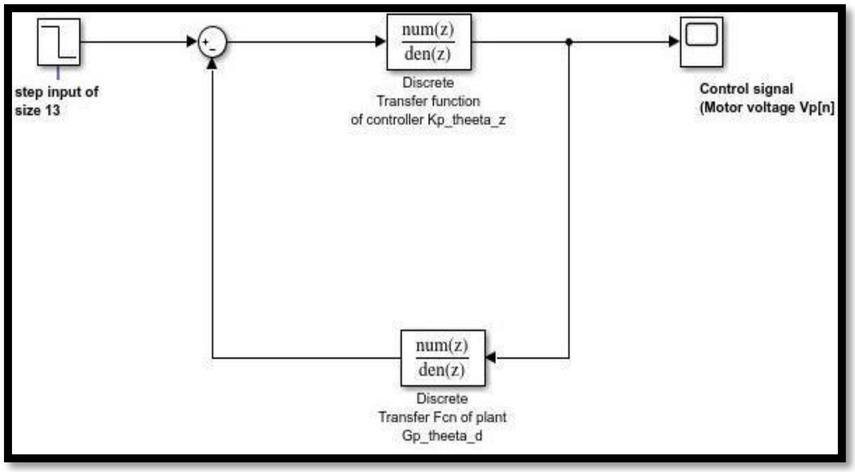

Figure- 7

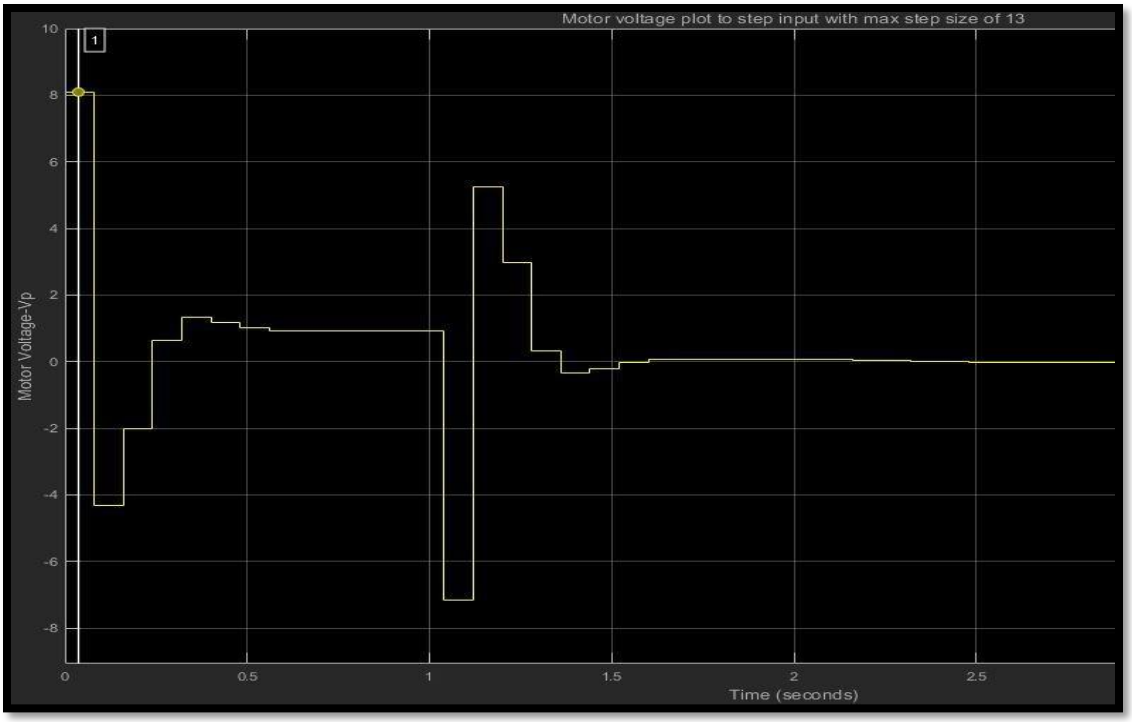

Figure- 8

From the above plot of motor voltage (control signal) it can be seen that for the step size of 13 it is approaching the value 8v hence it can be concluded that Maximum step size should be 13.

