Concordia University

Engineering and Computer Science

A Project Report on

MODELLING OF CHUA'S CIRCUIT

For requirement of ENCS 6021 (Engineering Analysis) course

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1. Abstract

The project makes use of Ordinary Differential Equations (ODEs) with Chua's circuit that exhibits chaotic behaviour. The circuit is analysed using Kirchhoff's circuit laws. The dynamics of Chua's circuit can be found and modelled by means of nonlinear ODEs to obtain the closed form and numerical solutions. The realistic Chua circuit equations and dimensionless Chua circuit equations are compared and validated by means of code and simulation using MATLAB and MULTISIM software. The voltage-current characteristics of the Chua's diode are also plotted.

2. Introduction

The concept of chaos is informally defined as the gradual loss of predictability as time progresses. The fundamental idea driving the circuit was to give a set of differential equations leading to chaos. The Chua's circuit is considered to be the simplest chaotic circuit.

For a circuit to be considered as a chaotic circuit it should meet three criteria as mentioned below:

1) Autonomous - The system should be independent of the external power source or system parameters
2) Degree of freedom - The system must have at least three degrees of freedom which is determined by the number of energy-carrying components
3) Non-linear device – The only non-linear device used in the circuit should be one or more operational amplifier

The Chua's circuit considered in this project does not have an external power source, has three energy storing elements which accounts for the three degrees of freedom and consists of two operational amplifiers, thereby qualifying as a chaotic circuit. The two operational amplifiers employed in the Chua's circuit acts as a negative resistance which causes non-linearity.

3. Model development

3.1. Chua's Circuit

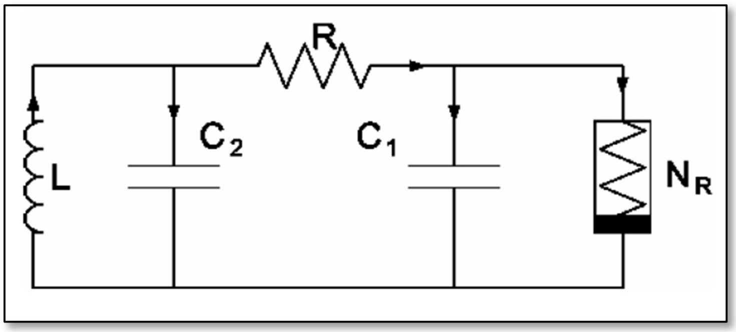

Figure 1: Chua's Circuit

(Source:[2])

The circuit consists of an inductor (L), two capacitors (C_1 and C_2), a locally active resistor (R) and a nonlinear negative resistance (N_R) also known as the Chua's diode. Where, L, C_1, C_2 and R are assumed to exhibit ideal linear characteristics whereas the diode is assumed to have a static nonlinearity consisting of a piecewise linear characteristic.

3.2. Chua's Diode

In order to obtain the Chua's diode, two negative resistance circuits are to be connected in parallel.

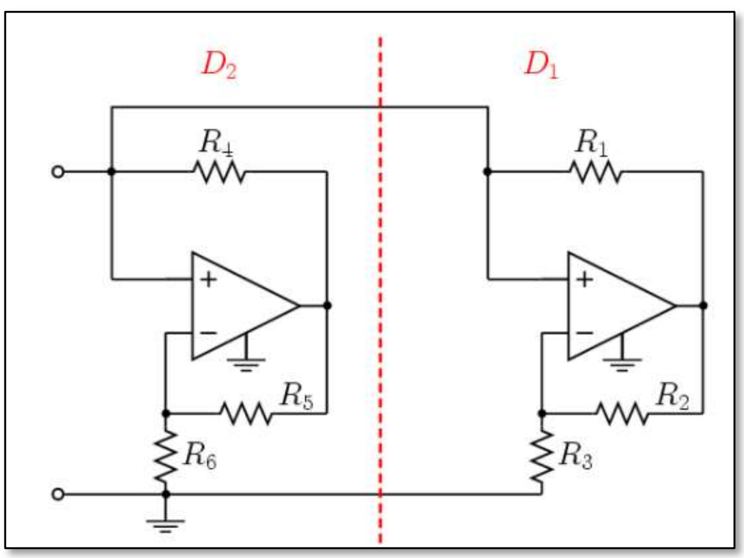

Figure 2: Chua's Diode

(Source:[8])

3.3. Components

Table 1: Table of Components

COMPONENT	VALUE
R_1	220Ω
R_2	220Ω
R_3	$2.2\ k\Omega$
R_4	$22\ k\Omega$
R_5	$22\ k\Omega$
R_6	$3.3\ k\Omega$
R	$Varied\ (1.7k\Omega - 2\ k\Omega)$
C_1	$10\ n\Omega$
C_2	$100\ n\Omega$
L	$18\ mH$
$Op - Amp$	$TL082CD$

3.4. Calculation of Negative Slopes G_a and G_b

The negative slopes G_a and G_b are calculated using the resistors of the negative resistance circuits. The equation is given by

$$G_a = G_{a1} + G_{a2}$$

$$G_b = G_{a1} + G_{b2}$$

Where,

$$G_{a1} = -\frac{1}{R_3} = -\frac{1}{2.2 \times 10^3} = -0.4545 \times 10^{-3}$$

$$G_{a2} = -\frac{1}{R_6} = -\frac{1}{3.3 \times 10^3} = -0.3030 \times 10^{-3}$$

$$G_{b2} = \frac{1}{R_4} = \frac{1}{22 \times 10^3} = 4.545 \times 10^{-5}$$

Therefore,

$$G_a = G_{a1} + G_{a2} = (-0.4545 \times 10^{-3}) + (-0.3030 \times 10^{-3})$$

$$\boldsymbol{G_a = -0.758 \times 10^{-3}\ S}$$

$$G_b = G_{a1} + G_{b2} = (-0.4545 \times 10^{-3}) + (4.545 \times 10^{-5})$$

$$\boldsymbol{G_b = -0.409 \times 10^{-3}\ S}$$

3.5. State Equations (Realistic Equations)

Consider v_1, v_2 and i_L as state variables where,

$$v_1 - \text{Voltage across the capacitor } C_1$$

$$v_2 - \text{Voltage across the capacitor } C_2$$

$$i_L - \text{ Current through the inductor L}$$

On applying Kirchoff's law, the state equations for the Chua's circuit (Figure 1) are shown below.

$$\dot{v_1} = \frac{dv_1}{dt} = \frac{1}{C_1}\left[\frac{1}{R}(v_2 - v_1) - g(v_1)\right] - - - - (\mathbf{1})$$

$$\dot{v_2} = \frac{dv_2}{dt} = \frac{1}{C_2}\left[\frac{1}{R}(v_1 - v_2) + i_L\right] - - - - (\mathbf{2})$$

$$\dot{i_L} = \frac{di_L}{dt} = -\frac{1}{L}(v_2) - - - - (\mathbf{3})$$

where $g(v_1)$ is the non-linear voltage-current characteristic of the nonlinear diode which is given by,

$$g(v_1) = G_b v_1 + \frac{1}{2}(G_a - G_b)\left\{\left|v_1 + B_p\right| - \left|v_1 - B_p\right|\right\}$$

$g(v_1)$ has negative slopes G_a and G_b (as calculated in section 3.4) and breakpoints $\pm B_p$ which can be obtained from the V-I characteristic plot of Chua's Diode (Refer Figure 3).

3.6. Simplified Solution (Dimensionless Equations)

The dimensionless state equations for the nonlinear model of Chua's circuit are shown below. The dimensionless parameters in the equations are $\alpha, \beta, \tau, x, y$ and z. The $\dot{x}, \dot{y}$ and $\dot{z}$ represents the first order derivates with respect to τ.

$$\dot{x} = \frac{dx}{d\tau} = \alpha\big(y - x - h(x)\big) - - - - (\mathbf{4})$$

where, $h(x) = m_0 x + \frac{1}{2}(m_1 - m_0)\{|x+1| - |x-1|\}$

$$\dot{y} = \frac{dy}{d\tau} = (x - y + z) - - - - (\mathbf{5})$$

$$\dot{z} = \frac{dz}{d\tau} = (-\beta y) - - - - (\mathbf{6})$$

where,

$$\alpha = \frac{C_2}{C_1}; \beta = \frac{R^2 C_2}{L}; \tau = \frac{t}{RC_2}; x = \frac{v_1}{B_P}; y = \frac{v_2}{B_P}; z = \frac{i_3}{B_P G};$$

$$m_0 = G_a R \text{ and } m_1 = G_b R$$

4. Manual Calculation:

The realistic equations (1 to 3) and dimensionless equations (4 to 6) are solved using Laplace Transforms and their corresponding solutions are obtained in equations (9) to (12).

