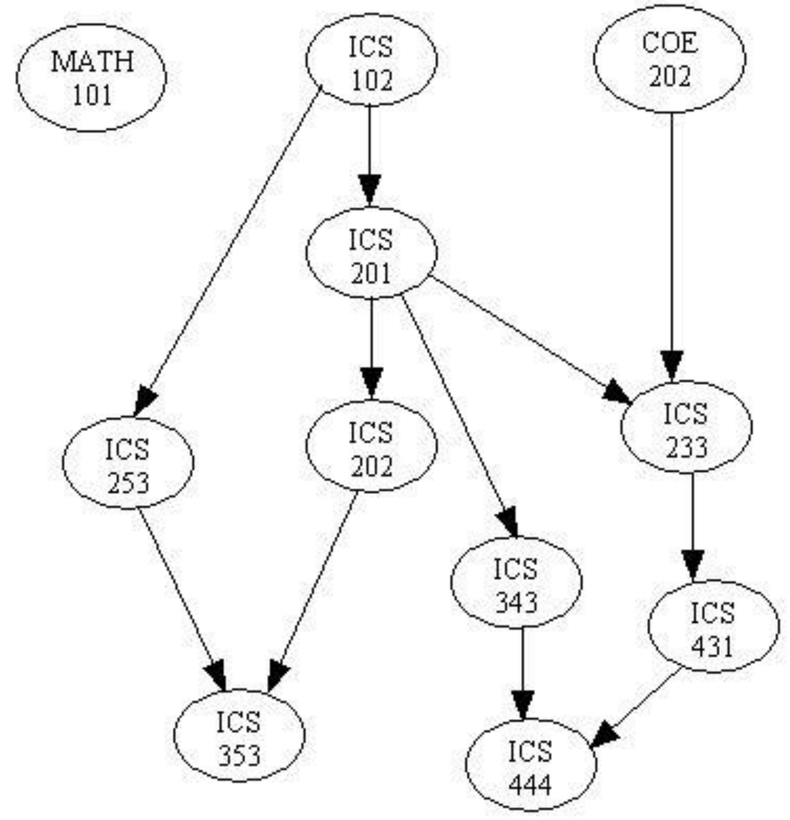
Topological sort

Motivation

Given a set of tasks with dependencies,

is there an order in which we can complete the tasks?

Cycles in dependencies can cause issues...



Definition of topological sorting

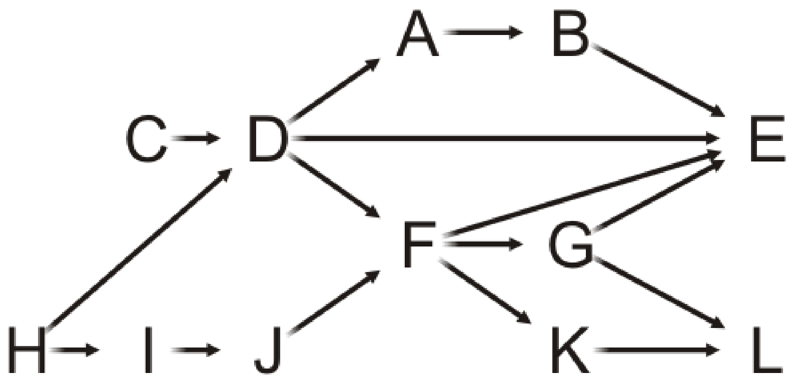
A topological sorting of the vertices in a DAG is an ordering

$$v_1, v_2, v_3, \ldots, v_{|V|}$$

such that v_j appears before v_k if there is a path from v_j to v_k

Given this DAG, a topological sort is

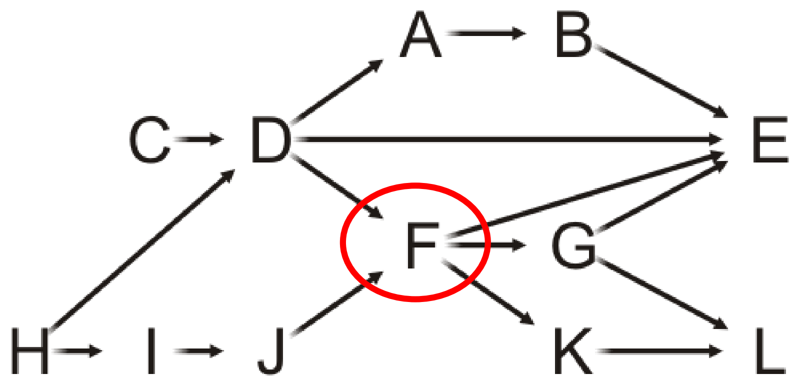
H, C, I, D, J, A, F, B, G, K, E, L



Example

For example, there are paths from H, C, I, D and J to F, so all these must come before F in a topological sort

H, **C**, **I**, **D**, **J**, A, **F**, B, G, K, E, L



Clearly, this sorting need not be unique

Applications

Consider someone is getting ready for a dinner out

He must wear the following:

- jacket, shirt, socks, tie, shoes etc.

There are certain constraints:

- the tie really should go on after the shirt,
- socks are put on before shoes

Applications

C++ header and source files have `#include` statements

- A change to an included file requires a recompilation of the current file
- On a large project, it is desirable to recompile only those source files that depended on those files which changed
- For large software projects, full compilations may take hours

Different scheduling programs in operating system

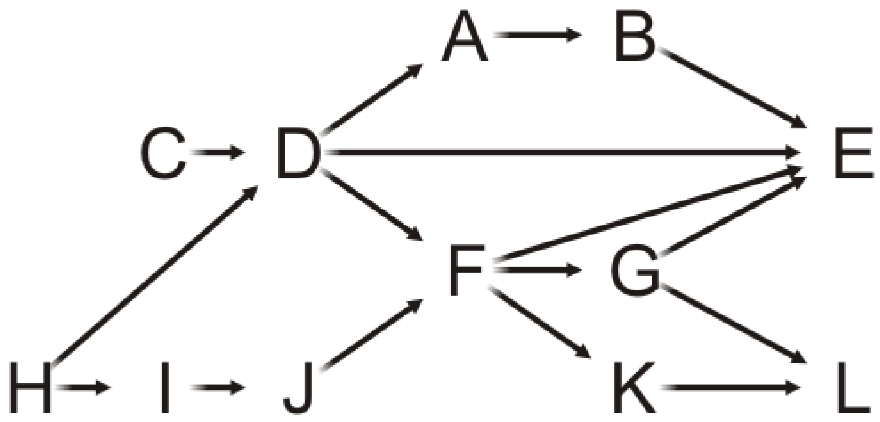
Topological Sort

Idea:

- Given a DAG V, make a copy W and iterate:
  - Find a vertex v in W with in-degree zero
  - Let v be the next vertex in the topological sort
  - Continue iterating with the vertex-induced sub-graph $W \setminus \{v\}$

On this graph, iterate the following $|V| = 12$ times

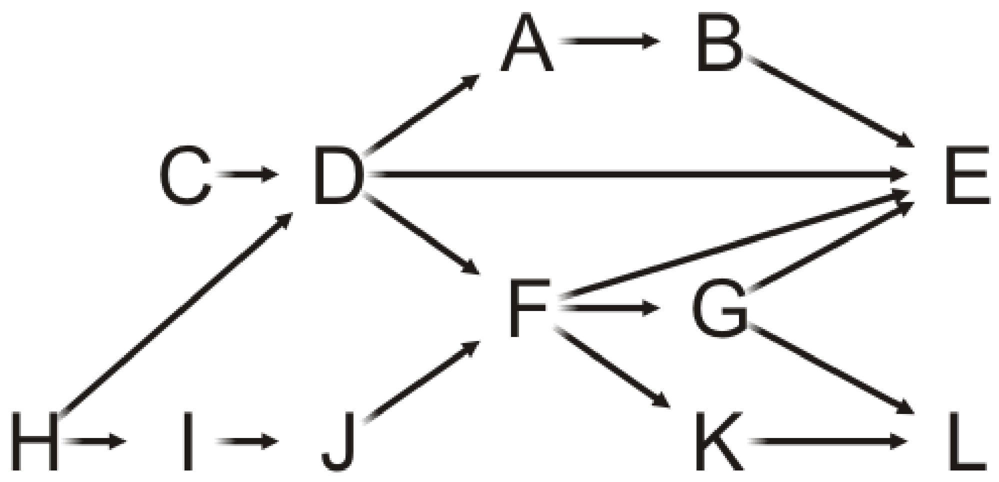
Choose a vertex v that has in-degree zero
Let v be the next vertex in our topological sort
Remove v and all edges connected to it



Example

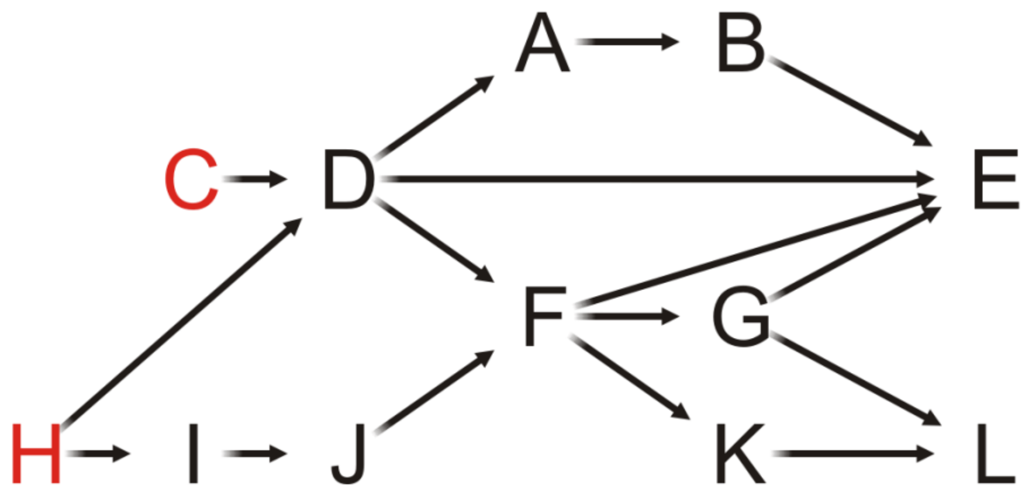
Let's step through this algorithm with this example

- Which task can we start with?



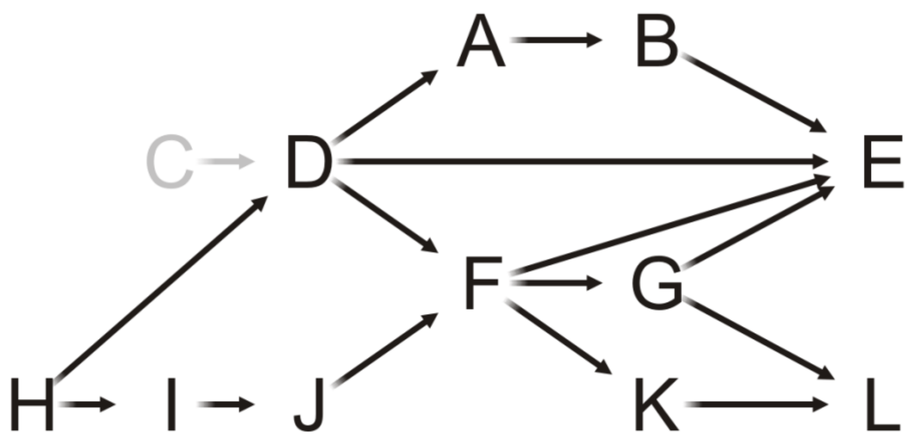
Example

Of Tasks C or H, choose Task C



Example

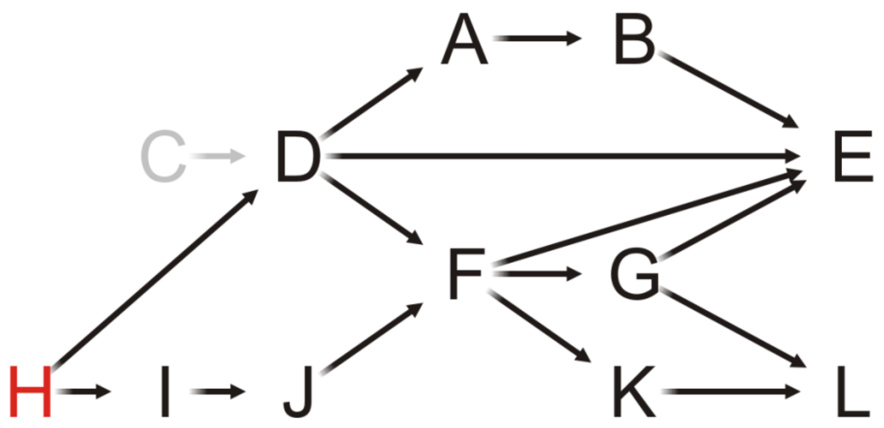
Having completed Task C, which vertices have in-degree zero?



C

Example

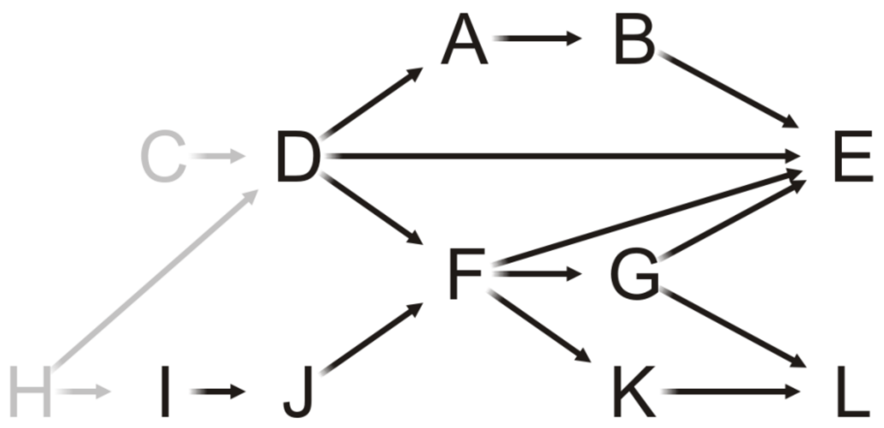
Only Task H can be completed, so we choose it



C

Example

Having removed H, what is next?

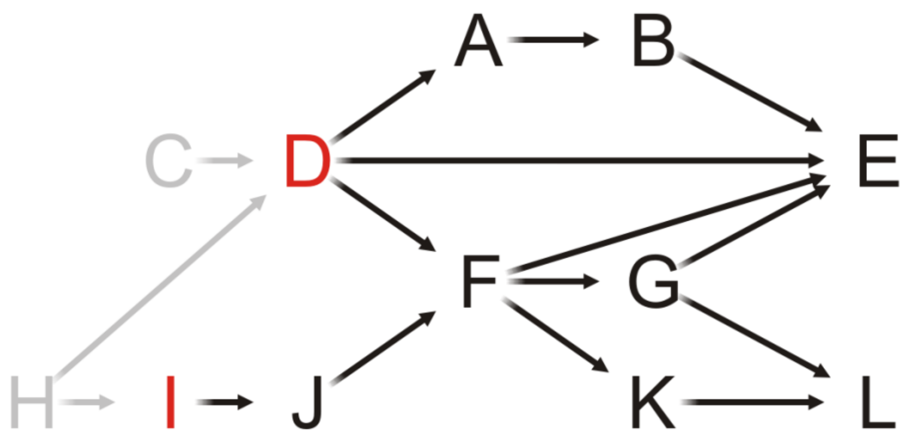


C, H

Example

Both Tasks D and I have in-degree zero

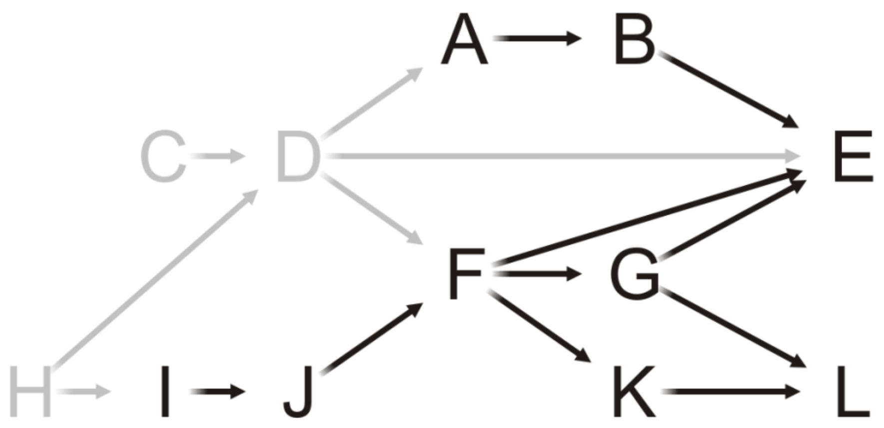
- Let us choose Task D



C, H

Example

We remove Task D, and now?

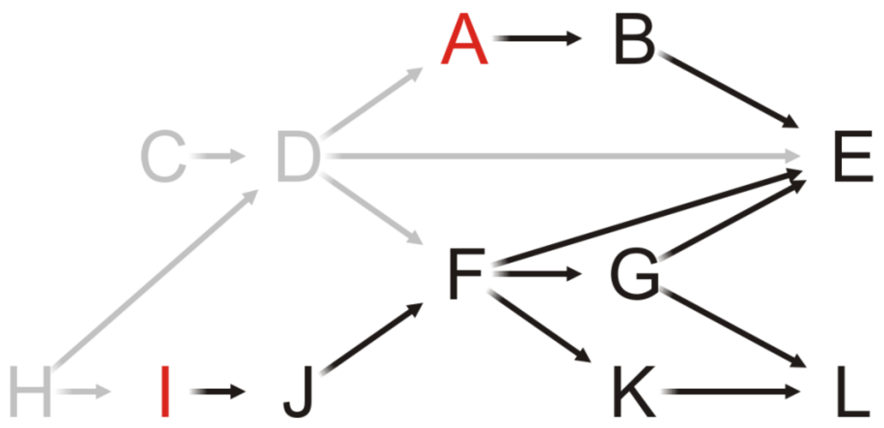


C, H, D

Example

Both Tasks A and I have in-degree zero

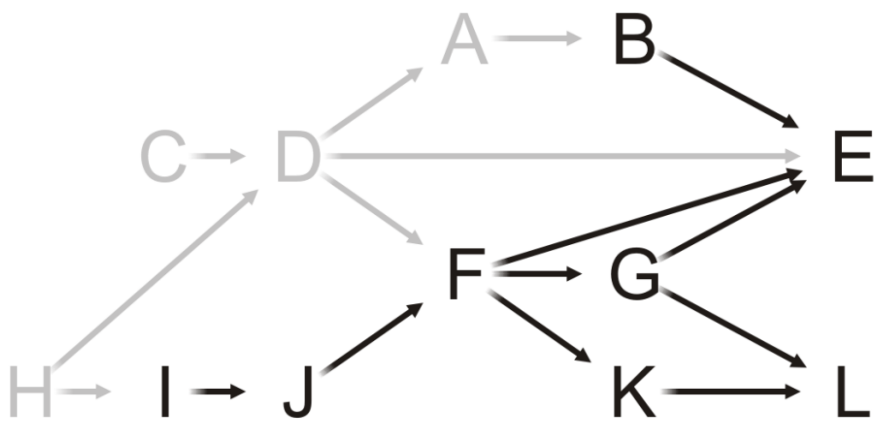
- Let's choose Task A



C, H, D

Example

Having removed A, what now?

