BRAC UNIVERSITY

Inspiring Excellence

MAT110

Assignment 2

SET:1

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Section:07

Assignment

Answer - 1

Constraints: $l + w + h \leq 62$ and $h = w$

objective: $v = lwh$ largest $\Rightarrow$ maximum

using constraints together and equality instead of inequality:

$l + w + h = 62$ becomes $l + w + w = 62$

or, $l + 2w = 62$

so $l = 62 - 2w$

substitute $v = lwh = (62 - 2w)\, w \cdot w$

$v = 62w^2 - 2w^3$

maximize $\Rightarrow$ $v' = 124w - 6w^2$

$0 = 2w(62 - 3w)$

$2w = 0$ or $62 - 3w = 0$

$w = 0$

not a max

$\frac{62}{3} = w$

must be max (check for sure).

If $w = \frac{62}{3} = 20\frac{2}{3}$ inches.

$h = w = 20\frac{2}{3}$ inches

and $l = 62 - 2\left(\frac{62}{3}\right) = 20\frac{2}{3}$ in.

The maximum volume will be cube shape.

(Ans)

Answer 2

At first,

$$y = \frac{x-1}{x+1}$$

$$\frac{dy}{dx} = \frac{(x+1)\frac{d}{dx}(x-1) - (x-1)\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$\frac{dy}{dx} = \frac{(x+1).1 - (x-1).1}{(x+1)^2}$$

$$\frac{dy}{dx} = \frac{x+1-x+1}{(x+1)^2}$$

$$\frac{dy}{dx} = \frac{2}{(x+1)^2}$$

Now

$$\frac{d^2y}{dx^2} = \frac{(x+1)^2 \frac{d}{dx}(2) - 2 \cdot \frac{d}{dx}(x+1)^2}{\{(x+1)^2\}^2}$$

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$$\Rightarrow \frac{d^2y}{dx^2} = \frac{(x+1)^2 - 2\cdot 2(x+1)}{(x+1)^4}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-4(x+1)}{(x+1)^4} = \frac{-4}{(x+1)^3}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{-4}{(x+1)^3} \text{ (Ans)}$$

Answer 3

Given,

$y = \ln(x^2+4) - x \arctan\left(\frac{x}{2}\right)$

Now,

$\Rightarrow \frac{dy}{dx} = \frac{1}{x^2+4} \times (2x+0) - \left[x \frac{1}{1+\left(\frac{x}{2}\right)^2} \times \frac{1}{2} + 1 . \arctan \frac{x}{2}\right.$

$\Rightarrow \frac{dy}{dx} = \frac{2x}{x^2+4} - \frac{2x}{x^2+4} - \arctan \frac{x}{2}$

$\Rightarrow \frac{dy}{dx} = - \arctan\left(\frac{x}{2}\right)$

$\Rightarrow \frac{dy}{dx}\Big|_{x=2} = - \arctan\left(\frac{2}{2}\right) = -\arctan 1$

$$\therefore \left.\frac{dy}{dx}\right|_{x=2} = -\frac{\pi}{4} \quad (\text{Ans})$$

Answer 4

Given

$$y = (\sin x)^{\cos x} + (\cos x)^{\sin x} - 5x$$

Let, $y_1 = (\sin x)^{\cos x}$ $\quad y_2 = (\cos x)^{\sin x}$ $\quad y_3 = -5x$

Now, $y_1 = (\sin x)^{\cos x}$

$$\log y_1 = \cos x \log(\sin x)$$

Now differentiating both sides with respect to y_1

$$\frac{1}{y_1}\frac{dy_1}{dx} = -\sin x \log \sin x + \frac{\cos x}{\sin y} \cdot \cos x$$

$$\frac{dy_1}{dx} = \left(\frac{\cos^2 x}{\sin x} - \sin x \log \sin x\right)(\sin x)^{\cos x}$$

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Again,

$$y_2 = (\cos x)^{\sin x}$$

Now,

$$\log y_2 = (\sin x) \log(\cos x)$$

Differentiating both sides with respect to y.

$$\frac{1}{y_2} \frac{dy_1}{dx} = (\cos x) \log(\cos x) + \frac{\sin x}{\cos x} \cdot (-\sin x)$$

$$\Rightarrow \frac{dy_2}{dx} = \left\{ (\cos x) \log(\cos x) - \frac{\sin^2 x}{\cos x} \right\} y_2$$

Now substituting y_2

$$\frac{dy_3}{dx} = \left\{ (\cos x) \log(\cos x) - \frac{\sin^2 x}{\cos x} \right\} (\cos x)^{\sin x}$$

Again,

$$y_3 = -5x$$

$$\frac{dy_3}{dx} = -5$$

Since, $y = y_1 + y_2 + y_3$

$$\therefore \frac{dy}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx} + \frac{dy_3}{dx}.$$

Therefore,

$$\frac{dy}{dx} = (\cot x \cos x - \sin x \log \sin x)(\sin x)^{\cos x} + (\cos x \log \cos x - \tan x \sin x)(\cos x)^{\sin x} - 5$$

Ans

Answer 5

Given,

$f(x) = x + 2\sin x$

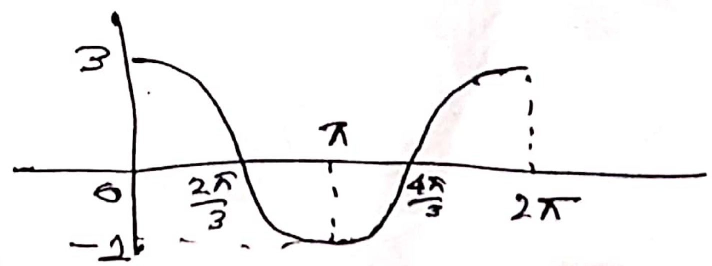
Now $f'(x) = \frac{d}{dx}(x + 2\sin x)$

$= 1 + 2\cos x$

$f''(x) = \frac{d}{dx}(f'(x))$

$= \frac{d}{dx}(1 + 2\cos x) = -2\sin x$

graph of $f'(x)$ in interval $[0, 2\pi]$



#If $f'(x) > 0$ then $f(x)$ is increasing and if $f'(x) < 0$. $f(x)$ is decreasing.

$f'(x) > 0$ in $\left[0, \frac{2\pi}{3}\right) \cup \left(\frac{4\pi}{3}, 2\pi\right]$

Hence $f(x)$ is increasing in interval $\left[0, \frac{2\pi}{3}\right) \cup \left(\frac{4\pi}{3}, 2\pi\right)$ and decreasing in interval $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$

#If $f''(x) > 0$ graph is concave up and if $f''(x) < 0$, graph is concave down.

$f''(x) < 0$ in $(0, \pi)$ and $f''(x) > 0$ in $(\pi, 2\pi)$

So graph is concave down in interval $(0, \pi)$ and concave up in interval $(\pi, 2\pi)$.

At point of inflection, $f''(x) = 0$ and graph changes its concavity

$f''(x) = 0$ at $x = \pi$ and here graph changes its concavity as well.

So, $x = \pi$ is a point of inflection. (Ans)

Answer 6

Given,

$f(x) = 3x^5 - 5x^3$

Now, $f'(x) = 15x^4 - 15x^2$

Relative extremas are the points where $f'(x) = 0$

$f'(x) = 0$ i.e $15x^4 - 15x^2 = 0$

$15x^2(x^2 - 1) = 0$

$15x^2(x+1)(x-1) = 0$

at, $x = \{0, 1, -1\}$

Hence for $x = (0, 1, -1)$ points, we get the relative extremas. (Ans),