BRAC University

MAT092: Final Examination

Please write your name, ID, and Set on the script.
Solve ***all the problems.***
You must show your detailed work to receive the full credits.
Answer the questions by yourself. Cheating and Copying will lead to cancellation of your final examination.

Total marks: 50.

SET: A

1. (a) (15 marks) Show that $\dfrac{\sin 5\theta}{\sin \theta} = 16\cos^4\theta - 12\cos^2\theta + 1$

   (b) (10 marks) Let z be a root of $z^5 - 1 = 0$ with $z \neq 1$. Compute

   $$z^{15} + z^{16} + z^{17} + \cdots + z^{50}$$

2. (a) (15 marks) A sequence $(a_1, b_1), (a_2, b_2), (a_3, b_3), \cdots$ of points in the coordinate plane satisfies $(a_{n+1}, b_{n+1}) = \left(\sqrt{3}a_n - b_n, \sqrt{3}b_n + a_n\right)$ for $n = 1, 2, 3, \cdots$. Suppose that $(a_{100}, b_{100}) = (2, 4)$, what is $a_1 + b_1 = ?$

   (b) (10 marks) We say that, a matrix M is *skew-symmetric* if it satisfies $M^T = -M$. Show that if a 3×3 matrix M is skew-symmetric, then $\det(M) = 0$.

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SET: B

1. (a) (15 marks) Determine $(MN)^{-1}$ where $M = \begin{pmatrix} 8 & 5 \\ 4 & 1 \end{pmatrix}$ and $N = \begin{pmatrix} 5 & -1 \\ -5 & 2 \end{pmatrix}$.

   (b) (10 marks) If $\mathbf{a}, \mathbf{b}$, and $\mathbf{c}$ are three non zero vectors, such that $\mathbf{a} \times \mathbf{a} = \mathbf{a} \times \mathbf{c}$ and $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$, then it is true that $\mathbf{b} = \mathbf{c}$. Can you prove or disprove it?

2. (a) (15 marks) Sketch each of the following sets of complex numbers z that satisfy the given equations/inequalities (show your work):
   i. $|z + 3 - i| = \sqrt{2}$
   ii. $|z + 3| \leq |z + 1|$.

   (b) (10 marks) If A is a *skew-symmetric* matrix then show that $AA^T = A^T A$ and A^2 is symmetric. [Note A^T denotes the *transpose* of the matrix].

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SET: C

1. (a) (15 marks) If $c + id = (1 - i\sqrt{3})^{100}$, find c and d.

   (b) (10 marks) Let $a = 1 + bi$ be a complex number, $b > 0$, such that a^3 is a real number. Evaluate

   $$1 + a + a^2 + \cdots + a^{11}.$$

2. (a) (15 marks) Let M and N be 3×3 matrices with $\det(M) = 64$ and $\det(N) = 81$. Calculate

   $$\det\left(\frac{3}{8}MN^{-1}\right)$$

   Also show whether N^TMN is symmetric or skew-symmetric, according as M is symmetric or skew-symmetric.

   (b) (10 marks) Use matrix multiplication to divide Tk.2000 in two parts such that the total annual interest at 5% on the first part and 10% on the second part amounts Tk.170.

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SET: D

1. (a) (15 marks) Let $f(x)$ is the polynomial of degree 4 whose coefficients are real numbers; two of its zeros are -3 and $4-2i$. Explain why one of the remaining zeros must be a real number. Write down one of the missing zeros.

   (b) (10 marks) Your friend claims
   "*If the determinant of the coefficient matrix is zero, then Cramer's Rule does not apply and the corresponding system of linear equations has no solution.*" He gives you the following example:

$$\begin{aligned} x + 4y - 3z &= 1 \\ 3x - y + 3z &= 1 \\ 10x + y + 6z &= -2. \end{aligned}$$

   Do you agree with him? Explain with example in support of your argument.

2. (a) (15 marks) Find all unit vectors orthogonal to both

$$\mathbf{u} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}, \quad \mathbf{v} = -3\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$$

   (b) (10 marks) If a, b, and $c \neq 0$ are real numbers with $ac = bc$, then $a = b$. Does this same property hold for matrices? In other words, if A, B, and $C \neq 0$ (zero matrix), are matrices and $AC = BC$, must $A = B$? Explain why or why not (in your own words).