$\underline{\underline{(1)}}$

The given equation,

$$5+6x+x^2-2y=0$$

$$\Rightarrow x^2+6x=2y-5$$

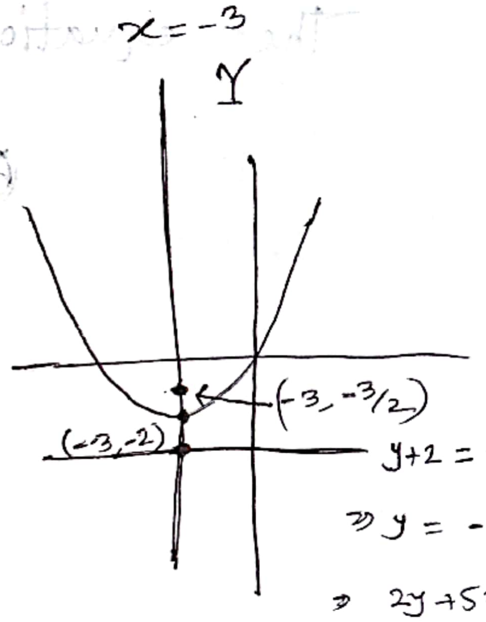
$$\Rightarrow x^2+2x.3+3^2=2y-5+9$$

$$\Rightarrow (x+3)^2=2y+4$$

$$\Rightarrow (x+3)^2=2(y+2)$$

$$\Rightarrow (x+3)^2=4.\tfrac{1}{2}(y+2)$$

This is the standard form of the equation of the parabola.



$y+2=-\frac{1}{2}$

$\Rightarrow y=-\frac{5}{2}$

$\Rightarrow 2y+5=0$

Now, $x+3=0$ and $y+2=0$

$\Rightarrow x=-3$ $\quad y=-2$

therefore, vertex $(-3,-2)$ (Ans)

focus $(-3,-2+\frac{1}{2})$

$\Rightarrow (-3,-\frac{3}{2})$ (Ans)

The equation of the directrix

$$y + 2 = -\frac{1}{2}$$

$$\Rightarrow y = -2 - \frac{1}{2}$$

$$\Rightarrow y = \frac{-5}{2}$$

$$\Rightarrow 2y + 5 = 0 \quad \text{(Ans)}$$

(2)

The given equation,

$$-24-24x+12x^2+3y^2=0$$

$$\Rightarrow 12x^2-24x+3y^2=24$$

$$\Rightarrow 12(x^2-2x)+3y^2=24$$

$$\Rightarrow 4(x^2-2x)+y^2=8$$

$$\Rightarrow 4(x^2-2x+1)+y^2=8+4$$

$$\Rightarrow 4(x-1)^2+y^2=12$$

$$\Rightarrow \frac{(x-1)^2}{3}+\frac{y^2}{12}=1$$

$$\Rightarrow \frac{(x-1)^2}{(\sqrt{3})^2}+\frac{y^2}{(2\sqrt{3})^2}=1 \quad \text{——— (1)}$$

This is the standard formed of the equation of ellipse.

From equation (1),

$x - 1 = 0$ and $y = 0$

$\therefore x = 1$

Therefore, centre $(1, 0)$

and $a = \sqrt{3}$ and $b = 2\sqrt{3}$

here, $b > a$

Therefore Y - is major axis.