4.1. Realistic Equations

4.1.1. Voltage across capacitor C_1

Consider equation (1),

$$\dot{v_1} = \frac{dv_1}{dt} = \frac{1}{C_1}\left[\frac{1}{R}(v_2 - v_1) - g(v_1)\right]$$

where, $g(v_1) = G_b v_1 + \frac{1}{2}(G_a - G_b)\left\{\left|v_1 + B_p\right| - \left|v_1 - B_p\right|\right\}$

$$\dot{v_1} = \frac{v_2 - v_1}{C_1 R} - \frac{1}{C_1}\left[G_b v_1 + \frac{1}{2}(G_a - G_b)\left\{\left|v_1 + B_p\right| - \left|v_1 - B_p\right|\right\}\right]$$

$$\dot{v_1} = \frac{v_2 - v_1}{C_1 R} - \frac{1}{C_1}\left[G_b v_1 + \frac{1}{2}(G_a v_1 - G_b v_1 + G_a B_P - G_b B_P - G_a v_1 + G_b v_1 + G_a B_P - G_b B_P)\right]$$

$$\dot{v_1} = \frac{v_2 - v_1}{C_1 R} - \frac{1}{C_1}\left[G_b v_1 + \frac{1}{2}2(G_a B_P - G_b B_P)\right]$$

$$\dot{v_1} = \frac{v_2}{C_1 R} - \frac{v_1}{C_1 R} - \frac{G_b v_1}{C_1} - \frac{(G_a B_P - G_b B_P)}{C_1}$$

$$\dot{v_1} = \frac{v_2}{C_1 R} - v_1\left[\frac{1}{C_1 R} + \frac{G_b}{C_1}\right] - \frac{(G_a B_P - G_b B_P)}{C_1}$$

$$\dot{v_1} + v_1\left[\frac{1}{C_1 R} + \frac{G_b}{C_1}\right] = \frac{v_2}{C_1 R} - \frac{(G_a B_P - G_b B_P)}{C_1}$$

Substituting the values of C_1, R, G_a, G_b and B_P

$$\dot{v_1} + v_1\left[\frac{1}{10\times10^{-9}\times1.75\times10^{3}} + \frac{-0.409\times10^{-3}}{10\times10^{-9}}\right] = \frac{v_2}{10\times10^{-9}\times1.75\times10^{3}} - \frac{(-0.758\times10^{-3}\times1) - (-0.409\times10^{-3}\times1)}{10\times10^{-9}}$$

$$\dot{v_1} + 16242.857 v_1 = 57142.857 v_2 + 34900$$

It is of the form

$$\dot{v_1} + a v_1 = r(t)$$

Taking Laplace transform on both sides, we get

$$[SV_1(S) - v_1(0)] + 16242.857 V_1(S) = 57142.857 V_2(S) + 34900$$

Re-grouping the terms, we get

$$V_1(S)[S + 16242.857] - v_1(0) = 57142.857 V_2(S) + 34900$$

$$V_1(S) = \frac{57142.857V_2(S) + 34900 + v_1(0)}{S + 16242.857}$$

Assuming $v_1(0)$ = 0, we get

$$V_1(S) = \frac{57142.857V_2(S) + 34900}{S + 16242.857}$$

$$V_1(S) = \frac{57142.857V_2(S)}{S - (-16242.857)} + \frac{34900}{S - (-16242.857)}$$

Taking Inverse Laplace transform on both sides, we get

$$v_1(t) = 57142.857v_2(t)e^{-162\quad .857t} + 34900e^{-16242.857t}$$

$$\boldsymbol{v_1(t) = e^{-16242.857t}[57142.857v_2(t) + 34900] - - - -(7)}$$

4.1.2. Voltage across capacitor C_2

Consider equation (2),

$$\dot{v_2} = \frac{dv_2}{dt} = \frac{1}{C_2}\left[\frac{1}{R}(v_1 - v_2) + i_L\right]$$

$$\dot{v_2} = \frac{v_1 - v_2}{C_2R} + \frac{i_L}{C_2}$$

$$\dot{v_2} = \frac{v_1}{C_2R} - \frac{v_2}{C_2R} + \frac{i_L}{C_2}$$

$$\dot{v_2} + \frac{v_2}{C_2R} = \frac{v_1}{C_2R} + \frac{i_L}{C_2}$$

Substituting the values of C_2 and R

$$\dot{v_2} + \frac{v_2}{100 \times 10^{-9} \times 1.75 \times 10^3} = \frac{v_1}{100 \times 10^{-9} \times 1.75 \times 10^3} + \frac{i_L}{100 \times 10^{-9}}$$

$$\dot{v_2} + 5714.285v_2 = 5714.285v_1 + (10 \times 10^6)i_L$$

It is of the form

$$\dot{v_2} + av_2 = r(t)$$

Taking Laplace transform on both sides, we get

$$[SV_2(S) - v_2(0)] + 5714.285V_2(S) = 5714.285V_1(S) + (10 \times 10^6)I_L(S)$$

Re-grouping the terms, we get

$$V_2(S)[S + 5714.285] - v_2(0) = 5714.285V_1(S) + (10 \times 10^6)I_L(S)$$

$$V_2(S) = \frac{5714.285V_1(S) + (10 \times 10^6)I_L(S) + v_2(0)}{S + 5714.285}$$

Assuming $v_2(0)$ = 0, we get

$$V_2(S) = \frac{5714.285V_1(S) + (10 \times 10^6)I_L(S)}{S + 5714.285}$$

$$V_2(S) = \frac{5714.285V_1(S)}{S-(-5714.285)} + \frac{(10\times10^6)I_L(S)}{S-(-5714.285)}$$

Taking Inverse Laplace transform on both sides, we get

$$v_2(t) = 5714.285v_1(t)e^{-5714.285t} + (10\times10^6)i_L(t)e^{-571\ \ .285t}$$

$$\boldsymbol{v_2(t) = e^{-5714.285t}\left[5714.285v_1(t) + (10\times10^6)i_L(t)\right] ----(8)}$$

4.1.3. Current through inductor L

Consider equation (3),

$$\dot{i_L} = \frac{di_L}{dt} = -\frac{1}{L}(v_2)$$

$$\dot{i_L} = -\frac{v_2}{L} = -\frac{v_2}{18\times10^{-3}}$$

$$\dot{i_L} = 55.555v_2$$

It is of the form

$$\dot{i_L} + ai_L = r(t)$$

Taking Laplace transform on both sides, we get

$$[SI_L(S) - i_L(0)] = 55.555V_2(S)$$

Re-grouping the terms, we get

$$I_L(S)[S] - i_L(0) = 55.555V_2(S)$$

$$I_L(S) = \frac{55.555V_2(S) + i_L(0)}{S}$$

Assuming $i_L(0)$ = 0, we get

$$I_L(S) = \frac{55.555V_2(S)}{S}$$

Taking Inverse Laplace transform on both sides, we get

$$\boldsymbol{i_L(t) = 55.555v_2(t) ----(9)}$$

4.2. Dimensionless Equations

4.2.1. Voltage across capacitor C_1

Consider equation (4),

$$\dot{x} = \frac{dx}{d\tau} = \alpha\big(y - x - h(x)\big)$$

where, $h(x) = m_1x + \frac{1}{2}(m_0 - m_1)\{|x+1| - |x-1|\}$

$$\dot{y} = \frac{dy}{d\tau} = (x - y + z) ----(\mathbf{5})$$

$$\dot{z} = \frac{dz}{d\tau} = \ (-\beta y - \gamma z) - - - - (\mathbf{6})$$

where, $\alpha = \frac{C_2}{C_1}; \beta = \frac{R^2 C_2}{L}; \gamma = \frac{RR_0 C_2}{L}; \tau = \frac{t}{RC_2}; x = \frac{v_1}{B_P}; y = \frac{v_2}{B_P}; z = \frac{i_3}{B_P G}$

$$\dot{x} = \frac{dx}{d\tau} = \ \alpha\big(y - x - h(x)\big) - - - - (\mathbf{4})$$

$$\dot{x} - \ \alpha y + \alpha x + \alpha h(x) = 0$$

$$\dot{x} - \ \alpha y + \alpha x + \alpha\left[m_1 x + \frac{1}{2}(m_0 - m_1)\ \{|x+1| - |x-1|\}\right] = 0$$

$$\dot{x} - \ \alpha y + \alpha x + \alpha\left[m_1 x + \frac{1}{2}(m_0 x - m_1 x + m_0 - m_1 - m_0 x + m_1 x + m_0 - m_1)\right] = 0$$

$$\dot{x} - \ \alpha y + \alpha x + \alpha\left[m_1 x + \frac{1}{2}(2\{m_0 - m_1\})\right] = 0$$

$$\dot{x} - \ \alpha y + \alpha x + \alpha m_1 x + \ \alpha m_0 - \alpha m_1 = 0$$

WKT

$$\alpha = \frac{C_2}{C_1} = \frac{100 \times 10^{-9}}{10 \times 10^{-9}} = 10$$

WKT

$$m_1 = G_b R$$

Taking

$$G_b = -0.409 \times 10^{-3}\ and\ R = 1.75 \times 10^3$$

$$m_1 = -0.71575$$

Similarly

$$m_0 = G_a R$$

Taking

$$G_a = -0.758 \times 10^{-3}\ and\ R = 1.75 \times 10^3$$

$$m_0 = -1.3265$$

Substituting the values of $\alpha, m_1 and\ m_0$,

$$\dot{x} - \ 10y + 10x + (10 \times -0.71575x) + (10 \times -1.3265) - (10 \times -0.71575) = 0$$

We get

$$\dot{x} - \ 10y + 10x - 7.1575x - -1.3265 + 7.1575 = 0$$

$$\dot{x} - \ 10y + 2.8425x + 5.831 = 0$$

$$\dot{x} + 2.8425x = 10y - 5.831$$

It is of the form

$$\dot{x} + ax = r(t)$$

Taking Laplace transform on both sides, we get

$$[SX(S) - x(0)] + 2.8425X(S) = 10Y(S) - 5.831$$

Re-grouping the terms, we get

$$X(S)[S + 2.8425] - x(0) = 10Y(S) - 5.831$$

$$X(S) = \frac{10Y(S) - 5.831 + x(0)}{S + 2.8425}$$

Assuming x(0) = 0, we get

$$X(S) = \frac{10Y(S) - 5.831}{S + 2.8425}$$

$$X(S) = \frac{10Y(S)}{S - (-2.8425)} - \frac{5.831}{S - (-2.8425)}$$

Taking Inverse Laplace transform on both sides, we get

$$x(\tau) = 10y(\tau)e^{-2.8425\tau} - 5.831e^{-2.8425\tau}$$

$$x(\tau) = e^{-2.842} \quad [10y(\tau) - 5.831]$$

WKT,

$$\tau = \frac{t}{RC_2} = \frac{t}{1.75 \times 10^3 \times 100 \times 10^{-9}} = 5714.285t$$