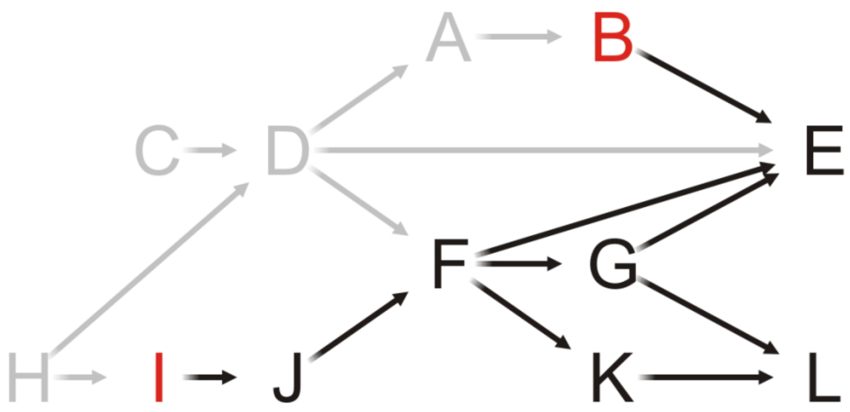


C, H, D, A

Example

Both Tasks B and I have in-degree zero

- Choose Task B

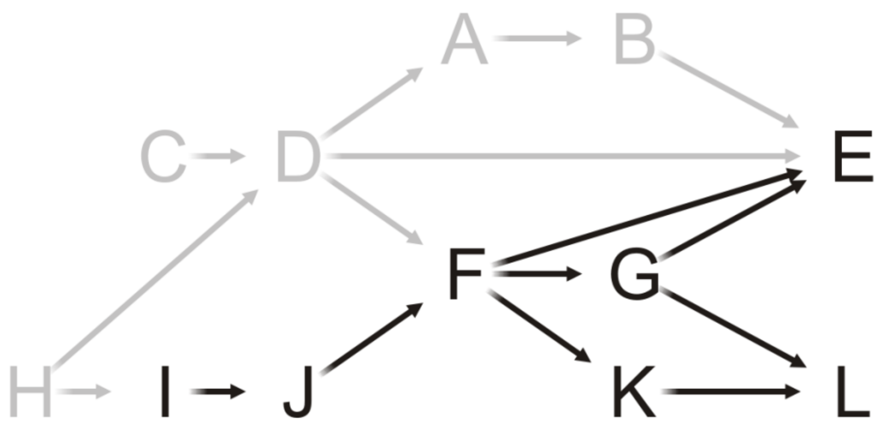


C, H, D, A

Example

Removing Task B, we note that Task E still has an in-degree of two

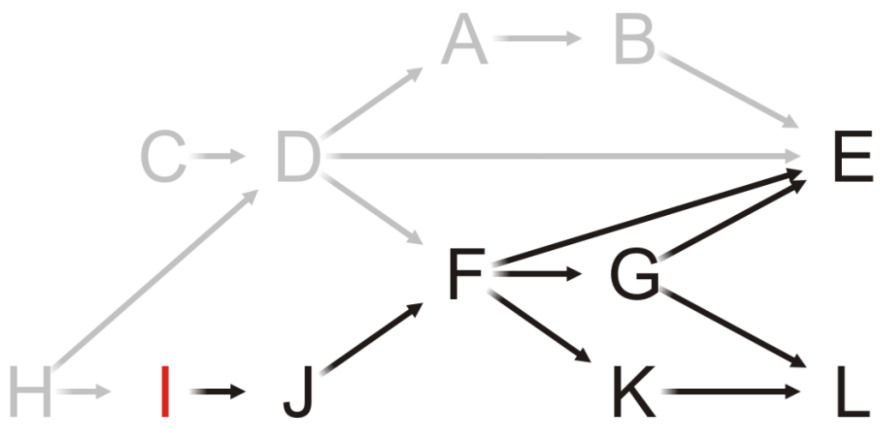
- Next?



C, H, D, A, B

Example

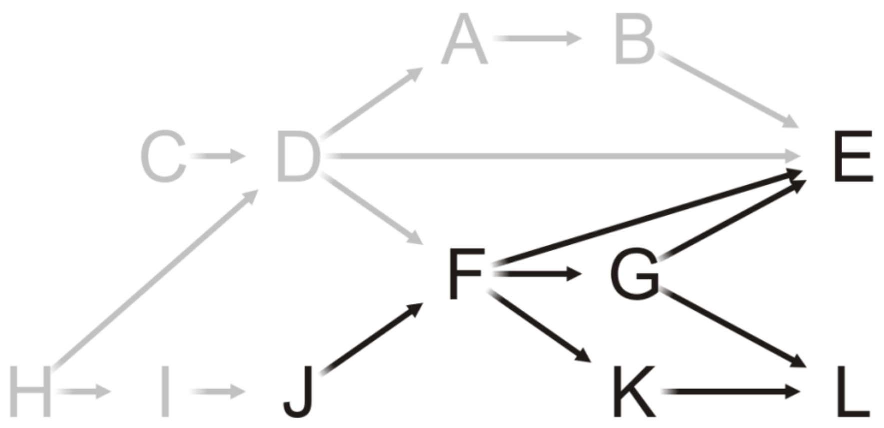
As only Task I has in-degree zero, we choose it



C, H, D, A, B

Example

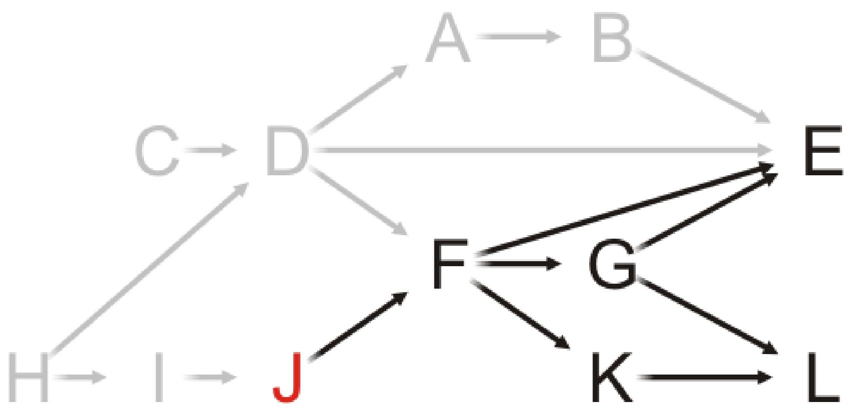
Having completed Task I, what now?



C, H, D, A, B, I

Example

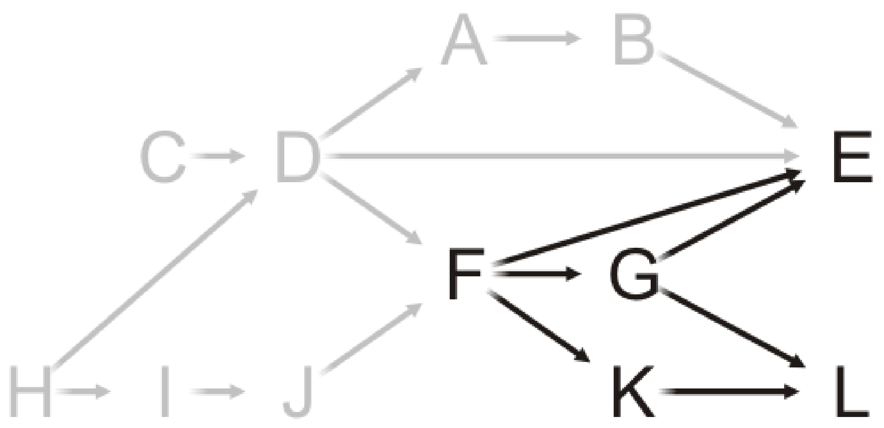
Only Task J has in-degree zero: choose it



C, H, D, A, B, I

Example

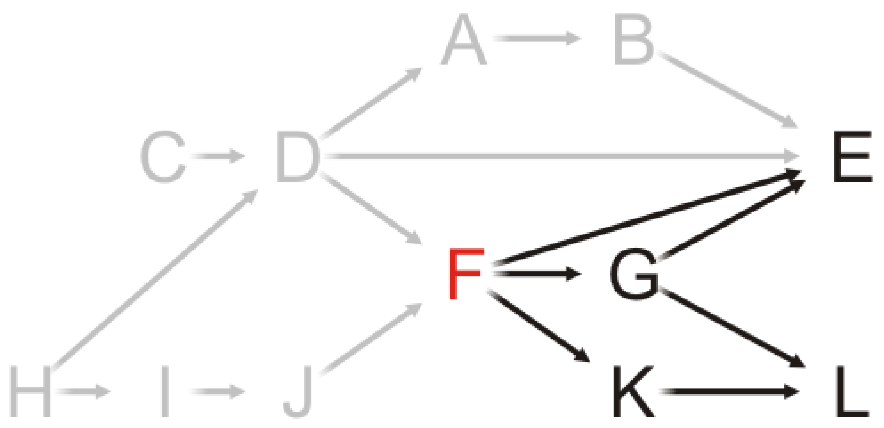
Having completed Task J, what now?



C, H, D, A, B, I, J

Example

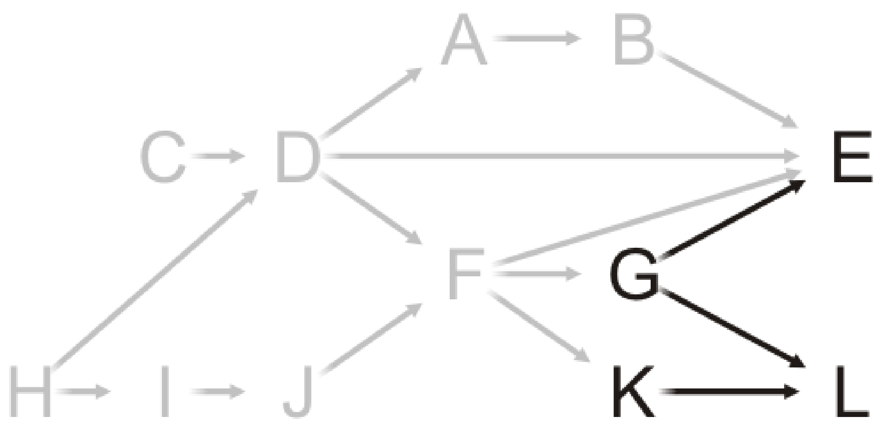
Only Task F can be completed, so choose it



C, H, D, A, B, I, J

Example

What choices do we have now?

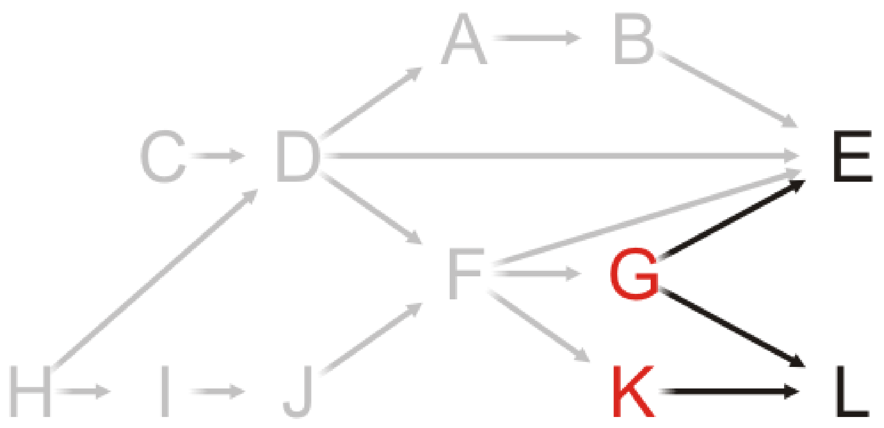


C, H, D, A, B, I, J, F

Example

We can perform Tasks G or K

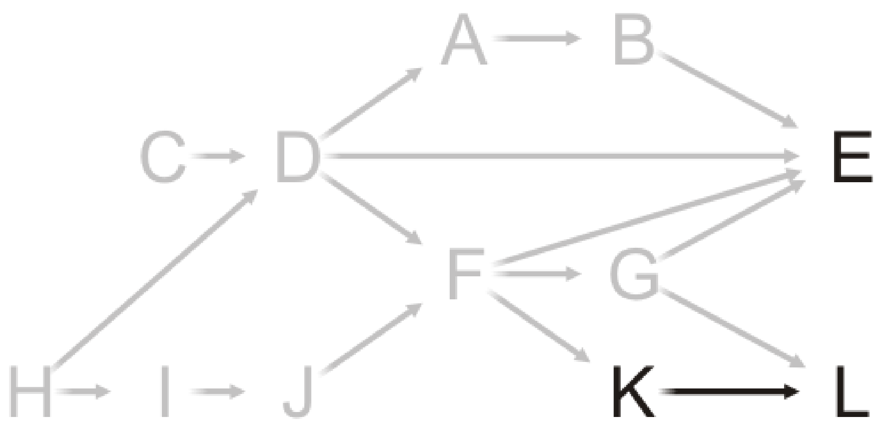
- Choose Task G



C, H, D, A, B, I, J, F

Example

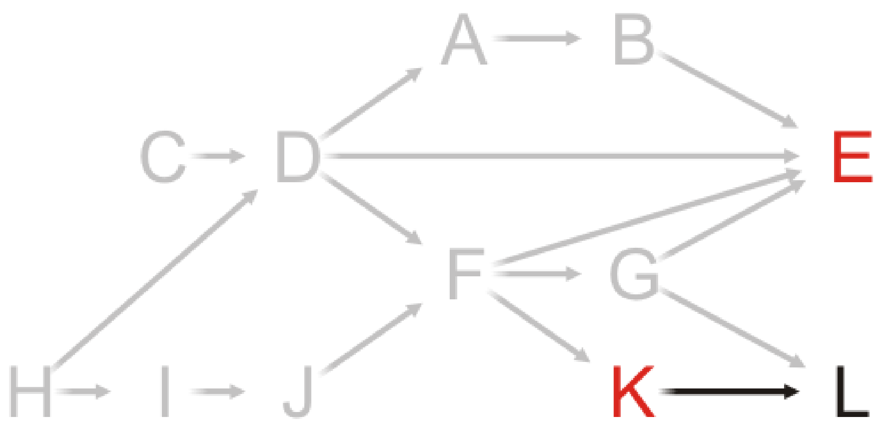
Having removed Task G from the graph, what next?



C, H, D, A, B, I, J, F, G

Example

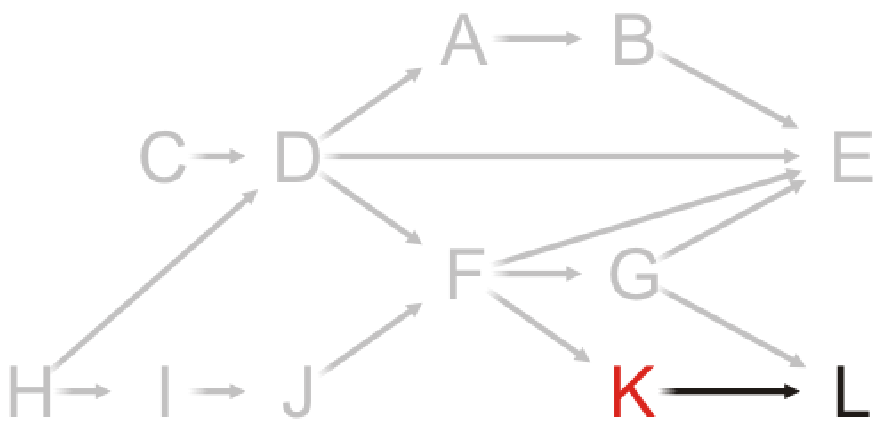
Choosing between Tasks E and K, choose Task E



C, H, D, A, B, I, J, F, G

Example

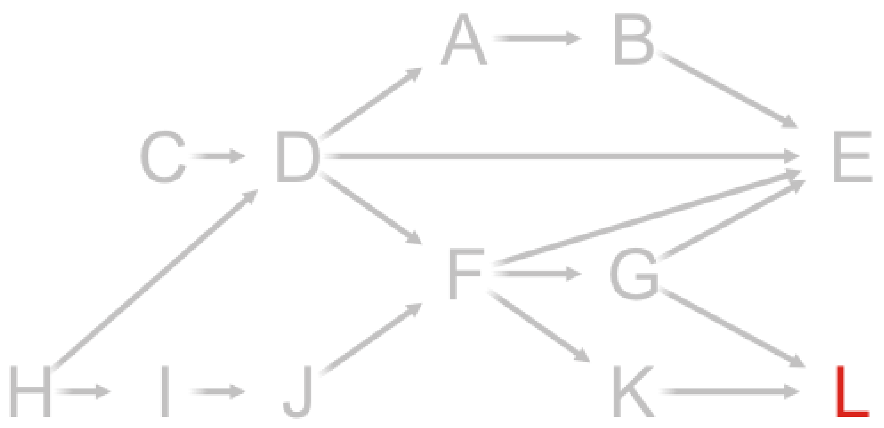
At this point, Task K is the only one that can be run



C, H, D, A, B, I, J, F, G, E

Example

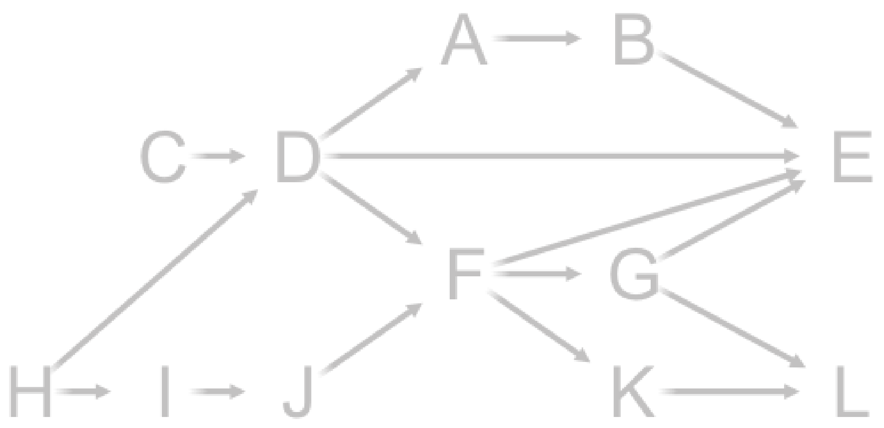
And now that both Tasks G and K are complete,
we can complete Task L



