



MAT110

Assignment:5

SET:26

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Section:07

Assignment 05

Answer 1

The given equation,

$$y^2 - 2y = 8x - 1$$

$$\Rightarrow y^2 - 2y + 1 = 8x - 1 + 1$$

$$\Rightarrow (y-1)^2 = 8x$$

$$\Rightarrow (y-1)^2 = 4 \cdot 2 \cdot x \quad \left(\begin{array}{l} \text{The standard form of the} \\ \text{equation of parabola} \end{array} \right) \quad (\text{Ans})$$

For Vertex,

$$x = 0, \quad y - 1 = 0$$

$$\Rightarrow y = 1$$

Therefore vertex $(0, 1)$ (Ans)

here, $a = 2$

For focus

$$x = 2 \quad \text{and}$$

$$y - 1 = 0$$

$$\Rightarrow y = 1$$

therefore focus $(2, 1)$ (Am)

Equation of the directrix,

$$x = -2$$

$$\therefore x + 2 = 0 \quad (Ans)$$

Answer-2

The given equation

$$-x^2 + 4y^2 - 2x - 16y + 11 = 0$$

$$\Rightarrow -(x^2 + 2x) + 4(y^2 - 4y) = -11$$

$$\Rightarrow -(x^2 + 2x + 1) + 4(y^2 - 4y + 4) = -11 - 1 + 16$$

$$\Rightarrow -(x+1)^2 + 4(y-2)^2 = 4$$

$$\Rightarrow \frac{-(x+1)^2}{4} + \frac{4(y-2)^2}{4} = 1$$

$$\Rightarrow -\frac{(x+1)^2}{4} + (y-2)^2 = 1$$

$$\therefore \frac{(y-2)^2}{1^2} - \frac{(x+1)^2}{4} = 1$$

This is the standard form of the equation of the hyperbola. (Ans)

For center,

$$(x+1) = 0 \quad \text{and} \quad y-2 = 0$$

$$\Rightarrow x = -1 \quad \text{and} \quad y = 2$$

\therefore centre $(-1, 2)$ (Ans)

For vertices,

$$x+1 = 0 \quad \text{and}$$

$$\therefore x = -1$$

$$y-2 = \pm 1$$

$$y = \pm 1 + 2$$

$$\therefore y = (3, 1)$$

therefore vertices $(-1, 3)$ and $(-1, 1)$ (Ans)

eccentricity, $e = \sqrt{1 + \frac{a^2}{b^2}}$

$$= \sqrt{1 + \frac{4}{1}}$$

$$= \sqrt{5}$$

for foci,

$$x+1=0 \quad \text{and} \quad y-2 = \pm 1 \times \sqrt{5}$$

$$\Rightarrow x = -1 \quad \Rightarrow y = \pm \sqrt{5} + 2$$

Therefore foci $(-1, \sqrt{5}+2)$ and $(-1, -\sqrt{5}+2)$

(Ans.)

Answer- 3

The given equation,

$$x^2 + 4x - 4y = 0$$

$$\Rightarrow x^2 + 4x = 4y$$

$$\Rightarrow x^2 + 4x + 4 = 4y + 4$$

$$\Rightarrow (x+2)^2 = 4 \cdot 1 (y+1)$$

This is the standard form of the equation of parabola. (Ans)

\therefore vertex $(-2, -1)$ (Ans)

For focus, $(-2, -1)$ and

$$x+2 = 0$$

$$\Rightarrow x = -2$$

$$y+4 = 4$$

$$\therefore y = 4 - 4 = 0$$

$$\therefore y = 0$$

\therefore focus $(-2, 0)$ (Ans)