Therefore,

$$x(5714.285t) = e^{-2.8425 \times 5714.285t}[10y(5714.285t) - 5.831]$$

$$5714.285 \times x(t) = e^{-2.8425 \times 5714.285t}[10 \times 5714.285y(t) - 5.831]$$

$$5714.285 \times \frac{v_1(t)}{B_P} = e^{-2.8425 \times 5714.285t}\left[10 \times 5714.285\frac{v_2(t)}{B_P} - 5.831\right]$$

As per the V-I graph plotted (Figure 3), $B_P = 1$, therefore,

$$\mathbf{5714.285} \times \boldsymbol{v_1(t)} = \boldsymbol{e^{-16242.857t}}[\mathbf{57142.857}\boldsymbol{v_2(t)} - \mathbf{5.831}] - - - -(\mathbf{10})$$

4.2.2. Voltage across capacitor C_2

Consider equation (5),

$$\dot{y} = (x - y + z)$$

$$\dot{y} + y = x + z$$

It is of the form

$$\dot{y} + ay = r(t)$$

Taking Laplace transform on both sides, we get

$$[SY(S) - y(0)] + Y(S) = X(S) + Z(S)$$

Re-grouping the terms, we get

$$Y(S)[S + 1] - y(0) = X(S) + Z(S)$$

$$Y(S) = \frac{X(S) + Z(S) + y(0)}{S + 1}$$

Assuming y(0) = 0, we get

$$Y(S) = \frac{X(S) + Z(S)}{S + 1}$$

$$Y(S) = \frac{X(S)}{S - (-1)} - \frac{Z(S)}{S - (-1)}$$

Taking Inverse Laplace transform on both sides, we get

$$y(\tau) = x(\tau)e^{-\tau} + z(\tau)e^{-\tau}$$

$$y(\tau) = e^{-\tau}[x(\tau) + z(\tau)]$$

WKT,

$$\tau = \frac{t}{RC_2} = \frac{t}{1.75 \times 10^3 \times 100 \times 10^{-9}} = 5714.285t$$

Therefore,

$$y(5714.285t) = e^{-571\ \ .285t}[x(5714.285t) + z(5714.285t)]$$

$$5714.285 \times y(t) = e^{-5714.285t}[5714.285x(t) + 5714.285z(t)]$$

$$5714.285 \times \frac{v_2(t)}{B_P} = e^{-5714.285t}\left[5714.285\frac{v_1(t)}{B_P} + 5714.285\frac{i_L(t)}{B_P}\right]$$

As per the V-I graph plotted (Figure 3), $B_P = 1$, therefore,

$$5714.285 \times \frac{v_2(t)}{B_P} = e^{-57\ \ \ .285t}\left[5714.285\frac{v_1(t)}{B_P} + 5714.285\frac{i_L(t)}{B_P}\right]$$

$$\mathbf{5714.285 \times \boldsymbol{v_2(t)} = \boldsymbol{e}^{-5714.285t}[5714.285\boldsymbol{v_1(t)} + 5714.285\boldsymbol{i_L(t)}] - - - -(11)}$$

4.2.3. Current through inductor L

Consider equation (6),

$$\dot{z} = \frac{dz}{d\tau} = (-\beta y - \gamma z)$$

For a classical Chua's circuit, $\gamma = 0$, therefore,

$$\dot{z} = -\beta y$$

It is of the form

$$\dot{z} + az = r(t)$$

Taking Laplace transform on both sides, we get

$$[SZ(S) - z(0)] = -\beta Y(S)$$

Re-grouping the terms, we get

$$Z(S)[S] - z(0) = -\beta Y(S)$$

$$Z(S) = \frac{-\beta Y(S) + z(0)}{S}$$

Assuming z(0) = 0, we get

$$Z(S) = \frac{-\beta Y(S)}{S}$$

Taking Inverse Laplace transform on both sides, we get

$$z(\tau) = -\beta y(\tau)$$

WKT

$$\beta = \frac{R^2 C_2}{L} = \frac{R^2 C_2}{L} = \frac{(1.75 \times 10^3)^2 \times 100 \times 10^{-9}}{18 \times 10^{-3}} = 17.0138$$

Therefore,

$$z(\tau) = -17.0138 y(\tau)$$

WKT,

$$\tau = \frac{t}{RC_2} = \frac{t}{1.75 \times 10^3 \times 100 \times 10^{-9}} = 5714.285t$$

Therefore,

$$z(5714.285t) = -17.0138 y(5714.285t)$$

$$5714.285 \times z(t) = -17.0138 \times 5714.285 y(t)$$

$$5714.285 \times z(t) = -97222.21 y(t)$$

$$5714.285 \times \frac{i_L(t)}{B_P} = -97222.21 \frac{v_2(t)}{B_P}$$

As per the V-I graph plotted (Figure 3), $B_P = 1$, therefore,

$$\mathbf{5714.285 \times i_L(t) = -97222.21 v_2(t) - - - - (12)}$$

4.3. Comparison of Realistic and Dimensionless Solutions

Consider the realistic equation solutions (7), (8) and (9)

$$v_1(t) = e^{-16242.857t}[57142.857 v_2(t) + 34900]$$

$$v_2(t) = e^{-5714.285t}[5714.285 v_1(t) + (10 \times 10^6) i_L(t)]$$

$$i_L(t) = 55.555 v_2(t)$$

Consider the dimensionless equation solutions (10), (11) and (12)

$$5714.285 \times v_1(t) = e^{-16242.857t}[57142.857 v_2(t) - 5.831]$$

$$5714.285 \times v_2(t) = e^{-5714.28} \quad [5714.285 v_1(t) + 5714.285 i_L(t)]$$

$$5714.285 \times i_L(t) = -97222.21 v_2(t)$$

Although the final solutions of the Realistic and dimensionless equations are not accurate, it can be observed that the solutions (10), (11) and (12) obtained by removing the simplifying hypothesis ($\alpha, \beta, \tau, x, y$ and z) from the dimensionless equations converge to the solutions of the realistic equations (7), (8) and (9) up to a reasonable extent.

5. Implementation and code validation

5.1. V-I Characteristics

Chaotic oscillators designed with Chua's diode are generally based on a single, three-segment, odd-symmetric, voltage-controlled piecewise-linear nonlinear resistor structure. Since the voltage V across the two negative resistance circuits are the same, the resulting characteristic is obtained by summing the currents for equal values of V. The value of R is set to 1.7kΩ.

5.1.1. MATLAB Code

```
%chuaplot
R = 1700;
C1=10*10^-9;
C2=100*10^-9;
L=18*(1E-3);

%Chua's Diode parameters
m0=-1.326;
m1=-0.716;

%Plotting Voltage vs Current Characteristics
syms x
h(x) = m1*x+0.5*(m0- m1)*(abs(x+1)-abs(x-1));
fplot(h)
grid on;
title('Chua Diode function h(x)')
xlabel('Voltage (V)')
ylabel('Current (A)')
```

5.1.2. Output

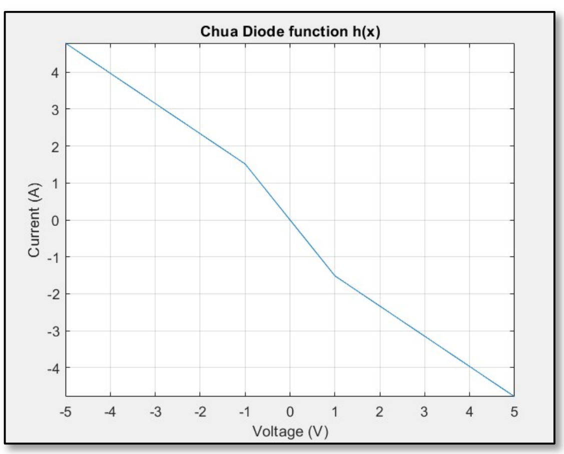

Figure 3: V-I Characteristic of Chua's Diode

The above V-I graph contains three linear sections, and it can be noted that the breakpoints B_P are +1 and -1.

5.2. Realistic Equations

The MATLAB codes to solve the realistic chua equations are presented below. The equations were solved using an in-built MATLAB function ODE45 solver yielding fourth order Runge-Kutta method. The value of R is set to 1.7kΩ.