7. RESULTS FOR THE YAW CONTROLLER

- **Root locus of the plant $G_{y\psi}(z)$**

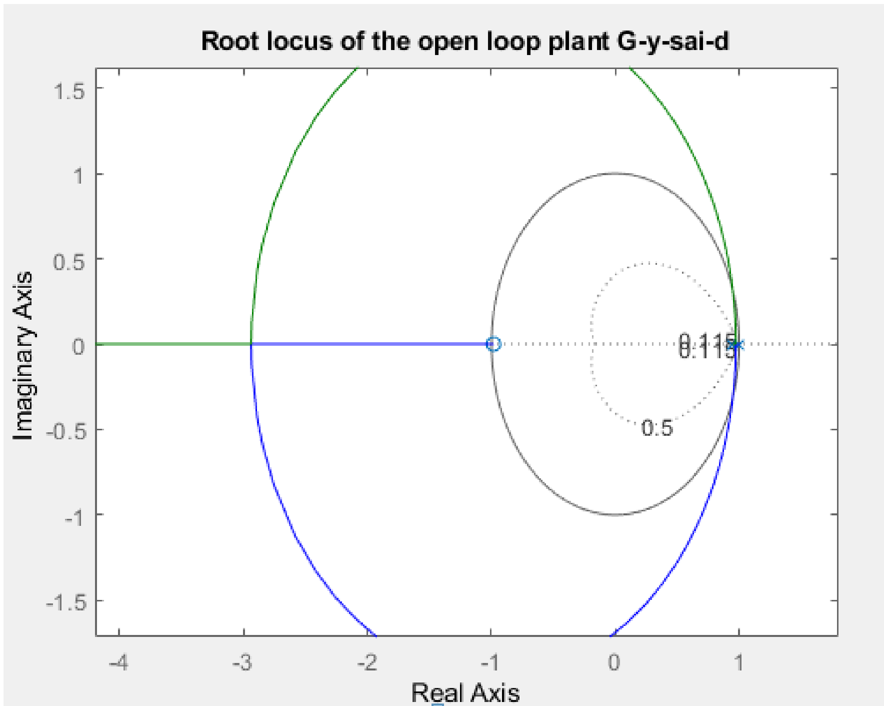

Figure- 9

From the curve it can be seen that root locus is not under the desired area (shown as dotted line), which can be made possible to enter into that region with the help of controller by adding the poles and zeros.

- **Root locus of the plant with controller $K_{\theta}(z)$**

Hence after designing Controller with additional two poles (one of them is at Z = 1) and complex zeroes it can be made possible to fall the root locus branches into the desired region as shown with the highlighted color in the graph.

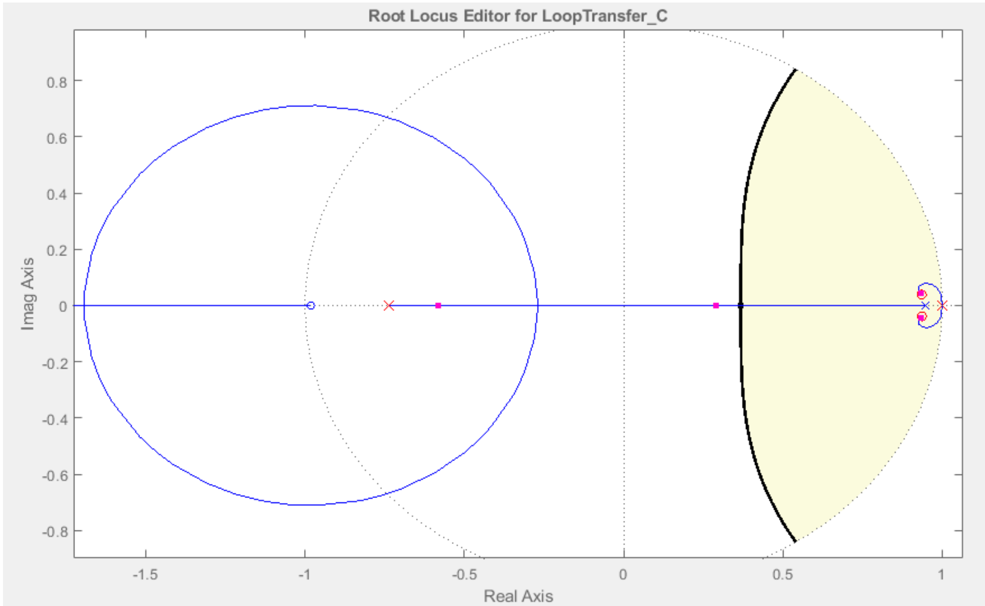

Figure- 10

- **Response of the closed loop system G_y_sai_cl_d [$G_{y,\psi,CL}(z)$] to step reference input R[n] = 1[n]**

