1 Find an expression for the derivative $\frac{dy}{dx}$ of the Parametric function:

$x = te^{2t} + \ln(t^2)$ and $y = t^2 e^t$ and evaluate it at $t = 1$. Express your answer in terms of e.

Answer:

Given that,

$x = te^{2t} + \ln(t^2)$

or, $\frac{dx}{dt} = e^{2t} + 2te^{2t} + \frac{2}{t}$

or, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

or, $\frac{dy}{dx} = \frac{2te^t + t^2 e^t}{e^{2t} + 2te^{2t} + 2/t}$

or, $\frac{dy}{dx} = \frac{2 \times 1 \times e^1 + (1)^2 e^1}{e^{2\times 1} + 2 \times 1 \times e^{2\times 1} + \frac{2}{1}}$

or, $$\frac{dy}{dx} = \frac{2e + e}{e^{2} + 2e^{2} + 2}$$

$$\therefore \frac{dy}{dx} = \frac{3e}{3e^{2} + 2} \quad \text{(ans)}$$

2. use logarithmic differentiation to find an expression for the derivative $\frac{dy}{dx}$ of

$$y = \frac{e^{-x}(\cos x)^{2}}{x^{2} + x + 1}$$

Answer:

$$y = \frac{e^{-x}(\cos x)^{2}}{x^{2} + x + 1}$$

take log both side,

$$\ln y = \ln\left(\frac{e^{-x}(\cos x)^{2}}{x^{2} + x + 1}\right)$$

or, $\ln y = \ln e^{-x} + \ln(\cos x)^2 - \ln(x^2+x+1)$

or, $\ln y = -x + 2\ln\cos x - \ln(x^2+x+1)$

<u>differentiate w.r.t x</u>

$$\frac{1}{y}\frac{dy}{dx} = -1 - \frac{2\sin x}{\cos x} - \frac{2x+1}{x^2+x+1}$$

or, $$\frac{dy}{dx} = y\left[-1 - 2\tan x - \frac{2x+1}{x^2+x+1}\right]$$

$\therefore$ $$\frac{dy}{dx} = \frac{e^{-x}(\cos x)^2}{x^2+x+1}\left[-1-2\tan x - \frac{2x+1}{x^2+x+1}\right]$$

(ans)

Using ~~Lab~~ Leibniz Product rule, find an expression for the 5th derivative of the function.

~~$y = (x^4)\sinh(x)$~~ $y = (x^2+3x)\cosh(x)$

Answer

$y = (x^2+3x)\cosh(x)$

using Product Rule

$$\frac{d}{dx}(u.v) = v\frac{du}{dx} + u\frac{dv}{dx}$$

$$y' = (2x+3)\cosh(x) + (x^2+3x)\sinh(x)$$

$$y'' = 2\cosh(x) + (2x+3)\sin(x) + (2x+3)\sinh(x) + (x^2+3x)\cosh(x)$$

$$y''' = 2\sinh(x) + 2(2\sinh(x) + (2x+3)\cosh(x) + (2x+3)\cosh(x) + (x^2+3x)\sinh(x)$$

$$y'''' = 2\cosh(x) + 4\cosh(x) + 4\{2\cosh(x) + (2x+3)\sinh(x)\} + (2x+3)\cosh(x) + (x^2+3x)\sinh(x)$$

$$y''''' = 14\cosh(x) + 4(2x+3)\sinh(x) + (2x+3)\cosh(x) + (x^2+3x)\sinh(x)$$

$$y'''''' = 14\sinh(x) + 4\{2\sinh(x) + (2x+3)\cosh(x)\} + 2\cosh(x) + (2x+3)\sinh(x) + (2x+3)\sinh(x) + (x^2+3x)\cosh(x)$$

$$y^5 = 22\sinh(x) + 4(2x+3)\cosh(x) + 2\cosh(x) + 2(2x+3)\sinh(x) + (x^2+3x)\cosh(x)$$

$$y^5(0) = 22\sinh(0) + 12\cosh(0) + 2\cosh(0) + 2\times 3\sinh(0) + (0+0)\cosh(0)$$

$$y^5(0) = 0 + 12 + 2 + 0+0$$

$$y^5(0) = 14 \text{ (Ans}$$

Find an expression for the first derivative of the function $y = \sqrt{e^{\arcsin(x+1)}}$

Answer:

$$y = \sqrt{e^{\sin^{-1}(x+1)}}$$

differentiate y ~~w.r.t~~ to x, using chain Rule.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{e^{\sin^{-1}(x+1)}}\right)$$

$$= \frac{1}{2}\,\frac{1}{\sqrt{e^{\sin^{-1}(x+1)}}}\,\frac{d}{dx}\left(e^{\sin^{-1}(x+1)}\right)$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{e^{\sin^{-1}(x+1)}}} \, e^{\sin^{-1}(x+1)} \frac{d}{dx}\left(\sin^{-1}(x+1)\right)$$

$$= \frac{e^{\sin^{-1}(x+1)}}{2\sqrt{e^{\sin^{-1}(x+1)}}} \cdot \frac{1}{\sqrt{1-(x+1)^2}} \cdot \frac{d}{dx}(x+1)$$

$$= \frac{e^{\sin^{-1}(x+1)}}{\sqrt{e^{\sin^{-1}(x+1)}}} \cdot \frac{1}{\sqrt{1-(x+1)^2}} \cdot \frac{d}{dx}(x+1)$$

$$= \frac{\sqrt{e^{\sin^{-1}(x+1)}}}{2\sqrt{1-x^2-2x-1}} \cdot (1+0)$$

$$= \frac{\sqrt{e^{\sin^{-1}(x+1)}}}{2\sqrt{1-(x+1)^2}}$$

using formula

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$$

First derivative of $y = \sqrt{e^{\sin^{-1}(x+1)}}$ is

$$y' = \frac{\sqrt{e^{\sin^{-1}(x+1)}}}{2\sqrt{1-(x+1)^2}}$$

(ans)