Deep Learning

1 The Artificial Neuron (MP Neuron and Perceptron)

Dr. Konda Reddy Mopuri
kmopuri@ai.iith.ac.in
Dept. of AI, IIT Hyderabad
Jan-May 2024

The Neuron

- About 100 billion neurons in human brain

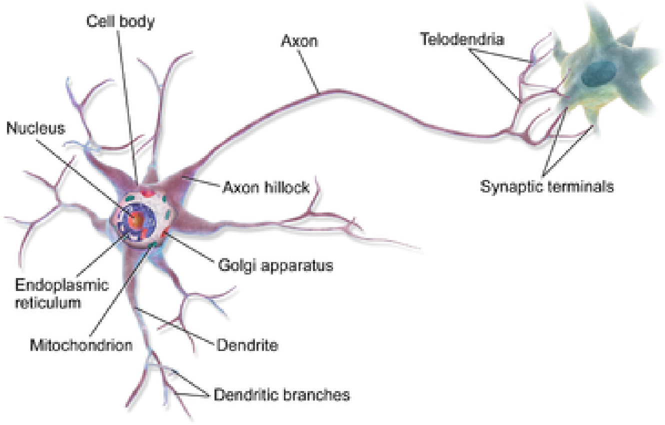
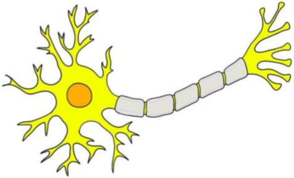


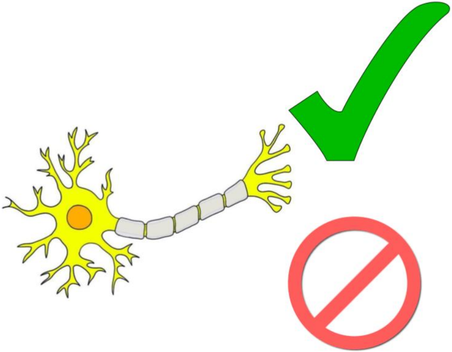
Figure credits: Wikipedia

The dilemma: To watch or not to watch?

Let's use our brain

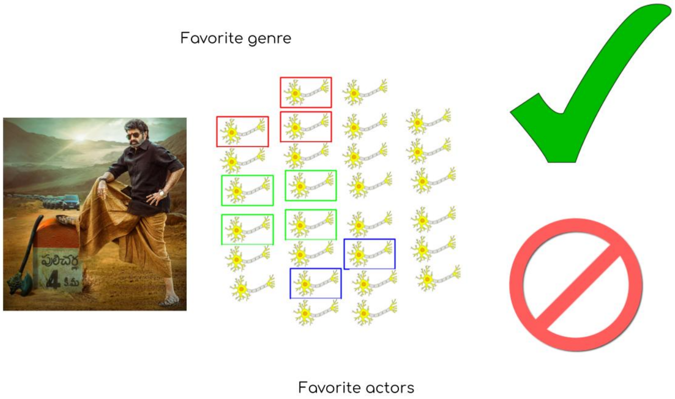


Let's use our brain

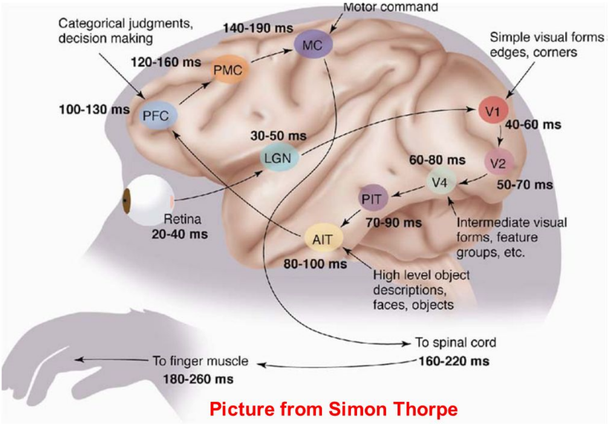


It’s a network of many neurons

There is a division of responsibilities



Neurons in the brain have a hierarchy



Picture from Simon Thorpe

Threshold Logic Unit

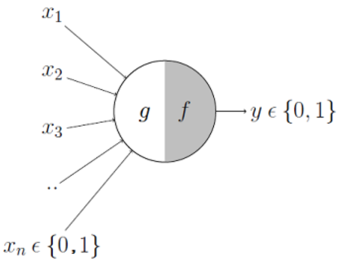
1. First Mathematical Model for a neuron

