Assingment-1

Sunday, August 22, 2021 9:18 PM

7. Determine the 1st and 2nd degree Taylor polynomials L(x,y) and Q(x,y) for $f(x,y) = x^2y + y^2$ for (x,y) near the point (1,3).

1st degree.

$$L(x,y) = f(a,b) + \underline{f_x(a,b)}(x-a) + f_y(a,b)(y-b)$$

$$(a,b) = (1,3)$$

$$f(x,y) = x^2y + y^2$$

$$f_x(x,y) = \frac{\partial}{\partial x}(x^2y+y^2) \qquad f_x(1,3) = 2.1.3$$

$$= y2x + 0 \qquad = \underline{6}.$$

$$= \underline{2xy}$$

$$f_y(x,y) = \frac{\partial}{\partial y}(x^2y+y^2) \qquad f_y(1,3) = 1^2+2.3$$

$$= x^2.1 + 2y \qquad = 7$$

$$= x^2+2y$$

$$f(1,3) = 1^2.3+3^2 = 12$$

$$L(x,y) = 12 + 6.(x-1) + 7(y-3)$$

$$= 12 + 6x - 6 + 7y - 21$$

$$= 6x + 7y - 15$$

2nd degree!

$$Q(x,y) = L(x,y) + \frac{f_{xx}(a,b)}{2}(x-a)^2 + f_{xy}(a,b)(x-a)(y-b) + \frac{f_{yy}(a,b)}{2}(y-b)^2$$

$$f_x = 2xy$$

$$f_{xx} = \frac{\partial}{\partial x}(2xy)$$

$$= 2y.1 \qquad f_{xx}(1,3) = 2.3$$

$$= 2y \cdot 1$$
$$= 2y$$

$f_{xx}(1,3) = 2.3$
$= 6$

$f_y = x^2 + 2y$

$$f_{yy} = \frac{\partial}{\partial y}(x^2+2y)$$
$$= 0 + 2.1$$
$$= 2$$

$f_{yy}(1,3) = 2$

$f_x = 2xy$

$$f_{xy} = \frac{\partial}{\partial y}(2xy)$$
$$= 2x \cdot 1$$
$$= 2x$$

$f_{xy}(1,3) = 2.1$
$= 2$

$$Q(x,y) = 6x + 7y - 15 + \frac{6}{2}(x-1)^2 + 2 \cdot (x-1)(y-3) + \frac{2}{2} \cdot (y-3)^2$$

$$= 6x + 7y - 15 + 3(x-1)^2 + 2(x-1)(y-3) + (y-3)^2$$

Ans

8. Determine the 1st and 2nd degree Taylor polynomials L(x,y) and Q(x,y) for $f(x, y) = \ln(x^2 + y^2 + 1)$ for (x,y) near the point (0,0).

$$f_x = \frac{\partial}{\partial x}\left\{\ln(x^2+y^2+1)\right\}$$
$$= \frac{1}{x^2+y^2+1} \cdot 2x$$
$$= \frac{2x}{x^2+y^2+1}$$

$f_x(0,0) = 0.$

$$f_y = \frac{1}{x^2+y^2+1} \cdot 2y$$

$f_y(0,0) = 0.$

$$f(0,0) = \ln(0^2+0^2+1)$$

$$f(0,0) = \ln(0^2+0^2+1)$$
$$= \ln 1$$
$$= 0$$

$$L(x) = 0 + 0.(x-0) + 0.(y-0)$$
$$= 0.$$

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{2x}{x^2+y^2+1}\right) \qquad \left[\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}\right]$$