5.2.1. MATLAB Code

```
t,y] = ode45(@RealChua,[0 0.05],[-0.5 -0.2 0]);
plot3(y(:,1),y(:,2),y(:,3))
plot(y(:,1),y(:,2))
title("Real Chua's circuit")
grid


function out = RealChua(t,in)

x = in(1); %v_1
y = in(2); %v_2
z = in(3); %i_L


C1   = 10*10^(-9);
C2   = 100*10^(-9);
R = 1700;
G = 1/R;

%Chua Diode
R1 = 220;
R2 = 220;
R3 = 2200;
R4 = 22000;
R5 = 22000;
R6 = 3300;
C = 100*10^(-9);
L = 18*10^(-3);


Esat = 9;
E1 = R3/(R2+R3)*Esat;
E2 = R6/(R5+R6)*Esat;

m12 = -1/R6;
m02 = 1/R4;
m01 = 1/R1;
m11 = -1/R3;

m1 = m12+m11;

if(E1>E2)
m0 = m11 + m02;
else
m0 = m12 + m01;
```

```
end

mm1 = m01 + m02;
Emax = max([E1 E2]);
Emin = min([E1 E2]);


if abs(x) < Emin
   g = x*m1;
elseif abs(x) < Emax
    g = x*m0;
    if x > 0
    g = g + Emin*(m1-m0);
    else
    g = g + Emin*(m0-m1);
    end

elseif abs(x) >= Emax
    g = x*mm1;
    if x > 0
        g = g + Emax*(m0-mm1) + Emin*(m1-m0);
    else
        g = g + Emax*(mm1-m0) +  Emin*(m0-m1);
    end
end


% Chua's Circuit Equations
xdot = (1/C1)*(G*(y-x)-g);
ydot = (1/C2)*(G*(x-y)+z);
zdot  = -(1/L)*y;

out = [xdot ydot zdot]';
```

5.2.2. Output

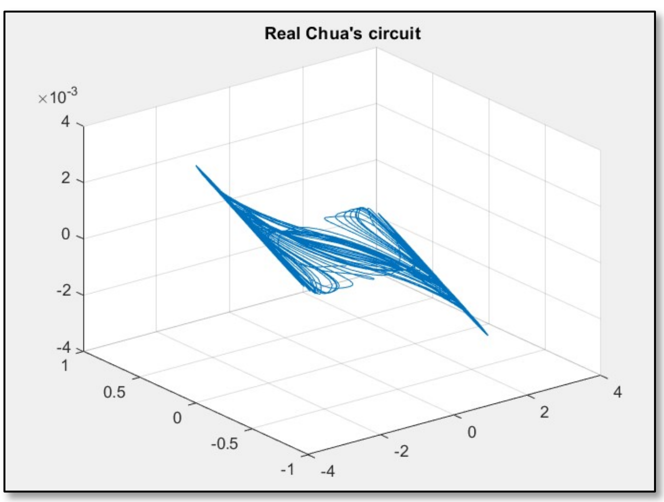

Figure 4: 3D MATLAB output of Realistic Chua Equations

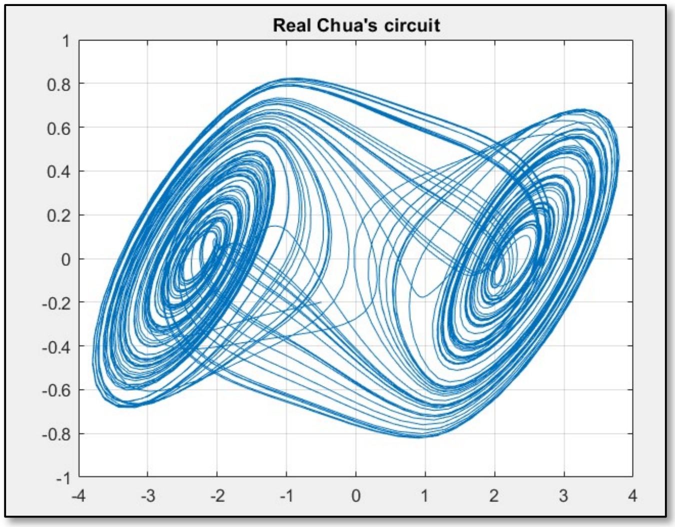

Figure 5: 2D MATLAB output of Realistic Chua Equations

5.3. Dimensionless Equations

The MATLAB codes to solve the realistic and dimensionless chua equations are presented below. The equations were solved using an in-built MATLAB function ODE45 solver yielding fourth order Runge-Kutta method. The value of R is set to 1.7kΩ. The dimensionless code presents 2 plots: V_{C2} versus V_{C1} and V_{C1} versus time

5.3.1. MATLAB Code

```
global R C1 C2 L

R = 1.70*(1E3);
C1 = 10*(1E-9);
C2 = 100*(1E-9);
L = 18*(1E-3);

[x,y]=ode45('sub_chua1' ,[0 100],[0.1 0.1 0.1]);

subplot(121), plot(y(:,1),y(:,2)),grid on
xlabel('x'), ylabel('y')
subplot(122), plot(x,y(:,1)), grid on
xlabel('Time'), ylabel('x')
title("Chua's Chaotic Oscillator")

function dy = sub_chua1(t,y)
global R C1 C2 L
dy = zeros(3,1);
alpha = C2/C1;
beta = ((C2*(R*R))/L);
Ga = -0.758*(1E-3);
Gb = -0.409*(1E-3);
a = R*Ga;
b = R*Gb;
```

```
h = (b*y(1))+0.5*(a-b)*(abs(y(1)+1)-abs(y(1)-1));

% Chua's Circuit Equations
dy(1) = alpha*(y(2)-y(1)-h);
dy(2) = y(1)-y(2)+y(3);
dy(3) = -beta*(y(2));

end
```

5.3.2. Output

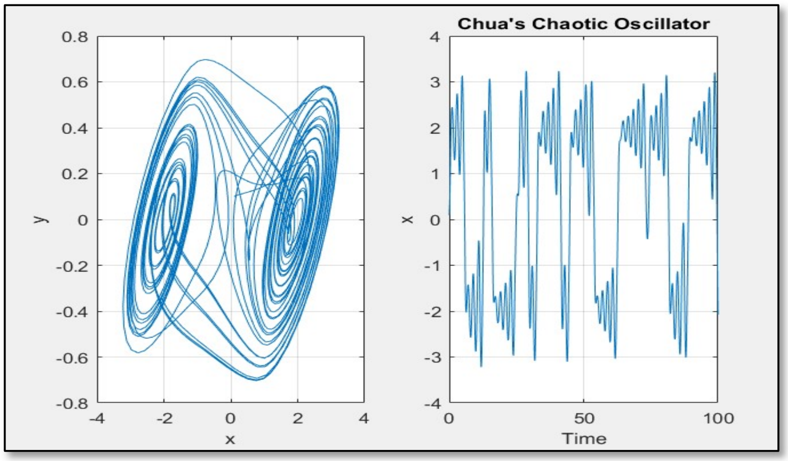

Figure 6: MATLAB output of Dimensionless Chua Equations

5.4. Multisim Simulation

To verify the Matlab code, Multisim simulation was carried out.