Threshold Logic Unit

1. First Mathematical Model for a neuron
2. McCulloch and Pitts, $1943 \rightarrow$ MP neuron

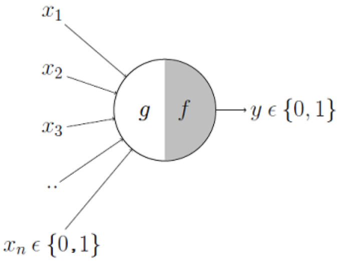
Threshold Logic Unit

1. First Mathematical Model for a neuron
2. McCulloch and Pitts, $1943 \rightarrow$ MP neuron
3. Boolean inputs and output



Threshold Logic Unit

1. First Mathematical Model for a neuron
2. McCulloch and Pitts, $1943 \rightarrow$ MP neuron
3. Boolean inputs and output



4. $$f(x) = \mathbb{1}(\sum_i x_i \geq \theta)$$

Threshold Logic Unit

1. Inputs can be of excitatory or inhibitory nature

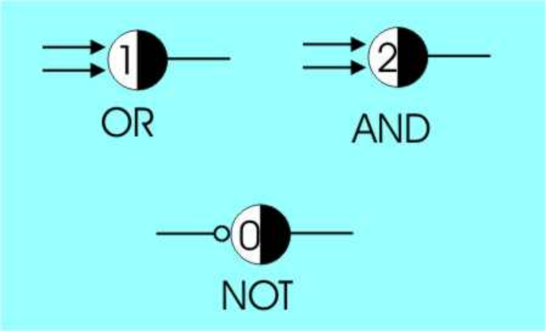
Threshold Logic Unit

1. Inputs can be of excitatory or inhibitory nature
2. When an inhibitory input is set ($=1$) output $\rightarrow 0$

Threshold Logic Unit

1. Inputs can be of excitatory or inhibitory nature
2. When an inhibitory input is set ($=1$) output $\rightarrow 0$
3. Counts the number of 'ON' signals on the excitatory inputs versus the inhibitory

Threshold Logic Unit



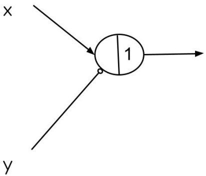
Example Boolean functions

Threshold Logic Unit

1. let's implement simple functions

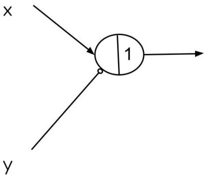
Threshold Logic Unit

1. let's implement simple functions
2. xy'

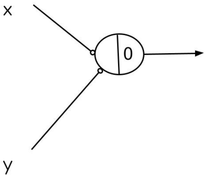


Threshold Logic Unit

1. let's implement simple functions
2. xy'



3. NOR



Threshold Logic Unit

1. What one unit does? - Learn linear separation

Threshold Logic Unit

1. What one unit does? - Learn linear separation
   - line in 2D, plane in 3D, hyperplane in higher dimensions

Threshold Logic Unit

1. What one unit does? - Learn linear separation
   - line in 2D, plane in 3D, hyperplane in higher dimensions
2. No learning; heuristic approach

Perceptron

1. Frank Rosenblatt 1957 (American Psychologist)

Perceptron

1. Frank Rosenblatt 1957 (American Psychologist)
2. Very crude biological model

Perceptron

1. Frank Rosenblatt 1957 (American Psychologist)
2. Very crude biological model
3. Similar to MP neuron - Performs linear classification

Perceptron

భారతీయ సాంకేతిక విజ్ఞాన సంస్థ హైదరాబాద్
भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

1. Frank Rosenblatt 1957 (American Psychologist)
2. Very crude biological model
3. Similar to MP neuron - Performs linear classification
4. Inputs can be real, weights can be different for different i/p components

