Department of Mathematics and Natural Sciences

MAT 110

PRACTICE SHEET

SUMMER 2021

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1. American Airlines requires that the total outside dimensions (length+width+height) of a checked bag not exceed 62 inches. Suppose you want to check a bag whose height is same as its width. What is the largest volume bag of this shape that you can check on an American Air Flight?
2. Find 2nd derivative($\dfrac{d^2y}{dx^2}$) of $\tan y = \dfrac{x-1}{x+1}$ in terms of x.
3. Find the equation of the tangent line to the graph of $y = \ln(x^2+4) - x\arctan(\dfrac{x}{2})$ at $x = 2$.
4. If $y = (\sin x)^{\cos x} + (\cos x)^{\sin x} - 5x$, find $\frac{dy}{dx}$.
5. Let $f(x) = x + 2\sin x$ over the interval $[0, 2\pi]$. Use the first and second derivatives of f to determine where f is increasing, decreasing, concave up, and concave down. Locate all inflection points,if they exist.

1) Constraints: $l + w + h \leq 62$ and $h = w$

objective: $V = lwh$ largest $\Rightarrow$ maximum

using Constraints together and equality instead of inequality:

$l+w+h = 62$ becomes $l + w + w = 62$

or $l + 2w = 62$

so $l = 62 - 2w$

Substitue $V = lwh = (62 - 2w)\, w\, w$

$V = 62w^2 - 2w^3$

maximize $\Rightarrow$ $V' = 124w - 6w^2$

$0 = 2w(62 - 3w)$

$2w = 0$ or $62 - 3w = 0$

$w = 0$ $\qquad\qquad \downarrow$

not a max $\qquad \frac{62}{3} = w \rightarrow$ must be max check to be sure

If $w = \frac{62}{3} = 20\frac{2}{3}$ inches

$h = w = 20\frac{2}{3}$ inches

and $l = 62 - 2\left(\frac{62}{3}\right) = 20\frac{2}{3}$ in

The maximum Volume will be cube shape

2) $y = \dfrac{x-1}{x+1}$

$$\frac{dy}{dx} = \frac{(x+1)\frac{d}{dx}(x-1) - (x-1)\frac{d}{dx}(x+1)}{(x+1)^2}$$

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$$\frac{dy}{dx} = \frac{(x+1)\cdot 1 - (x-1)\cdot 1}{(x+1)^2}$$

$$\frac{dy}{dx} = \frac{x+1 - x+1}{(x+1)^2}.$$

$$\frac{dy}{dx} = \frac{2}{(x+1)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(x+1)^2 \frac{d}{dx}(2) - 2\frac{d}{dx}(x+1)^2}{\left((x+1)^2\right)^2}$$

$$= \frac{(x+1)^2 \cdot 0 - 2\,2(x+1)}{(x+1)^4}$$

$$\frac{d^2y}{dx^2} = -\frac{4(x+1)}{(x+1)^4} = \frac{-4}{(x+1)^3}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{-4}{(x+1)^3}}$$

3) $y = \ln(x^2+4) - x \arctan\left(\frac{x}{2}\right)$

$$\frac{dy}{dx} = \frac{1}{(x^2+4)} \times (2x+0) - \left[x \frac{1}{1+\left(\frac{x}{2}\right)^2} \times \frac{1}{2} + 1\cdot \arctan\left(\frac{x}{2}\right)\right.$$

$$= \frac{\cancel{2x}}{x^2+4} - \frac{\cancel{2x}}{x^2+4} - \arctan\left(\frac{x}{2}\right)$$

$$\frac{dy}{dx} = -\arctan\left(\frac{x}{2}\right)$$

$$\left.\frac{dy}{dx}\right|_{x=2} = -\arctan\left(\frac{2}{2}\right) = -\arctan 1$$

$$\left.\frac{dy}{dx}\right|_{x=2} = -\frac{\pi}{4} \quad \text{Ans}$$

4) $y = (\sin x)^{\cos x} + (\cos x)^{\sin x} - 5x$

let $y_1 = (\sin x)^{\cos x}$ $\quad y_2 = (\cos x)^{\sin x}$ $\quad y_3 = -5x$

$y_1 = (\sin x)^{\cos x}$

$\log y_1 = (\cos x)\log(\sin x)$

differentiating both sides with respect to x

$$\frac{1}{y_1}\frac{dy_1}{dx} = -\sin x \log \sin x + \frac{\cos x}{\sin x}\cdot \cos x$$

$$\frac{dy_1}{dx} = \left(\frac{\cos^2 x}{\sin x} - \sin x \log \sin x\right) y_1$$