$$= \frac{2(x^2+y^2+1) - 2x.(2x)}{(x^2+y^2+1)^2}$$

$$f_{xx}(0,0) = \frac{2(0+0+1) - 0}{(0+0+1)^2} = 2$$

$$f_{yy} = \frac{2(x^2+y^2+1) - 2y(2y)}{(x^2+y^2+1)^2}$$

$$f_{yy}(0,0) = 2.$$

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{2x}{x^2+y^2+1}\right)$$

$$= 2x\frac{\partial}{\partial y}\left(\frac{1}{x^2+y^2+1}\right)$$

$$= 2x. -1 \frac{1}{(x^2+y^2+1)^2} .2y$$

$$= -\frac{4xy}{(x^2+y^2+1)^2}$$

Rough

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dy}\left(\frac{1}{2+y^2}\right)$$

$$= \frac{d}{dy}\left[(2+y^2)^{-1}\right]$$

$$= -1\,(2+y^2)^{-1-1}$$

$$= -1$$

$$f_{xy}(0,0) = 0.$$

$$Q(x,y) = 0 + \frac{2}{2}(x-0)^2 + 0.(x-0)(y-0) + \frac{2}{2}(y-0)^2$$

$$Q(x,y) = x^2+y^2 \text{ (Ans).}$$

9. Locate all relative maxima, relative minima and saddle points (if any) for $f(x,y) = x^2 + y^2 + \frac{2}{xy}$

$= \frac{1}{x}$

$$f_x = 2x + \frac{2}{y} \cdot \left(-\frac{1}{x^2}\right)$$

$$f_x = 2x - \frac{2}{x^2 y}$$

$f_x = 0.$

$$2x - \frac{2}{x^2 y} = 0.$$

$$\Rightarrow x = \frac{1}{x^2 y}$$

$$\Rightarrow x^3 = \frac{1}{y}$$

$$\Rightarrow y = \frac{1}{x^3} \quad \text{—(i)}$$

$$f_y = 2y - \frac{2}{xy^2}$$

$f_y = 0$

$$2y - \frac{2}{xy^2} = 0$$

from (i) we get

$$2 \cdot \frac{1}{x^3} - \frac{2}{x \cdot \left(\frac{1}{x^3}\right)^2} = 0.$$

$$\frac{2}{x^3} - \frac{2}{x \cdot \frac{1}{x^6}} = 0.$$

$$\frac{1}{x^3} - x^5 = 0.$$

$$\Rightarrow \frac{1 - x^8}{x^3} = 0.$$

$$\Rightarrow 1 - x^8 = 0.$$

$$\Rightarrow x^8 = 1$$

$$\Rightarrow x = \pm 1.$$

$$\therefore y = \frac{1}{\pm 1} = \pm 1$$

$$(x,y) = (1,1), (-1,-1)$$

$$f_{xx} = \frac{\partial}{\partial x}\left(2x - \frac{2}{x^2 y}\right)$$

$$= 2.1 - \frac{2}{y}(-2)\frac{1}{x^3}$$

$$= 2 + \frac{4}{x^3 y}$$

$$f_{yy} = \frac{\partial}{\partial y}\left(2y - \frac{2}{xy^2}\right)$$

$$= 2 - \frac{2}{x} \cdot (-2) \frac{1}{y^3}$$

$$= 2 + \frac{4}{xy^3}$$

$$f_{xy} = \frac{\partial}{\partial y}\left(2x - \frac{2}{x^2 y}\right)$$

$$= 0 - \frac{2}{x^2} \cdot \frac{-1}{y^2}$$

$$= \frac{2}{x^2 y^2}$$

$$D = f_{xx} + f_{yy} - f_{xy}^2$$

$$= 2 + \frac{4}{x^3 y} + 2 + \frac{4}{xy^3} - \frac{4}{x^4 y^4}$$

In $(1,1)$,

$$D = 2 + \frac{4}{1^3 . 1} + 2 + \frac{4}{1 . 1^3} - \frac{1}{1^4 . 1^4}$$

$$= 2 + 4 + 2 + 4 - 1$$
$$= 11$$

at, $(1,1)$

$$f_{xx} = 2 + \frac{4}{x^3 y}$$

$$= 2 + \frac{4}{1^3 . 1}$$
$$= 2 + 4 = 6$$

$\therefore (1,1)$ is a relative minimum.

