Department of Mathematics and Natural Sciences

MAT 110

ASSIGNMENT 3

SUMMER 2021

Please write your name and ID on the first page of the assignment answer script - you have to do this for both handwritten or LaTeX submission. The last date of submission is 8-8-2021, 1159 pm. Solve all problems.

You can only submit a PDF file - image or doc files won't be accepted. Before submitting the PDF, please rename the PDF file in the format -SET_ID_SECTION.

If you use LaTeX, you must add a screenshot of the raw code and compiled pdf side by side, in order to earn your bonus.

This set was prepared by MMRU. If you have any questions, please text MMRU on Slack.

1. In Einstein's theory of special relativity the kinetic energy of an object E moving with velocity v is

$$E = mc^2(\gamma - 1),$$

$$\text{where} \quad \gamma = \frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$

where c is the speed of light (a constant). **Show** using **Maclaurin expansion** of $\dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$, that $E \approx \dfrac{1}{2}mv^2$ (the known formula for kinetic energy with concerning everyday objects) **when** $v \ll c$.

2. The electric field E at the point P in the figure is

$$E = \frac{q}{D^2} - \frac{q}{(D+d)^2}$$

By expanding this expression for E as a series in powers of $\dfrac{d}{D}$, show that E is approximately proportional to $\dfrac{1}{D^3}$ when P is far away from the dipole.

3. **Approximate** the function $f(x) = x^{1/4}$ by a Taylor polynomial of degree 2 around $x = 10$. **Approximate** the value of $x^{1/4}$ when $x = 9.9$ and $x = 8$ **using** your series. Now **find** the values of both $9.9^{1/4}$ and $8^{1/4}$ **using** your **calculator**. **Compare** the values you obtained from your **series** and your **calculator**. **Comment** on the accuracy of your Taylor series if you want to approximate $x^{1/4}$ when x is close to 10 **and** when x is further from 10.

4. Use the chain rule to find out the total derivative dv/dt, when

$$v = xe^{y/z}, \quad \text{where} \quad x = t^2, y = \cos t, z = \tan t$$

5. Use the Chain Rule to find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$.

$$v = \ln(2x+3y), \quad \text{where} \quad x = s\sin t, \quad y = t\cos s$$

6. Let $p(t) = f(g(t), h(t))$, where f is differentiable, $g(2) = 1$, $g'(2) = -3$, $h(2) = 7$, $h'(2) = 8$, $f_x(1,7) = 2$, $f_y(1,7) = 8$. Find $p'(2)$.

7) Calculate the Maclaurin polynomials p_0, p_1, and p_2 for the function

$$f(x) = e^{\sin x}$$

8) Find expressions for f_{xx} and f_{yy} for the multivariable function

$$f(x, y) = \ln(x^2 y) + y^3 x^2$$

9) Given that $x(t) = t^2 + 2$, $y(t) = t$ and $f(x, y) = y^2 \sin(xy) + x^2 y$. Using the chain rule for partial derivatives find an expression for $\dfrac{df}{dt}$ and evaluate it when $t = 0$.

10) Find all the first order partial derivatives of the function

$$g(u, v, w) = \cos\left(\frac{u}{v^2 + u}\right) - \frac{6u^2 + v}{w^2 - v^2}$$

11) If $g(x, y, z) = z^3 x^3 \sin(y^2) + x^3 \cos(y^3)$, find an expression for g_{zyx} and evaluate it at the point $(1, \sqrt{2\pi}, 1)$. Write your answer in terms of π.