~~Vertices are~~

eccentricity, $e = \sqrt{1 - \frac{a^2}{b^2}}$

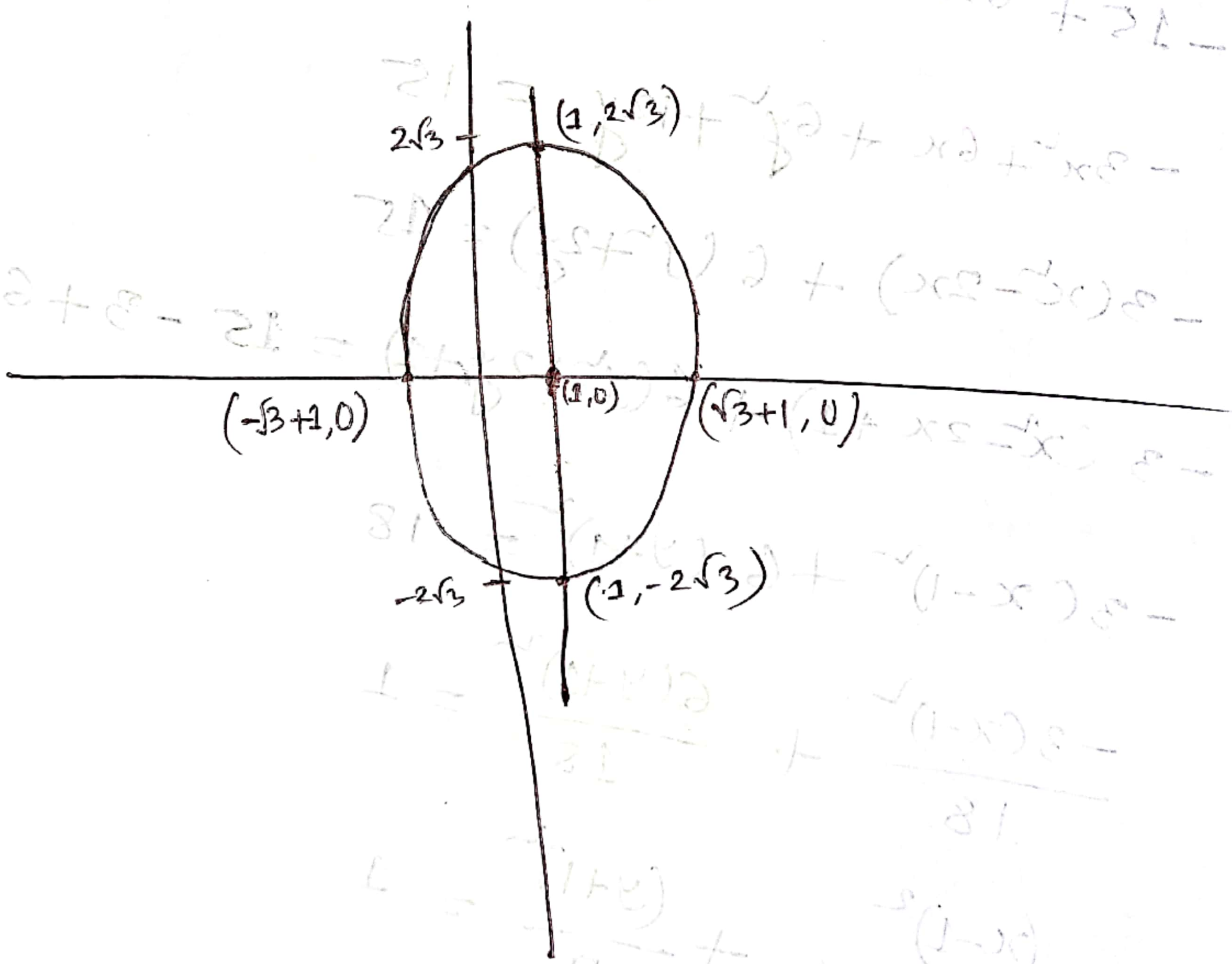
$= \sqrt{1 - \frac{3}{12}}$

$= \sqrt{\frac{12-3}{12}}$

$= \sqrt{\frac{9}{12}}$

$= \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

Vertices $(1, \pm 2\sqrt{3})$



(3)