5.4.1. Circuit used for simulation

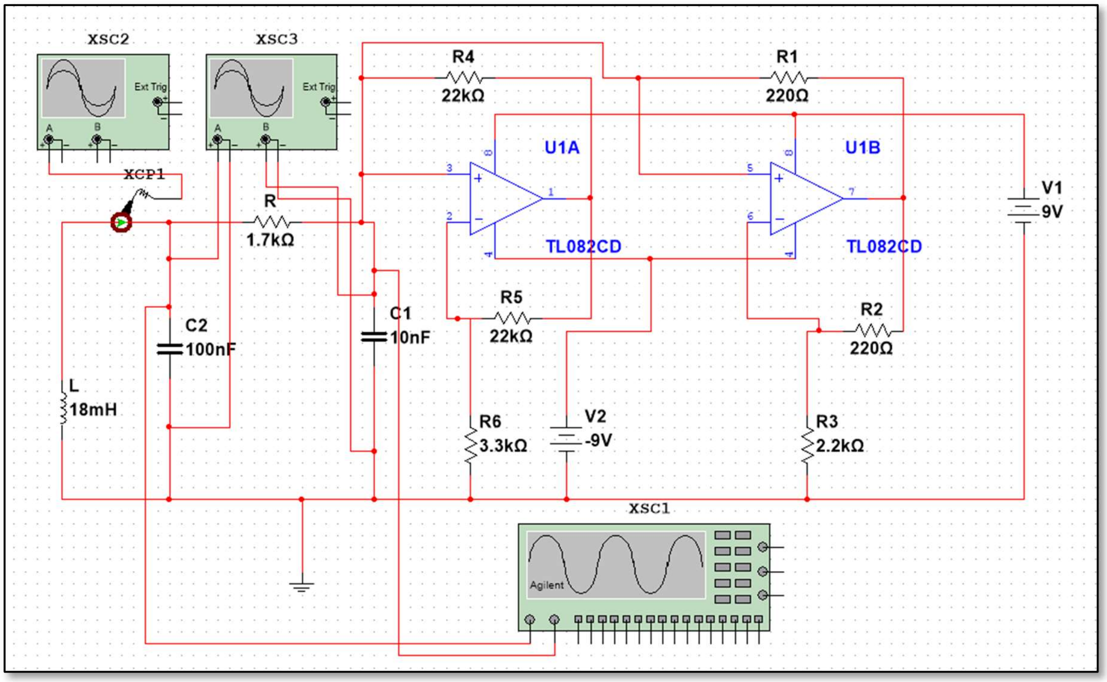

Figure 7: Chua's Circuit in developed in Multisim

5.4.2. Output

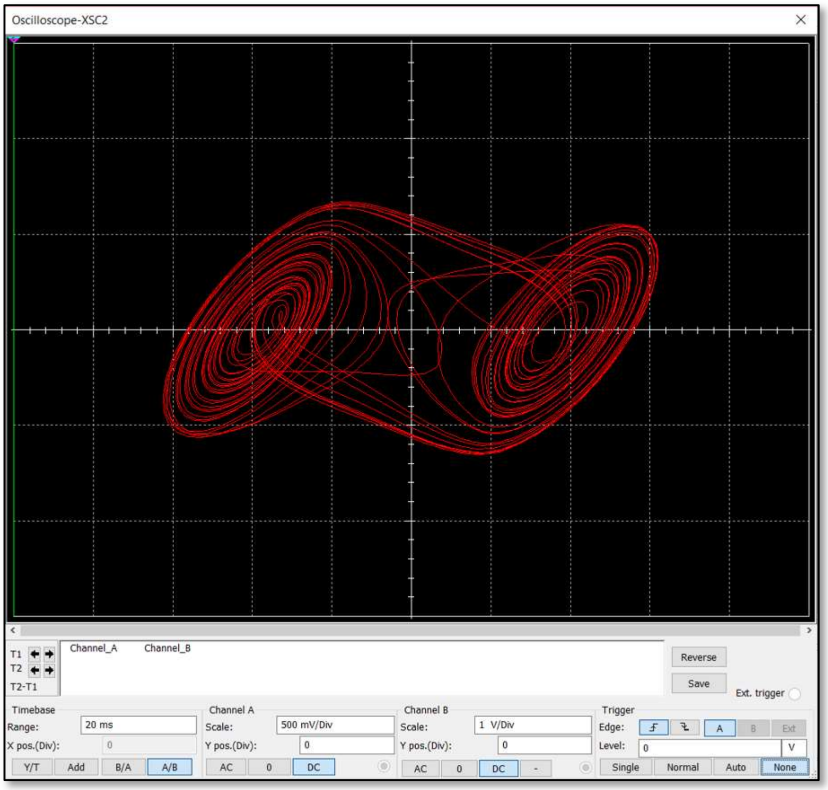

Figure 8: Simulation Output of Chua Circuit

Scale: I_L 1A/div and time/div = 1ms/div

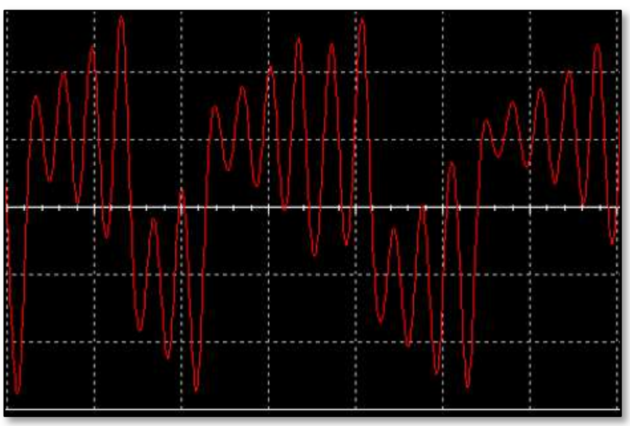

Figure 9: Waveform of current through the inductor L

Scale: V_{C2} = 1V/div and time = 1ms/div

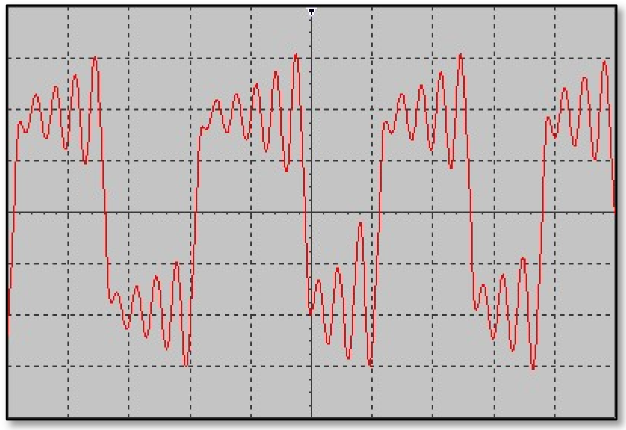

Figure 10: Waveform of voltage across the capacitor C_2

Scale: V_{C1} = 200mV/div and time = 1ms/div

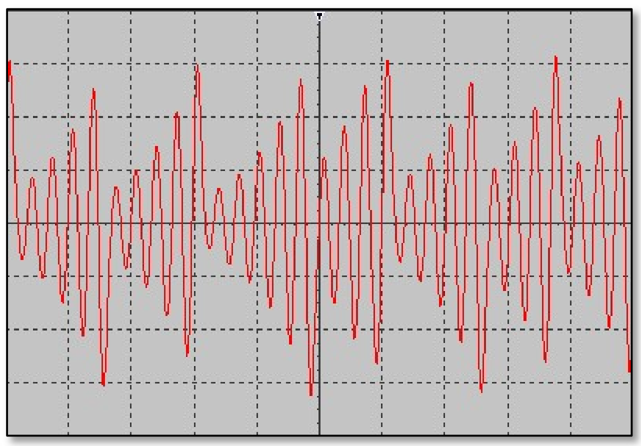

Figure 11: Waveform of voltage across the capacitor C_1

6. Results and Discussion

The R bifurcation sequences in Chua's circuit when varied from R = 2k to R = 1.7K are shown below:

- **R=2kΩ**

By setting R=2kΩ, the realistic and dimensionless MATLAB code and MULTISIM simulation displays a limit cycle bifurcation.

Realistic

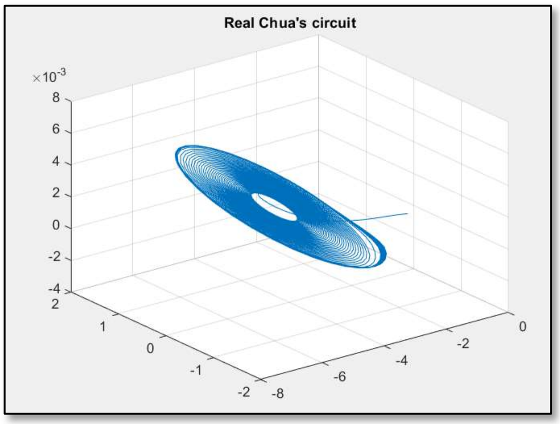

Figure 12: 3D MATLAB output of Realistic Chua Equations for R=2k ohms

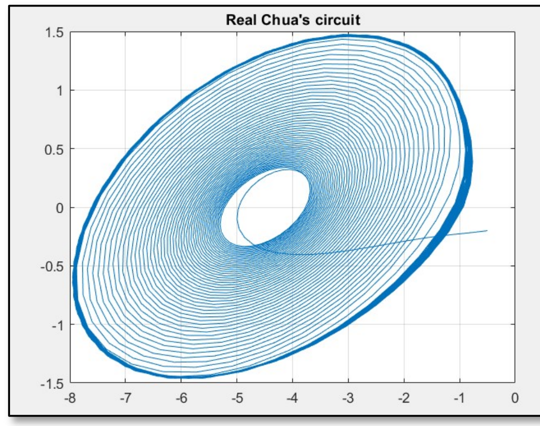

Figure 13: 2D MATLAB output of Realistic Chua Equations for R=2k ohms

Dimensionless

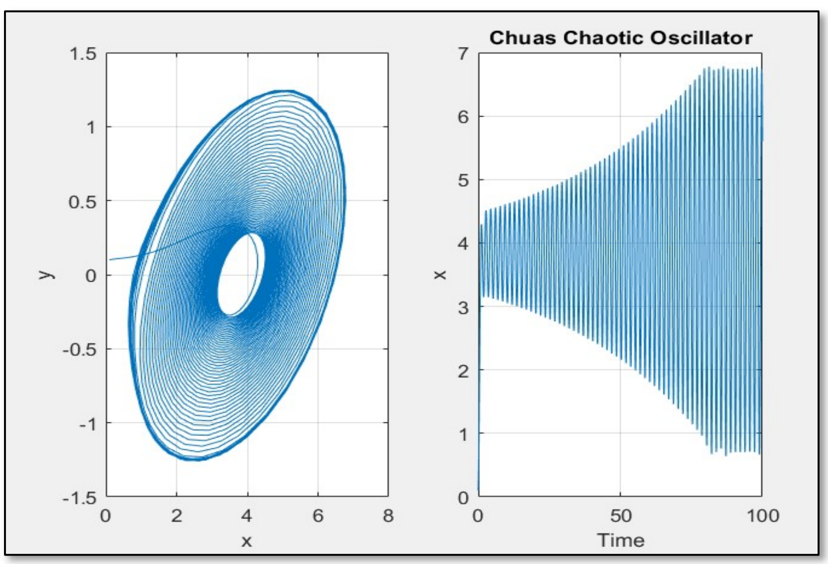

Figure 14: MATLAB output of Dimensionless Chua Equations for R=2k ohms

Multisim

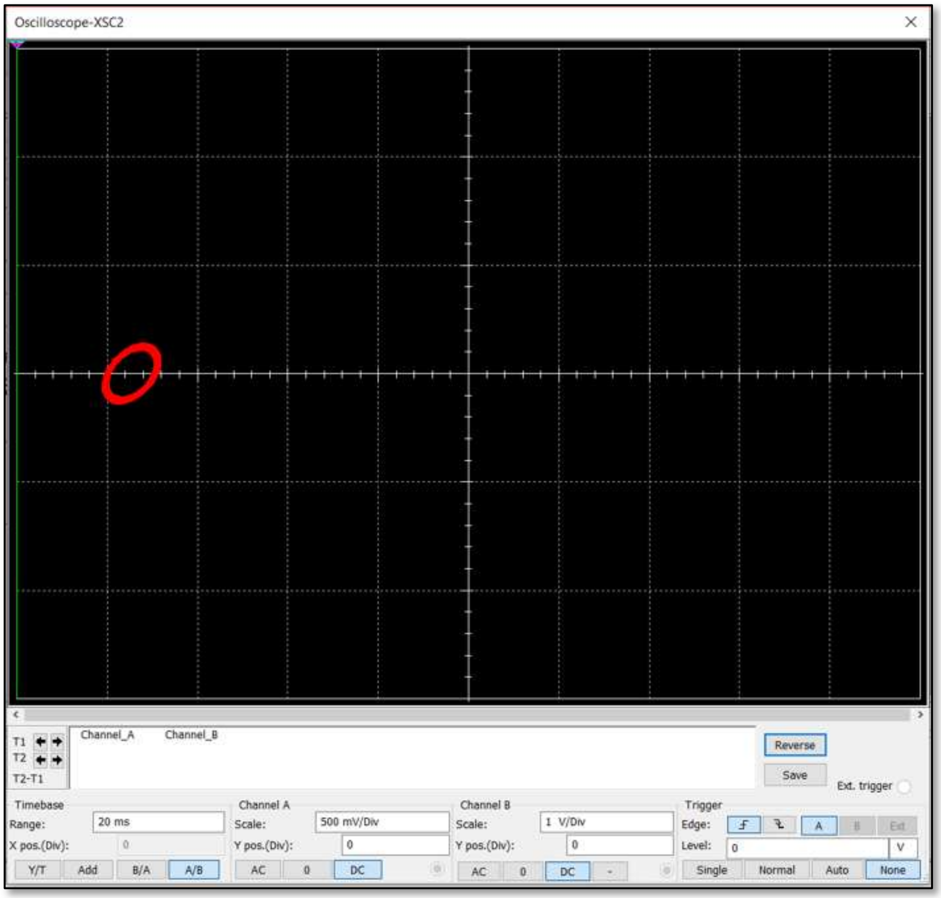

Figure 15: Simulation output of Chua Circuit for R=2k ohms

- **R=1.95kΩ**

By setting R=1.95kΩ, the realistic and dimensionless MATLAB code and MULTISIM simulation displays a Rossler type attractor.

