BRAC University **SET: D**

MAT092: Final Exam

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Total marks: 40.

1. (10 marks) Let $A = \begin{pmatrix} -1 & 0 & -3 \\ -2 & 2 & -2 \\ 5 & -4 & 7 \end{pmatrix}$. Find all the values of λ such that $\det(A - \lambda I) = 0$, where $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is the 3×3 identity matrix and λ is some constant. (The values of λ are known as *eigenvalues* of A).

2. (10 marks) Using **Cramer's Rule**, find the solution to the following system of linear equations:

$$-x + 3y - 2z = 5$$
$$4x - y - 3z = -8$$
$$2x + 2y - 5z = 7$$

3. (10 marks)

   (a) Find the inverse matrix of $A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 2 & 1 & 1 \end{pmatrix}$.

   (b) Find a nonsingular 2×2 matrix A such that $A^3 = A^2B - 3A^2$, where $B = \begin{pmatrix} 4 & 1 \\ 2 & 6 \end{pmatrix}$

4. (10 marks)

   (a) For what values of x and y is $\begin{pmatrix} 1 \\ x \\ y \end{pmatrix}$ orthogonal to both $\begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} -4 \\ 1 \\ 2 \end{pmatrix}$?

   (b) Find the area of a triangle with the points $A(1, -1, 2)$, $B(2, 1, -1)$, $C(3, -1, 2)$ as it vertices by using the **cross product**.

Assignment 4

Answer 1

Given, $A = \begin{pmatrix} -1 & 0 & -3 \\ -2 & 2 & -2 \\ 5 & -4 & 7 \end{pmatrix}$ $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\lambda I = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}$

Now, $A - \lambda I = \begin{pmatrix} -1 & 0 & -3 \\ -2 & 2 & -2 \\ 5 & -4 & 7 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}$

$= \begin{pmatrix} -1-\lambda & 0 & -3 \\ -2 & 2-\lambda & -2 \\ 5 & -4 & 7-\lambda \end{pmatrix}$

$= (-1-\lambda)\{(7-\lambda)(2-\lambda) - 8\} - 0 - 3(8 - 10 + 5\lambda)$

$= (-1-\lambda)\{(14 - 7\lambda - 2\lambda + \lambda^2 - 8)\} - 3(-2 + 5\lambda)$

$= (-1-\lambda)(6 - 9\lambda + \lambda^2) + 6 - 15\lambda$

$= -6 + 9\lambda - \lambda^2 - 6\lambda + 9\lambda^2 - \lambda^3 + 6 - 15\lambda$

$= -12\lambda + 8\lambda^2 - \lambda^3$

Now $\Rightarrow -\lambda^3 + 8\lambda^2 - 12\lambda = 0$ [According to question]

$\Rightarrow \lambda^3 - 8\lambda^2 + 12\lambda = 0$

$\Rightarrow \lambda(\lambda^2 - 6\lambda - 2\lambda + 12) = 0$

$\Rightarrow \lambda ($

$$\Rightarrow \lambda\{\lambda(\lambda-6)-2(\lambda-6)\}=0$$

$$\Rightarrow \lambda\{(\lambda-6)(\lambda-2)\}=0$$

$$\Rightarrow \lambda(\lambda-6)(\lambda-2)=0$$

$$\therefore \lambda=0,\ \lambda=6,\ \lambda=2$$

$$\lambda=(0,6,2) \text{ (Ans)}$$

Answer-2

The given three equations,

$$-x+3y-2z=5 \quad \text{--- (i)}$$

$$4x-y-3z=-8 \quad \text{--- (ii)}$$

$$2x+2y-5z=7 \quad \text{--- (iii)}$$

Determinants formed by the three equations,

$$D=\begin{vmatrix} -1 & 3 & -2 \\ 4 & -1 & -3 \\ 2 & 2 & -5 \end{vmatrix}$$

$$=-1(5+6)-3(-20+6)-2(8+2)$$

$$=-11+42-20$$

$$=11$$

$\therefore$ The equations have solution.