Substituting y_1

$$\frac{dy_1}{dx} = \left(\frac{\cos^2 x}{\sin x} - \sin x \log \sin x\right)(\sin x)^{\cos x}$$

$y_2 = (\cos x)^{\sin x}$

$\log y_2 = (\sin x)\log(\cos x)$

differentiating both side with respect to x

$$\frac{1}{y_2}\frac{dy_2}{dx} = (\cos x)\log(\cos x) + \frac{\sin x}{\cos x}\cdot(-\sin x)$$

$$\frac{dy_2}{dx} = \left((\cos x)\log(\cos x) - \frac{\sin^2 x}{\cos x}\right) y_2$$

Substituting y_2

$$\frac{dy_2}{dx} = \left((\cos x)\log(\cos x) - \frac{\sin^2 x}{\cos x}\right)(\cos x)^{\sin x}$$

$y_3 = -5x$

$$\frac{dy_3}{dx} = -5$$

Since, $y = y_1 + y_2 + y_3$

$$\frac{dy}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx} + \frac{dy_3}{dx}$$

Hence

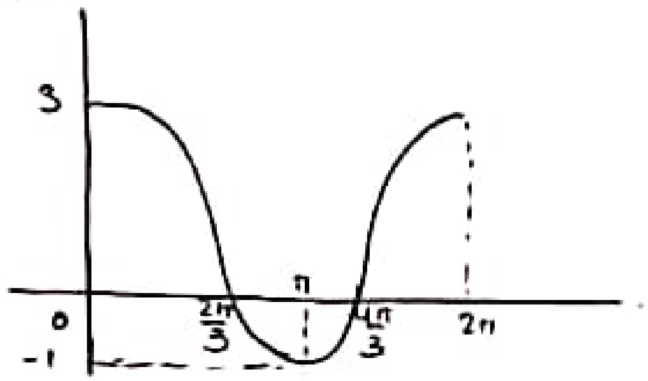
$$\boxed{\frac{dy}{dx} = (\cot x \cos x - \sin x \log \sin x)(\sin x)^{\cos x} + (\cos x \log \cos x - \ldots \sin x)(\cos x)^{\sin x} - 5}$$

5) $f(x) = x + 2\sin x$

$$f'(x) = \frac{d}{dx}(x + 2\sin x)$$
$$= 1 + 2\cos x$$

$$f''(x) = \frac{d}{dx}(f'(x))$$
$$= \frac{d}{dx}(1 + 2\cos x) = -2\sin x$$

graph of $f'(x)$ is Interval $[0, 2\pi]$



- If $f'(x) > 0$ then $f(x)$ is Increasing and if $f'(x) < 0$, $f(x)$ is decreasing.

$f'(x) > 0$ in $[0, \frac{2\pi}{3}) \cup (\frac{4\pi}{3}, 2\pi]$

Hence $f(x)$ is increasing in interval $[0, \frac{2\pi}{3}) \cup (\frac{4\pi}{3}, 2\pi)$ and decreasing in interval $(\frac{2\pi}{3}, \frac{4\pi}{3})$

- If $f''(x) > 0$, graph is concave up and if $f''(x) < 0$, graph is concave down.

$f''(x) < 0$ in $(0, \pi)$ and $f''(x) > 0$ in $(\pi, 2\pi]$

So, graph is concave down in interval $(0, \pi)$ and concave up in interval $(\pi, 2\pi)$

- At point of inflection, $f''(x) = 0$ and graph changes its concavity

$f''(x) = 0$ at $x = \pi$ and here graph changes its concavity as well so $x = \pi$ is point of inflection