Department of Mathematics and Natural Sciences

MAT 110

ASSIGNMENT 1

SUMMER 2021

SET: 16 (AII)

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Set:16

1. Determine whether $f(x)$ is continuous at $x = 0$, where

$$f(x) = \begin{cases} x - 5; & x < 0 \\ x^2 - 7; & 0 \leq x \leq 2 \\ x^3; & x > 2. \end{cases}$$

2. Check whether $\lim_{x\to 0} f(x)$ exists or not, where

$$f(x) = \begin{cases} x^2 + 5x + 6; & x < 0 \\ e^x; & 0 \le x \le 1 \\ x - 2x^5; & x > 1. \end{cases}$$

3. Evaluate $\frac{d^2}{dx^2}\left(\frac{5x-9x^2}{x+1}\right)$.

4. Find $\frac{dy}{dx}$ from $(x+2y)^2 = 2x + 3y^2 - 1$.

5. Evaluate $\frac{d}{dx}\left(\cos^2\left(e^{x+1}\right)\right)$.

6. If an object with mass m is dropped from rest, one model for its speed v after t seconds, taking air resistance into account, is

$$v = \frac{mg}{c}\left(1 - e^{-\frac{ct}{m}}\right)$$

where g is the acceleration due to gravity and c is a positive constant (which is known as the proportionality constant since the air resistance is proportional to the speed of the object).

   (a) Calculate $\lim_{t\to\infty}(v)$.What is the meaning of this limit?

   (b) For fixed t, calculate $\lim_{c\to 0^+}(v)$. What can you conclude about the velocity of a falling object in a vacuum?

Answer (1)

Given, $f(x) = \begin{cases} x-5 & ; \quad x < 0 \\ x^2-7 & ; \quad 0 \leq x \leq 2 \\ x^3 & ; \quad x > 2 \end{cases}$

Now, for x^2-7

$f(0) = 0^2 - 7 = -7$

$\lim_{x \to 0^-} (x-5) = 0 - 5 = -5$

$\lim_{x \to 0^+} x^2 - 7 = 0 - 7 = -7$

$\therefore$ The function $f(x)$ is not continuous.

Answer 2

Given $f(x) = \begin{cases} x^2 + 5x + 6, & x < 0 \\ e^x, & 0 \leq x \leq 1 \\ x - 2x^5 & x > 1 \end{cases}$

Now,

$$\lim_{x \to 0^-} (x^2 + 5x + 6) = 0 + 0 + 6 = 6$$

$$\lim_{x \to 0^+} e^x = e^0 = 1$$

$\therefore$ The $\lim_{x \to 0} f(x)$ exists.

(Answer-3)

$$\frac{d^2}{dx^2}\left(\frac{5x-9x^3}{x+1}\right)$$

$$= \frac{d}{dx}\left\{\frac{d}{dx}\left(\frac{5x-9x^3}{x+1}\right)\right\}$$

$$= \frac{d}{dx}\left\{\frac{(x+1)\frac{d}{dx}(5x-9x^2) - (5x-9x^2)\frac{d}{dx}(x+1)}{(x+1)^2}\right\}$$

$$= \frac{d}{dx}\left\{\frac{(x+1)(5-18x)-(5x-9x^2)(1+0)}{(x+1)^2}\right\}$$

$$= \frac{d}{dx}\left\{\frac{\cancel{5x}-18x^2+5-18x\cancel{-5x}+9x^2}{(x+1)^2}\right\}$$

$$= \frac{d}{dx}\left\{\frac{-9x^2-18x+5}{(x+1)^2}\right\}$$

$$= \frac{(x+1)^2 \frac{d}{dx}(-9x^2-18x+5) - (-9x^2-18x+5)\frac{d}{dx}(x+1)^2}{\{(x+1)^2\}^2}$$

