BRAC UNIVERSITY

Inspiring Excellence

Department of Mathematics and Natural Sciences

MAT 110

Final Examination

SUMMER 2021

SET: 12

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1. Given that $f(x) = \frac{2}{1-x}$, find $f^{(n)}(0)$ using Maclaurin series.
2. Find points P and Q on the parabola $y = 1 - x^2$ so that the triangle ABC formed by the x-axis and the tangent lines at P and Q is an equilateral triangle (see figure 1).

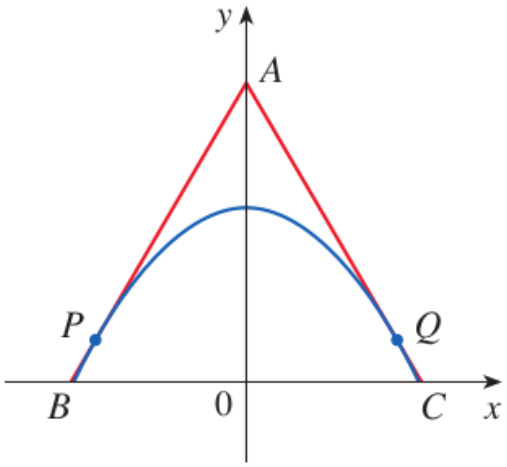

Figure 1: Figure for question 1

3. Let
$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, x \neq 0, \\ 0, x = 0. \end{cases} .$$
Find $f_x(x,y), f_y(x,y)$ at all points. Leave your answer as a piecewise function.

4. If the length of the diagonal of a rectangular box must be L, what is the largest possible volume?

5. If a vector function $\mathbf{F}$ depends on both space coordinates (x,y,z) and time t, show that $d\mathbf{F} = (d\mathbf{r} \cdot \nabla)\mathbf{F} + \frac{\partial \mathbf{F}}{\partial t}$.

6. Given the parameter τ, constants α and c, show that the parametric equations:
$$X(\tau) = \frac{c^2}{\alpha}\Big(\cosh\big(\frac{\alpha\tau}{c}\big) - 1\Big), T(\tau) = \frac{c}{\alpha}\sinh\Big(\frac{\alpha\tau}{c}\Big)$$
describe a hyperbola on the $X-T$ plane. This is hyperbolic motion in special relativity.

7. In three-dimensional cylindrical coordinates, sketch the surfaces of constant ρ, constant φ and constant z. Add a brief explanation.