$$D_x = \begin{vmatrix} 5 & 3 & -2 \\ -8 & -1 & -3 \\ 7 & 2 & -5 \end{vmatrix}$$

$$= 5(5+6) - 3(40+21) - 2(-16+7)$$

$$= 55 - 183 + 18$$

$$= -110$$

$$x = \frac{D_x}{D} = \frac{-110}{11} = -10$$

$$D_y = \begin{vmatrix} -1 & 5 & -2 \\ 4 & -8 & -3 \\ 2 & 7 & -5 \end{vmatrix}$$

$$= -1(40+21) - 5(-20+6) - 2(28+16)$$

$$= -61 + 70 - 88$$

$$= -79$$

$$\therefore y = \frac{D_y}{D} = \frac{-79}{11} = \frac{-79}{11}$$

$$D_z = \begin{vmatrix} -1 & 3 & 5 \\ 4 & -1 & -8 \\ 2 & 2 & 7 \end{vmatrix}$$

$$= -1(-7+16) - 3(28+16) + 5(8+2)$$

$$= -9 - 132 + 50$$

$$= -91$$

$$z = \frac{D_z}{D} = \frac{-91}{11} = \frac{-91}{11}$$

$$\therefore (x, y, z) = \left(-10, \frac{-79}{11}, \frac{-91}{11}\right) \text{ (Ans)}$$

Answer 3(a)

Given $A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 2 & 1 & 1 \end{pmatrix}$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 2 & 1 & 1 \end{vmatrix}$$

$$= 1(0-0) - 0 + 1(1-0)$$

$$= 0 \times 1$$

$$= 1$$

$\therefore$ A has inverse.

The co-factors are,

$$A_{11} = \begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix} = 0 - 0 = 0$$

$$A_{12} = -\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = -(1-0) = -1$$

$$A_{13} = \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = (1-0) = 1$$

$$A_{21} = -\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -(0-1) = 1$$

$$A_{22} = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1-2 = -1$$

$$A_{23} = -\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = -(1-0) = -1$$

$$A_{31} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$$

$$A_{32} = -\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -(0-1) = 1$$

$$A_{33} = \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} = 0 - 0 = 0$$

$$\therefore A^{-1} = \frac{1}{|A|} \begin{bmatrix} 0 & -1 & 1 \\ 1 & -1 & -1 \\ 0 & 1 & 0 \end{bmatrix}^T$$

$$= \frac{1}{1} \begin{bmatrix} 0 & 1 & 0 \\ -1 & -1 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ -1 & -1 & 1 \\ 1 & -1 & 0 \end{bmatrix} \text{ (Ans)}$$

Answer 3(b)

Given, $B = \begin{pmatrix} 4 & 1 \\ 2 & 6 \end{pmatrix}$

Now the equation, $A^3 = A^2 B - 3A^2$

$\Rightarrow A = B - 3I$ [Multiplying both side by A^{-2}]

$$= \begin{pmatrix} 4 & 1 \\ 2 & 6 \end{pmatrix} - 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 1 \\ 2 & 6 \end{pmatrix} - \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} \text{ (Ans)}$$

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Answer 4(a)

Let $A = \begin{pmatrix} 1 \\ x \\ y \end{pmatrix}$, $B = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}$, $C = \begin{pmatrix} -4 \\ 1 \\ 2 \end{pmatrix}$

Now, $A \cdot B = \begin{pmatrix} 1 \\ x \\ y \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = 0$ [$\because$ Orthogonal Projection]

$\Rightarrow 3 - x + 4y = 0$

$\Rightarrow -x + 4y = -3$

$\Rightarrow x - 4y = 3$

$\Rightarrow x = 3 + 4y$ — (I)

$A \cdot C = \begin{pmatrix} 1 \\ x \\ y \end{pmatrix} \begin{pmatrix} -4 \\ 1 \\ 2 \end{pmatrix}$

$\Rightarrow -4 + x + 2y = 0$ [$\because$ orthogonal projection]

$\Rightarrow x + 2y = 4$ — (II)

Putting value of x in (II)

$3 + 4y + 2y = 4$

$\Rightarrow 3 + 6y = 4$

$\Rightarrow 6y = 4 - 3$

$\Rightarrow y = \frac{1}{6}$ — (III)

Putting value of y in (I)

$x = 3 + \cancel{4}^{2} \times \frac{1}{\cancel{6}\,3}$

$= \frac{9+2}{3} = \frac{11}{3}$ $\therefore (x, y) = \left(\frac{11}{3}, \frac{1}{6}\right)$ Ans

Answer 4 (b)

$\because$ Given, $A = (1, -1, 2)$

$B = (2, 1, -1)$

$C = (3, -1, 2)$

$\vec{AB} = (1, 2, -3)$

$\vec{AC} = (2, 0, 0)$

Now, $\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 1 & 2 & -3 \\ 2 & 0 & 0 \end{vmatrix}$

$i(0-0) - j(0+6) + k(0-4)$

$= -6j - 4k$

Area $= \frac{1}{2} \| \vec{AB} \times \vec{AC} \|$

$= \frac{1}{2} \left(\sqrt{6^2 + 4^2} \right)$

$= \frac{1}{2} \times 2\sqrt{13}$

$= \sqrt{13}$ unit (Ans)