(Ans).

At $(-1,-1)$,

$$D = 2 + \frac{4}{(-1)^3 . (-1)} + 2 + \frac{4}{(-1)^3 . (-1)} - \frac{1}{(-1)^3 (-1)^3}$$

$= 2 + \quad 4 + 2 + \quad 4 \quad - 1$

$= 11$

At $(-1, -1)$

$f_{xx} = 2 + \dfrac{4}{x^3 y} = 2 + \dfrac{4}{(-1)^3(-1)}$

$= 2 + 4$

$= 6.$

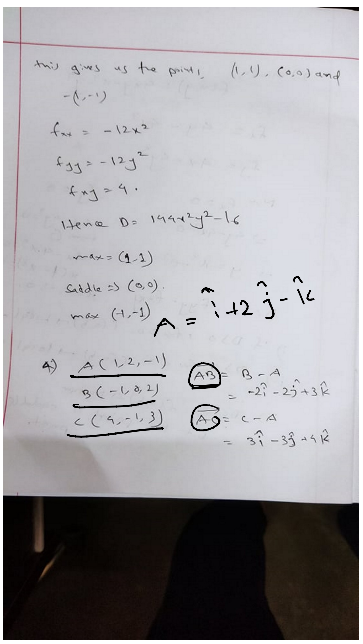
$\therefore f(1, -1)$ is relative minimum.

An

10. Locate all relative maxima, relative minima and saddle points (if any) for $f(x, y) = xy + \dfrac{2}{x} + \dfrac{4}{y}$.

I will send.

4).



this gives us the points $(1,1)$, $(0,0)$ and $-(1,-1)$

$f_{xx} = -12x^2$

$f_{yy} = -12y^2$

$f_{xy} = 4.$

Hence $D = 144x^2y^2 - 16$

max $= (1,1)$

Saddle $\Rightarrow (0,0)$.

max $(-1,-1)$

$A = \hat{i} + 2\hat{j} - \hat{k}$

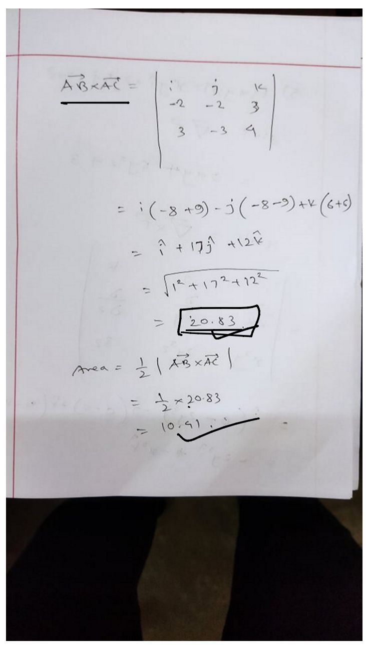
4) $A(1, 2, -1)$

$B(-1, 0, 2)$

$C(4, -1, 3)$

$AB = B - A$
$= -2\hat{i} - 2\hat{j} + 3\hat{k}$

$AC = C - A$
$= 3\hat{i} - 3\hat{j} + 4\hat{k}$



$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ -2 & -2 & 3 \\ 3 & -3 & 4 \end{vmatrix}$

$= i(-8+9) - j(-8-9) + k(6+6)$

$= \hat{i} + 17\hat{j} + 12\hat{k}$

$= \sqrt{1^2 + 17^2 + 12^2}$

$= \boxed{20.83}$

Area $= \frac{1}{2} |\vec{AB} \times \vec{AC}|$

$= \frac{1}{2} \times 20.83$

$= 10.41.$

11. Compute the Divergence and Curl of the following vector $\vec{F}$
$\vec{F} = yze^{xy}\vec{i} + xze^{xy}\vec{j} + (e^{xy} + 3\cos 3z)\vec{k}$