Realistic

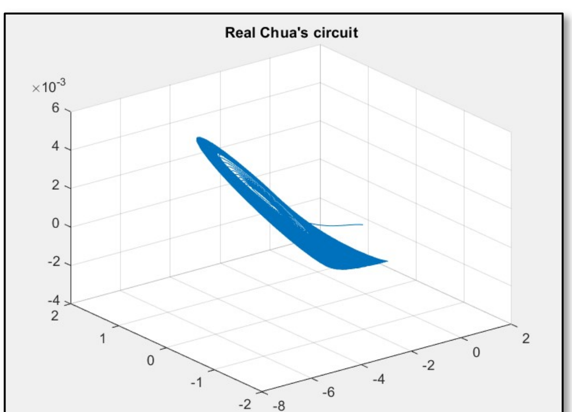

Figure 16: 3D MATLAB output of Realistic Chua Equations for R=1.95k ohms

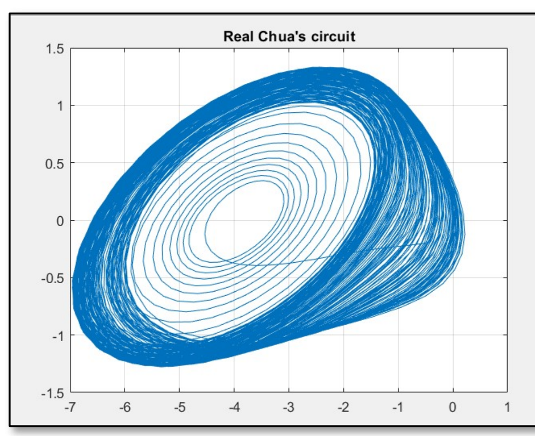

Figure 17: 2D MATLAB output of Realistic Chua Equations for R=1.95k ohms

Dimensionless

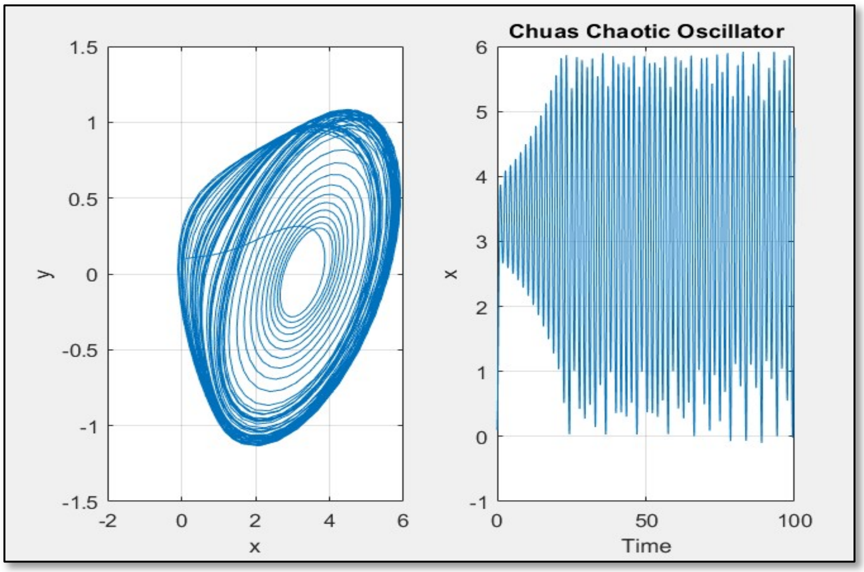

Figure 18: MATLAB output of Dimensionless Chua Equations for R=1.95k ohms

Multisim

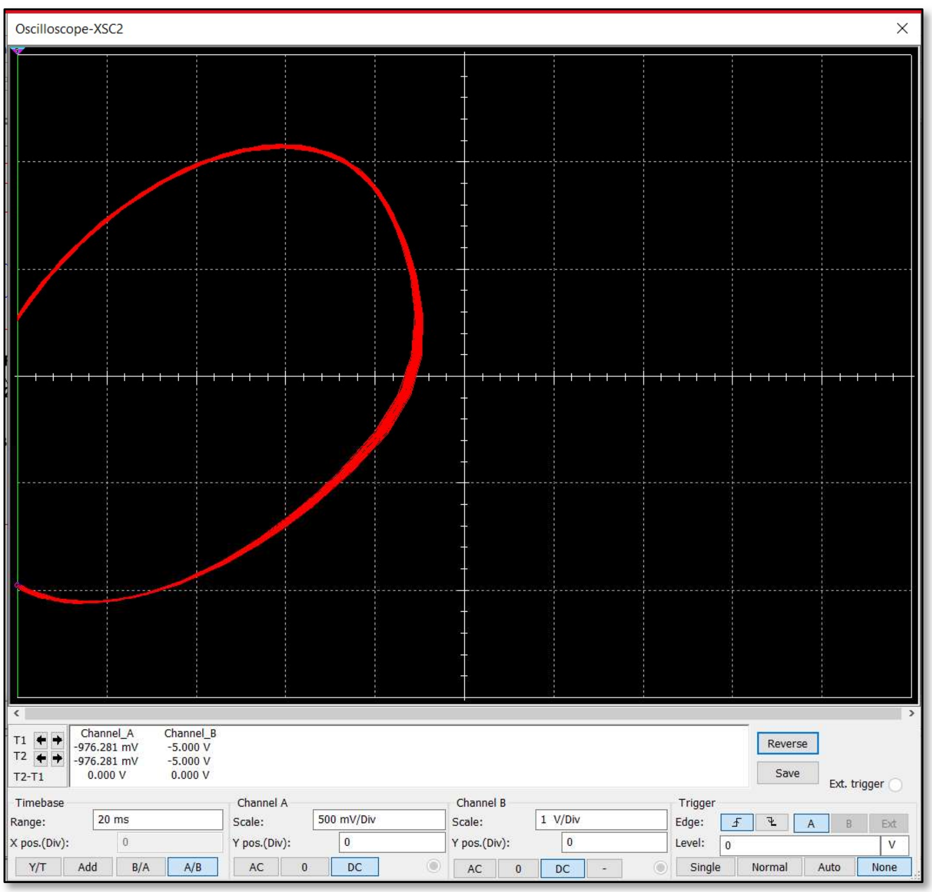

Figure 19: Simulation output of Chua Circuit for R=1.95k ohms

- **R=1.9kΩ**

By setting R=1.9kΩ, the realistic and dimensionless MATLAB code displays a Double scroll attractor and MULTISIM simulation displays a Single scroll attractor.