Equation of directrix,

$$y+1 = -1$$

$$y = -1 - 1$$

$$\therefore y = -2$$

$$\therefore y+2 = 0 \quad (\text{Ans})$$

Answer 1

The given equation,

$$-4x^2 + y^2 - 16x - 2y - 19 = 0$$

$$\Rightarrow -4(x^2 + 4x) + (y^2 - 2y) = 19$$

$$\Rightarrow -4(x^2 + 4x + 4) + (y^2 - 2y + 1) = 19 - 16 + 1$$

$$\Rightarrow -4(x+2)^2 + (y-1)^2 = 4$$

$$\Rightarrow \frac{-(x+2)^2}{1^2} + \frac{(y-1)^2}{2^2} = 1$$

$$\therefore \frac{(y-1)^2}{2^2} - \frac{(x+2)^2}{1^2} = 1$$

This is the standard form of the equation of hyperbola. (Ans)

$$\therefore \text{centre } (-2, 1) \text{ (Ans)}$$

for vertices,

$$x+2=0 \quad \text{and} \quad y-1 = \pm 2$$

$$\therefore x = -2 \quad \Rightarrow y = \pm 2 + 1$$

$$\therefore \text{vertices } (-2, 3) \text{ and } (-2, -1) \text{ (Ans)}$$

for foci,

$$x+2=0 \quad \text{and} \quad y-1 = \pm 2 \times \sqrt{1 + \frac{1}{4}}$$

$$x = -2$$

$$\Rightarrow y-1 = \pm 2 \times \frac{\sqrt{5}}{2}$$

$$\Rightarrow y-1 = \pm \sqrt{5}$$

$$\therefore y = \pm \sqrt{5} + 1$$

$$\text{Therefore, foci, } (-2, \sqrt{5}+1) \text{ and } (-2, -\sqrt{5}+1) \text{ (Ans)}$$

Answer 5

The given equation

$$y^2 + 12y = 1 - x$$

$$\Rightarrow y^2 + 2y \cdot 6 + 36 = 1 - x + 36$$

$$\Rightarrow (y+6)^2 = 37 - x$$

$$\Rightarrow (y+6)^2 = -(x-37)$$

$$\Rightarrow (y+6)^2 = -4 \cdot \frac{1}{4} (x-37)$$

This is the standard form of the equation of parabola. (Ans)

vertex $(37, -6)$ (Ans)

for focus,

$$x - 37 = -\frac{1}{4}$$

$$\text{and } y + 6 = 0$$

$$y = -6$$

$$\Rightarrow x = -\frac{1}{4} + 37$$

$$\therefore x = \frac{147}{4}$$

therefore, focus $\left(\frac{147}{4}, -6\right)$ (Ans)

the equation of directrix

$$x - 37 = \frac{1}{4}$$

$$\therefore x = 37 + \frac{1}{4}$$

$$\therefore x = \frac{149}{4}$$

$$\therefore 4x - 149 = 0 \quad (\text{Ans})$$

Answer - 6 :

The given equation,

$$-x^2 + 2y^2 + 2x + 8y + 3 = 0$$

$$\Rightarrow -(x^2 - 2x) + 2(y^2 + 4y) = -3$$

$$\Rightarrow -(x^2 - 2x + 1) + 2(y^2 + 4y + 4) = -3 - 1 + 8$$

$$\Rightarrow -(x-1)^2 + 2(y+2)^2 = 4$$

$$\Rightarrow -\frac{(x-1)^2}{4} + \frac{(y+2)^2}{2} = 1$$

$$\Rightarrow \frac{(y+2)^2}{(\sqrt{2})^2} - \frac{(x-1)^2}{2^2} = 1$$

This is the standard form of the equation of hyperbola. (Ans)

\therefore centre $(1, -2)$ (Ans)

For vertices,

$$(x-1) = 0$$

$$\therefore x = 1$$

$$\text{and } y+2 = \pm\sqrt{2}$$

$$y = \pm\sqrt{2} - 2$$

\therefore vertices $(1, \pm\sqrt{2} - 2)$ (Ans)

for foci,

$$x-1 = 0$$

$$\therefore x = 1$$

$$\text{and, } y+2 = \pm\sqrt{2} \times \sqrt{1 + \frac{4}{2}}$$

$$\Rightarrow y+2 = \pm\sqrt{2} \times \frac{\sqrt{6}}{\sqrt{2}}$$

$$\therefore y+2 = \pm\sqrt{6}$$

$$\therefore y = \pm\sqrt{6} - 2$$

foci $(1, \pm\sqrt{6} - 2)$ (Ans)