Mid Assignment (set2)

Answer 1

Given,

$$f(t) = \begin{cases} \sqrt{t-4}, & t \geq 4 \\ 8-2t, & t < 4 \end{cases}$$

Now, putting the values

$f(-10) = 8 - 2(-10) = 28$

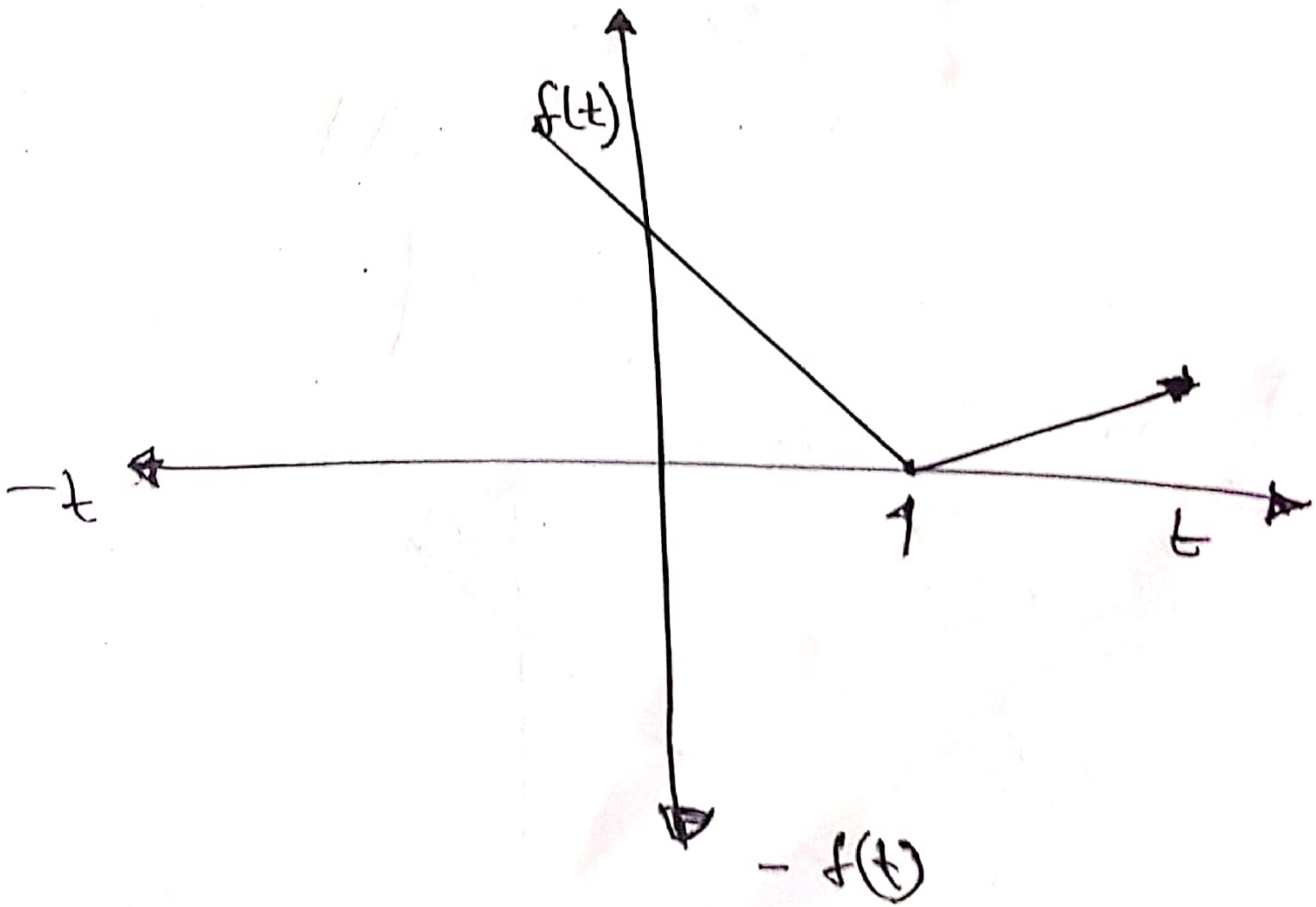
$f(1) = 8 - 2 \cdot 1 = 6$

$f(10) = \sqrt{10-4} = \sqrt{6}$

Dom $f = \mathbb{R}$

Range $f = [0, \infty)$.