Realistic

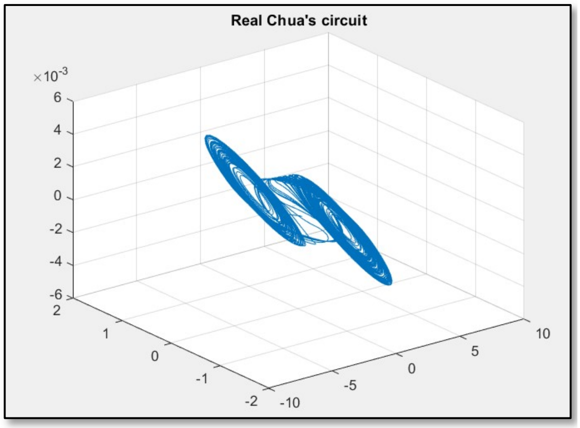

Figure 20: 3D MATLAB output of Realistic Chua Equations for R=1.9k ohms

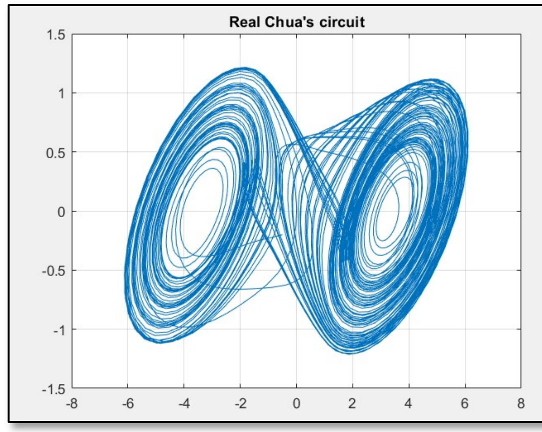

Figure 21: 2D MATLAB output of Realistic Chua Equations for R=1.9k ohms

Dimensionless

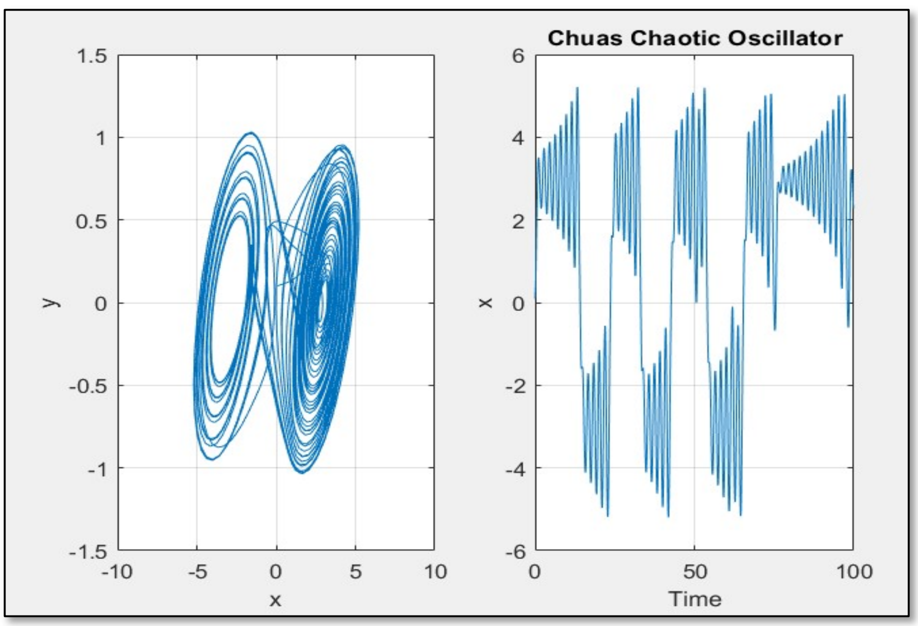

Figure 22: MATLAB output of Dimensionless Chua Equations for R=1.9k ohms

Multisim

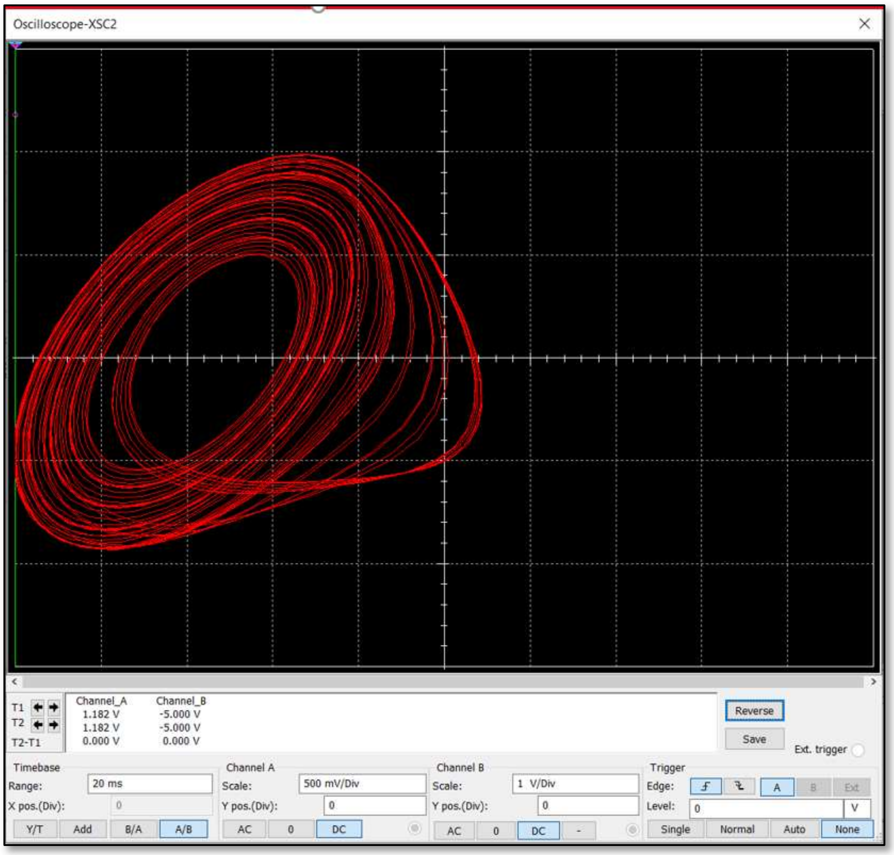

Figure 23: Simulation output of Chua Circuit for R=1.9k ohms

- **R=1.85kΩ**

By setting R=1.85kΩ, the realistic and dimensionless MATLAB code and MULTISIM simulation displays a Double scroll attractor.

Realistic

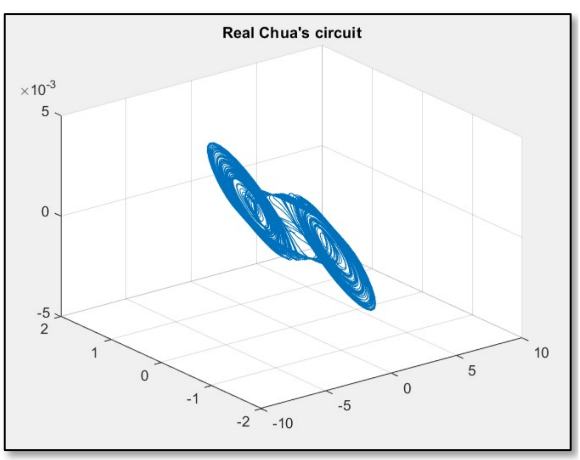

Figure 24: 3D MATLAB output of Realistic Chua Equations for R=1.85k ohms

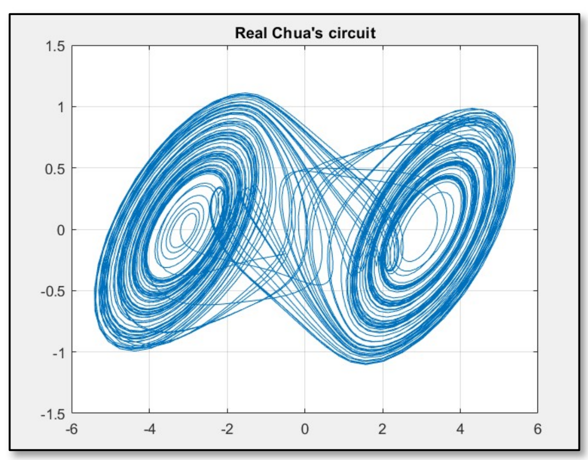

Figure 25: 2D MATLAB output of Realistic Chua Equations for R=1.85k ohms

Dimensionless

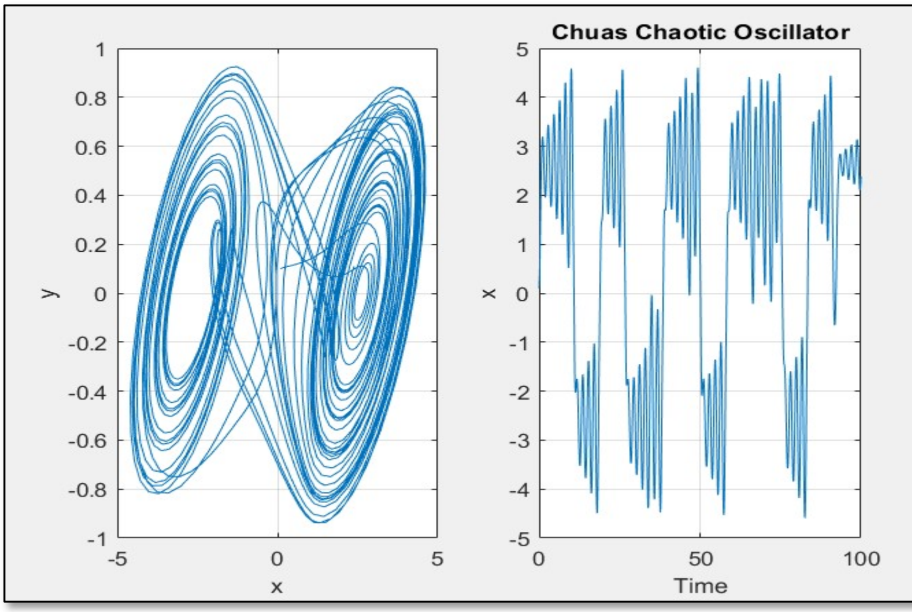

Figure 26: MATLAB output of Dimensionless Chua Equations for R=1.85k ohms

Multisim

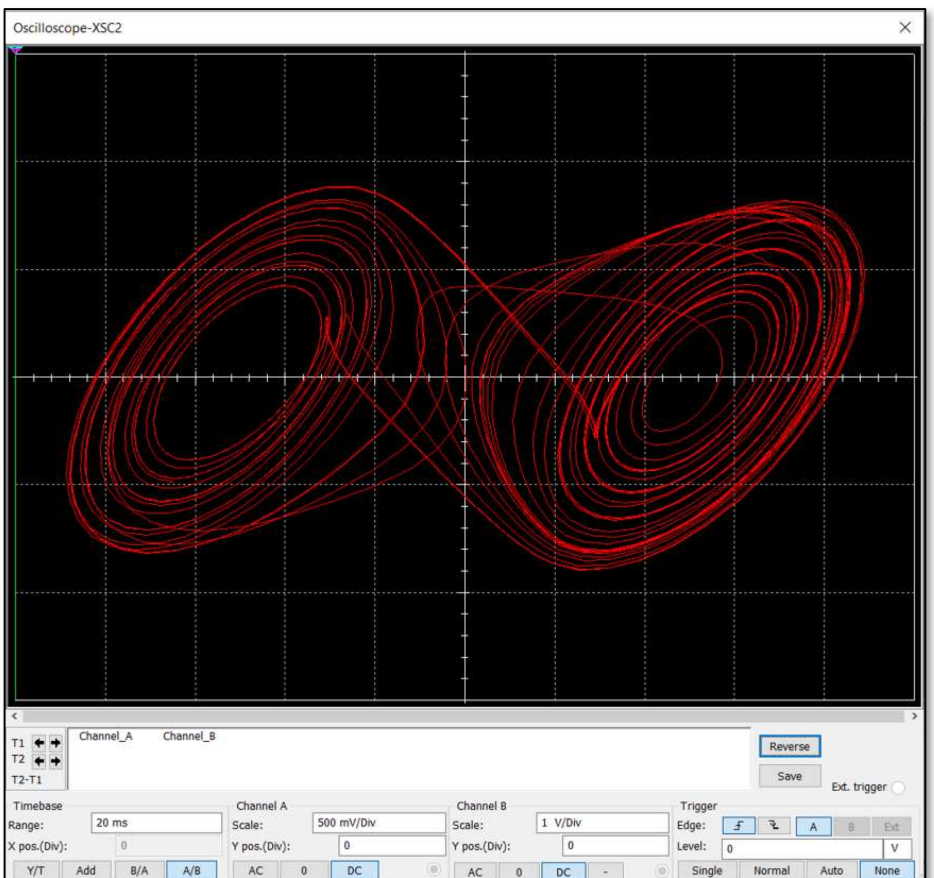

Figure 27: Simulation output of Chua Circuit for R=1.85k ohms

- **R=1.7kΩ**

By setting R=1.7kΩ, the realistic and dimensionless MATLAB code and MULTISIM simulation displays a Double scroll attractor.

