(1)

The given equation,

$$5+6x+x^2-2y=0$$

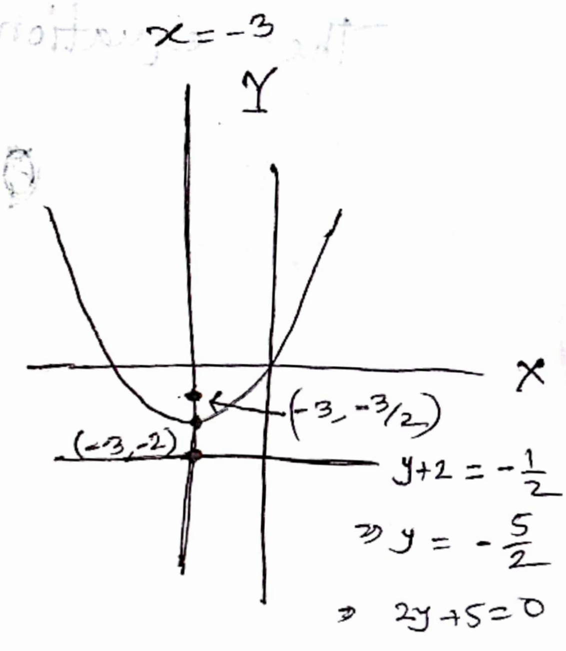
$$\Rightarrow x^2+6x=2y-5$$

$$\Rightarrow x^2+2x.3+3^2=2y-5+9$$

$$\Rightarrow (x+3)^2=2y+4$$

$$\Rightarrow (x+3)^2=2(y+2)$$

$$\Rightarrow (x+3)^2=4.\tfrac{1}{2}(y+2)$$



This is the standard form of the equation of the parabola.

Now,

$x+3=0$ and $y+2=0$

$\Rightarrow x=-3$ $\quad y=-2$

therefore, vertex $(-3,-2)$ (Ans)

focus $(-3,-2+\frac{1}{2})$

$\Rightarrow (-3,-\frac{3}{2})$ (Ans)

The equation of the directrix

$$y+2=-\frac{1}{2}$$

$$\Rightarrow y=-2-\frac{1}{2}$$

$$\Rightarrow y=\frac{-5}{2}$$

$$\Rightarrow 2y+5=0 \text{ (Ans)}$$

(2)

The given equation,

$$-24-24x+12x^2+3y^2=0$$

$$\Rightarrow 12x^2-24x+3y^2=24$$

$$\Rightarrow 12(x^2-2x)+3y^2=24$$

$$\Rightarrow 4(x^2-2x)+y^2=8$$

$$\Rightarrow 4(x^2-2x+1)+y^2=8+4$$

$$\Rightarrow 4(x-1)^2+y^2=12$$

$$\Rightarrow \frac{(x-1)^2}{3}+\frac{y^2}{12}=1$$

$$\Rightarrow \frac{(x-1)^2}{(\sqrt{3})^2}+\frac{y^2}{(2\sqrt{3})^2}=1 \quad \text{——— (1)}$$

This is the standard formed of the equation of ellipse.

From equation (1),

$x - 1 = 0$ and $y = 0$

$\therefore x = 1$

Therefore, centre $(1, 0)$

$a = \sqrt{3}$ and $b = 2\sqrt{3}$

here, $b > a$

Therefore Y - is major axis.