Now the graph of the function



Answer 2

Given $S(t) = 2\sin(\pi t) + 3\cos(\pi t)$

Now, $s(t) = 2\sin(\pi t) + 3\cos(\pi t)$

At $t = 2s$,

$v = 2\pi\cos(2\pi) - 3\pi\sin(2\pi) = 2\pi$

$= 6.28\ ms^{-1}$ (Ans)

Answer 3

Given,

$$\lim_{t\to 1} \sin^{-1}\left(\frac{1-\sqrt{t}}{1-t}\right)$$

Now, $$\lim_{t\to 1} \sin^{-1}\left(\frac{1-\sqrt{t}}{1-t}\right)$$

$$= \lim_{t\to 1} \sin^{-1}\left(\frac{1-\sqrt{t}}{1^2-(\sqrt{t})^2}\right)$$

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$$= \lim_{t \to 1} \sin^{-1}\left(\frac{1-\sqrt{t}}{(1+\sqrt{t})(1-\sqrt{t})}\right)$$

$$= \lim_{t \to 1} \sin^{-1}\left(\frac{1}{1+\sqrt{t}}\right)$$

$$= \sin^{-1}\left(\frac{1}{1+1}\right)$$

$$= \sin^{-1}\left(\frac{1}{2}\right)$$

$$= \frac{\pi}{6}$$ (Ans).

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~~Answer to~~

Answer 4

We know–

Taylor series, $f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \dots$

$f(x) = \sin x$; $f(0) = \sin 0 = 0$, $f(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$

$f'(x) = \cos x$; $f'(0) = 1$, $f'(\frac{\pi}{3}) = \frac{1}{2}$

$f''(x) = -\sin x$; $f''(0) = 0$ $f''(\frac{\pi}{3}) = -\frac{\sqrt{3}}{2}$

$f'''(x) = -\cos x$; $f'''(0) = -1$, $f'''(\frac{\pi}{3}) = -\frac{1}{2}$.

$\therefore \sin x = 0 + 1(x-0) + 0(x-0)^2 - 1(x-0)^3$

$= 0 + x + 0 - x + \dots$

$\therefore \sin x = \frac{\sqrt{3}}{2} + \frac{1}{2}\left(x - \frac{\pi}{3}\right) - \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right)^2 - \frac{1}{2}\left(x - \frac{\pi}{3}\right)^3 + \dots$ [centered at $x = \frac{\pi}{3}$]

(Ans).

Answer 5

Given,

$x = 2\sin(t)$

$y = \cos(2t) + 2\sin t$

$\left[\text{for } -\frac{\pi}{2} \leq t \leq \frac{\pi}{2} \right]$

Now,

$x = 2\sin t$

$\Rightarrow \frac{dx}{dt} = 2\cos t$ — (I)

Again,

$y = \cos(2t) + 2\sin t$

$\Rightarrow \frac{dy}{dt} = -2\sin 2t + 2\cos t$ — (II)

Dividing (II) by (I)

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\sin 2t + 2\cos t}{2\cos t}$$

$$\Rightarrow \frac{dy}{dt} = \frac{2\cos t - 2\sin 2t}{2\cos t}$$

$$\Rightarrow \frac{dy}{dt} = \frac{2\cos t - 4\sin t\cos t}{2\cos t}$$

$$= \frac{2\cos t(1 - 2\sin t)}{2\cos t}$$

$$= 1 - 2\sin t.$$

Now at the point of stationary,

$$\frac{dy}{dt} = 0$$

$$\therefore 1 - 2\sin t = 0$$

$$\Rightarrow 2\sin t = 1$$

$$\Rightarrow t = \frac{\pi}{6} < \frac{\pi}{2}$$

$$\therefore x = 2\sin\frac{\pi}{6}$$

$$= 1$$