BRAC UNIVERSITY

Inspiring Excellence

# MAT110

Assignment 4

SET:24

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Section:07

# Assignment 4

## Answer - 1

Given, $f(x,y) = x^2y + y^2$

Now,

$f_x(x,y) = 2xy$

$f_y(x,y) = x^2 + 2y$

$f_{xx}(x,y) = 2y$

$f_{yy}(x,y) = 2$

$f_{xy}(x,y) = 2x$

Now at point $(1,3)$

$f(1,3) = 1^2 \times 3 + 3^2 = 12$

$f_x(1,3) = 2 \times 1 \times 3 = 6$

$f_y(1,3) = 1^2 + 2 \times 3 = 7$

$f_{xx}(1,3) = 2 \times 3 = 6$

$f_{yy}(1,3) = 2$

$f_{xy}(1,3) = 2\times 1 = 2$

Now 1st degree Taylor polynomial of $f(x,y)$ near at point $(1,3)$

$$L(x,y) = f(1,3) + f_x(1,3)(x-1) + f_y(1,3)(y-3)$$
$$= 12 + 6(x-1) + 7(y-3)$$
$$= 12 + 6x - 6 + 7y - 21$$
$$= 6x + 7y - 15$$

$\therefore L(x,y) = 6x + 7y - 15$

Again, 2nd degree Taylor polynomial of $f(x,y)$ near at point $(1,3)$

$$Q(x,y) = L(x,y) + \frac{f_{xx}(1,3)}{2}(x-1)^2 + f_{xy}(1,3)(x-1)(y-3) + \frac{f_{yy}(1,3)}{2}(y-3)^2$$

$$= 6x + 7y - 15 + \frac{6}{2}(x-1)^2 + 2(x-1)(y-3) + \frac{2}{2}(y-3)^2$$

$$= 6x + 7y - 15 + 3(x-1)^2 + 2(x-1)(y-3) + (y-3)^2$$

$$= 6x + 7y - 15 + 3(x^2 - 2x + 1) + 2(xy - 3x - y + 3) + y^2 - 6y + 9$$

$$= 6x + 7y - 15 + 3x^2 - 6x + 3 + 2xy - 6x - 2y + 6 + y^2 - 6y + 9$$

$$= 3x^2 + y^2 - 6x - y + 2xy + 3$$

$\therefore Q(x,y) = 3x^2 + y^2 - 6x - y + 2xy + 3$ (Ans)

## Answer - 2

Given,

$$f(x,y) = \ln(x^2+y^2+1)$$

Now,

$$f_x(x,y) = \frac{2x}{x^2+y^2+1} = \frac{2x}{x^2+y^2+1}$$

$$f_y(x,y) = \frac{1.2y}{x^2+y^2+1} = \frac{2y}{x^2+y^2+1}$$

$$f_{xx}(x,y) = \frac{(x^2+y^2+1).2 - 2x(2x+0+0)}{(x^2+y^2+1)^2}$$

$$= \frac{2x^2+2y^2+2-4x^2}{(x^2+y^2+1)^2}$$

$$= \frac{2y^2-2x^2+2}{(x^2+y^2+1)^2}$$

$$f_{xy}(x,y) = -\frac{2x}{(x^2+y^2+1)^2}(0+2y+0)$$

$$= -\frac{4xy}{(x^2+y^2+1)^2}$$

$$f_{yy}(x,y) = \frac{(x^2+y^2+1)^2 - 2y(0+2y+0)}{(x^2+y^2+1)^2}$$

$$= \frac{2x^2+2y^2+2-4y^2}{(x^2+y^2+1)^2}$$

$$= \frac{2x^2-2y^2+2}{(x^2+y^2+1)^2}$$

Now,

$f(0,0) = \ln(0+0+1) = \ln 1 = 0$

$f_x(0,0) = 0$

$f_y(0,0) = 0$

$f_{xy}(0,0) = 0$

$f_{xx}(0,0) = 2$

$f_{yy}(0,0) = 2$

$\therefore$ 1st degree Taylor polynomial of $f(x,y)$ near at point $(0,0)$.