Realistic

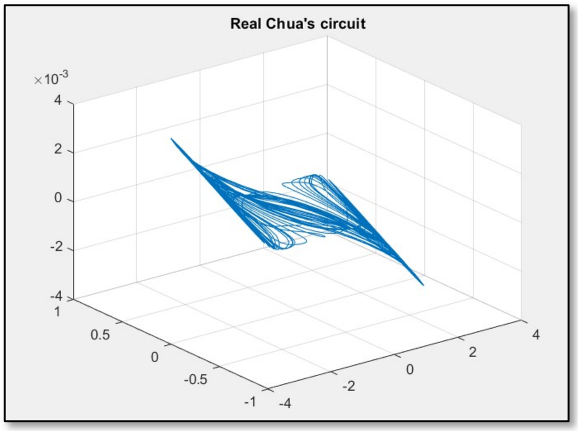

Figure 28: 3D MATLAB output of Realistic Chua Equations for R=1.7k ohms

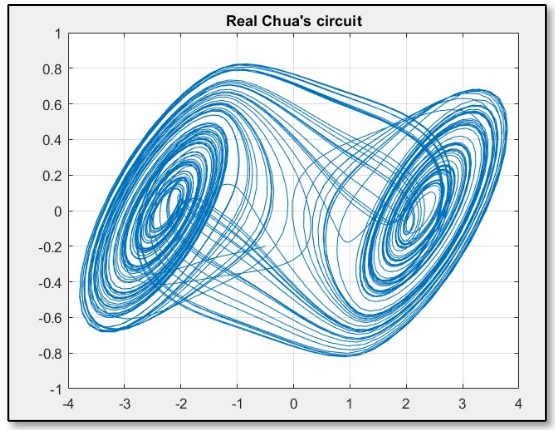

Figure 29: 2D MATLAB output of Realistic Chua Equations for R=1.7k ohms

Dimensionless

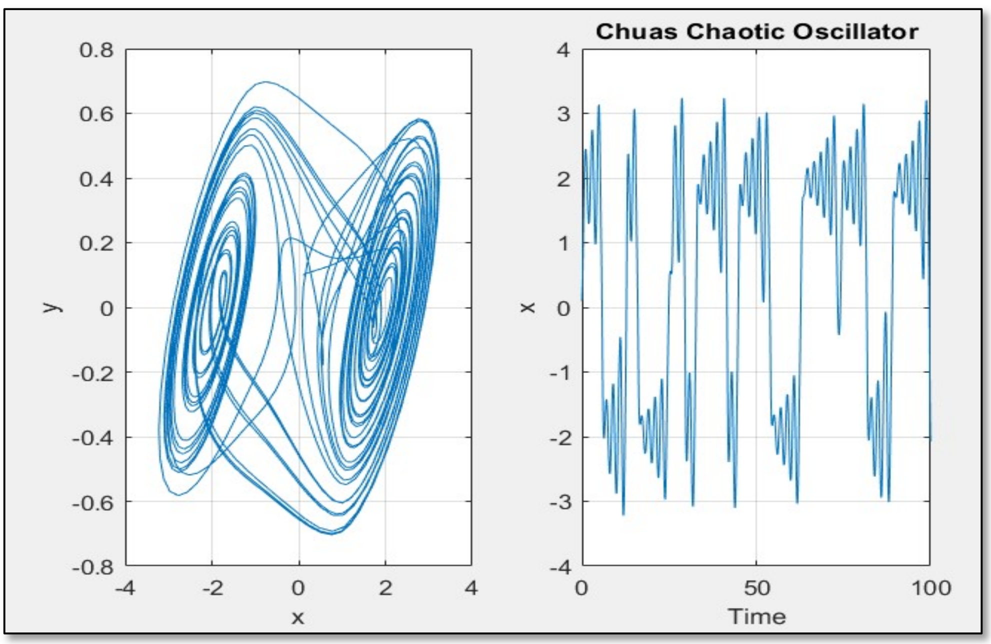

Figure 30: MATLAB output of Dimensionless Chua Equations for R=1.7k ohms

Multisim

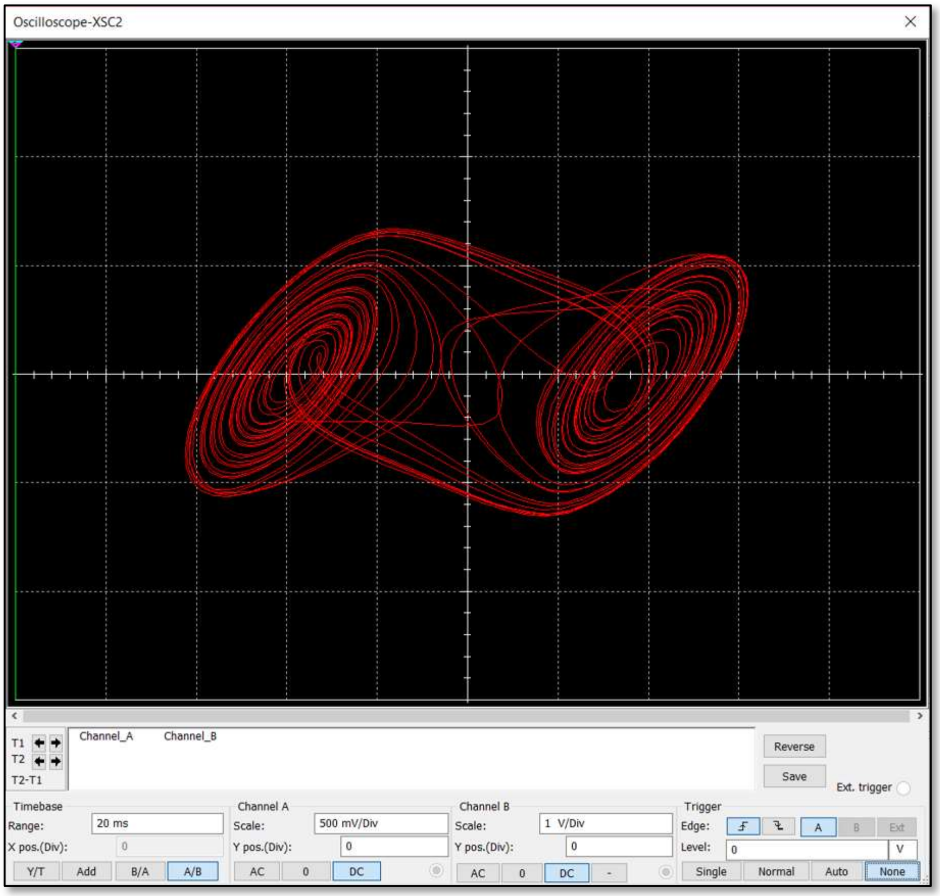

Figure 31: Simulation output of Chua Circuit for R=1.7k ohms

- By carrying out the MATLAB code and MULTISIM circuit simulation for the value of R in a range between 1.7kΩ to 2kΩ, the bifurcation results of the realistic and dimension less Chua circuit and MULTISIM Chua circuit are obtained as per figures 12 - 31. From these figures, two basic attractor types observed are single scroll and doble scroll Chua attractors. The double scroll attracter was observed for a small range of R.
- It can be observed that solving the dimensionless equations of the Chua's circuit results in a solution that converges (to an extent) to the realistic Chua equation solution.
- Considering the case when R=1.9kΩ, it is observed that the MATLAB results obtained for realistic and dimension less Chua circuit shows a double scroll attracter, whereas the MULTISIM result shows a single scroll (Rossler type attracter). The difference in these results is caused due to the difference in the initial conditions.

7. Conclusion

In this project, the differential equations governing Chua's circuit was developed and the analytical solution for the same was obtained. The Chua's chaotic circuit system was studied by varying the linear resistance value R which resulted in various attractors. These resulting attractors were not only demonstrated by MATLAB code (numerical solution) but also using MULTISIM simulation. The Chua's diode (N_R) was modelled by a piecewise linear V-I characteristics. The combination of MATLAB and MULTISIM is useful to model and simulate the non-linear behaviour of Chua circuit.

By studying the obtained attractors, it was noted that the realistic, dimensionless and MULTISIM solutions were noticeably similar. However, the variation in the results is due to the initial conditions, as even a slight change in the initial condition can cause larger differences in the result.

8. References

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