The given equation,

$$-15 + 6x - 3x^2 + 12y + 6y^2 = 0$$

$$\Rightarrow -3x^2 + 6x + 6y^2 + 12y = 15$$

$$\Rightarrow -3(x^2 - 2x) + 6(y^2 + 2y) = 15$$

$$\Rightarrow -3(x^2 - 2x + 1) + 6(y^2 + 2y + 1) = 15 - 3 + 6$$

$$\Rightarrow -3(x-1)^2 + 6(y+1)^2 = 18$$

$$\Rightarrow \frac{-3(x-1)^2}{18} + \frac{6(y+1)^2}{18} = 1$$

$$\Rightarrow \frac{(x-1)^2}{-6} + \frac{(y+1)^2}{3} = 1$$

$$\Rightarrow \frac{(y+1)^2}{(\sqrt{3})^2} - \frac{(x-1)^2}{(\sqrt{6})^2} = 1$$

This is the standard form of hyperbola

Now,

$x-1=0$ and $y+1=0$

$\therefore x=1$ $\therefore y=-1$

Therefore, center $(1,-1)$

$$\text{ecentricity} = \sqrt{1+\frac{a^2}{b^2}}$$

$$= \sqrt{1+\frac{6}{3}}$$

$$= \sqrt{1+2}$$

$$= \sqrt{3} \quad \text{(Ans)}$$

~~Vertices $(1, -1+\sqrt{3})$ and $(1, -1-\sqrt{3})$~~

~~(Ans)~~

Again, for vertices

$x-1=0$ $y+1=\pm\sqrt{3}$

$x=1$ $y=-1\pm\sqrt{3}$

vertices $(1, -1+\sqrt{3})$ and $(1, -1-\sqrt{3})$

(Ans)

for foci,

$x-1=0$ and $y+1=\pm be$

$\therefore x=1$ $\Rightarrow y+1=\pm\sqrt{3}\times\sqrt{3}$

$\Rightarrow y=-1\pm 3$

$\therefore$ therefore, foci $(1,2)$ and $(1,-4)$

equation of the directrix

$\because y+1=\pm\frac{b}{e}$

$\Rightarrow y+1=\pm\frac{\sqrt{3}}{\sqrt{3}}$

$\Rightarrow y+1=\pm 1$

$\Rightarrow y=\pm 1-1$

$\therefore y=0$ and $y=-2$

(Ans)

(7)

The given equation,

$y^2 - 2y = 8x - 1$

$\Rightarrow y^2 - 2y + 1 = 8x - 1 + 1$

$\Rightarrow (y-1)^2 = 8x$

$\Rightarrow (y-1)^2 = 4.2x$

This is the standard form of the equation of parabola

For vertex, $x = 0$, $y - 1 = 0$
$y = 1$

Therefore, vertex $(0, 1)$

here, $a = 2$

for focus

$x = 2$ and $y - 1 = 0$

$\Rightarrow y = 1$

therefore, focus $(2, 1)$

equation of the directrix,

$x = -2$

$\therefore x + 2 = 0$ (Ans)

(8)

The given equation,

$$-x^2+4y^2-2x-16y+11=0$$

$$\Rightarrow -(x^2+2x)+4(y^2-4y)=-11$$

$$\Rightarrow -(x^2+2x+1)+4(y^2-4y+4)=-11-1+16$$

$$\Rightarrow -(x+1)^2+4(y-2)^2=4$$

$$\Rightarrow -\frac{(x+1)^2}{4}+\frac{4(y-2)^2}{4}=1$$

$$\Rightarrow -\frac{(x+1)^2}{4}+(y-2)^2=1$$

$$\therefore \frac{(y-2)^2}{1^2}-\frac{(x+1)^2}{4}=1$$

This is the standard form of the equation of the hyperbola.

for center,

$x+1=0$ and $y-2=0$

$\therefore x=-1$ $y=2$

$\therefore$ center$(-1,2)$

for vertices,

$x+1=0$

$\Rightarrow x=-1$

and,

$y-2=\pm 1$

$\Rightarrow y=\pm 1+2$

$\therefore y=3,1$

therefore, verticess $(-1,3)$ an $(-1,1)$

~~for foci~~

$\cancel{x+1=}$

eccentricity, $e=\sqrt{1+\frac{a^2}{b^2}}$

$=\sqrt{1+\frac{4}{1}}$

$=\sqrt{5}$

for foci,

$x+1=0$

$\therefore x=-1$

and $y-2=\pm 1\times\sqrt{5}$

$\Rightarrow y=\pm\sqrt{5}+2$

therefore foci $(-1,\sqrt{5}+2)$ and $(-1,-\sqrt{5}+2)$

(Ans)