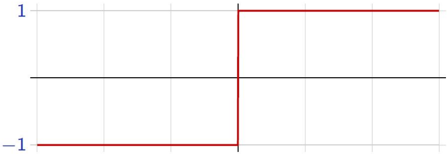
Perceptron

1. Frank Rosenblatt 1957 (American Psychologist)
2. Very crude biological model
3. Similar to MP neuron - Performs linear classification
4. Inputs can be real, weights can be different for different i/p components
5. $$f(x) = \begin{cases} 1 & \text{when } \sum_i w_i x_i + b \geq 0 \\ 0 & \text{else} \end{cases}$$

Perceptron

1. For simplicity we consider +1 and -1 responses

$$\sigma(x) = \begin{cases} 1 & \text{when } x \geq 0 \\ -1 & \text{else} \end{cases}$$

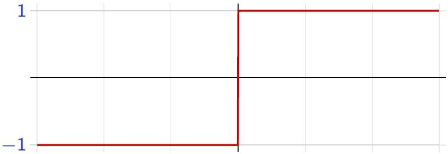


$$f(\mathbf{x}) = \sigma(\mathbf{w}^{\mathbf{T}} \cdot \mathbf{x} + \mathbf{b})$$

Perceptron

1. For simplicity we consider +1 and -1 responses

$$\sigma(x) = \begin{cases} 1 & \text{when } x \geq 0 \\ -1 & \text{else} \end{cases}$$



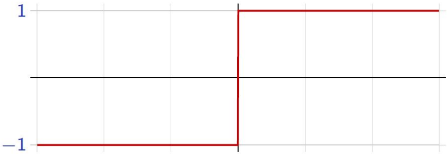
$$f(\mathbf{x}) = \sigma(\mathbf{w}^{\mathbf{T}} \cdot \mathbf{x} + \mathbf{b})$$

2. In general, $\sigma(\cdot)$ that follows a linear operation is called an activation function

Perceptron

1. For simplicity we consider +1 and -1 responses

$$\sigma(x) = \begin{cases} 1 & \text{when } x \geq 0 \\ -1 & \text{else} \end{cases}$$



$$f(\mathbf{x}) = \sigma(\mathbf{w}^{\mathbf{T}} \cdot \mathbf{x} + \mathbf{b})$$

2. In general, $\sigma(\cdot)$ that follows a linear operation is called an activation function
3. $\mathbf{w}$ are referred to as weights and b as the bias

Perceptron vs. MP neuron

1. Perceptron is more general computational model

Perceptron vs. MP neuron

1. Perceptron is more general computational model
2. Inputs can be real

Perceptron vs. MP neuron

1. Perceptron is more general computational model
2. Inputs can be real
3. Weights are different on the input components

Perceptron vs. MP neuron

1. Perceptron is more general computational model
2. Inputs can be real
3. Weights are different on the input components
4. Mechanism for learning the weights