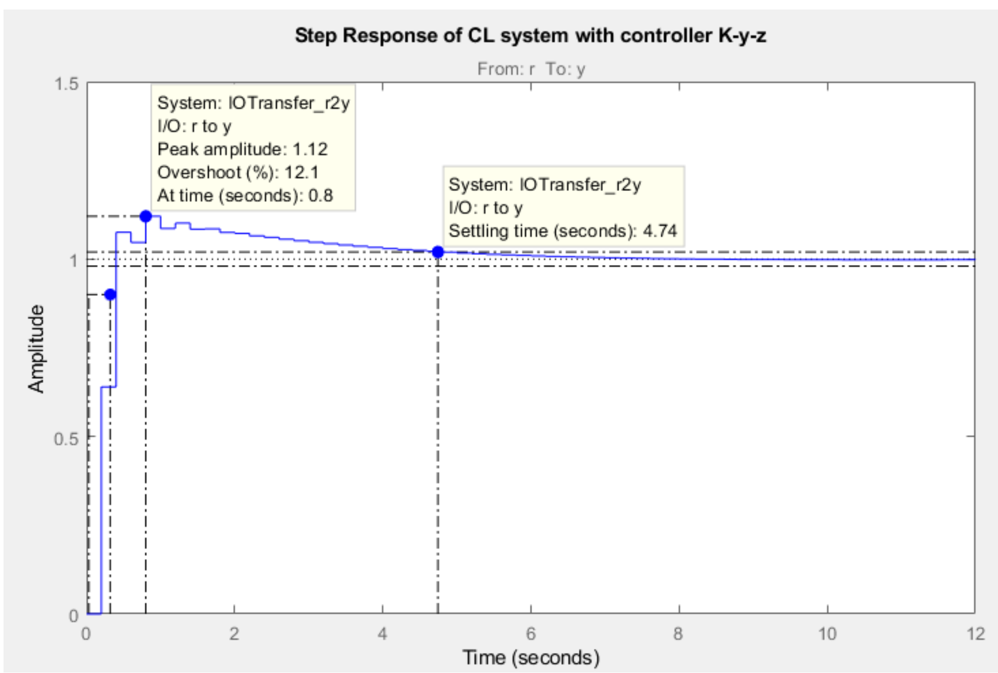

Figure- 11

Observing the step response its clear that desired specifications are met and also we can obtain transient specification about the closed loop system G_y_sai_cl_d by the use of MATLAB command stepinfo().

Obtained specifications are as follows :-

RiseTime: 0.2000

SettlingTime: 4.8000

Overshoot: 12.0145

Peak: 1.1201

- **Response of G_y_sai_cl_d [$G_{y,\psi,CL}(z)$] to step disturbance in d[n] = 1[n]**