$$L(x,y) = f(0,0) + f_x(0,0)(x-0) + f_y(0,0)(y-0)$$

$$= 0 + 0 + 0$$

$$= 0$$

$\therefore L(x,y) = 0$ (Ans)

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Again 2nd degree Taylor polynomial of $f(x,y)$ near at point $(0,0)$.

$$Q(x,y) = L(x,y) + \frac{f_{xx}(0,0)}{2}(x-0)^2 + f_{xy}(0,0)(x-0)(y-0) + \frac{f_{yy}(0,0)}{2}(y-0)^2$$

$$= 0 + \frac{2}{2}x^2 + 0 \times xy + \frac{2}{2}y^2$$

$$= x^2 + y^2$$

$\therefore Q(x,y) = x^2 + y^2$ (Ans)

Ans-3

Given,

$f(x,y) = x^2 + y^2 + \frac{2}{xy}$

$f_x(x,y) = 2x + \frac{2}{y}\left(-\frac{1}{x^2}\right) = 2x - \frac{2}{x^2 y}$

$f_y(x,y) = 2y - \frac{2}{xy^2}$

Now for critical point,

$f_x(x,y) = 0$ and

$\Rightarrow 2x - \frac{2}{x^2 y} = 0$ $\qquad$ $2y - \frac{2}{xy^2} = 0$

$\Rightarrow x - \frac{1}{x^2 y} = 0$ $\qquad$ $\Rightarrow y = \frac{1}{xy^2}$

$\Rightarrow x = \frac{1}{x^2 y}$ $\qquad$ $\Rightarrow y^3 = \frac{1}{x}$

$\Rightarrow x^3 = \frac{1}{y}$ $\qquad$ $\Rightarrow x = \frac{1}{y^3}$

$\therefore y = \frac{1}{x^3}$

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Thus the critical points are $(1, 1)$ ; $(-1, -1)$.

$$f_{xx}(x,y) = 2 - \frac{2}{y}(-2)\frac{1}{x^3} = 2 + \frac{4}{x^3 y}$$

$$f_{yy}(x,y) = 2 - \frac{2}{x}(-2)\frac{1}{y^3} = 2 + \frac{4}{x y^3}$$

$$f_{xy}(x,y) = 0 - \frac{2}{x^2}(-1)\frac{1}{y^2} = \frac{2}{x^2 y^2}$$

$$D(x,y) = f_{xx}(x,y)\, f_{yy}(x,y) - \left(f_{xy}(x,y)\right)^2$$

$$= \left(2 + \frac{4}{x^3 y}\right)\left(2 + \frac{4}{x y^3}\right) - \left(\frac{2}{x^2 y^2}\right)^2$$

At point $(1, 1)$

$$D(1, 1) = (2+4)\left(2 + \frac{4}{1}\right) - \left(\frac{2}{1\times 1}\right)^2$$

$$= 6 \times 6 - 4$$

$$= 36 - 4$$

$$= 32 > 0$$

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$\therefore f_{xx}(1,1) = 2+4 = 6 > 0$

Therefore $f(x,y)$ have a relative minimum at point $(1,1)$

Again at point $(-1,-1)$

$$D(-1,-1) = \left\{2+\frac{4}{(-1)^3(-1)}\right\}\left\{2+\frac{4}{(-1)\cdot(-1)^3}\right\} - \left\{\frac{2}{(-1)^2(-1)^2}\right\}^2$$

$$= (2+4)(2+4) - 4$$

$$= 6\times 6 - 4$$

$$= 32 > 0$$

$f(x,y)$ has a relative minimum at point $(1,1)$ (Ans)

# Answer-4

$$f(x,y) = xy + \frac{2}{x} + \frac{4}{y}$$

$$f_x(x,y) = y - \frac{2}{x^2}$$

$$f_y(x,y) = x - \frac{4}{y^2}$$

Let, $y - \frac{2}{x^2} = 0$

$\Rightarrow y = \frac{2}{x^2}$

~~and~~ $\Rightarrow x^2 y = 2$ —— (I)

Again, $x - \frac{4}{y^2} = 0$

$\Rightarrow x = \frac{4}{y^2}$

$\Rightarrow xy^2 = 4$ —— (II)