$\nabla = \dfrac{\partial}{\partial x}\hat{i} + \dfrac{\partial}{\partial y}\hat{j} + \dfrac{\partial}{\partial z}\hat{k}$

$\text{div } F = \vec{\nabla} \cdot \vec{F}$

$$= \frac{\partial}{\partial x}\left(yze^{xy}\right) + \frac{\partial}{\partial y}\left(xze^{xy}\right) + \frac{\partial}{\partial z}\left(e^{xy} + 3\cos 3z\right)$$

$$= yze^{xy}\cdot y + zxe^{xy}\cdot x + 0 - 3\sin 3z \cdot 3$$

$$= y^2ze^{xy} + zx^2e^{xy} - 9\sin 3z$$

$$\operatorname{curl}\vec{F} = \vec{\nabla} \times \vec{F}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yze^{xy} & \underbrace{x}_{u}\underbrace{ze^{xy}}_{v} & e^{xy} + 3\cos 3z \end{vmatrix}$$

$$\boxed{\frac{d}{dx}(u\cdot v) = u\frac{dv}{dx} + v\frac{du}{dx}}$$

$$= \hat{i}\left\{\left(\cancel{xe^{xy}} + 0\right) - \cancel{xe^{xy}}\right\} - \hat{j}\left\{\left(\cancel{ye^{xy}} + 0\right) - \cancel{ye^{xy}}\right\} + \hat{k}\left\{z\left(\cancel{xe^{xy}\cdot y} + \cancel{e^{xy}\cdot 1}\right) - z\left(\cancel{ye^{xy}\cdot x} + \cancel{e^{xy}}\right)\right\}$$

$$= \hat{i}\cdot 0 - \hat{j}\cdot 0 - 0$$

$$= 0.$$

5) $F(x,y,z) = x^2y\,\hat{i} + 2y^3z\,\hat{j} + 3z\,\hat{k}$

$\operatorname{div} F = \vec{\nabla}\cdot\vec{F}$

$= 2xy + 6y^2z + 3$

$\operatorname{curl} F = \vec{\nabla}\times\vec{F}$

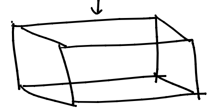
$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & 2y^3z & 3z \end{vmatrix}$$

$= \hat{i}(0 - 2y^3) - \hat{j}(0-0) + \hat{k}(0 - x^2)$

$= -2y^3\hat{i}\ \cancel{+} - x^2\hat{k}$

6. The volume V of the parallelopiped that has $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ as adjacent edges is given by: $V = \boxed{|\mathbf{u}.(\mathbf{v}\times\mathbf{w})|}$. If $\mathbf{u}.(\mathbf{v}\times\mathbf{w}) = \mathbf{0}$, then $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ lie in the same plane. Thus, find the volume of the parallelopiped formed by the followings: $\mathbf{u} =< 1, 2, 3 >, \mathbf{v} =< 4, 5, 6 >,\ \mathbf{w} =< 2, -3, 4 >$

paraallelopiped



$uv = v - u$

$= <4-1, 5-2, 6-3>$

$= <3, 3, 3>$

$uw = w - u$

$= <2-1, -3-2, 4-3>$

$= <1, -5, 1>$

$vw = w - v$

$= <2-4, -3-5, 4-6>$

$= <-2, -8, -2>$

$$\text{Volume} = \begin{vmatrix} 3 & 3 & 3 \\ -2 & -8 & -2 \\ 1 & -5 & 1 \end{vmatrix}$$

$= 3(-8-10) - 3(-2+2) + 3(10+8)$

$= 3\times -18 - 0 + 3\times 18$

$V = 0.$

10. Locate all relative maxima, relative minima and saddle points (if any) for $f(x, y) = xy + \frac{2}{x} + \frac{4}{y}$.

12. Compute the Divergence and Curl of the following vector $\vec{F} = (xyz)\,\vec{i} + y \sin z \vec{j} + (y \cos x)\,\vec{k}$