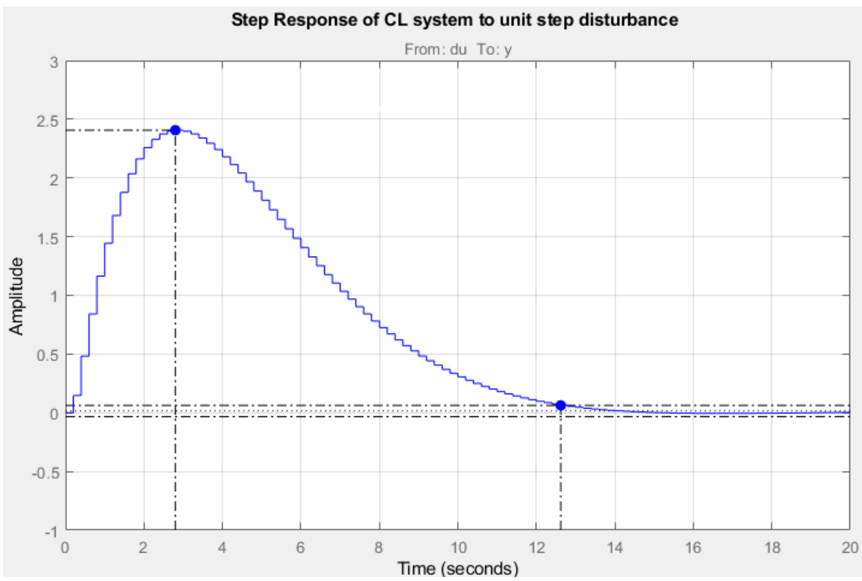

Figure- 12

- Response of the system depicts that maximum value it approaches is approximately 6 and as per the given specification it should be settled within 16 seconds.
- Now $|\theta(t)|$ = 0.02*θ_{max}, it is used to define settling time of response to step disturbance.
- Hence from the obtained graph, $|\theta(t)|$ = 0.05 which means settling time can be observed around point when the amplitude of the graph becomes 0.05 and it is nothing but at nearly 12.6 seconds.
- So, desired specification is met as response of closed loop system to disturbance settled within 16 sec.