$$= \frac{(x+1)^2(-18-18+0) - (-9x^2-18x+5)\,2(x+1)}{(x+1)^4}$$

$$= \frac{(x+1)\{(x+1)(-36) - 2(-9x^2-18x+5)\}}{(x+1)^4}$$

$$= \frac{-36x-36+18x^2+36x-10}{(x+1)^3}$$

$$= \frac{18x^2-46}{(x+1)^3} \quad \text{(Ans)}$$

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Answer 4

$$(x+2y)^2 = 2x + 3y^2 - 1$$

$$\frac{d}{dx}(x+2y)^2 = \frac{d}{dx}(2x + 3y^2 - 1)$$

$$\Rightarrow 2(x+2y)\frac{d}{dx}(x+2y) = 2\cdot 1 + 3.2y\frac{dy}{dx}$$

$$\Rightarrow (2x+4y)\left(1 + 2\frac{dy}{dx}\right) = 2 + 6y\frac{dy}{dx}$$

$$\Rightarrow 2x + 4y + (2x+4y)\,2\frac{dy}{dx} = 2 + 6y\frac{dy}{dx}$$

$$\Rightarrow 2x + 4y + (4x+8y)\frac{dy}{dx} = 2 + 6y\frac{dy}{dx}$$

$$\Rightarrow (4x+8y)\frac{dy}{dx} - 6y\frac{dy}{dx} = 2 - 2x - 4y$$

$$\Rightarrow \cancel{\frac{dy}{dx}}\,(4x + 8y - 6y)\frac{dy}{dx} = 2 - 2x - 4y$$

$$\Rightarrow \frac{dy}{dx} = \frac{2 - 2x - 4y}{4x + 2y}$$

$$= \frac{\cancel{7}(1-x-2y)}{\cancel{7}(2x+y)}$$

$$= \frac{1-x-2y}{2x+y} \quad \text{(Ans)}$$

Answer 5

$$\frac{d}{dx}\left(\cos^2\left(e^{x+1}\right)\right)$$

$$= \frac{d}{dx}\left(\cos\left(e^{x+1}\right)\right)^2$$

$$= 2\cos\left(e^{x+1}\right)\frac{d}{dx}\cos\left(e^{x+1}\right)$$

$$= 2\cos e^{x+1}\left\{-\sin\left(e^{x+1}\right)\right\}\frac{d}{dx}\left(e^{x+1}\right)$$

$$= -2\sin e^{x+1}\cdot\cos e^{x+1}\cdot e^{x+1}(1+0)$$

$$= -\sin\left(2\times e^{x+1}\right)\cdot e^{x+1}$$

$$= -e^{x+1}\sin\left(2e^{x+1}\right)$$

Answer 6 (a)

$$\lim_{t \to \alpha} (v)$$

$$= \lim_{t \to \alpha} \frac{mg}{c}\left(1 - e^{-\frac{ct}{m}}\right)$$

$$= \frac{mg}{c}\left(1 - e^{-\frac{c \times \alpha}{m}}\right)$$

$$= \frac{mg}{c}\left(1 - e^{-\alpha}\right)$$

$$= \frac{mg}{c}\left(1 - \frac{1}{e^{\alpha}}\right)$$

$$= \frac{mg}{c}(1 - 0)$$

$$= \frac{mg}{c}$$

This limit means the velocity will be constant after a long time and the magnitude will be $\frac{mg}{c}$. (Ans).

Answer 6 (b)

$$\lim_{c \to 0^+} (v)$$

$$= \lim_{c \to 0^+} \frac{mg}{c} \left(1 - e^{-\frac{ct}{m}}\right)$$

$$= \lim_{c \to 0^+} \frac{mg}{c} \left\{ 1 - \left(1 - \frac{ct}{m.1!} + \frac{c^2t^2}{m^2.2!} - \frac{c^3t^3}{m^3.3!} + \cdots\right) \right\}$$

$$= \lim_{c \to 0^+} \frac{mg}{c} \left(1 - 1 + \frac{ct}{m.1!} - \frac{c^2t^2}{m^2.2!} + \frac{c^3t^3}{m^3 3!} - \cdots\right)$$

$$= \lim_{c \to 0^+} \frac{mg}{c} \cdot c \left(\frac{t}{m \cdot 1!} - \frac{c t^2}{m^2 2!} + \frac{c^2 t^3}{m^3 \cdot 3!} \cdots \right)$$

$$= mg \left(\frac{t}{m} - 0 + 0 - 0 \cdots \right)$$

$$= mg \times \frac{t}{m}$$

$$= gt$$

So we can say that the velocity will be proportioned to the time of a falling object in vaccum..