Now,

$1 \times y - 11 \times x$

$$x^2y^2 = 2y$$
$$x^2y^2 = 4x$$
$$(-) \qquad (-)$$

$$2y - 4x = 0$$

$$\Rightarrow y = 2x$$

Now from eq (1)

$$x^3 \cdot 2x = 2$$
$$\Rightarrow x^3 = 1$$
$$\therefore x = 1$$

$$y = \frac{2}{x^2} = \frac{2}{1^2}$$

$$\therefore y = 2$$

~~Stationary point (1, 2).~~

critical point (1, 2).

$$f_{xx}(x,y) = -2(-2)\frac{1}{x^3} = \frac{4}{x^3}$$

$$f_{yy}(x,y) = -4(-2)\frac{1}{y^3} = \frac{8}{y^3}$$

$$f_{xy}(x,y) = 1+0 = 1$$

$$D(x,y) = f_{xx}(x,y) \times f_{yy}(x,y) - \left(f_{xy}(x,y)\right)^2$$

$$= \frac{4}{x^3} \times \frac{8}{y^3} - 1^2$$

At point $(1,2)$

$$D(1,2) = \frac{4}{1^3} \times \frac{8}{2^3} - 1$$

$$= 4 - 1 = 3 > 0$$

and $f_{xx}(1,2) = \frac{4}{1^3} = 4 > 0$

Thus $f(x,y)$ has a Relative minimum at point $(1,2)$ Ans

# Answer 5

Given,

$$\vec{F} = yze^{xy}\hat{i} + xze^{xy}\hat{j} + \left(e^{xy} + 3\cos 3z\right)\hat{k}$$

Therefore the divergence of $\vec{F}$,

$$\text{div}\,\vec{F} = \vec{\nabla}\cdot\vec{F}$$

$$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left\{yze^{xy}\hat{i} + xze^{xy}\hat{j} + \left(e^{xy} + 3\cos 3z\right)\hat{k}\right\}$$

$$= \frac{\partial}{\partial x}\left(yze^{xy}\right) + \frac{\partial}{\partial y}\left(xze^{xy}\right) + \frac{\partial}{\partial z}\left(e^{xy} + 3\cos 3z\right)$$

$$= yze^{xy}y + xze^{xy}x + \left(0 - 3\sin 3z \cdot 3\right)$$

$$\therefore \text{div}\,\vec{F} = y^2ze^{xy} + x^2ze^{xy} - 9\sin 3z.$$

Ans

$$\text{curl}\,\vec{F} = \vec{\nabla} \times \vec{F}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yze^{xy} & xze^{xy} & e^{xy} + 3\cos 3z \end{vmatrix}$$

$$= \hat{i}\left(e^{xy}x + 0 - xe^{xy}\right) - \hat{j}\left(e^{xy}y + 0 - ye^{xy}\right)$$
$$+ \hat{k}\left(zx\,e^{xy}y + e^{xy}z - yz\,e^{xy}x - e^{xy}z\right)$$

$$= \hat{i} \times 0 - \hat{j} \times 0 + \hat{k} \times 0$$

$$= 0 \quad \text{Ans}$$

## Answer 6

Given

$\vec{F} = xyz\,\hat{i} + y\sin z\,\hat{j} + (y\cos x)\,\hat{k}$

$\text{div}\,\vec{F} = \vec{\nabla}\cdot\vec{F}$

$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\left(xyz\,\hat{i} + y\sin z\,\hat{j} + y\cos x\,\hat{k}\right)$

$= \frac{\partial}{\partial x}(xyz) + \frac{\partial}{\partial y}(y\sin z) + \frac{\partial}{\partial z}(y\cos x)$

$= yz + \sin z + 0$

$= yz + \sin z$

$\text{curl}\,\vec{F} = \vec{\nabla}\times\vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xyz & y\sin z & y\cos x \end{vmatrix}$$

$= \hat{i}(\cos x - y\cos z) - \hat{j}(-y\sin x - xy) + \hat{k}(0 - zx)$

$\therefore \text{curl}\,\vec{F} = (\cos x - y\cos z)\,\hat{i} + (xy + y\sin x)\,\hat{j} - zx\hat{k}$ (Ans)

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