8. Results due to the effect of cross coupling

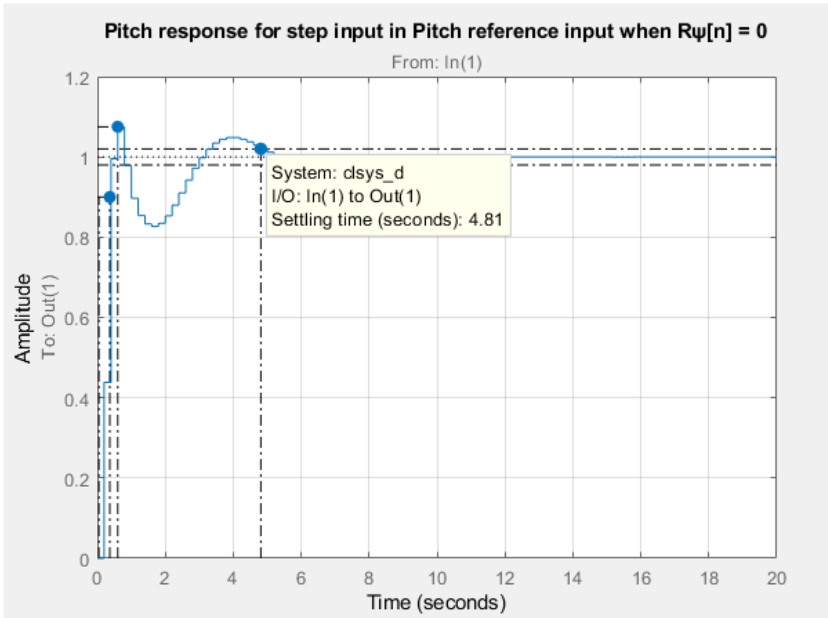

Figure - 13

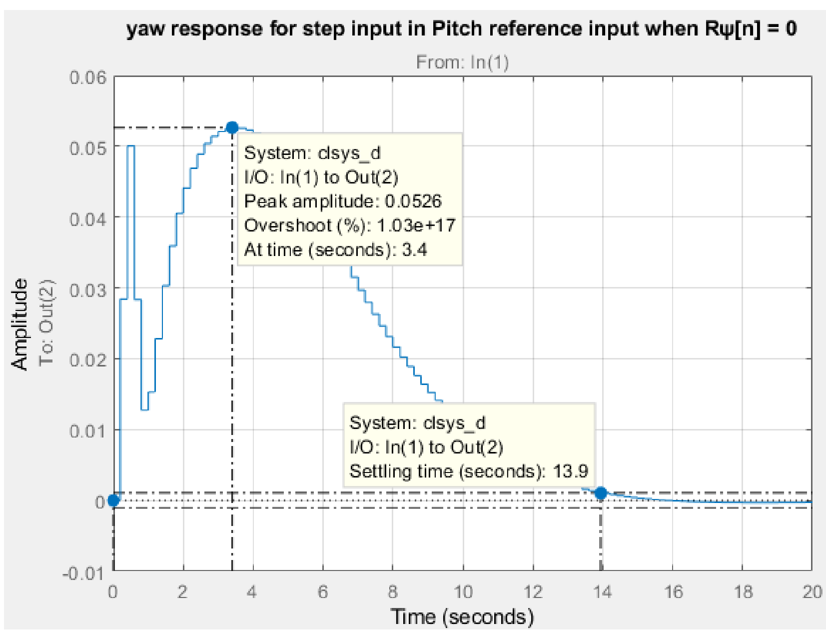

Figure - 14

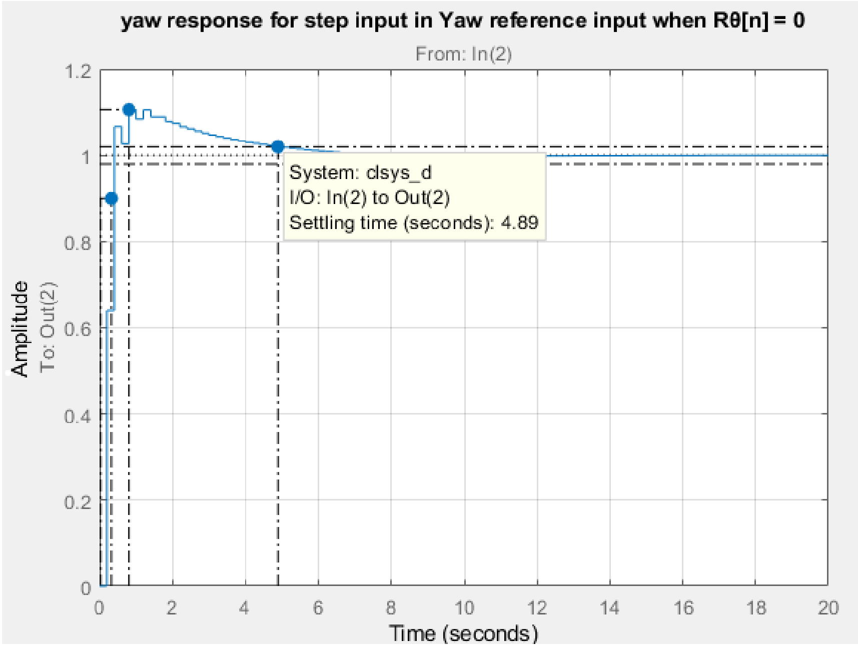

Figure - 15

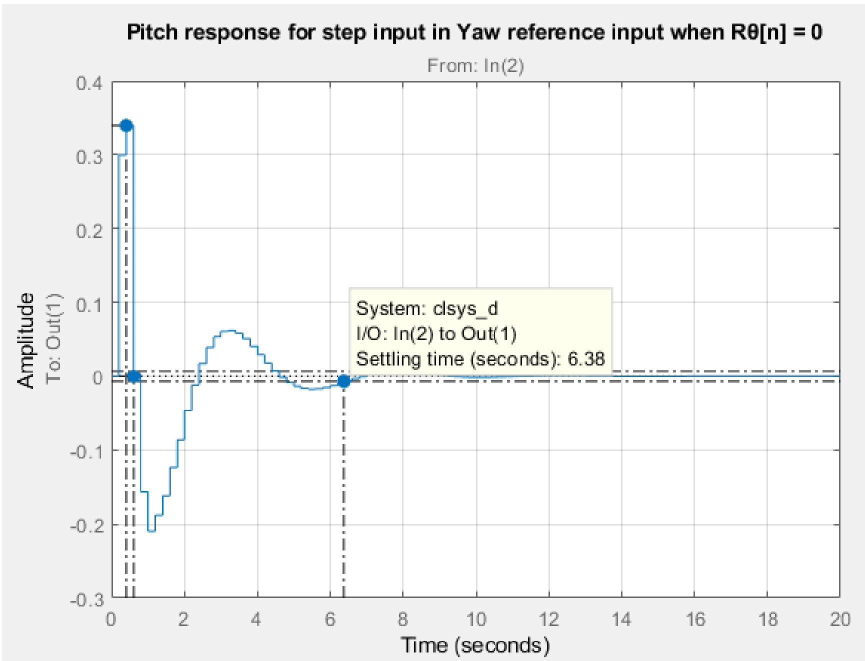

Figure - 16

9. Assessing the effect of cross coupling

Step responses obtained for the whole closed loop MIMO system are shown as just in above slides. Four step responses are observed for two different inputs and outputs. Different criteria such as Disturbance, Settling time and Rise time are observed and explained for the effect of cross coupling.