~~Vertices are~~

eccentricity, $e = \sqrt{1 - \frac{a^2}{b^2}}$

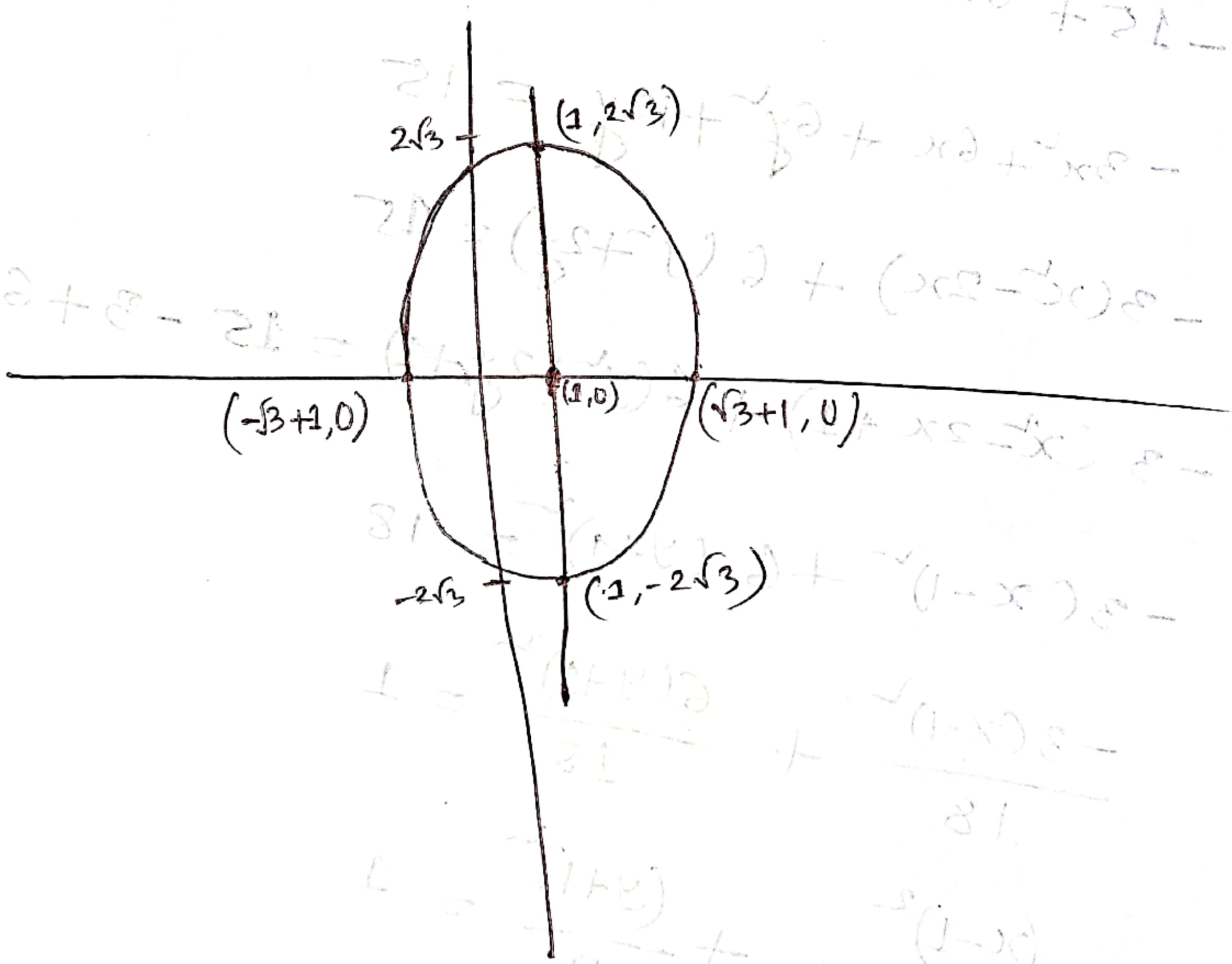
$= \sqrt{1 - \frac{3}{12}}$

$= \sqrt{\frac{12 - 3}{12}}$

$= \sqrt{\frac{9}{12}}$

$= \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

Vertices $(1, \pm 2\sqrt{3})$



(3)