C, H, D, A, B, I, J, F, G, E, K

Example

There are no more vertices left

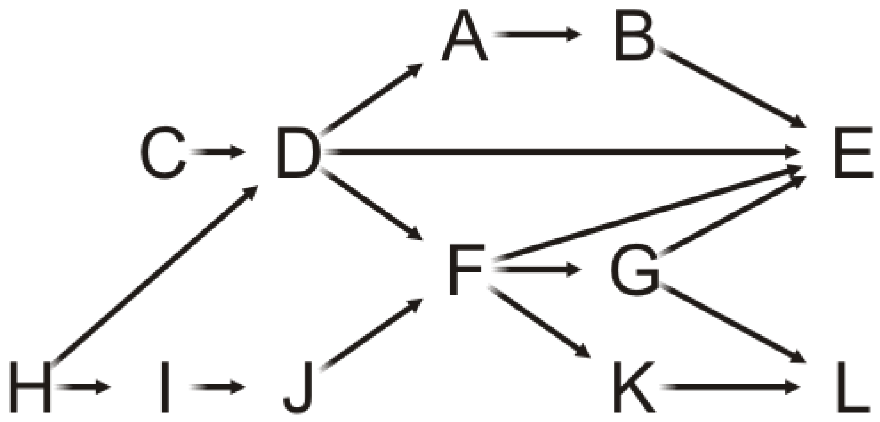


C, H, D, A, B, I, J, F, G, E, K, L

Example

Thus, one possible topological sort would be:

C, H, D, A, B, I, J, F, G, E, K, L

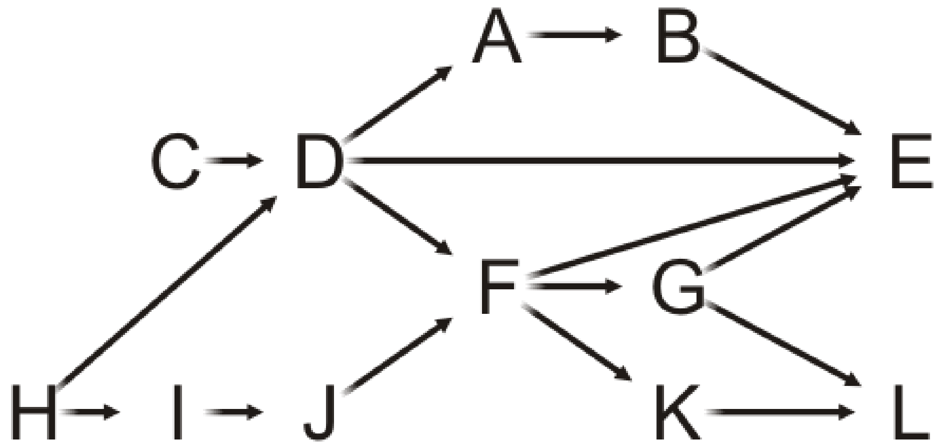


Example

Note that topological sorts need not be unique:

C, H, D, A, B, I, J, F, G, E, K, L

H, I, J, C, D, F, G, K, L, A, B, E

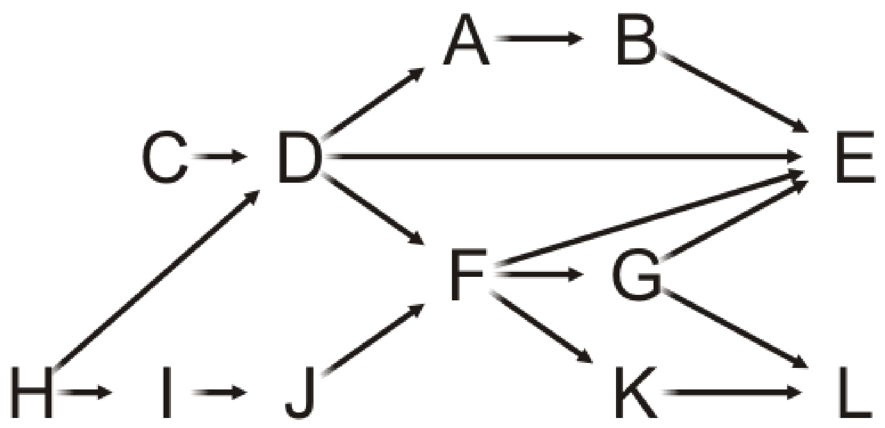


Analysis

What are the tools necessary for a topological sort?

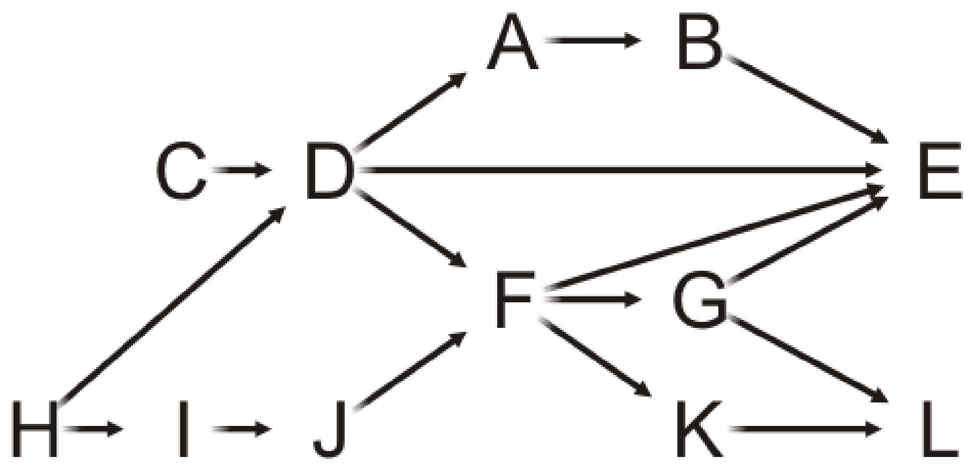
- We must know and be able to update the in-degrees of each of the vertices
- We could do this with a table of the in-degrees of each of the vertices
- This requires $\Theta(|V|)$ memory

A	1
B	1
C	0
D	2
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2



Analysis

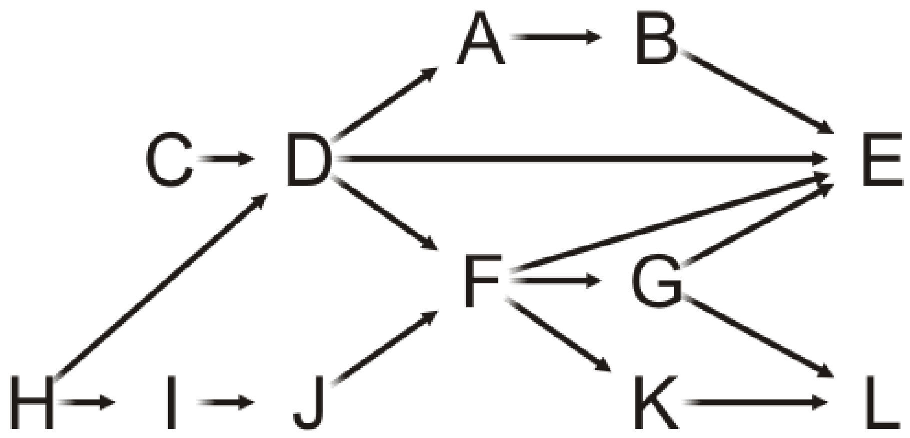
We must iterate at least $|V|$ times, so the run-time must be $\Omega(|V|)$



A	1
B	1
C	0
D	2
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2

Analysis

We need to find vertices with in-degree zero
- We could loop through the array with each iteration
- The run time would be $O(|V|^2)$

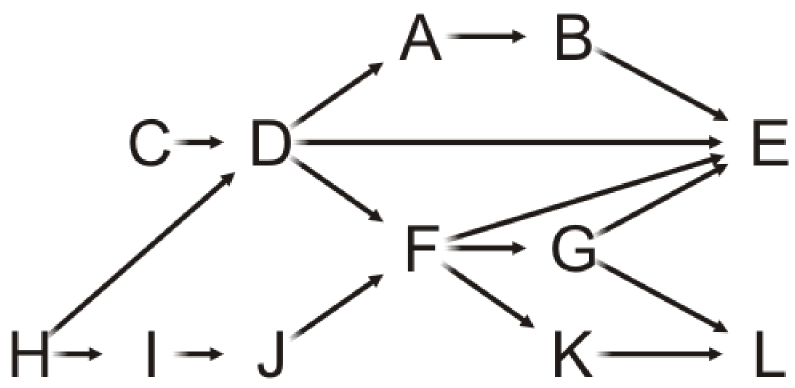


A	1
B	1
C	0
D	2
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2

Analysis

What did we do with tree traversals?

- Use a queue (or other container) to temporarily store those vertices with in-degree zero
- Each time the in-degree of a vertex is decremented to zero, push it onto the queue

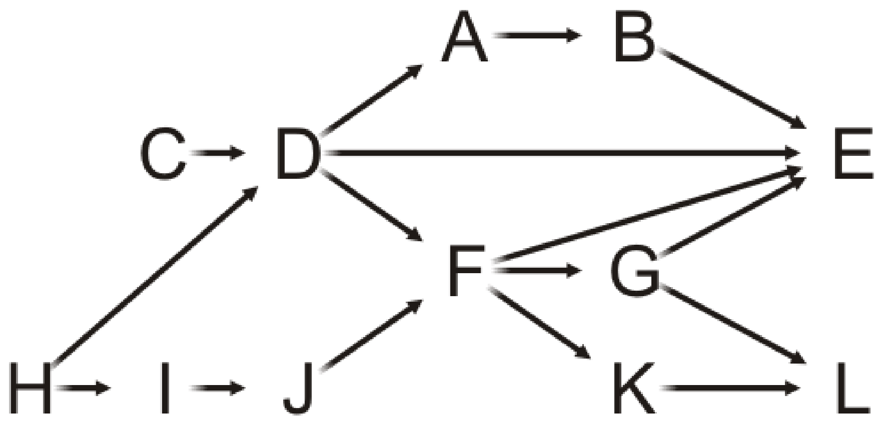


A	1
B	1
C	0
D	2
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2

Analysis

What are the run times associated with the queue?

- Initially, we must scan through each of the vertices: $\Theta(|V|)$
- For each vertex, we will have to push onto and pop off the queue once, also $\Theta(|V|)$

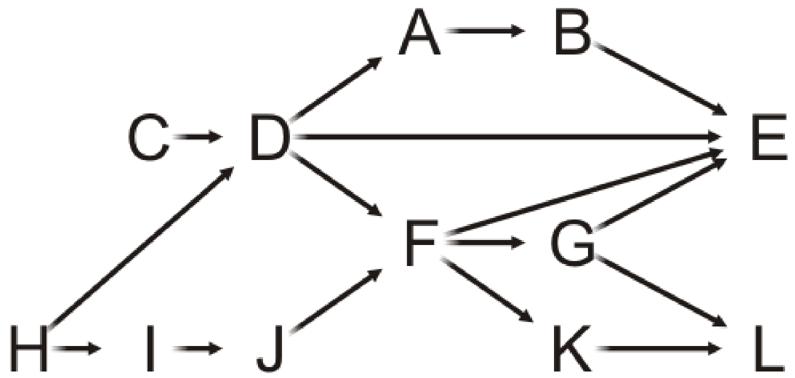


A	1
B	1
C	0
D	2
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2

Analysis

Finally, each value in the in-degree table is associated with an edge

- Here, $|E| = 16$
- Each of the in-degrees must be decremented to zero
- The run time of these operations is $\Omega(|E|)$
- If we are using an adjacency matrix: $\Theta(|V|^2)$
- If we are using an adjacency list: $\Theta(|E|)$



A	1
B	1
C	0
D	2
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2

+

16

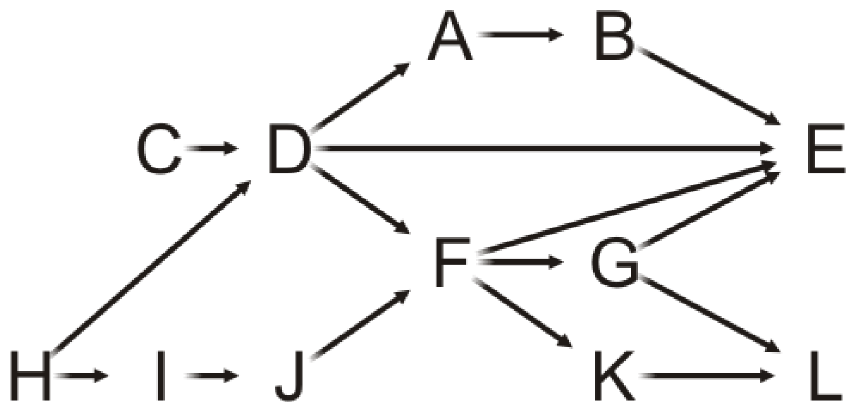
Analysis

Therefore, the run time of a topological sort is:

$\Theta(|V| + |E|)$ if we use an adjacency list

$\Theta(|V|^2)$ if we use an adjacency matrix

and the memory requirements is $\Theta(|V|)$

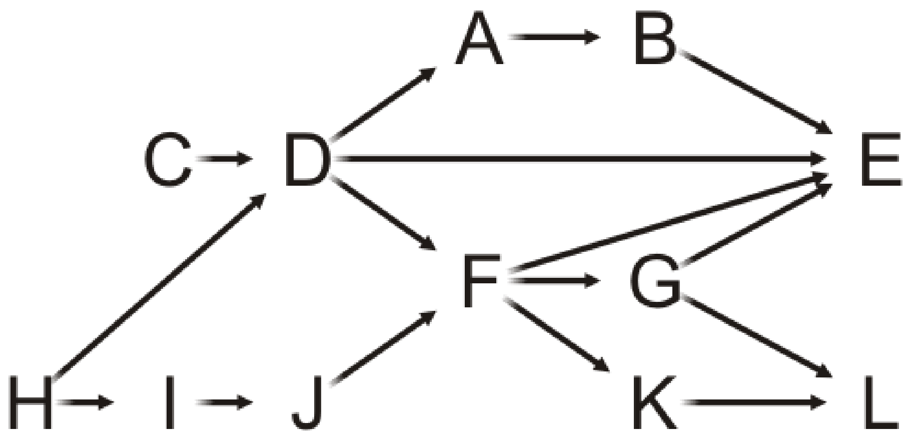


A	1
B	1
C	0
D	2
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2

Analysis

What happens if at some step, all remaining vertices have an in-degree greater than zero?

Consequence: we now have an $\Theta(|V| + |E|)$ algorithm for determining if a graph has a cycle



Implementation

Thus, to implement a topological sort:

- Allocate memory for and initialize an array of in-degrees
- Create a queue and initialize it with all vertices that have in-degree zero

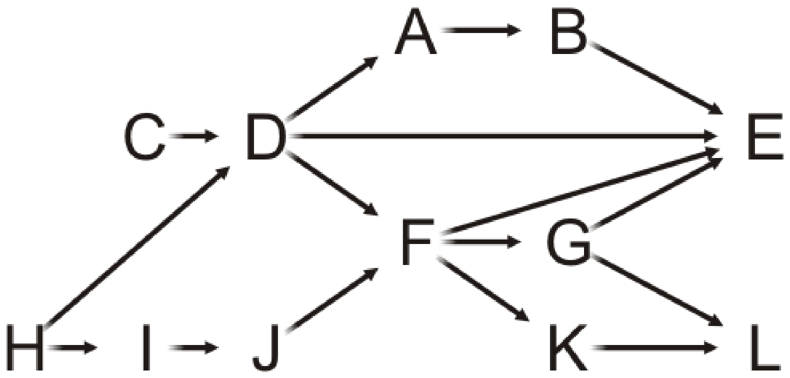
While the queue is not empty:

- Pop a vertex from the queue
- Decrement the in-degree of each neighbor
- Those neighbors whose in-degree was decremented to zero are pushed onto the queue

Example

With the previous example, we initialize:

- The array of in-degrees
- The queue



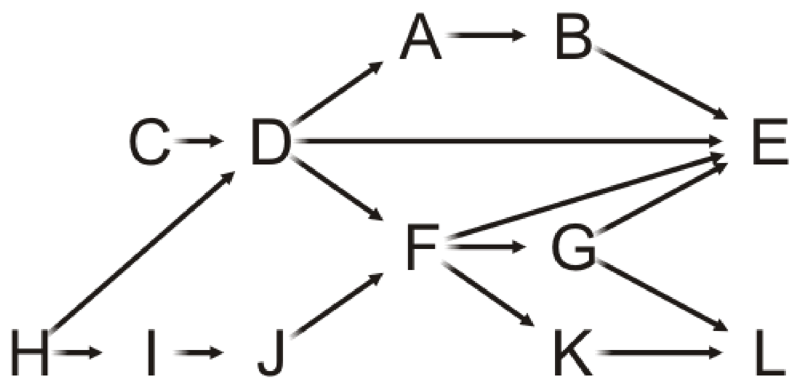
A	1
B	1
C	0
D	2
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2

Queue:

The queue is empty

Example

Stepping through the table, push all source vertices into the queue



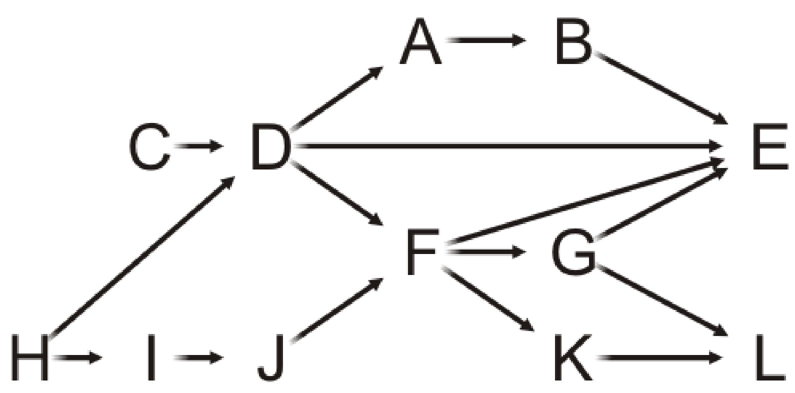
A	1
B	1
C	0
D	2
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2

Queue:

The queue is empty

Example

Stepping through the table, push all source vertices into the queue



A	1
B	1
C	**0**
D	2
E	4
F	2
G	1
H	**0**
I	1
J	1
K	1
L	2

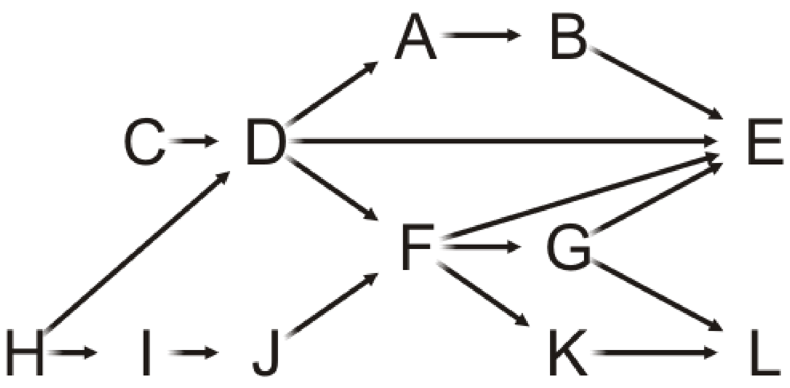
Queue:

C	**H**										

The queue is empty

Example

Pop the front of the queue



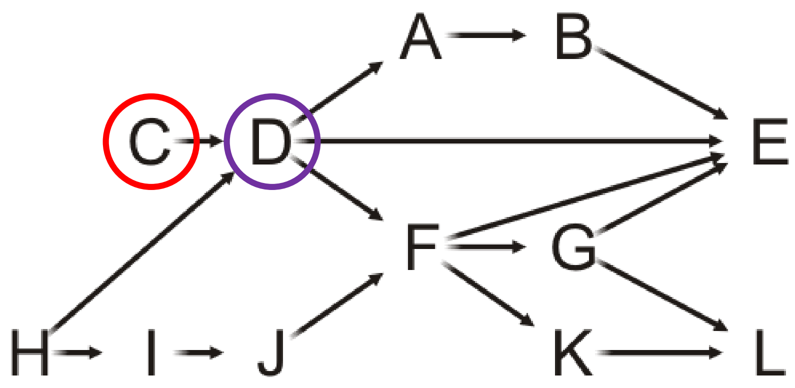
A	1
B	1
C	0
D	2
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2

Queue:	C	H										

Example

Pop the front of the queue

- C has one neighbor: D



A	1
B	1
C	0
D	2
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2

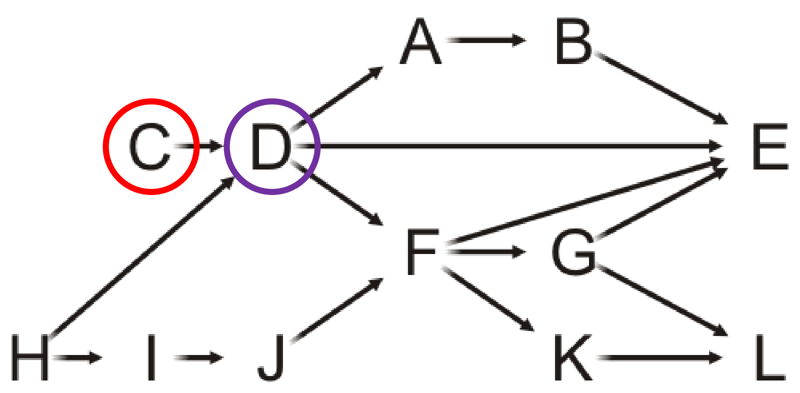
Queue:

C	H										

Example

Pop the front of the queue

- C has one neighbor: D
- Decrement its in-degree



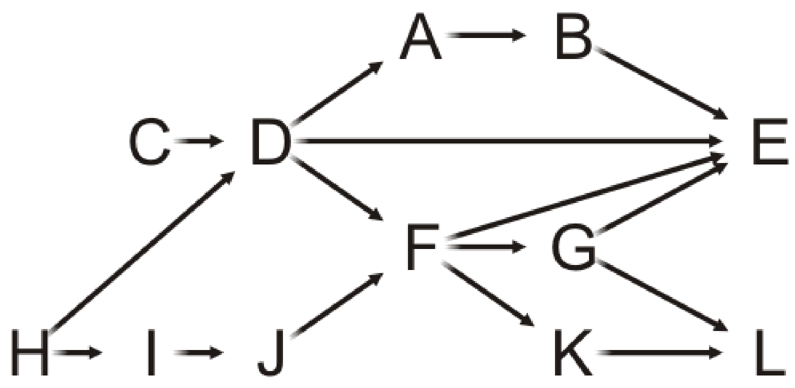
Queue:

C	H										

A	1
B	1
C	**0**
D	**1**
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2

Example

Pop the front of the queue



A	1
B	1
C	0
D	1
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2

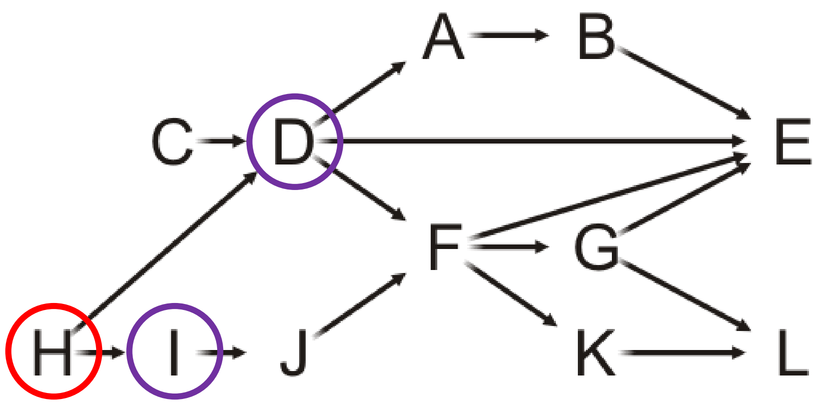
Queue:

C	H										

Example

Pop the front of the queue

- H has two neighbors: D and I



Queue:

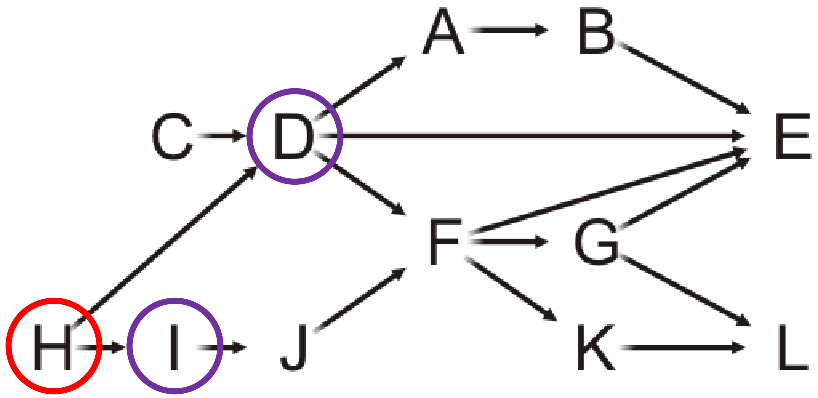
C	H										

A	1
B	1
C	0
D	**1**
E	4
F	2
G	1
H	**0**
I	**1**
J	1
K	1
L	2

Example

Pop the front of the queue
- H has two neighbors: D and I
- Decrement their in-degrees

A	1
B	1
C	0
D	**0**
E	4
F	2
G	1
H	**0**
I	**0**
J	1
K	1
L	2



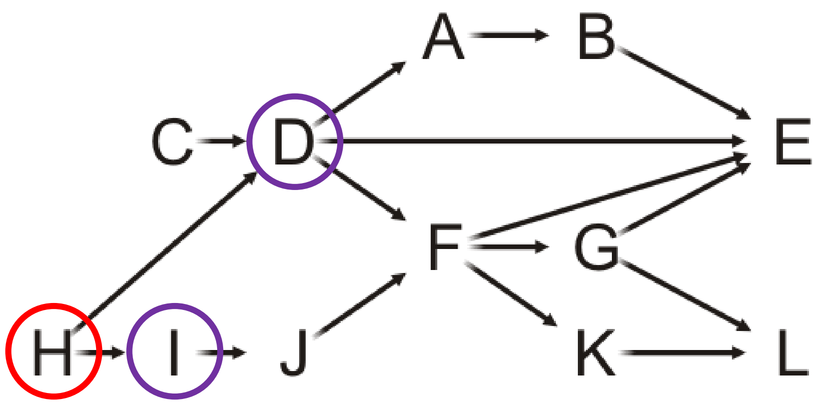
Queue:

C	H										

Example

Pop the front of the queue

- H has two neighbors: D and I
- Decrement their in-degrees
  - Both are decremented to zero, so push them onto the queue

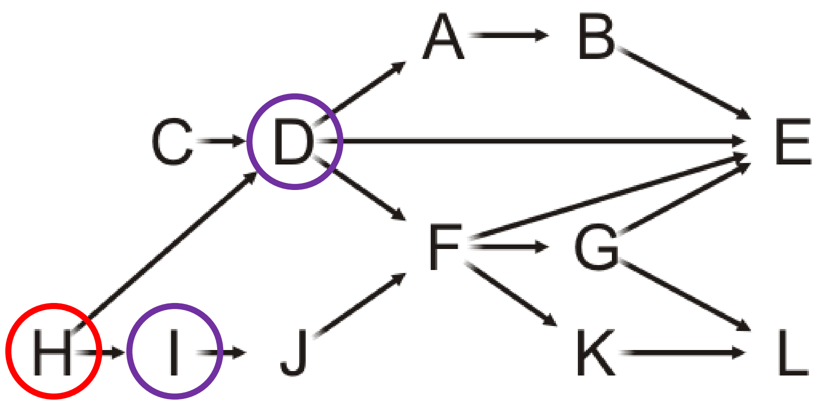


A	1
B	1
C	0
D	**0**
E	4
F	2
G	1
H	**0**
I	**0**
J	1
K	1
L	2

Example

Pop the front of the queue

- H has two neighbors: D and I
- Decrement their in-degrees
  - Both are decremented to zero, so push them onto the queue



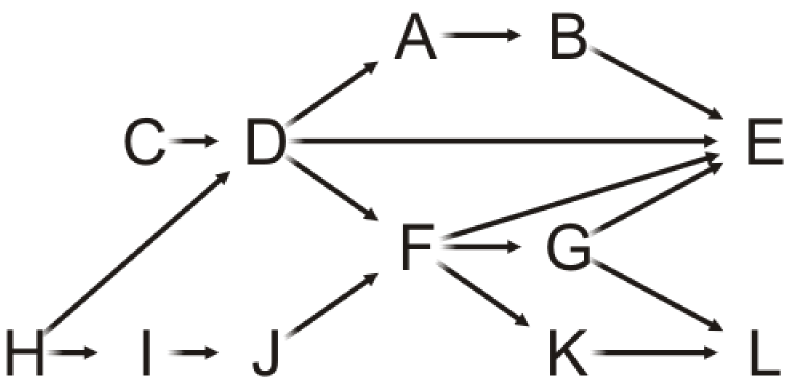
A	1
B	1
C	0
D	**0**
E	4
F	2
G	1
H	**0**
I	**0**
J	1
K	1
L	2

Queue:

C	H	D	I								

Example

Pop the front of the queue



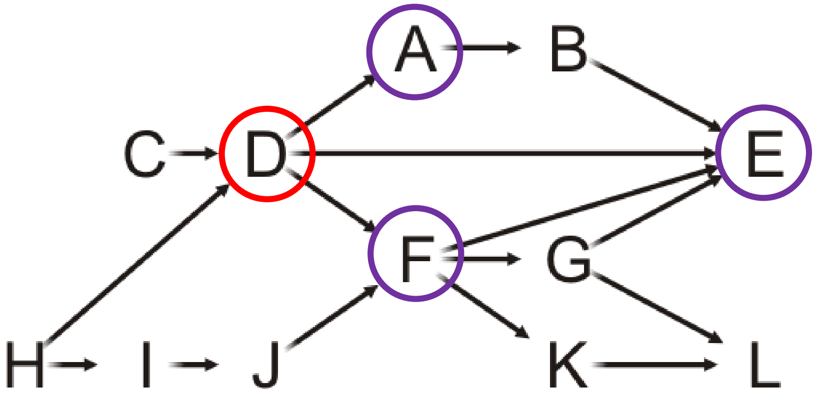
A	1
B	1
C	0
D	0
E	4
F	2
G	1
H	0
I	0
J	1
K	1
L	2

Queue:	C	H	D	I								

Example

Pop the front of the queue

- D has three neighbors: A, E and F



Queue:

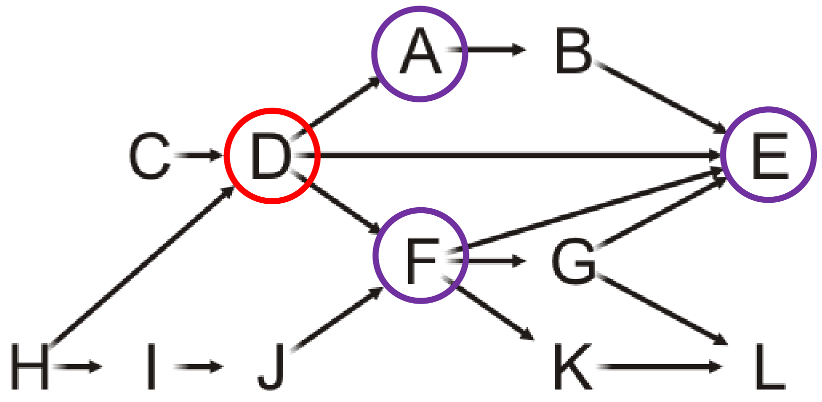
C	H	D	I								

A	**1**
B	1
C	0
D	**0**
E	**4**
F	**2**
G	1
H	0
I	0
J	1
K	1
L	2

Example

Pop the front of the queue

- D has three neighbors: A, E and F
- Decrement their in-degrees



A	0
B	1
C	0
D	0
E	3
F	1
G	1
H	0
I	0
J	1
K	1
L	2

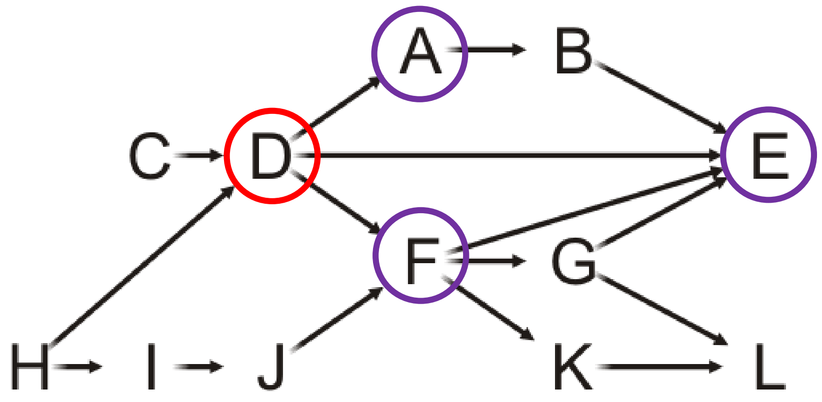
Queue:

C	H	D	I								

Example

Pop the front of the queue

- D has three neighbors: A, E and F
- Decrement their in-degrees
  - A is decremented to zero, so push it onto the queue



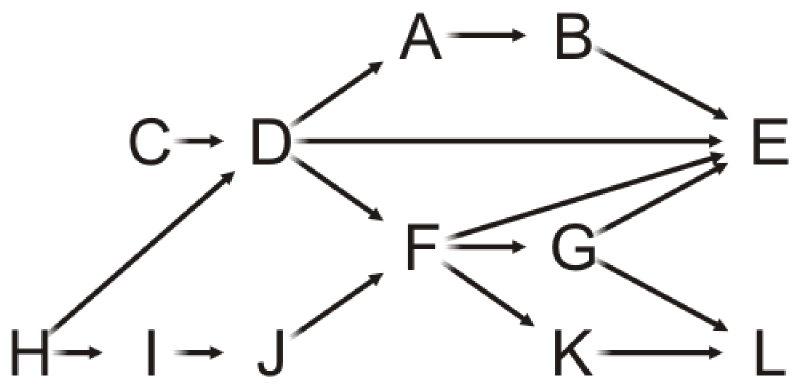
Queue:

C	H	D	I	A							

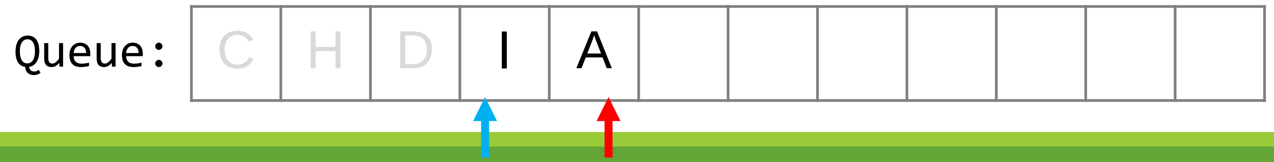
A	**0**
B	1
C	0
D	**0**
E	**3**
F	**1**
G	1
H	0
I	0
J	1
K	1
L	2

Example

Pop the front of the queue



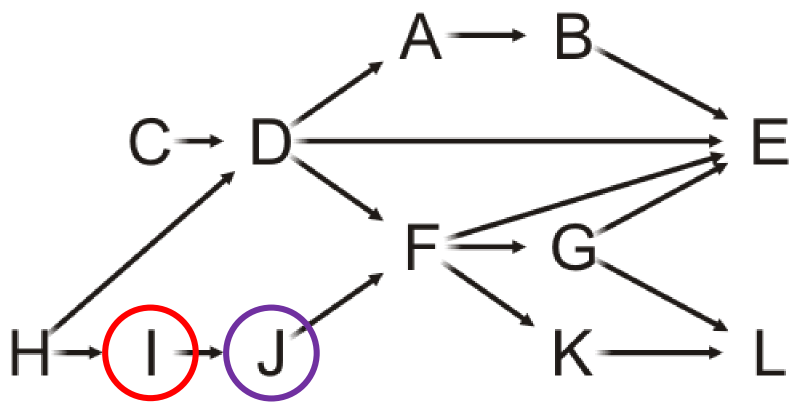
A	0
B	1
C	0
D	0
E	3
F	1
G	1
H	0
I	0
J	1
K	1
L	2



Example

Pop the front of the queue

- I has one neighbor: J



Queue:

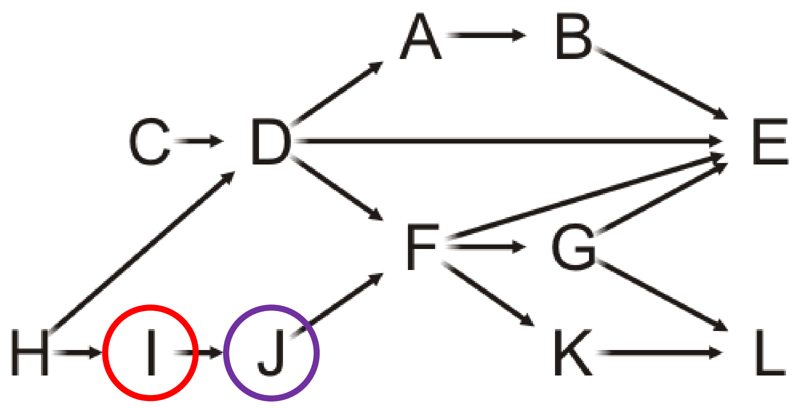
C	H	D	I	A							

A	0
B	1
C	0
D	0
E	3
F	1
G	1
H	0
I	**0**
J	**1**
K	1
L	2

Example

Pop the front of the queue

- I has one neighbor: J
- Decrement its in-degree



Queue:

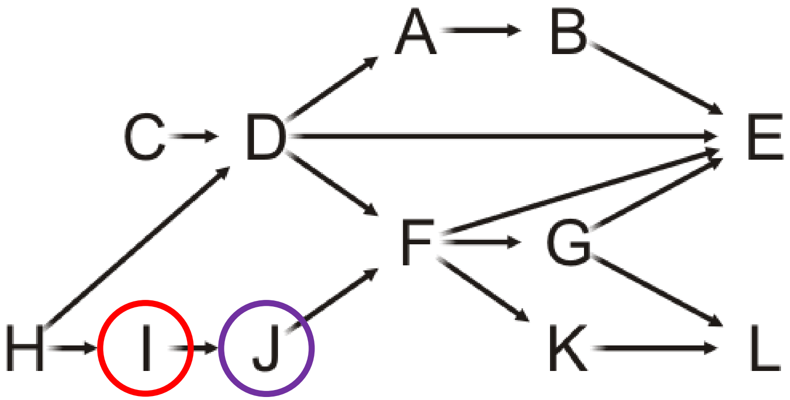
C	H	D	I	A							

A	0
B	1
C	0
D	0
E	3
F	1
G	1
H	0
I	**0**
J	**0**
K	1
L	2

Example

Pop the front of the queue

- I has one neighbor: J
- Decrement its in-degree
  - J is decremented to zero, so push it onto the queue



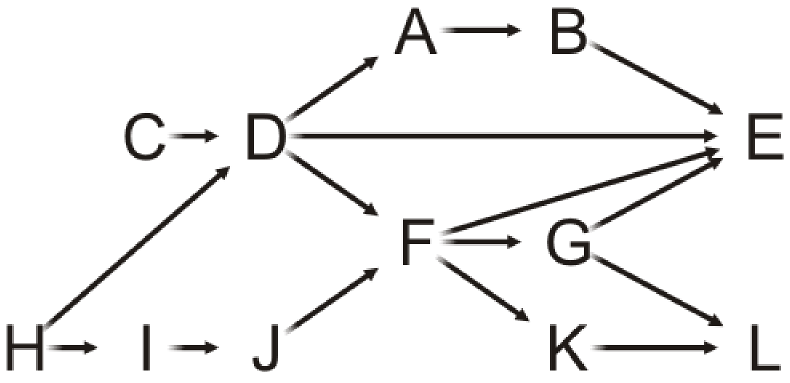
A	0
B	1
C	0
D	0
E	3
F	1
G	1
H	0
I	**0**
J	**0**
K	1
L	2

Queue:

C	H	D	I	A	J						

Example

Pop the front of the queue



A	0
B	1
C	0
D	0
E	3
F	1
G	1
H	0
I	0
J	0
K	1
L	2

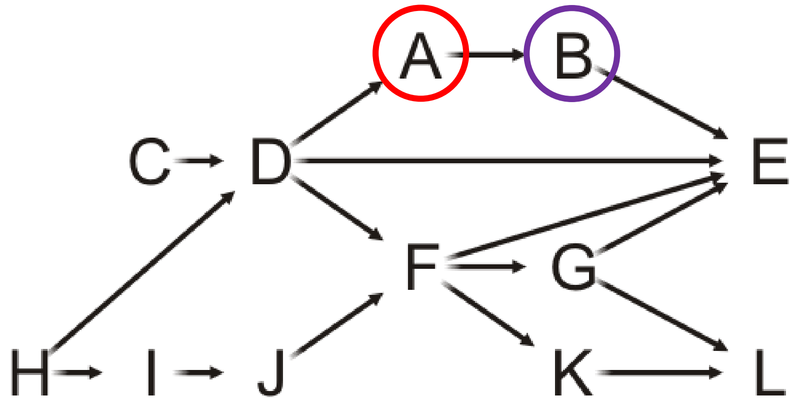
Queue:

C	H	D	I	A	J						

Example

Pop the front of the queue

- A has one neighbor: B



A	0
B	1
C	0
D	0
E	3
F	1
G	1
H	0
I	0
J	0
K	1
L	2

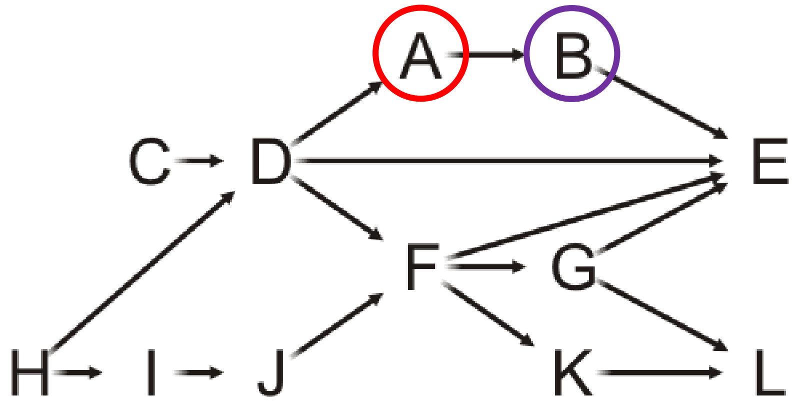
Queue:

C	H	D	I	A	J						

Example

Pop the front of the queue

- A has one neighbor: B
- Decrement its in-degree



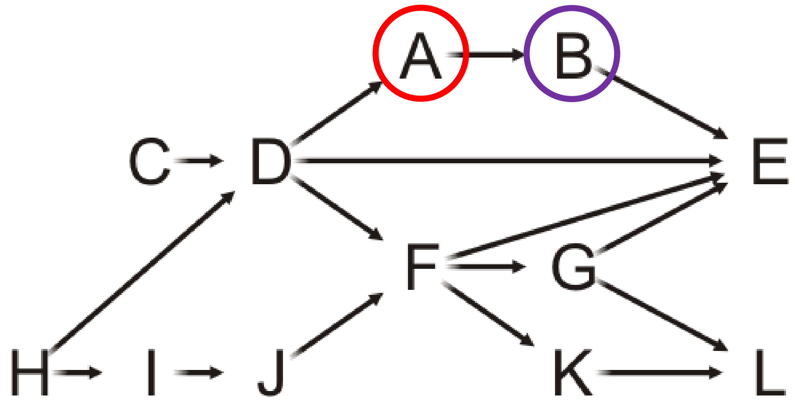
Queue:

C	H	D	I	A	J						

A	0
B	0
C	0
D	0
E	3
F	1
G	1
H	0
I	0
J	0
K	1
L	2

Example

Pop the front of the queue
- A has one neighbor: B
- Decrement its in-degree
  - B is decremented to zero, so push it onto the queue



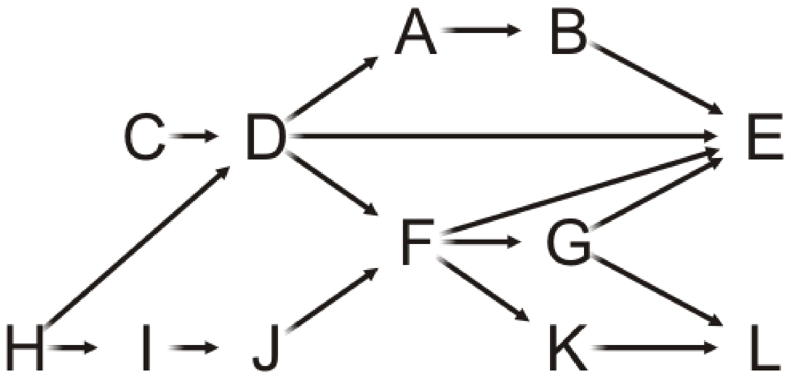
A	0
B	0
C	0
D	0
E	3
F	1
G	1
H	0
I	0
J	0
K	1
L	2

Queue:

C	H	D	I	A	J	B					

Example

Pop the front of the queue



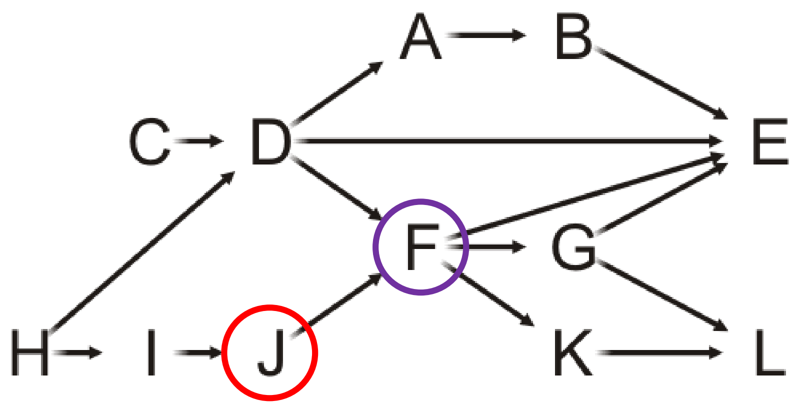
Queue:	C	H	D	I	A	J	B					

A	0
B	0
C	0
D	0
E	3
F	1
G	1
H	0
I	0
J	0
K	1
L	2

Example

Pop the front of the queue

- J has one neighbor: F



Queue:

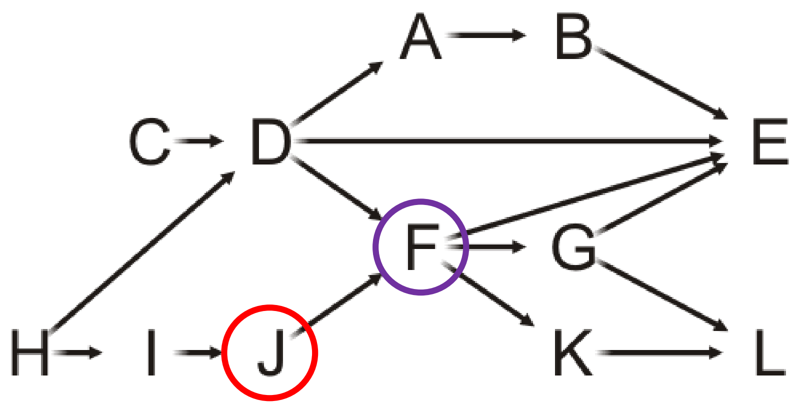
C	H	D	I	A	J	B					

A	0
B	0
C	0
D	0
E	3
F	**1**
G	1
H	0
I	0
J	**0**
K	1
L	2

Example

Pop the front of the queue

- J has one neighbor: F
- Decrement its in-degree



Queue:

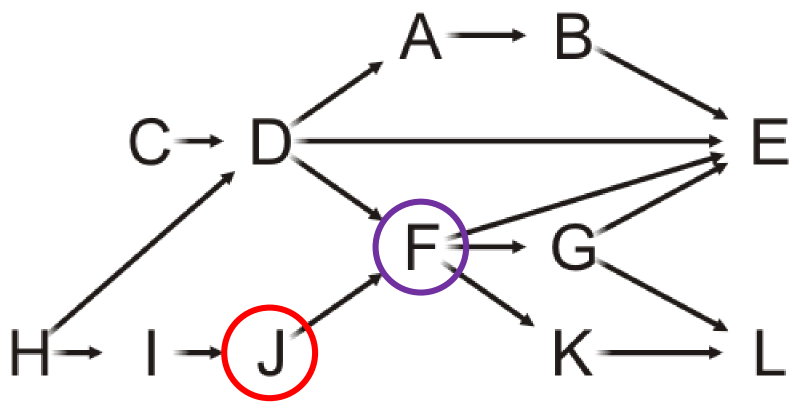
C	H	D	I	A	J	B					

A	0
B	0
C	0
D	0
E	3
F	**0**
G	1
H	0
I	0
J	**0**
K	1
L	2

Example

Pop the front of the queue

- J has one neighbor: F
- Decrement its in-degree
  - F is decremented to zero, so push it onto the queue



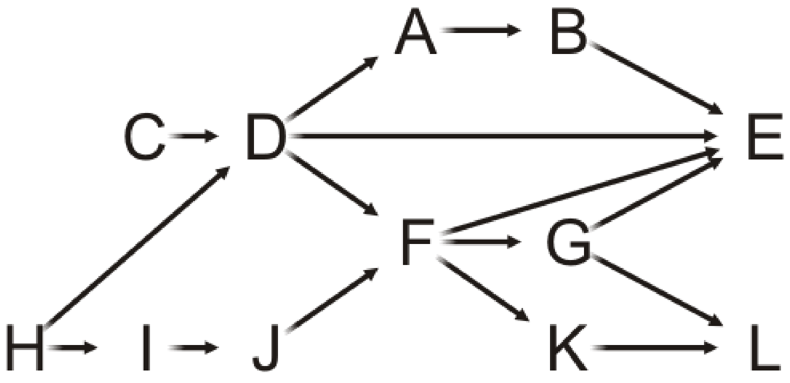
A	0
B	0
C	0
D	0
E	3
F	**0**
G	1
H	0
I	0
J	**0**
K	1
L	2

Queue:

C	H	D	I	A	J	B	F				

Example

Pop the front of the queue



A	0
B	0
C	0
D	0
E	3
F	0
G	1
H	0
I	0
J	0
K	1
L	2

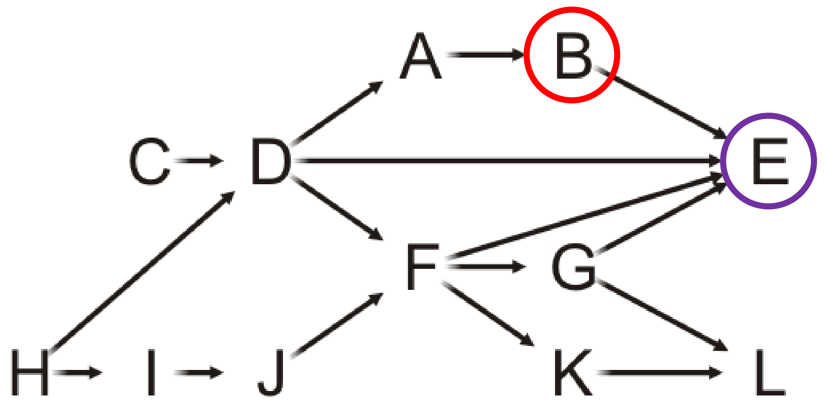
Queue:

C	H	D	I	A	J	B	F				

Example

Pop the front of the queue

- B has one neighbor: E



A	0
B	**0**
C	0
D	0
E	**3**
F	0
G	1
H	0
I	0
J	0
K	1
L	2

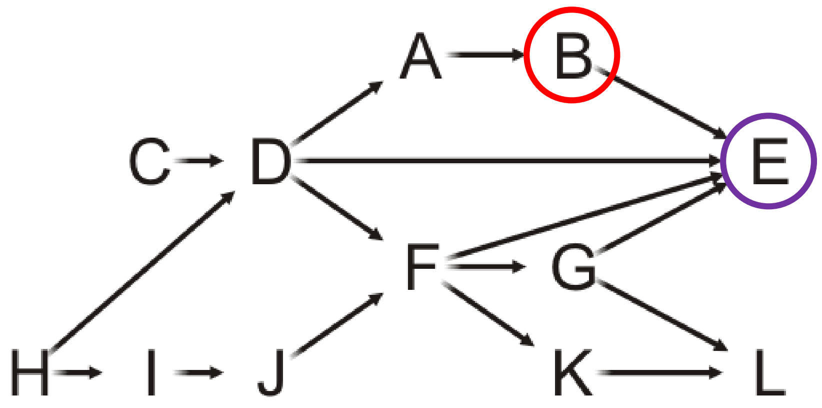
Queue:

C	H	D	I	A	J	B	F				

Example

Pop the front of the queue

- B has one neighbor: E
- Decrement its in-degree



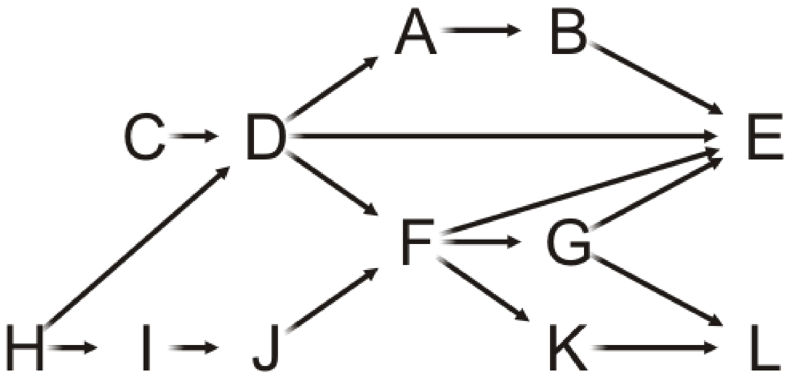
Queue:

C	H	D	I	A	J	B	F				

A	0
B	**0**
C	0
D	0
E	**2**
F	0
G	1
H	0
I	0
J	0
K	1
L	2

Example

Pop the front of the queue



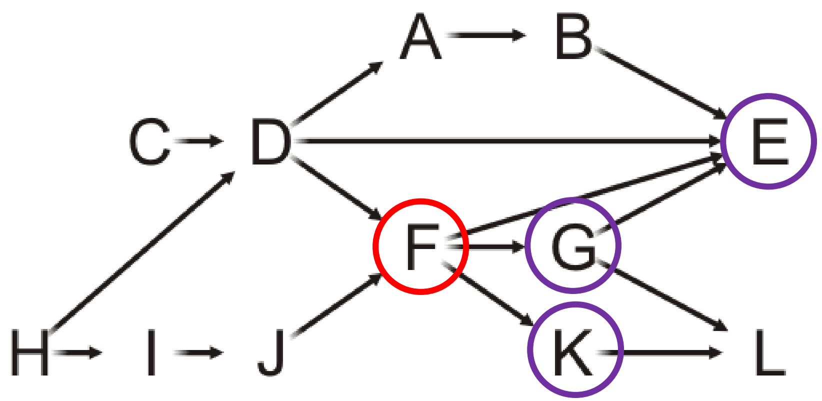
Queue:	C	H	D	I	A	J	B	F				

A	0
B	0
C	0
D	0
E	2
F	0
G	1
H	0
I	0
J	0
K	1
L	2

Example

Pop the front of the queue

- F has three neighbors: E, G and K



Queue:

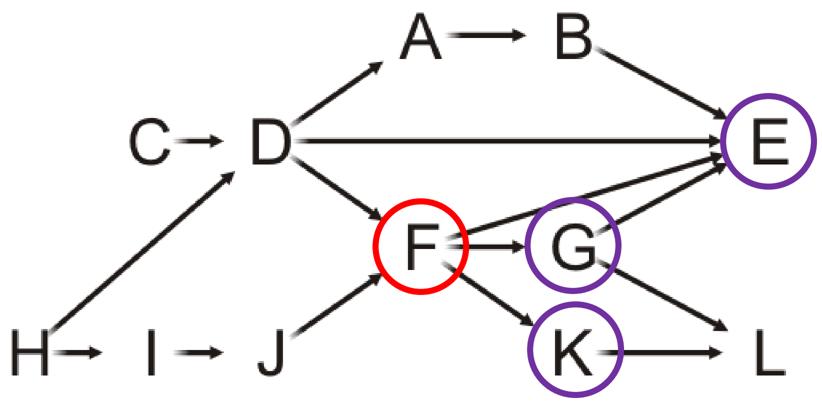
C	H	D	I	A	J	B	F				

A	0
B	0
C	0
D	0
E	**2**
F	**0**
G	**1**
H	0
I	0
J	0
K	**1**
L	2

Example

Pop the front of the queue

- F has three neighbors: E, G and K
- Decrement their in-degrees



Queue:

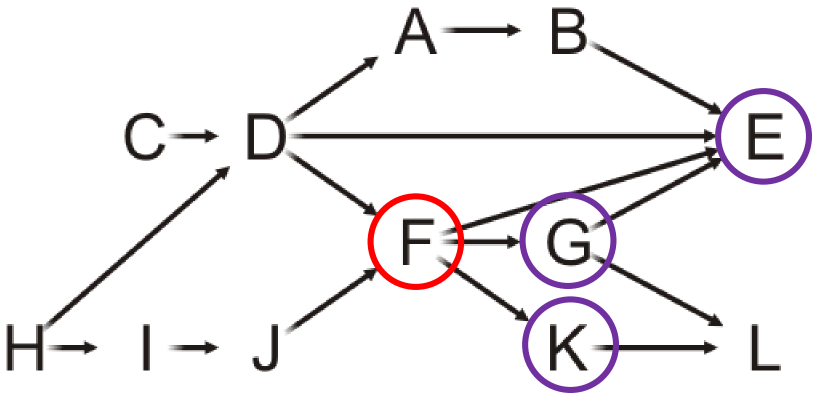
C	H	D	I	A	J	B	F				

A	0
B	0
C	0
D	0
E	**1**
F	**0**
G	**0**
H	0
I	0
J	0
K	**0**
L	2

Example

Pop the front of the queue

- F has three neighbors: E, G and K
- Decrement their in-degrees
  - G and K are decremented to zero,
    so push them onto the queue

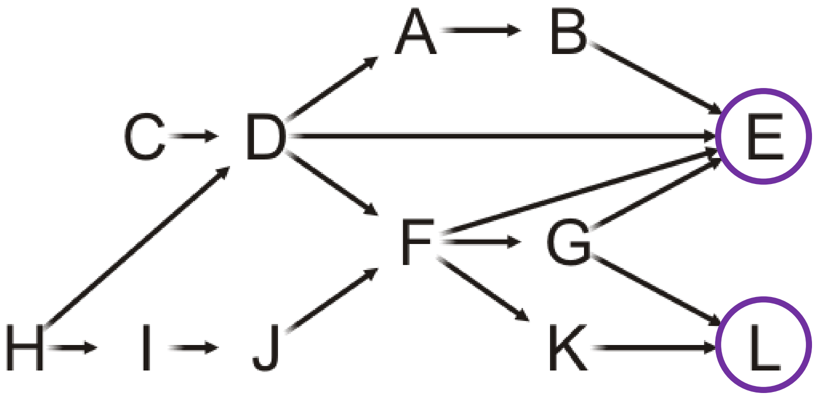


Queue:	C	H	D	I	A	J	B	F	G	K		

A	0
B	0
C	0
D	0
E	**1**
F	**0**
G	**0**
H	0
I	0
J	0
K	**0**
L	2

Example

Pop the front of the queue



Queue:

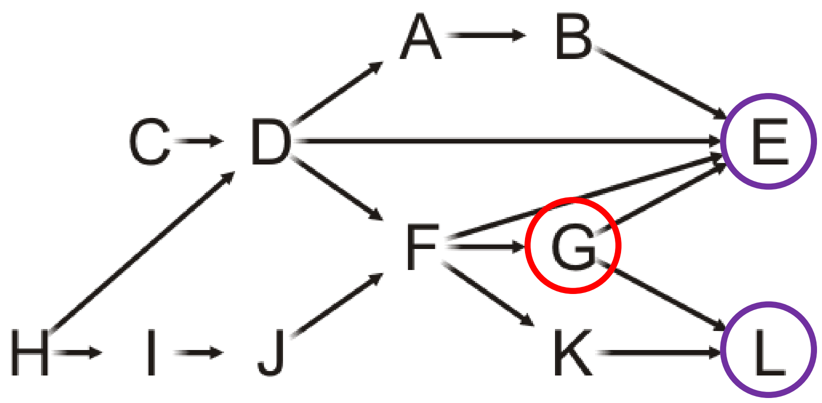
C	H	D	I	A	J	B	F	G	K		

A	0
B	0
C	0
D	0
E	1
F	0
G	0
H	0
I	0
J	0
K	0
L	2

Example

Pop the front of the queue

- G has two neighbors: E and L



A	0
B	0
C	0
D	0
E	**1**
F	0
G	**0**
H	0
I	0
J	0
K	0
L	**2**

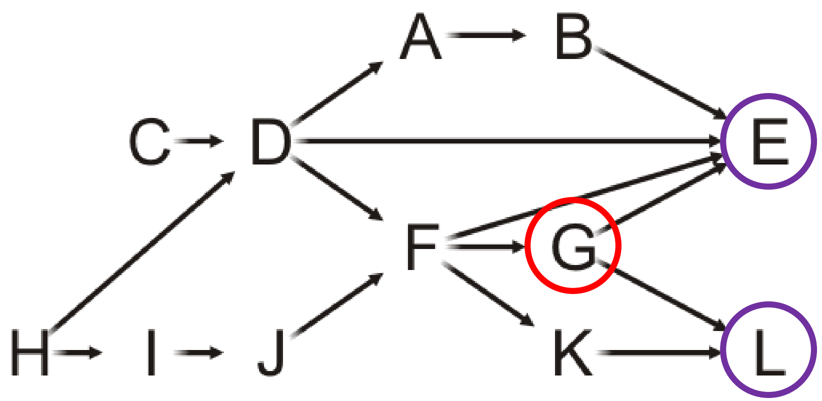
Queue:

C	H	D	I	A	J	B	F	G	K		

Example

Pop the front of the queue

- G has two neighbors: E and L
- Decrement their in-degrees



Queue:

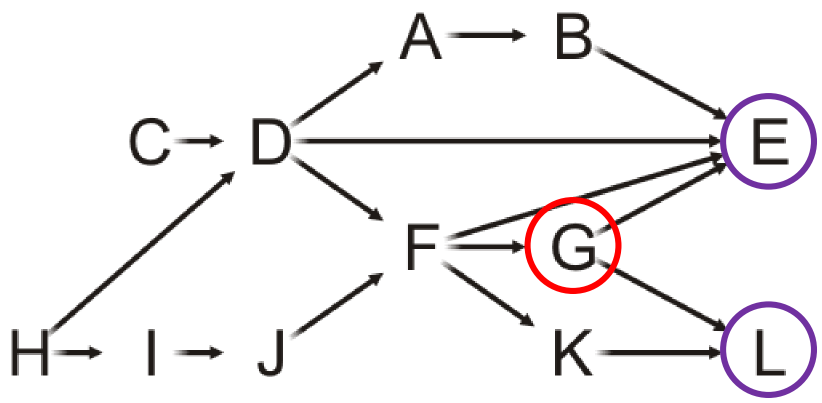
C	H	D	I	A	J	B	F	G	K		

A	0
B	0
C	0
D	0
E	**0**
F	0
G	**0**
H	0
I	0
J	0
K	0
L	**1**

Example

Pop the front of the queue

- G has two neighbors: E and L
- Decrement their in-degrees
  - E is decremented to zero, so push it onto the queue



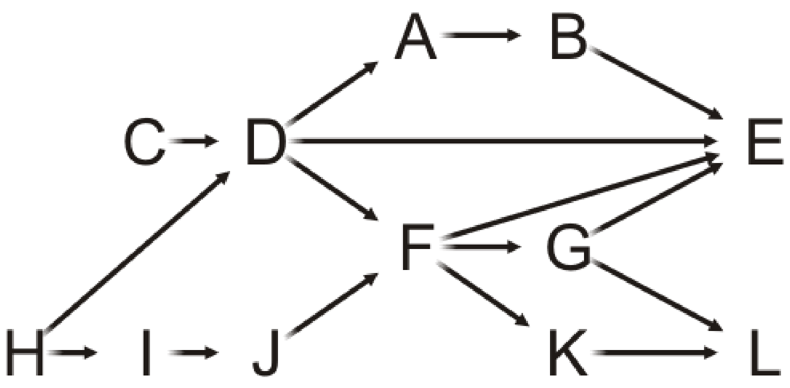
Queue:

C	H	D	I	A	J	B	F	G	K	E	

A	0
B	0
C	0
D	0
E	**0**
F	0
G	**0**
H	0
I	0
J	0
K	0
L	**1**

Example

Pop the front of the queue



A	0
B	0
C	0
D	0
E	0
F	0
G	0
H	0
I	0
J	0
K	0
L	1

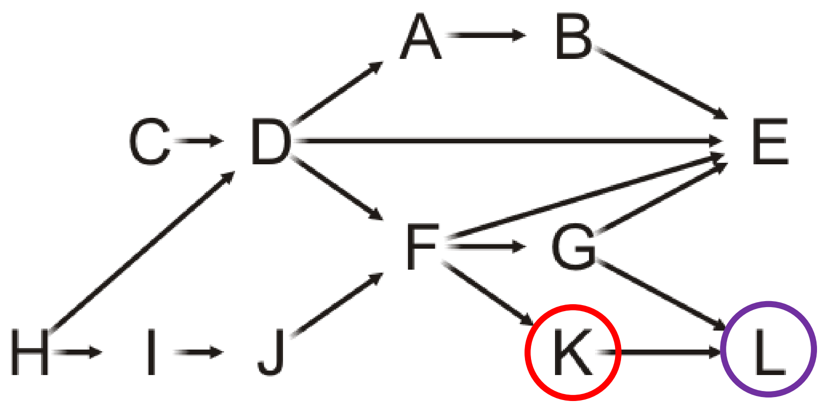
Queue:

C	H	D	I	A	J	B	F	G	K	E	

Example

Pop the front of the queue

- K has one neighbors: L



Queue:

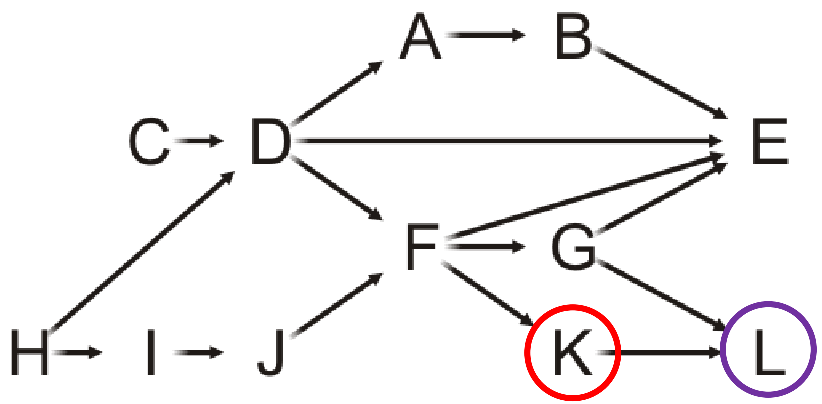
C	H	D	I	A	J	B	F	G	K	E	

A	0
B	0
C	0
D	0
E	0
F	0
G	0
H	0
I	0
J	0
K	**0**
L	**1**

Example

Pop the front of the queue

- K has one neighbors: L
- Decrement its in-degree



A	0
B	0
C	0
D	0
E	0
F	0
G	0
H	0
I	0
J	0
K	**0**
L	**0**

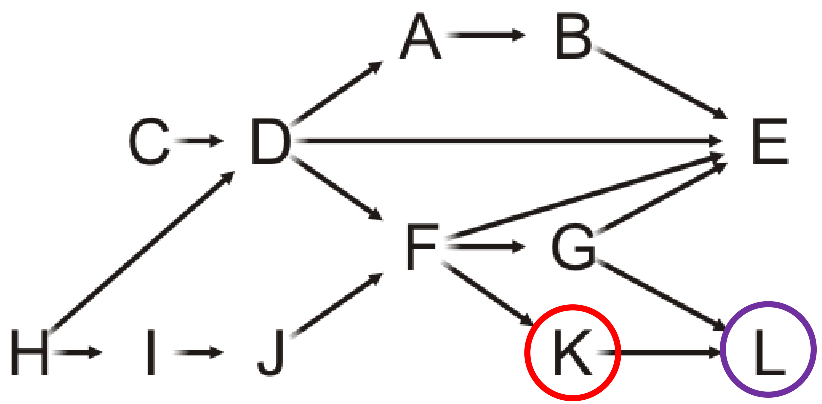
Queue:

C	H	D	I	A	J	B	F	G	K	E	

Example

Pop the front of the queue

- K has one neighbors: L
- Decrement its in-degree
  - L is decremented to zero, so push it onto the queue



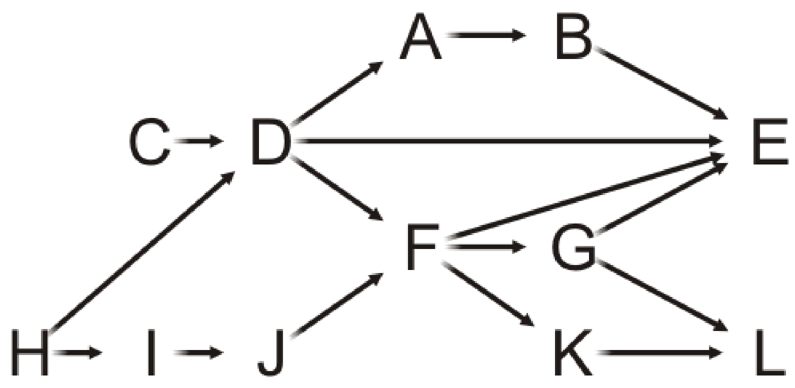
A	0
B	0
C	0
D	0
E	0
F	0
G	0
H	0
I	0
J	0
K	**0**
L	**0**

Queue:

C	H	D	I	A	J	B	F	G	K	E	L

Example

Pop the front of the queue

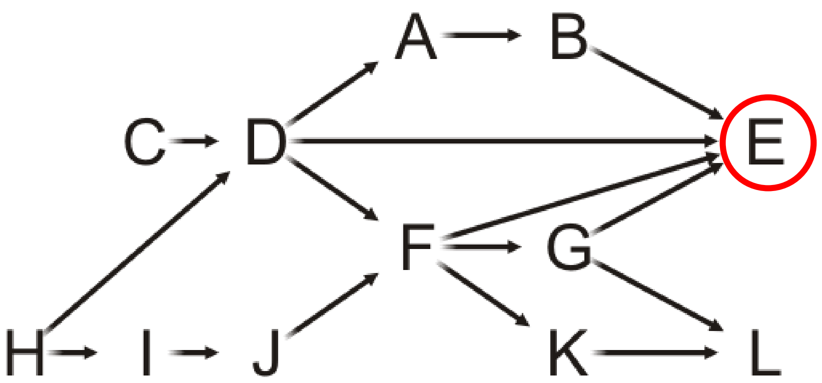


A	0
B	0
C	0
D	0
E	0
F	0
G	0
H	0
I	0
J	0
K	0
L	0

Example

Pop the front of the queue

- E has no neighbors—it is a *sink*



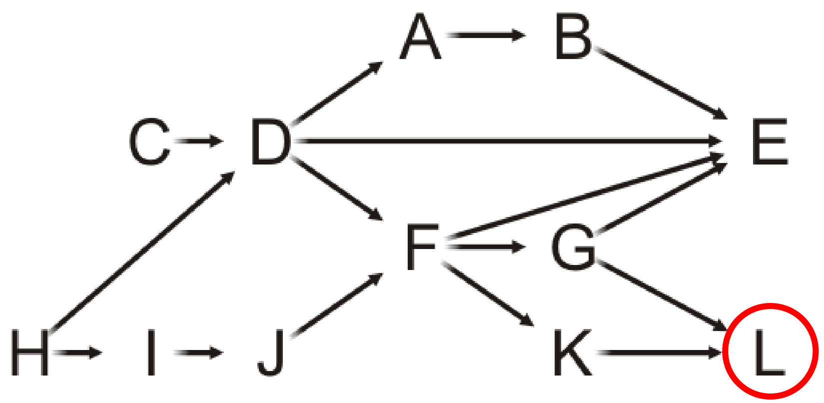
Queue:

C	H	D	I	A	J	B	F	G	K	E	L

A	0
B	0
C	0
D	0
E	**0**
F	0
G	0
H	0
I	0
J	0
K	0
L	0

Example

Pop the front of the queue



A	0
B	0
C	0
D	0
E	0
F	0
G	0
H	0
I	0
J	0
K	0
L	0

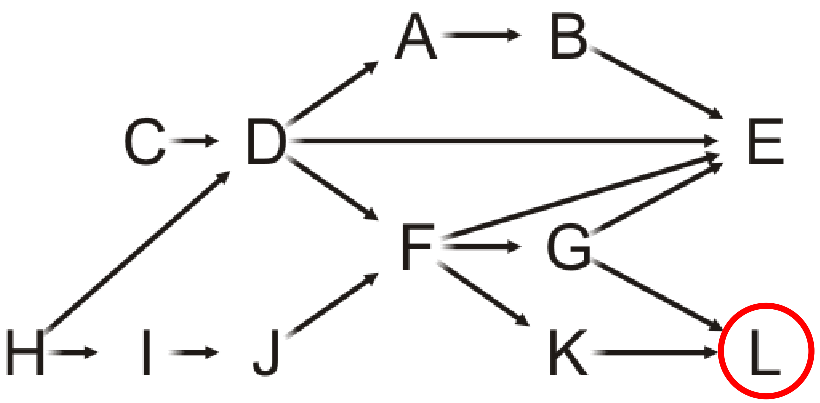
Queue:

C	H	D	I	A	J	B	F	G	K	E	L

Example

Pop the front of the queue

- L has no neighbors—it is also a *sink*



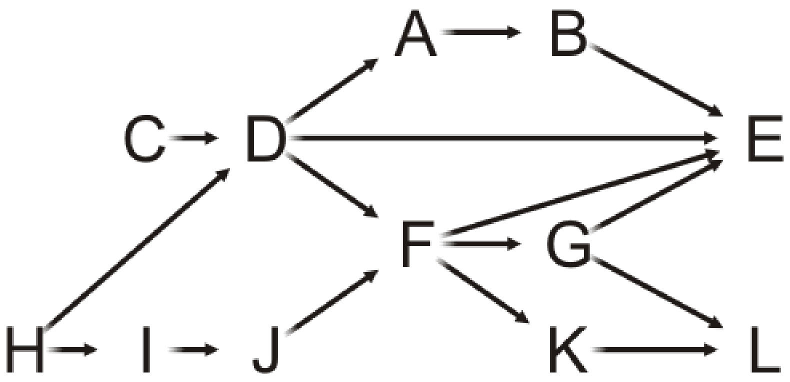
Queue:

C	H	D	I	A	J	B	F	G	K	E	L

A	0
B	0
C	0
D	0
E	0
F	0
G	0
H	0
I	0
J	0
K	0
L	**0**

Example

The queue is empty, so we are done



Queue:

C	H	D	I	A	J	B	F	G	K	E	L

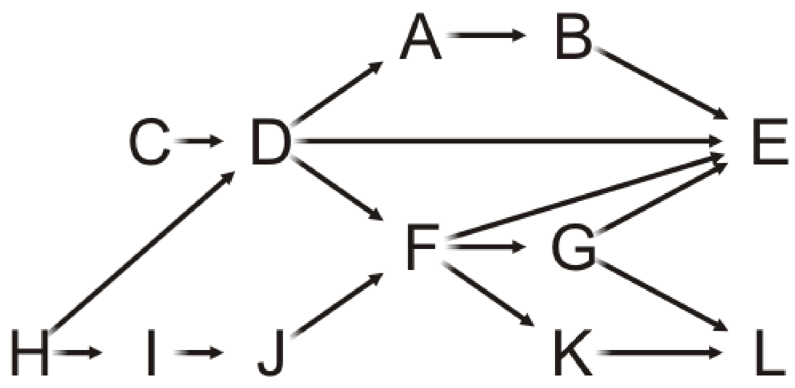
A	0
B	0
C	0
D	0
E	0
F	0
G	0
H	0
I	0
J	0
K	0
L	0

Example

We deallocate the memory for the temporary in-degree array

The array stores the topological sorting

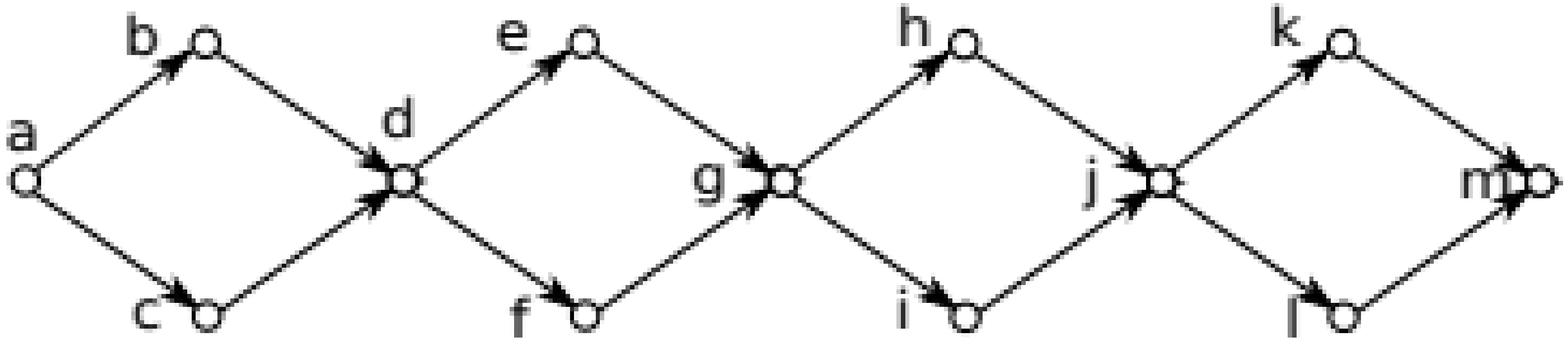
A	0
B	0
C	0
D	0
E	0
F	0
G	0
H	0
I	0
J	0
K	0
L	0



C	H	D	I	A	J	B	F	G	K	E	L

Question

Give a topological sort of the following graph

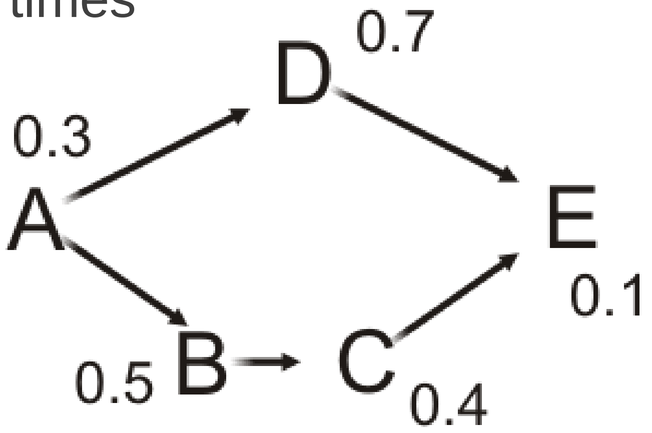


What is the number of different topological sorts of this graph? (Bonus Mark Question) (Negative mark for wrong answer)

Critical path

Suppose each task has a performance time associated with it

- If the tasks are performed serially, the time required to complete the last task equals to the sum of the individual task times

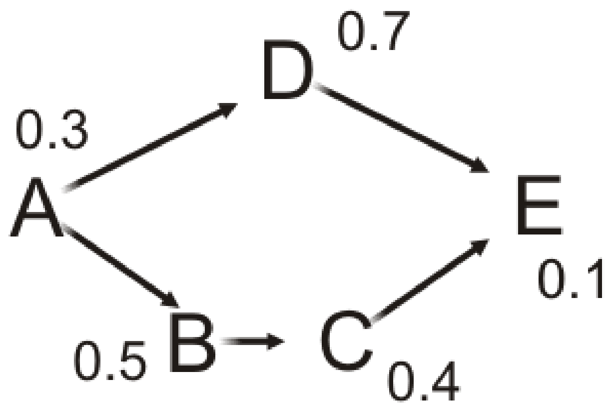


- These tasks require $0.3 + 0.7 + 0.5 + 0.4 + 0.1 = 2.0$ s to execute serially

Critical path

Suppose two tasks are ready to execute

- We could perform these tasks in parallel

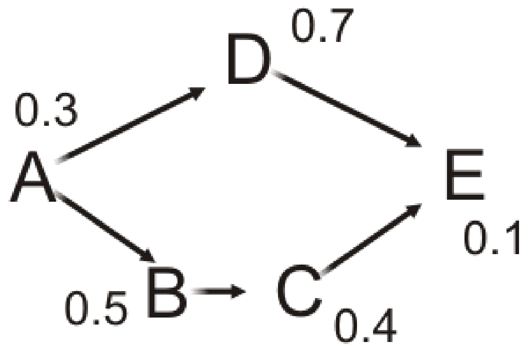


- Computer tasks can be executed in parallel (multi-processing)
- Different tasks can be completed by different teams in a company

Critical path

Suppose Task A completes

- We can now execute Tasks B and D in parallel



- However, Task E cannot execute until Task C completes, and Task C cannot execute until Task B completes
  - The least time in which these five tasks can be completed is
    $$0.3 + 0.5 + 0.4 + 0.1 = 1.3 \text{ s}$$
  - This is called the *critical time of all tasks*
  - The path (A, B, C, E) is said to be the *critical path*

Finding the critical path

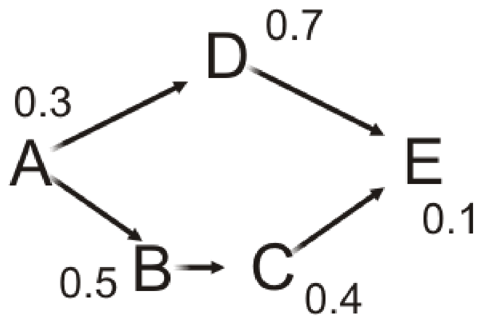
Tasks that have no prerequisites have a critical time equal to the time it takes to complete that task

For tasks that depend on others, the critical time will be:

- The maximum critical time that it takes to complete a prerequisite
- Plus the time it takes to complete this task

In this example, the critical times are:

- Task A completes in 0.3 s
- Task B must wait for A and completes after 0.8 s
- Task D must wait for A and completes after 1.0 s
- Task C must wait for B and completes after 1.2 s
- Task E must wait for both C and D, and completes after $\max(1.0, 1.2) + 0.1 = 1.3$ s



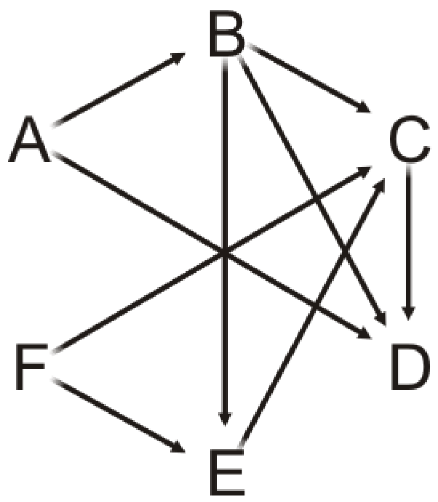
Finding the critical path

Thus, we require more information:

- We must know the execution time of each task
- We will have to record the critical time for each task
  - Initialize these to zero
- We will need to know the previous task with the longest critical time to determine the critical path
  - Set these to null

Finding the critical path

Suppose we have the following times for the tasks



Queue

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	0.0	Ø
B	1	6.1	0.0	Ø
C	3	4.7	0.0	Ø
D	3	8.1	0.0	Ø
E	2	9.5	0.0	Ø
F	0	17.1	0.0	Ø

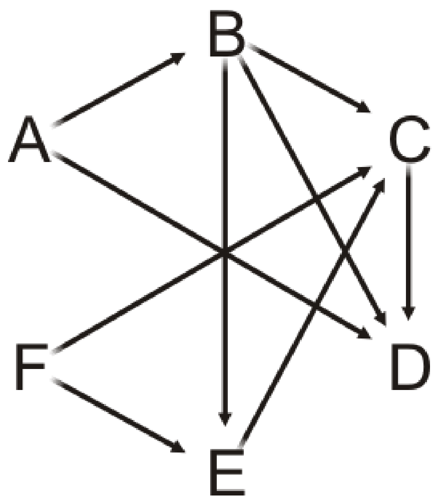
Finding the critical path

Each time we pop a vertex v, in addition to what we already do:

- For v, add the task time onto the critical time for that vertex:
  - That is the critical time for v
- For each adjacent vertex w:
  - If the critical time for v is greater than the currently stored critical time for w
    - Update the critical time with the critical time for v
    - Set the previous pointer to the vertex v

Finding the critical path

So we initialize the queue with those vertices with in-degree zero



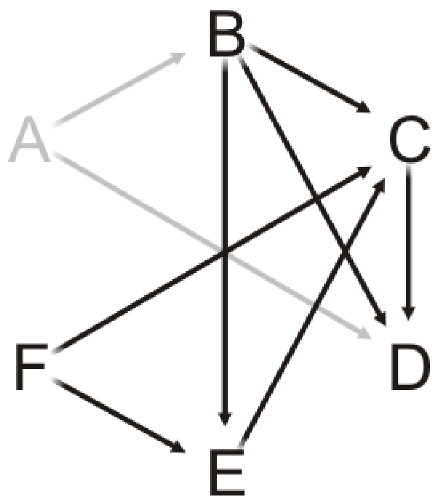
Queue

A	**F**		

Task	In-degree	Task Time	Critical Time	Previous Task
A	**0**	5.2	0.0	∅
B	1	6.1	0.0	∅
C	3	4.7	0.0	∅
D	3	8.1	0.0	∅
E	2	9.5	0.0	∅
F	**0**	17.1	0.0	∅

Finding the critical path

Pop Task A and update its critical time 0.0 + 5.2 = 5.2



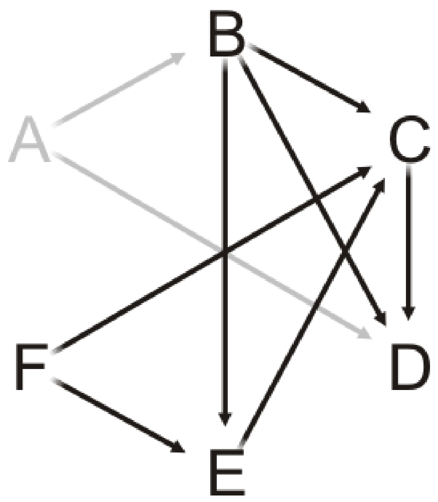
Queue

F			

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	0.0	Ø
B	1	6.1	0.0	Ø
C	3	4.7	0.0	Ø
D	3	8.1	0.0	Ø
E	2	9.5	0.0	Ø
F	0	17.1	0.0	Ø

Finding the critical path

Pop Task A and update its critical time 0.0 + 5.2 = 5.2



Queue

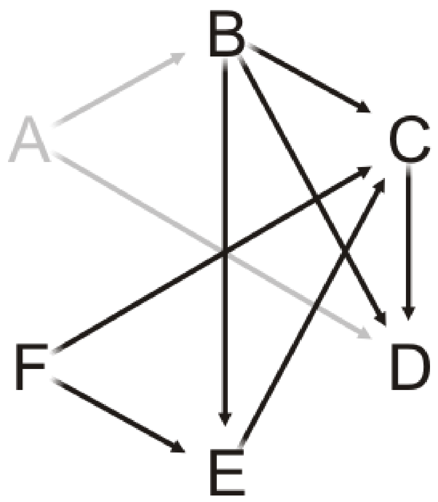
F			

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	**5.2**	Ø
B	1	6.1	0.0	Ø
C	3	4.7	0.0	Ø
D	3	8.1	0.0	Ø
E	2	9.5	0.0	Ø
F	0	17.1	0.0	Ø

Finding the critical path

For each neighbor of Task A:

- Decrement the in-degree, push if necessary, and check if we must update the critical time



Queue

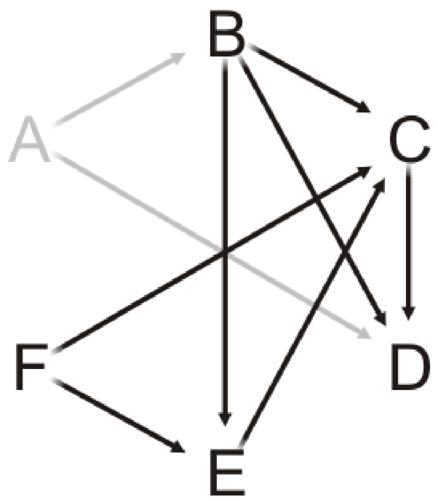
F			

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	1	6.1	0.0	Ø
C	3	4.7	0.0	Ø
D	3	8.1	0.0	Ø
E	2	9.5	0.0	Ø
F	0	17.1	0.0	Ø

Finding the critical path

For each neighbor of Task A:

- Decrement the in-degree, push if necessary, and check if we must update the critical time



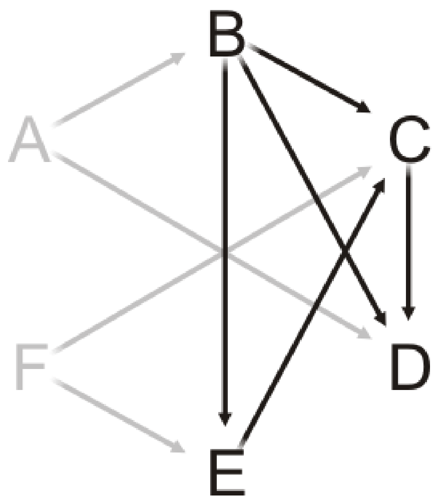
Queue

F	**B**		

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	**0**	6.1	**5.2**	**A**
C	3	4.7	0.0	Ø
D	**2**	8.1	**5.2**	**A**
E	2	9.5	0.0	Ø
F	0	17.1	0.0	Ø

Finding the critical path

Pop Task F and update its critical time 0.0 + 17.1 = 17.1



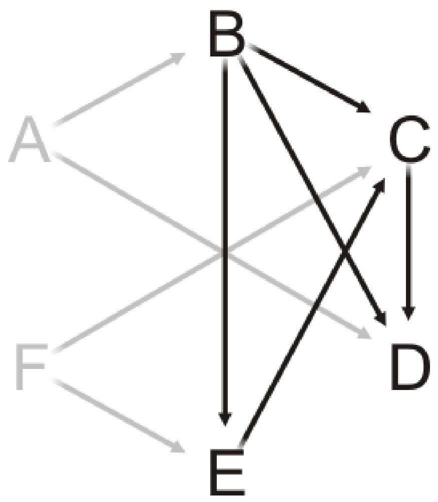
Queue

B			

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	5.2	A
C	3	4.7	0.0	Ø
D	2	8.1	5.2	A
E	2	9.5	0.0	Ø
F	0	17.1	0.0	Ø

Finding the critical path

Pop Task F and update its critical time 0.0 + 17.1 = 17.1



Queue

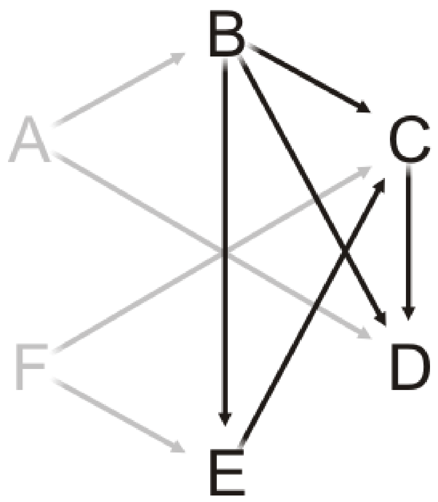
B			

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	5.2	A
C	3	4.7	0.0	Ø
D	2	8.1	5.2	A
E	2	9.5	0.0	Ø
F	0	17.1	**17.1**	Ø