1) Percent Overshoot and settling time

Firstly, Looking at the response of pitch to step pitch input in figure 13 the overshoot is slightly declined (7.1%) compared to the step response of pitch as in figure 4. Secondly, the disturbance in the yaw channel while applying the step pitch input is settling at more or less same amount of time (nearly 14 secs) with that of when the system is analyzed individually, it is settling at around 13.9 seconds in the latter case.

As shown in Figure 15, step response of yaw channel has slightly reduced overshoot (10.1%) in comparison to step response of yaw channel when analyzed individually(12.6%) shown in Figure 11. However, settling time is similar. Apart from this, Disturbance in the pitch channel due to step input at yaw channel is settling within considerably small amount of time (6.38 secs) and the same is the case when the disturbance is applied at pitch channel individually.

2) Rise time

When there is coupling of the yaw and pitch channel together, step responses, when giving step input to of yaw and pitch channel, system is taking a little more time to rise, which are 0.291sec and 0.32 sec respectively. However when the system is analyzed separately it is 0.2 secs for both of them.

- **Effects of Cross Coupling:**

In terms of effects of cross coupling, consideration must be given to the stability of total system. This is done to establish whether the system is stable or not; gain and phase margin exist or not and also to provide basis for manipulating response of system so as to secure acceptable overall response characteristics in presence of cross coupling which cannot be eliminated.

- **Verdict on Cross Coupling:**

The presence of cross coupling aids in making an unstable two-DOF helicopter system to be stable. Also the effect of cross coupling on the different specifications are acceptable according to us as the transient specifications which are desired, do not differ much in comparison with the system when evaluated lonely.

10. CONCLUSION

In this project, we have designed a controller for proposed two degree of freedom helicopter system by means of root locus method. We have also provided with calculations for design specifications and controller design. We have also made use of Matlab for poles and zeros placement in altering the stability and controllability of the system and attached m-files and simulation results for validation purpose. We also compare the step responses of both the channels and examine the effects of cross coupling.

11. APPENDIX

1) MATLAB commands for the Pitch controller are as shown below.

```
%Plant TF in Time domain with zeros and poles
G_p_theeta = tf(37.2021,[1 0.2830 2.7452]);
pole(G_p_theeta)
zero(G_p_theeta)

%Desired closed loop specification
zeta = 0.5;
Wn = 0.575;

%Sampling time
tr = 2;

%Ts <= tr/10;
Ts = 0.2;

%plant TF in Discrete domain
G_p_theta_d = c2d(G_p_theeta,Ts)
pole(G_p_theta_d)
zero(G_p_theta_d)

%Root locus of the open loop TF
rlocus(G_p_theta_d)
title('Root locus of the open loop plant G-p-theta-d')
zgrid(zeta,Wn*Ts)
figure
grid

%designing of a controller
controlSystemDesigner(G_p_theta_d);
%adding poles and zeros to root locus..
%..and adjusting them as well the gain ..
%..desired specifications are achieved by observing the step response

%controller K_theeta_z
K_theeta_z = tf(0.60575*[1 -1.812 0.8778],[1 -0.7206 -0.2794],Ts)
pole(K_theeta_z)
zero(K_theeta_z)


%closed loop system_TF
Gol_theeta = series(K_theeta_z,G_p_theta_d);
G_p_theeta_CL_d = feedback(Gol_theeta,1)
pole(G_p_theeta_CL_d)
zero(G_p_theeta_CL_d)

%root locus of plant with controller K_theeta_z
```

```
rlocus(Gol_theeta)
title('Root locus of the open loop plant G-p-theta-d with controller K-
theeta-z ')
zgrid(zeta,Wn*Ts)
figure

%Response of G_p_theeta_CL_d to step reference if r[n] = 1[n]
[y,t] = step(G_p_theeta_CL_d);
plot(t,y,'*')
title(' Step response of CL system with controller K-theeta-z')
grid
xlabel('Time(sec)')
figure
stepinfo(G_p_theeta_CL_d)

% Response of closed loop system to step disturbance d[n] = r[n]
G_d2y = feedback(G_p_theta_d,K_theeta_z);
[y_d,t] = step(G_d2y);
plot(t,y_d,'*')
grid
title(' Step response of CL system to unit step disturbance d[n]')
xlabel('Time(sec)')
figure
stepinfo(G_d2y)

%control signal Vp_z for step input
G_r2u = feedback(K_theeta_z,G_p_theta_d);
[u,t] = step(G_r2u);
plot(t,u,'*')
grid
title('control signal Vp-z to step input')
figure
stepinfo(G_r2u)
```

