Department of Mathematics and Natural Sciences

MAT 110

ASSIGNMENT 4

SUMMER 2021

SET: 30 (SADT)

Please write your name and ID on the first page of the assignment answer script - you have to do this for both handwritten or LaTeX submission. The last date of submission is 25/8/2021, 1159 pm. Solve all problems.

You can only submit a PDF file - image or doc files won't be accepted. Before submitting the PDF, please rename the PDF file in the format - SET_ID_SECTION.

Answer the questions by yourself. Plagiarism will lead to an F grade in the course. ***Total marks is 300. Each question is worth 50 marks.*** *If you do your work using LaTeX you will get a mark which will be added as a LaTeXbonus to your course grade.*

If you use LaTeX, you must add a screenshot of the raw code and compiled pdf side by side, in order to earn your bonus.

This set was prepared by SADT. If you have any questions, please text SADT on Slack.

1. Prove that the maximum and minimum distances from the origin to the curve of intersection defined by $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ and $Ax + By + Cz = 0$ is given by the values of d which solve the following equation:

$$\frac{A^2a^2}{a^2 - d^2} + \frac{B^2b^2}{b^2 - d^2} + \frac{C^2c^2}{c^2 - d^2} = 0.$$

2. Given that
$$f(x,y,z) = y^2 e^{xyz}$$
Find the rate of greatest increase of f at $P(0,1,-1)$ and find the direction of greatest increase (unit vector).

3. If $\mathbf{A}(u)$ is a differentiable vector function of u and $||\mathbf{A}(u)|| = 1$, prove that $\frac{d\mathbf{A}}{du}$ is perpendicular to $\mathbf{A}$.

4. If $\mathbf{A}$ and $\mathbf{B}$ are vector fields, prove the following:
$$\nabla.(\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B}).$$

5. Find the third order Taylor polynomial about $(0,0)$ of $\sin(x^2 + y^2)$.

6. Find the absolute maximum and minimum values of the function $f(x,y) = x^2 - 2xy + 2y$ on the rectangle $D = \{(x,y)|0 \leq x \leq 3, 0 \leq y \leq 2\}$.