Weights and Bias

1. Why are the weights important?

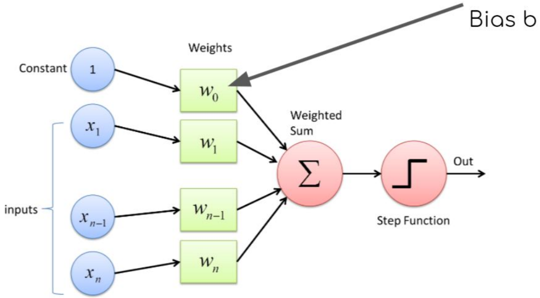


Figure credits: DeepAI

Weights and Bias

1. Why are the weights important?
2. Why is it called 'bias'? What does it capture?

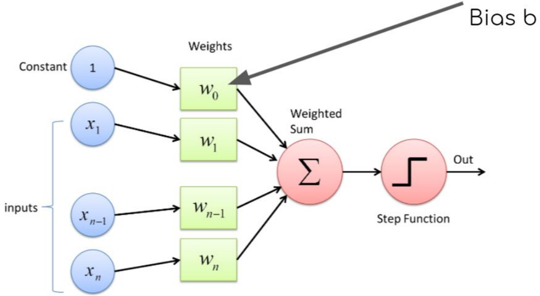


Figure credits: DeepAI

Perceptron

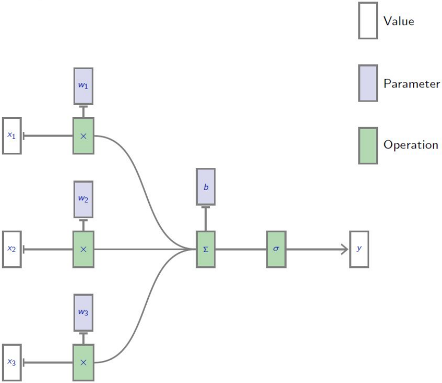


Figure credits: François Fleuret

Perceptron

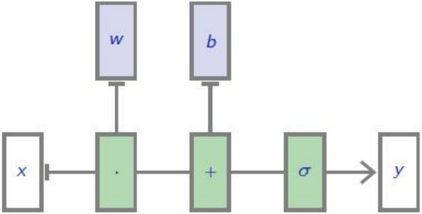


Figure credits: François Fleuret

Perceptron Learning algorithm

1. Training data $(x^i, y^i) \in \mathcal{R}^D \times \{-1, 1\}, i = 1, \ldots, N$

Perceptron Learning algorithm

1. Training data $(x^i, y^i) \in \mathcal{R}^D \times \{-1, 1\}, i = 1, \ldots, N$
2. Start with $k \leftarrow 1$ and $\mathbf{w_k} = \mathbf{0}$

Perceptron Learning algorithm

1. Training data $(x^i, y^i) \in \mathcal{R}^D \times \{-1, 1\}, i = 1, \ldots, N$
2. Start with $k \leftarrow 1$ and $\mathbf{w_k} = \mathbf{0}$
3. While $\exists\ i \in \{1, 2 \ldots N\}$ such that $y^i(\mathbf{w_k^T} \cdot \mathbf{x^i}) \leq \mathbf{0}$, update
   $\mathbf{w_{k+1}} = \mathbf{w_k} + \mathbf{y^i} \cdot \mathbf{x^i}$
   $k \leftarrow k + 1$

Perceptron Learning algorithm

1. Training data $(x^i, y^i) \in \mathcal{R}^D \times \{-1, 1\}, i = 1, \ldots, N$
2. Start with $k \leftarrow 1$ and $\mathbf{w_k} = \mathbf{0}$
3. While $\exists\ i \in \{1, 2 \ldots N\}$ such that $y^i(\mathbf{w_k^T} \cdot \mathbf{x^i}) \leq \mathbf{0}$, update
   $\mathbf{w_{k+1}} = \mathbf{w_k} + \mathbf{y^i} \cdot \mathbf{x^i}$
   $k \leftarrow k + 1$
4. Note that the bias b is absorbed as a component of $\mathbf{w}$ and $\mathbf{x}$ is appended with 1 suitably

Perceptron Learning Algorithm

▸ Colab Notebook: Perceptron-learning

Perceptron Learning Algorithm

1. Convergence result: For linearly separable dataset, the algorithm converges after finite iterations (refer to suggested readings)

Perceptron Learning Algorithm

1. Convergence result: For linearly separable dataset, the algorithm converges after finite iterations (refer to suggested readings)
2. Stops as soon as it finds a separating boundary

Perceptron Learning Algorithm

1. Convergence result: For linearly separable dataset, the algorithm converges after finite iterations (refer to suggested readings)
2. Stops as soon as it finds a separating boundary
3. Other algorithms maximize the margin from the boundary to the samples

So far...

1. Generalized to non-binary (real) input

So far...

1. Generalized to non-binary (real) input
2. Considered unequal importance to the inputs

So far...

1. Generalized to non-binary (real) input
2. Considered unequal importance to the inputs
3. Avoided heuristics with ‘learning’ the Threshold

So far...

1. Generalized to non-binary (real) input
2. Considered unequal importance to the inputs
3. Avoided heuristics with 'learning' the Threshold
4. What if the data is not linearly separable?