Finding the critical path

For each neighbor of Task F:

- Decrement the in-degree, push if necessary, and check if we must update the critical time



Queue

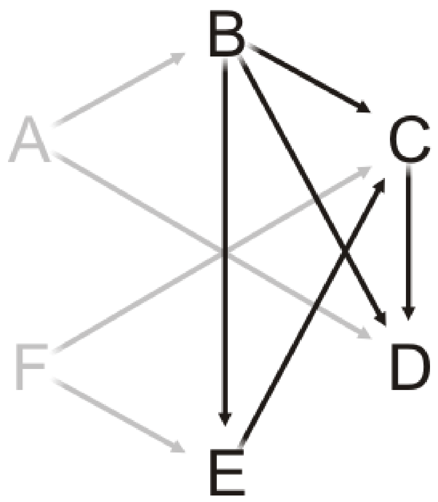
B			

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	5.2	A
C	3	4.7	0.0	Ø
D	2	8.1	5.2	A
E	2	9.5	0.0	Ø
F	0	17.1	17.1	Ø

Finding the critical path

For each neighbor of Task F:

- Decrement the in-degree, push if necessary, and check if we must update the critical time



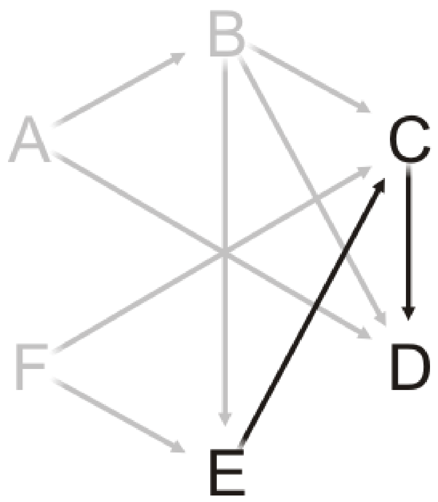
Queue

B			

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	5.2	A
C	**2**	4.7	**17.1**	**F**
D	2	8.1	5.2	A
E	**1**	9.5	**17.1**	**F**
F	0	17.1	17.1	Ø

Finding the critical path

Pop Task B and update its critical time 5.2 + 6.1 = 11.3

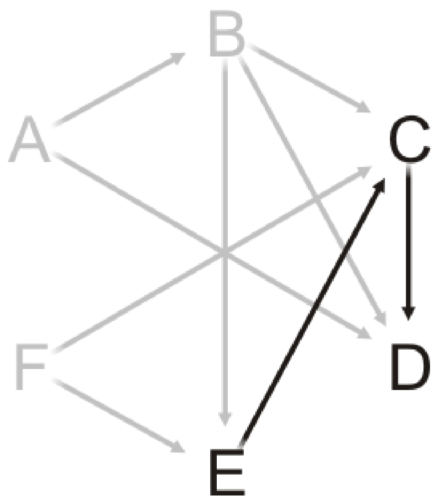


Queue

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	5.2	A
C	2	4.7	17.1	F
D	2	8.1	5.2	A
E	1	9.5	17.1	F
F	0	17.1	17.1	Ø

Finding the critical path

Pop Task B and update its critical time 5.2 + 6.1 = 11.3



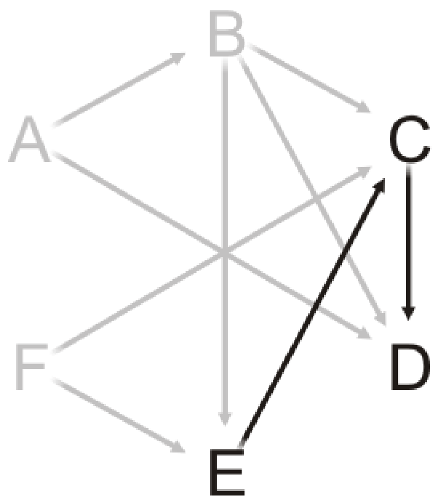
Queue

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	**11.3**	A
C	2	4.7	17.1	F
D	2	8.1	5.2	A
E	1	9.5	17.1	F
F	0	17.1	17.1	Ø

Finding the critical path

For each neighbor of Task B:

- Decrement the in-degree, push if necessary, and check if we must update the critical time



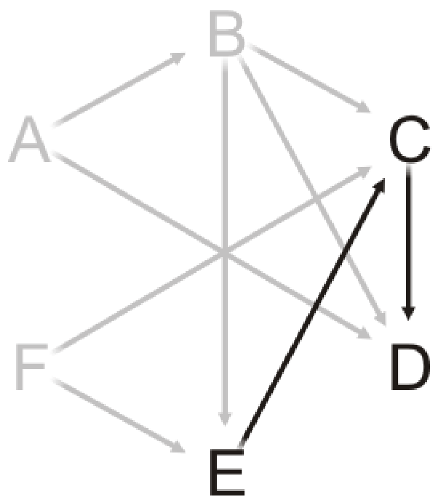
Queue

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	2	4.7	17.1	F
D	2	8.1	5.2	A
E	1	9.5	17.1	F
F	0	17.1	17.1	Ø

Finding the critical path

For each neighbor of Task B:

- Decrement the in-degree, push if necessary, and check if we must update the critical time
- Both C and E are waiting on F



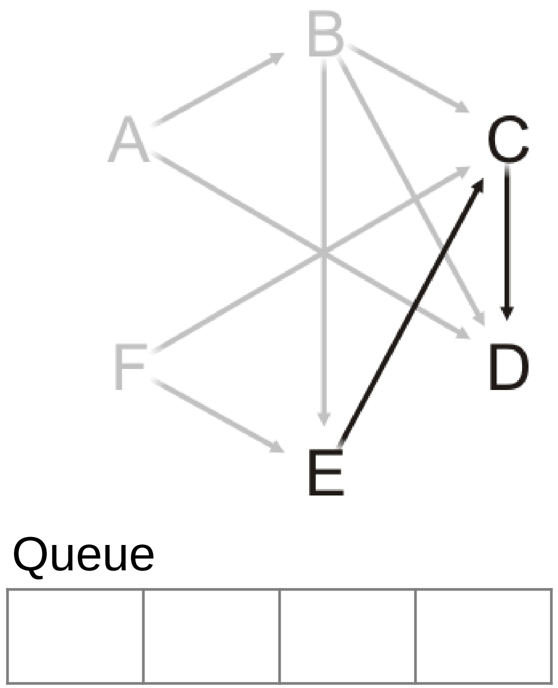
Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	1	4.7	17.1	F
D	1	8.1	**11.3**	**B**
E	0	9.5	17.1	F
F	0	17.1	17.1	Ø

Queue

E			

Finding the critical path

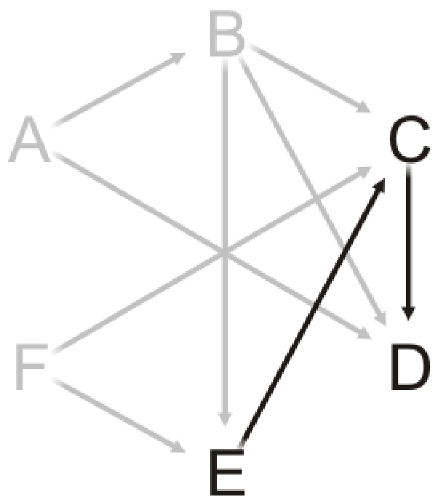
Pop Task E and update its critical time 17.1 + 9.5 = 26.6



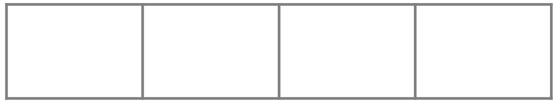
Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	1	4.7	17.1	F
D	1	8.1	11.3	B
E	0	9.5	17.1	F
F	0	17.1	17.1	Ø

Finding the critical path

Pop Task E and update its critical time 17.1 + 9.5 = 26.6



Queue

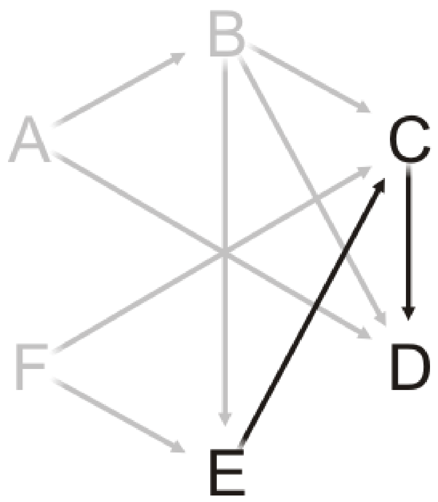


Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	1	4.7	17.1	F
D	1	8.1	11.3	B
E	0	9.5	**26.6**	F
F	0	17.1	17.1	Ø

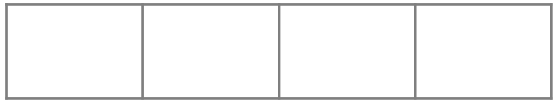
Finding the critical path

For each neighbor of Task E:

- Decrement the in-degree, push if necessary, and check if we must update the critical time



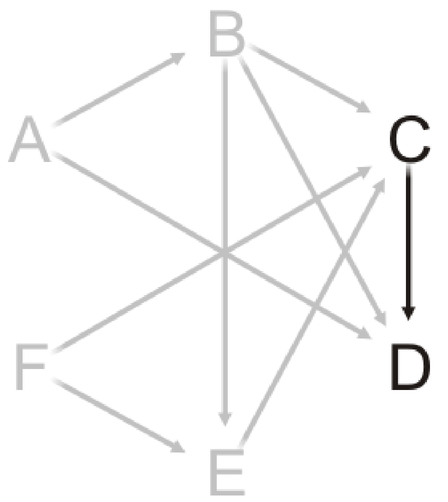
Queue



Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	1	4.7	17.1	F
D	1	8.1	11.3	B
E	0	9.5	26.6	F
F	0	17.1	17.1	Ø

Finding the critical path

For each neighbor of Task E:
- Decrement the in-degree, push if necessary, and check if we must update the critical time



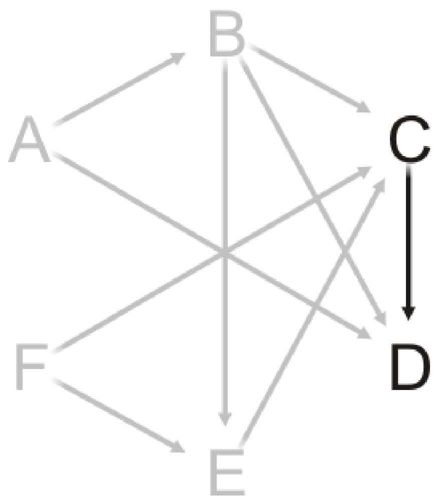
Queue

C			

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	0	4.7	26.6	E
D	1	8.1	11.3	B
E	0	9.5	26.6	F
F	0	17.1	17.1	Ø

Finding the critical path

Pop Task C and update its critical time 26.6 + 4.7 = 31.3

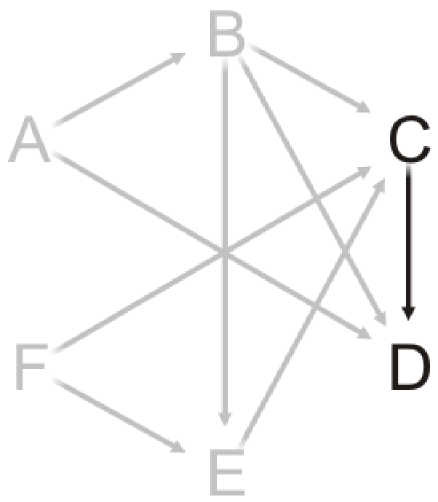


Queue

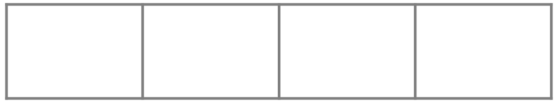
Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	0	4.7	26.6	E
D	1	8.1	11.3	B
E	0	9.5	26.6	F
F	0	17.1	17.1	Ø

Finding the critical path

Pop Task C and update its critical time 26.6 + 4.7 = 31.3



Queue

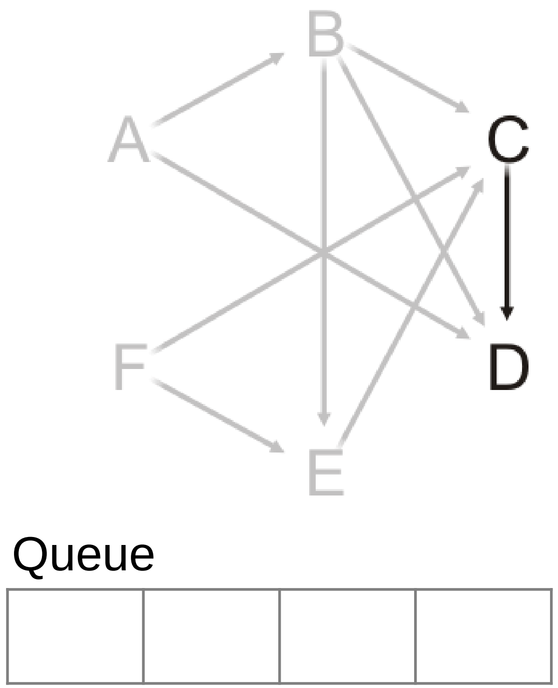


Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	0	4.7	**31.3**	E
D	1	8.1	11.3	B
E	0	9.5	26.6	F
F	0	17.1	17.1	Ø

Finding the critical path

For each neighbor of Task C:

- Decrement the in-degree, push if necessary, and check if we must update the critical time

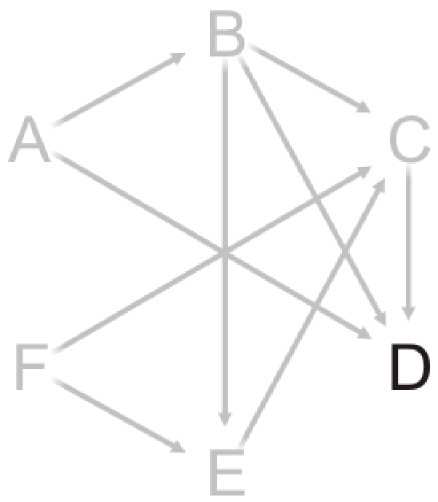


Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	0	4.7	31.3	E
D	1	8.1	11.3	B
E	0	9.5	26.6	F
F	0	17.1	17.1	Ø

Finding the critical path

For each neighbor of Task C:

- Decrement the in-degree, push if necessary, and check if we must update the critical time



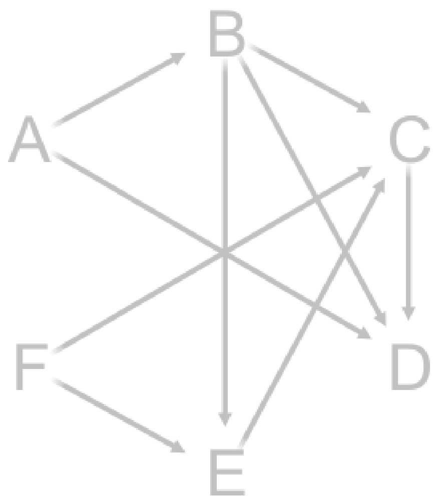
Queue

D			

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	0	4.7	31.3	E
D	**0**	8.1	**31.3**	**C**
E	0	9.5	26.6	F
F	0	17.1	17.1	Ø

Finding the critical path

Pop Task D and update its critical time 31.3 + 8.1 = 39.4

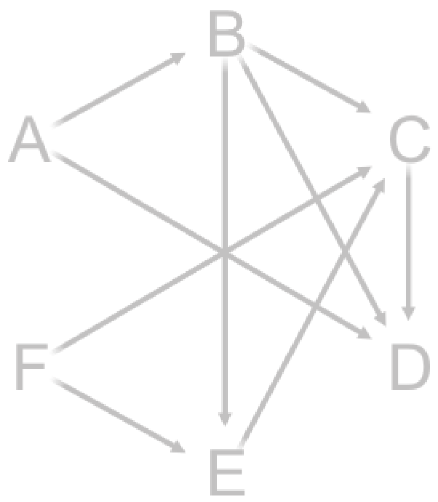


Queue

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	0	4.7	31.3	E
D	0	8.1	31.3	C
E	0	9.5	26.6	F
F	0	17.1	17.1	Ø

Finding the critical path

Pop Task D and update its critical time 31.3 + 8.1 = 39.4



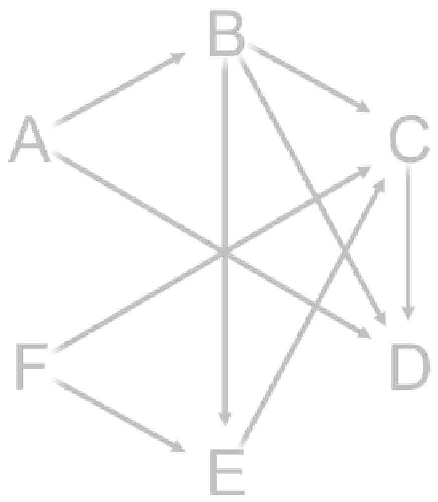
Queue

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	0	4.7	31.3	E
D	0	8.1	**39.4**	C
E	0	9.5	26.6	F
F	0	17.1	17.1	Ø

Finding the critical path

Task D has no neighbors and the queue is empty

- We are done



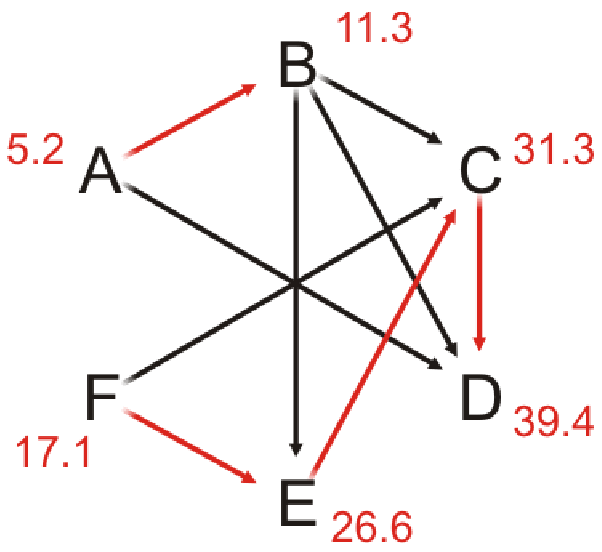
Queue

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	0	4.7	31.3	E
D	0	8.1	39.4	C
E	0	9.5	26.6	F
F	0	17.1	17.1	Ø

Finding the critical path

Task D has no neighbors and the queue is empty

- We are done



Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	0	4.7	31.3	E
D	0	8.1	39.4	C
E	0	9.5	26.6	F
F	0	17.1	17.1	Ø