Answer-1

Given,

$$f(x) = \frac{2}{1-x}$$

$$f'(x) = \frac{d}{dx}\left\{2(1-x)^{-1}\right\}$$

$$= -2(1-x)^{-2}(-1)$$

$$= \frac{2}{(1-x)^2}$$

$$\therefore f'(0) = \frac{2}{(1-0)^2} = 2$$

$$f''(x) = \frac{d}{dx}\left\{2(1-x)^{-2}\right\}$$

$$= -4(1-x)^{-3}(-1)$$

$$= \frac{4}{(1-x)^3}$$

$$\therefore f(0) = \frac{2}{1-0} = 2$$

$\therefore f'''(0) = \frac{12}{(1-0)^4} = 12$

. . . .

and so on.

Therefore using Maclaurin series, we have,

$$f^{(n)}(0) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots$$

$$= 2 + 2x + \frac{4x^2}{2!} + \frac{12}{3!}x^3 + \ldots$$

$\therefore$ ~~$f^{(n)}(0) = 2 + 2x^2$~~

$$f^{(n)}(0) = 2 + 2x + 2x^2 + 2x^3 + \ldots$$

(Ans)

Answer-2

The equation of the parabolla $y = 1 - x^2$

Now differentiating we get,

$$\frac{d}{dx}(y) = \frac{d}{dx}(1 - x^2)$$

$$\Rightarrow \frac{dy}{dx} = -2x.$$

At P we have the tangent AB.

Now, $\angle B = 60^\circ$ [$\triangle ABC$ is equilateral triangle]

$\therefore$ slope of AB $= \tan 60^\circ$

$= \sqrt{3}$

$$\therefore -2x_1 = \sqrt{3}$$

$$\Rightarrow x_1 = -\frac{\sqrt{3}}{2}$$

$$\therefore y_1 = 1 - x_1^2 = 1 - \frac{3}{4} = \frac{1}{4}$$

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$\therefore$ P is $\left(-\frac{\sqrt{3}}{2}, \frac{1}{4}\right)$, Q is $\left(\frac{\sqrt{3}}{2}, \frac{1}{4}\right)$

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Answer-3

Given,

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & x \neq 0 \\ 0, & x = 0 \end{cases}$$

when, $x \neq 0$

$$f(x,y) = \frac{xy}{x^2+y^2}$$

$$\therefore f_x(x,y) = \frac{(x^2+y^2)y - xy(2x+0)}{(x^2+y^2)^2}$$

$$= \frac{x^2y + y^3 - 2x^2y}{(x^2+y^2)^2}$$

$$\therefore f_x(x,y) = \frac{y^3 - x^2y}{(x^2+y^2)^2} \quad \text{(Ans)}$$

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again,

$$f(x,y) = \frac{xy}{x^2+y^2}$$

$$\therefore f_y(x,y) = \frac{(x^2+y^2)x - xy(0+2y)}{(x^2+y^2)^2}$$

Ans

Answer 5

$$\vec{F} = \vec{F}(x, y, z, t)$$

$$\therefore \frac{dF}{dt} = \frac{\partial \vec{F}}{\partial x}\frac{dx}{dt} + \frac{\partial \vec{F}}{\partial y}\frac{dy}{dt} + \frac{\partial \vec{F}}{\partial z}\frac{dz}{dt} + \frac{\partial \vec{F}}{\partial t}\frac{dt}{dt}$$

$$\Rightarrow d\vec{F} = \frac{\partial \vec{F}}{\partial x}dx + \frac{\partial \vec{F}}{\partial y}dy + \frac{\partial \vec{F}}{\partial z}dz + \frac{\partial \vec{F}}{\partial t}dt$$

[Multiplying both sides by dt]

$$\Rightarrow d\vec{F} = dx\frac{\partial \vec{F}}{\partial x} + dy\frac{\partial \vec{F}}{\partial y} + dz\frac{\partial \vec{F}}{\partial z} + \frac{\partial \vec{F}}{\partial t}dt$$

$$\Rightarrow d\vec{F} = dx\hat{i}\left(\hat{i}\frac{\partial}{\partial x}\vec{F}\right) + dy\hat{j}\left(\hat{j}\frac{\partial}{\partial y}\vec{F}\right) + dz\hat{k}\left(\hat{k}\frac{\partial}{\partial z}\vec{F}\right) + \frac{\partial \vec{F}}{\partial t}dt$$

$$\Rightarrow d\vec{F} = dx\hat{i}\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial t}\right]\vec{F} + dy\hat{j}\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial t}\right]\vec{F} + \frac{\partial \vec{F}}{\partial t}dt$$

$$= dx\hat{i}\,\nabla\vec{F} + dy\hat{j}\,\nabla\vec{F} +$$

$$dt\hat{k}\,\vec{\nabla}F + \frac{\partial\vec{F}}{\partial t}\,dt$$

$$= dx\hat{i} + dy\hat{j} + dz\,\hat{k}\,.\,\nabla\vec{F} +$$

$$\frac{\partial\vec{F}}{\partial t}\,dt$$

$$= \partial\vec{r}\,.\,\nabla\vec{F} + \frac{\partial\vec{F}}{\partial t}\,dt$$

Answer - 6

Given,

$$h(\tau) = \frac{c^2}{\alpha}\left(\cosh\left(\frac{\alpha\tau}{c}\right) - 1\right)$$

$$T(\tau) = \frac{c}{\alpha}\sinh\left(\frac{\alpha\tau}{c}\right)$$

Now $cT(\tau) = \frac{c^2}{\alpha}\sinh\left(\frac{\alpha\tau}{c}\right)$

This gives,

$$\left(h + \frac{c^2}{\alpha}\right)^2 - c^2T^2 = \frac{c^4}{\alpha^2}$$

$\therefore$ it is a hyperbola. (Ans)