(9)

The given equation,

$$x^2+4x-4y=0$$

$$\Rightarrow x^2+4x = 4y$$

$$\Rightarrow x^2+4x+4 = 4y+4$$

$$\Rightarrow (x+2)^2 = 4(y+1)$$

This is the standard form of the equation of parabola.

$\therefore$ vertex $(-2,-1)$

for focus

$$x+2=0$$

$$\Rightarrow x=-2$$

and

$$y+1=1$$

$$y=0$$

$\therefore$ focus $(-2,0)$

equation of directrix,

$$y+1=-1$$

$$\Rightarrow y=-1-1$$

$$\Rightarrow y=-2$$

$$\Rightarrow y+2=0$$ (Ans)

(10)

The given equation,

$$-4x^2+y^2-16x-2y-19=0$$

$$\Rightarrow -4(x^2+4x)+(y^2-2y)=19$$

$$\Rightarrow -4(x^2+4x+4)+(y^2-2y+1)=19-16+1$$

$$\Rightarrow -4(x+2)^2+(y-1)^2=4$$

$$\Rightarrow -\frac{(x+2)^2}{1^2}+\frac{(y-1)^2}{2^2}=1$$

$$\therefore \frac{(y-1)^2}{2^2}-\frac{(x+2)^2}{1^2}=1$$

This is the standard form of the equation of hyperbola.

$\therefore$ center$(-2, 1)$

for vertices,

$x+2=0$

$\Rightarrow x=-2$

and $y-1=\pm 2$

$\Rightarrow y=\pm 2+1$

$\therefore$ vertices $(-2, 3)$ and $(-2, -1)$

(10)

for foci,

$x+2=0$

$x=-2$

and $y-1 = \pm 2 \times \sqrt{1+\frac{1}{4}}$

$\Rightarrow y-1 = \pm 2 \times \frac{\sqrt{5}}{2}$

$\Rightarrow y-1 = \pm\sqrt{5}$

$\therefore y = \pm\sqrt{5}+1$

Therefore, foci $(-2, \sqrt{5}+1)$ and $(-2, -\sqrt{5}+1)$

(11)

The given equation,

$y^2+12y = 1-x$

$\Rightarrow y^2+2y.6+36 = 1-x+36$

$\Rightarrow (y+6)^2 = 37-x$

$\Rightarrow (y+6)^2 = -(x-37)$

~~This is the standard~~ form of the equation of the parabola.

~~vertex (37, -6)~~

$\Rightarrow (y+6)^2 = -4.\frac{1}{4}(x-37)$

This is the standard form of the equation of parabola.

vertex $(37, -6)$

for focus,

$x - 37 = -\frac{1}{4}$ and $y + 6 = 0$

$y = -6$

$\Rightarrow x = -\frac{1}{4} + 37$

$\therefore \cancel{x = 36.75}$

$\therefore x = \frac{147}{4}$

therefore, focus $\left(\frac{147}{4}, -6\right)$

the equation of directrix

$x - 37 = \frac{1}{4}$

$\Rightarrow x = 37 + \frac{1}{4}$

$\Rightarrow x = \frac{149}{4}$

$\Rightarrow 4x - 149 = 0$ (Ans)

12

The given equation,

$$-x^2+2y^2+2x+8y+3=0$$

$$\Rightarrow -(x^2-2x)+2(y^2+4y)=-3$$

$$\Rightarrow -(x^2-2x+1)+2(y^2+4y+4)=-3-1+8$$

$$\Rightarrow -(x-1)^2+2(y+2)^2=4$$

$$\Rightarrow -\frac{(x-1)^2}{4}+\frac{(y+2)^2}{2}=1$$

$$\Rightarrow \frac{(y+2)^2}{(\sqrt{2})^2}-\frac{(x-1)^2}{2^2}=1$$

This is the standard form of the equation of hyperbola.