2) MATLAB commands for the Yaw controller are as below

```
%TF of the yaw channel in Time domain with zeros and poles
G_y_sai = tf(7.461,[1 0.2701 0]);
pole(G_y_sai)
zero(G_y_sai)

%Desired closed loop specification
zeta = 0.5;
Wn = 0.575;

%Sampling time
tr = 2;

%Ts <= tr/10;
Ts = 0.2;

%ZOH discrete equivalent of yaw channel G_y_sai
G_y_sai_d= c2d(G_y_sai,Ts)
pole(G_y_sai_d)
```

```
zero(G_y_sai_d)

%Root locus of the open loop TF of yaw channel
rlocus(G_y_sai_d)
title('Root locus of the open loop plant G-y-sai-d')
zgrid(zeta,Wn*Ts)
figure
grid

%designing of a controller
controlSystemDesigner(G_y_sai_d);
%adding poles and zeros to root locus..
%..and adjusting them as well the gain ..
%..desired specifications are achieved by observing the step response

%controller K_y_z yaw channel
K_y_z = tf(4.364*[1 -1.872 0.8774],[1 -0.2633 -0.7367],Ts);
pole(K_y_z)
zero(K_y_z)

%controller K_theeta_z for pitch channel
K_theeta_z = tf(0.60575*[1 -1.812 0.8778],[1 -0.7206 -0.2794],Ts)


 %open loop system_TF
Gol_y = series(K_y_z,G_y_sai_d);
rlocus(Gol_y)
figure
grid
pole(Gol_y)
zero(Gol_y)

%closed loop system_TF
G_y_sai_CL_d = feedback(Gol_y,1)
Pole(G_y_sai_CL_d)
zero(G_y_sai_CL_d)

%root locus of plant with controller K_y_z
rlocus(Gol_y)
title('Root locus of the open loop plant G-p-theta-d with controller K-y-z ')
zgrid(zeta,Wn*Ts)
figure

%Response of closed loop response to step reference if r[n] = 1[n]
[y,t] = step(G_p_sai_CL_d);
plot(t,y,'*')
title(' Step response of CL system with controller K-y-z')
grid
xlabel('Time(sec)')
figure
stepinfo(G_y_sai_CL_d)

% Response of closed loop system to step disturbance d[n] = 1[n]
G_d2y = feedback(G_p_sai_CL_d,K_y_z);
```

```
[y_d,t] = step(G_d2y);
plot(t,y_d,'*')
grid
title(' Step response of CL system to unit step disturbance d[n]')
xlabel('Time(sec)')
figure
stepinfo(G_d2y)

%control signal Vy_z for step input
G_r2u = feedback(K_y_z,G_y_sai_CL_d);
[u,t] = step(G_r2u);
plot(t,u,'*')
grid
title('control signal Vy-z to step input')
figure
stepinfo(G_r2u)

%coupling of pitch and yaw channels
ss(K_y_z);
ss(K_theeta_z);
sys_k = append(K_theeta_z,K_y_z);
A = [0 1 0 0;-2.7451 -0.2829 0 0;0 0 0 1;0 0 0 -0.2701];
B = [0 0;37.2021 3.5306;0 0;2.3892 7.461];
C = [1 0 0 0;0 0 1 0];
D = [0 0;0 0];
sys_plant = ss(A,B,C,D);
sys_plant_d = c2d(sys_pant,0.2);
openloopsys_d = series(sys_k,sys_plant_d);
clsys_d = feedback(openloopsys_d,eye(2)); %eye(2) will generate identity
matrix
step(clsys_d)
```