The given equation,

$$-15 + 6x - 3x^2 + 12y + 6y^2 = 0$$

$$\Rightarrow -3x^2 + 6x + 6y^2 + 12y = 15$$

$$\Rightarrow -3(x^2 - 2x) + 6(y^2 + 2y) = 15$$

$$\Rightarrow -3(x^2 - 2x + 1) + 6(y^2 + 2y + 1) = 15 - 3 + 6$$

$$\Rightarrow -3(x-1)^2 + 6(y+1)^2 = 18$$

$$\Rightarrow \frac{-3(x-1)^2}{18} + \frac{6(y+1)^2}{18} = 1$$

$$\Rightarrow \frac{(x-1)^2}{-6} + \frac{(y+1)^2}{3} = 1$$

$$\Rightarrow \frac{(y+1)^2}{(\sqrt{3})^2} - \frac{(x-1)^2}{(\sqrt{6})^2} = 1$$

This is the standard form of hyperbola

Now,

$x-1=0$ and $y+1=0$

$\therefore x=1$ $\therefore y=-1$

Therefore, center $(1,-1)$

$$\text{ecentricity} = \sqrt{1+\frac{a^2}{b^2}}$$

$$= \sqrt{1+\frac{6}{3}}$$

$$= \sqrt{1+2}$$

$$= \sqrt{3} \quad \text{(Ans)}$$

~~Vertices $(1, -1+\sqrt{3})$ and $(1, -1-\sqrt{3})$~~

~~(Ans)~~

Again, for vertices

$x-1=0$ $y+1=\pm\sqrt{3}$

$x=1$ $y=-1\pm\sqrt{3}$

vertices $(1,-1+\sqrt{3})$ and $(1,-1-\sqrt{3})$

(Ans)

for foci,

$x-1=0$ and $y+1=\pm be$

$\therefore x=1$ $\Rightarrow y+1=\pm\sqrt{3}\times\sqrt{3}$

$\Rightarrow y=-1\pm 3$

$\therefore$ therefore, foci $(1,2)$ and $(1,-4)$

equation of the directrix

$\therefore y+1=\pm\frac{b}{e}$

$\Rightarrow y+1=\pm\frac{\sqrt{3}}{\sqrt{3}}$

$\Rightarrow y+1=\pm 1$

$\Rightarrow y=\pm 1-1$

$\therefore y=0$ and $y=-2$

(Ans)

(4)

(a) The given equation,

$$r = \frac{12}{5 - 8\cos\theta}$$

$$= \frac{\frac{12}{5}}{1 - \frac{8}{5}\cos\theta}$$

Therefore, eccentricity $e = \frac{8}{5}$

(b) since, the eccentricity $e = \frac{8}{5} > 1$

thus, the conic is a hyperbola. (Ans)

(c) here, $ed = \frac{12}{5}$

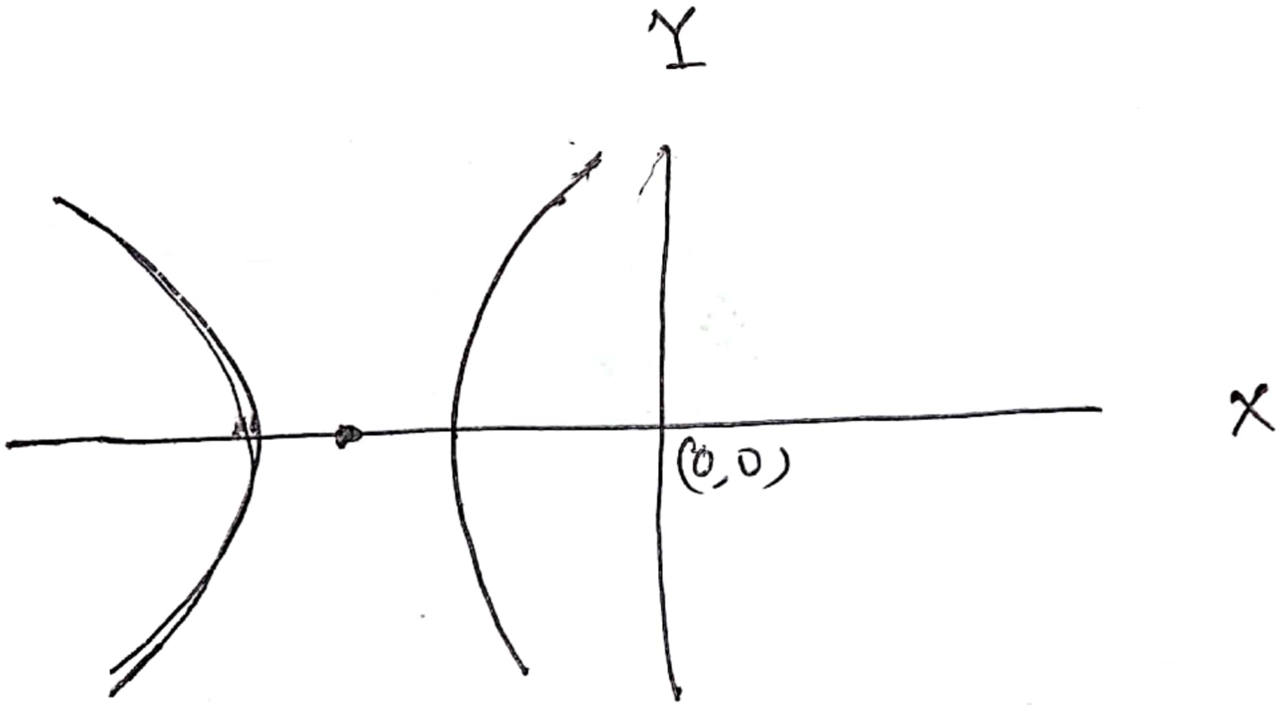
$$\Rightarrow \frac{8}{5} d = \frac{12}{5}$$

$$\Rightarrow d = \frac{12}{8}$$

$$\therefore d = \frac{3}{2}$$ (Ans)

(d)

eccentricity $e = 8/5$ and directrix $3/2$ unit:



(5)

Given the point $\left(\frac{4}{5}, \frac{2\pi}{3}, -2\right)$

which is in cylindrical co-ordinates

Now,
$$x = r\cos\theta$$
$$= \frac{4}{5}\cos\frac{2\pi}{3}$$
$$= \frac{4}{5} \times \left(-\frac{1}{2}\right)$$
$$\therefore x = -\frac{2}{5}$$

$$y = r\sin\theta$$
$$= \frac{4}{5}\sin\frac{2\pi}{3}$$
$$= \frac{4}{5} \times \frac{\sqrt{3}}{2}$$
$$= \frac{2\sqrt{3}}{5}$$

and $z = -2$

Therefore, the rectangular co-ordinates of the given point $\left(-\frac{2}{5}, \frac{2\sqrt{3}}{5}, -2\right)$ (Ans)

Notes

$(\rho, \theta, \phi) \to (x, y, z)$

$x = \rho \sin\phi \cos\theta$

$y = \rho \sin\phi \sin\theta$

$z = \rho \cos\phi$

(6)

The given point $\left(4, \frac{3\pi}{4}, \frac{\pi}{4}\right)$ which is in spherical coordinates.

Now,

$$x = \rho \sin\phi \cos\theta$$

$$= 4 \sin\frac{\pi}{4} \cos\frac{3\pi}{4}$$

$$= 4 \times \frac{1}{\sqrt{2}} \times \left(-\frac{1}{\sqrt{2}}\right)$$

$$= -2$$

$$y = \rho \sin\phi \sin\theta$$

$$= 4 \times \sin\frac{\pi}{4} . \sin\frac{3\pi}{4}$$

$$= 4 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$= 2$$

$$z = \rho \cos\phi$$

$$= 4 \cos\frac{\pi}{4}$$

$$= 4 \times \frac{1}{\sqrt{2}}$$

$$= 2\sqrt{2}$$

Therefore, the rectangular coordinates of the given point $(-2, 2, 2\sqrt{2})$ (Ans)