$\therefore$ center$(1,-2)$

For vertices,

$x-1=0$ and $y+2=\pm\sqrt{2}$

$\therefore x=1$ $\therefore y=\pm\sqrt{2}-2$

$\therefore$ vertices $(1, \pm\sqrt{2}-2)$ (Ans) for foci,

$x-1=0$

$\therefore x=1$

and $y+2=\pm\sqrt{2}\times\sqrt{1+\frac{4}{2}}$

$\Rightarrow y+2=\pm\sqrt{2}\times\frac{\sqrt{6}}{\sqrt{2}}$

$\therefore y+2=\pm\sqrt{6}$

$\therefore y=\pm\sqrt{6}-2$

$\therefore$ foci $(1, \pm\sqrt{6}-2)$ (Ans)

s·notes

(Ans)

notes

$(r, \theta, z) \to (x, y, z)$

$x = r\cos\theta$
$y = r\sin\theta$
$z = z$

(5)

Given the point $\left(\frac{4}{5}, \frac{2\pi}{3}, -2\right)$ which is in cylindroical co-ordinates

Now,

$$x = r\cos\theta$$
$$= \frac{4}{5}\cos\frac{2\pi}{3}$$
$$= \frac{4}{5} \times \left(-\frac{1}{2}\right)$$
$$\therefore x = -\frac{2}{5}$$

$$y = r\sin\theta$$
$$= \frac{4}{5}\sin\frac{2\pi}{3}$$
$$= \frac{4}{5} \times \frac{\sqrt{3}}{2}$$
$$\therefore = \frac{2\sqrt{3}}{5}$$

and $z = -2$

Therefore, the rectangular co-ordinates of the given point $\left(-\frac{2}{5}, \frac{2\sqrt{3}}{5}, -2\right)$ (Ans)

Notes
$(\rho, \theta, \phi) \to (x, y, z)$
$x = \rho \sin\phi \cos\theta$
$y = \rho \sin\phi \sin\theta$
$z = \rho \cos\phi$

(6)

The given point $\left(4, \frac{3\pi}{4}, \frac{\pi}{4}\right)$ which is in spherical coordinates.

Now, $x = \rho \sin\phi \cos\theta$

$= 4 \sin\frac{\pi}{4} \cos\frac{3\pi}{4}$

$= 4 \times \frac{1}{\sqrt{2}} \times \left(-\frac{1}{\sqrt{2}}\right)$

$= -2$

$y = \rho \sin\phi \sin\theta$

$= 4 \times \sin\frac{\pi}{4} . \sin\frac{3\pi}{4}$

$= 4 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

$= 2$

$z = \rho \cos\phi$

$= 4 \cos\frac{\pi}{4}$

$= 4 \times \frac{1}{\sqrt{2}}$

$= 2\sqrt{2}$

Therefore, the rectangular coordinates of the given point $(-2, 2, 2\sqrt{2})$ (Ans)

(4)

(a) The given equation,

$$r = \frac{12}{5 - 8\cos\theta}$$

$$= \frac{\frac{12}{5}}{1 - \frac{8}{5}\cos\theta}$$

Therefore, eccentricity $e = \frac{8}{5}$

(b) since, the eccentricity $e = \frac{8}{5} > 1$ thus, the conic is a hyperbola. (Ans)

(c) here, $ed = \frac{12}{5}$

$$\Rightarrow \frac{8}{5} d = \frac{12}{5}$$

$$\Rightarrow d = \frac{12}{8}$$

$$\therefore d = \frac{3}{2} \quad \text{(Ans)}$$

(d)

eccentricity $e = 8/5$ and directrix $3/2$ unit: