BRAC UNIVERSITY

Inspiring Excellence

MAT110

Assignment 3

SET:20

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Section:07

Assignment 2

Answer - 1

Given,

Let $-\frac{v^2}{c^2} = x$

$$\therefore \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1+x}} = (x+1)^{-1/2}$$

From Maclaurin expansion

$$(1+x)^{-1/2} = 1 - \frac{1}{2}x + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}x^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}x^3 + \ldots$$

$$= 1 - \frac{x}{2} + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \ldots$$

$$= 1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \frac{5}{16}\frac{v^6}{c^6} + \ldots \quad \left[\text{Replacing } x \text{ with } -\frac{v^2}{c^2}\right].$$

Now, kinetic energy, $E = mc^2(\gamma - 1)$

$$= mc^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$$

$$= mc^2\left(\cancel{1} + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \frac{5}{16}\frac{v^6}{c^6} + \dots - \cancel{1}\right)$$

$$= mc^2\left(\frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \frac{5}{16}\frac{v^6}{c^6} + \dots\right)$$

As $v \ll c$ so neglecting higher order of $\frac{v}{c}$, we get,

$$E \simeq m\cancel{c^2} \times \frac{1}{2}\frac{v^2}{\cancel{c^2}}$$

$$\simeq \frac{1}{2}mv^2 \text{ (showed)}$$

Answer 2

$$E = \frac{q}{D^2} - \frac{q}{(D+d)^2}$$

$$= \frac{q}{D^2} - \frac{q}{D^2\left(1+\frac{d}{D}\right)^2}$$

$$= \frac{q}{D^2} - \frac{q}{D^2\left(1-\frac{d}{D}\right)^2}$$

$$= \frac{q}{D^2}\left\{1-\left(1-2\frac{d}{D}+3\frac{d^2}{D^2}-\frac{4d^3}{D^3}+5\frac{d^4}{D^4}\dots\right)\right\}$$

$$= \frac{q}{D^2}\left(2\frac{d}{D}-3\frac{d^2}{D^2}+4\frac{d^3}{D^3}-5\frac{d^4}{D^4}+\dots\right)$$

If P is far away from the dipole then $D >> d$. So neglecting higher order of $\frac{d}{D}$ we get_

$$E \approx \frac{2}{D^2} \cdot 2\frac{d}{D}$$

$$\approx 22d\frac{1}{D^3}$$

As 2, 2 and d are constant

$\therefore$ E is approximately proportional to $\frac{1}{D^3}$.

Answer 3

$f(x) = x^{1/4}$ $\therefore$ $f(10) = 10^{1/4}$

$f'(x) = \frac{1}{4}x^{-\frac{3}{4}}$ $\therefore$ $f'(10) = \frac{1}{4}(10)^{-3/4}$

$f''(x) = -\frac{3}{16}x^{-\frac{7}{4}}$ $\therefore$ $f''(10) = -\frac{3}{16}(10)^{-\frac{7}{4}}$

Now two-degree Taylor series of $f(x)$ at $x = 10$,

$$f(x) = f(10) + f'(10)(x-10) + \frac{f''(10)}{2!}(x-10)^2$$

$$= (10)^{1/4} + \frac{1}{4}(10)^{-\frac{3}{4}}(x-10) - \frac{3}{16}(10)^{-\frac{7}{4}}(9.9-10)^2$$

$$= 1.773800369$$

when $x = 8$, then,

$$f(x) = (10)^{\frac{1}{4}} + \frac{1}{4}(10)^{-\frac{3}{4}}(8-10) - \frac{3}{16}(10)^{-\frac{7}{4}}(x-10)^2$$

$$= 1.67602834 .$$

Using calculator, $9.9^{\frac{1}{4}} = 1.773816942$.

$8^{\frac{1}{4}} = 1.681792831$

We see that values obtained from series is slightly different from original values obtained by using calculator. We can also see that it flactuates more when x is far from 10. So our Taylor series is ~~totally~~ accurate at $x=10$ and accuracy decreases when x is further from 10.

Answer 4

$x = t^2 \quad y = \cos t \quad z = \tan t \quad v = x e^{y/z}$

$\Rightarrow \frac{dx}{dt} = 2t \Rightarrow \frac{dy}{dt} = -\sin t \Rightarrow \frac{dz}{dt} = \sec^2 t$

Now,

$$\frac{dv}{dt} = \frac{\partial v}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial v}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial v}{\partial z} \cdot \frac{dz}{dt}$$

$$= e^{y/z} \cdot 1 \cdot 2t + x e^{y/z} \frac{1}{z} \cdot 1(-\sin t) + x e^{y/z} \cdot y(-1) z^{-2} \cdot \sec^2 t$$

$$= 2te^{y/z} - \frac{x}{z} e^{\frac{y}{z}} \sin t - xy \cdot \frac{1}{z^2} e^{\frac{y}{z}} \sec^2 t$$

$$= 2t \cdot e^{\cos t / \tan t} - \frac{t^2}{\tan t} e^{\cos t / \tan t} \sin t - t^2 \cos t \cdot \frac{1}{\tan^2 t} e^{\cos t / \tan t} \sec^2 t$$

$$= 2te^{\cos t \cdot \cot t} - t^2 \cos t \, e^{\cos t \cdot \cot t}$$

$$\frac{t^3}{\cos t \cdot \tan^2 t} \quad e^{\cos t \cdot \cot t}$$

Answer 5

$r = \ln(2x+3y)$, where $x = s\sin t$, $y = t\cos s$

$$\therefore \frac{\partial r}{\partial s} = \frac{\partial r}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial r}{\partial y}\frac{\partial y}{\partial s}$$

$$= \left(\frac{1}{2x+3y}\cdot 2\right)(\sin t) + \left(\frac{1}{2x+3y}\cdot 3\right)\left(t(-\sin s)\right)$$

$$= \frac{2\sin t}{2s\sin t + 3t\cos s} - \frac{3t\sin s}{2s\sin t + 3t\cos s}$$

$$\therefore \frac{\partial r}{\partial t} = \frac{\partial r}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial r}{\partial y}\frac{\partial y}{\partial t}$$

$$= \frac{2}{2x+3y} \cdot s\cos t + \frac{3}{2x+3y} \cos s$$

$$= \frac{2s\cos t}{2s\sin t + 3t\cos s} + \frac{3\cos s}{2s\sin t + 3t\cos s}$$

Answer 6

$$p'(t) = \frac{dp}{dt} = f_x(1,7)g'(2) + f_y(1,7)h'(2)$$

$$= 2\cdot(-3) + 8\cdot 8$$

$$= 58 \text{ Ans.}$$