FIM 500-001 Project Report – Fall 2021

Career Development for Quants

Group #1

Subject: Critical Stress Testing for Time Series Forecasting in Python

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1 Introduction

This report is being created to help the reader better comprehend the project scope, process, results, and observations as researched by the working group. We would like to extend a special thanks to our mentors and faculty Dr. Tao Pang and Mr. Jonathan Page for their support and guidance through the project.

The purpose of the project was to perform stress testing on US Equity and Debt Indices using certain macroeconomic variables which were shortlisted from a larger universe of variables available. The working group was able to leverage their learnings from subjects like Fixed Income, Credit Risk, and Statistics to research and apply the knowledge in a near real-world project scenario. The advice we received from our mentors helped us navigate through the project and attain better results. The project presented a great opportunity for every one of us to understand and apply what we're learning as part of the Financial Mathematics program.

We chose this project because of our shared interest in the financial markets, macroeconomics, and data analytics. We were able to employ and grow our skills like research, programming, data analysis, and project management through this opportunity. This allowed us to perform rigorous research on macroeconomic factors and how they impact our financial markets on a day over day basis. It required us to be able to bridge the knowledge we garnered in different verticals to achieve a meaningful output. Through the course of this project, we regularly tracked financial markets and indices to see how the news related to rates, employment reports, and other macroeconomic variables were impacting the markets. Using these observations, we narrowed down on our variable selection which we felt was most relevant and impactful. Further to this, we performed extensive research by analyzing data which was sourced from the FRED database which helped us solidify our selected macroeconomic variables. Additionally, we also referred to the latest Federal Reserve publications for Stress Testing to understand what the industry is observing in terms of macroeconomic trends.

2 Dependent Variable Selection

To set a base for stress testing, the first thing that was required was to find relevant, and impactful dependent variables. To do so, we performed some research on how well a variable represents the economy of the country as best as possible. With this, we shortlisted the following 2 variables:

- S&P 500 (SPY)
- 10-Year Treasury Yield (TNX)

We selected these variables for the following reasons.

2.1 S&P 500

S&P 500 is widely regarded as the single best gauge of large-cap U.S. equities. The index is composed using the market-cap-weighted method, which is used by a number of indices around the globe and considered to be a fair method to create an index.

S&P 500 consists of the 500 largest corporations in the United States, and the index provider follows a robust methodology to check if an organization qualifies for inclusion in the index or not. The methodology used is as follows:

- Universe: All constituents must be U.S. companies
- Eligibility Market Cap: Companies must have an unadjusted market cap of 13.1 billion dollars or greater
- Financial Viability: Companies must have positive as-reported earnings for the most recent quarter, as well as over the most recent 4 quarters (summed together)

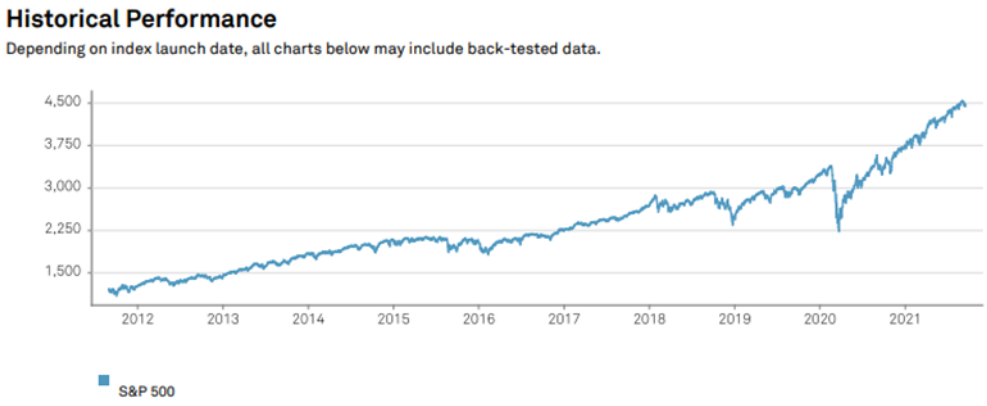

Figure 1: S&P 500 Historical Performance

The S&P 500 is composed of the 11 sectors which widely cover every aspect of the U.S economy, the sector participants in the index are as follows:

Sector	Market Cap
Communication Services	$6.93T
Consumer Discretionary	$9.06T
Consumer Staples	$4.58T
Energy	$2.77T
Financials	$8.70T
Health Care	$8.35T
Industrials	$5.85T
Information Technology	$16.04T
Materials	$2.67T
Real Estate	$1.73T
Utilities	$1.64T

Table 1: S&P 500 Sector Participants

We chose the S&P 500 as one of our dependent variables due to the wide coverage of the sectors in the U.S economy, index methodology, inclusion of large-cap organizations, and acceptance of index among industries making it a useful variable to be stressed against macroeconomic variables and get insights.

2.2 10-Year Treasury Yield

The 10 Year Treasury Note is frequently issued and is a highly liquid debt instrument issued by the U.S. Treasury to raise funds for the U.S. Government. The instrument has the explicit US government guarantee and hence has no credit risk, making it immune to any movements caused by the risk of default or credit rating degradation. Further, interest rates on Treasury securities are the benchmark interest rates throughout the U.S. economy as well as in international capital markets.

The TNX instrument is our choice for the reason that it is an intermediate-term maturity instrument capturing short and medium-term policy impacts. They are usually auctioned every three months and are based on 10 times the yield-to-maturity on the most recently auctioned 10-year Treasury note.

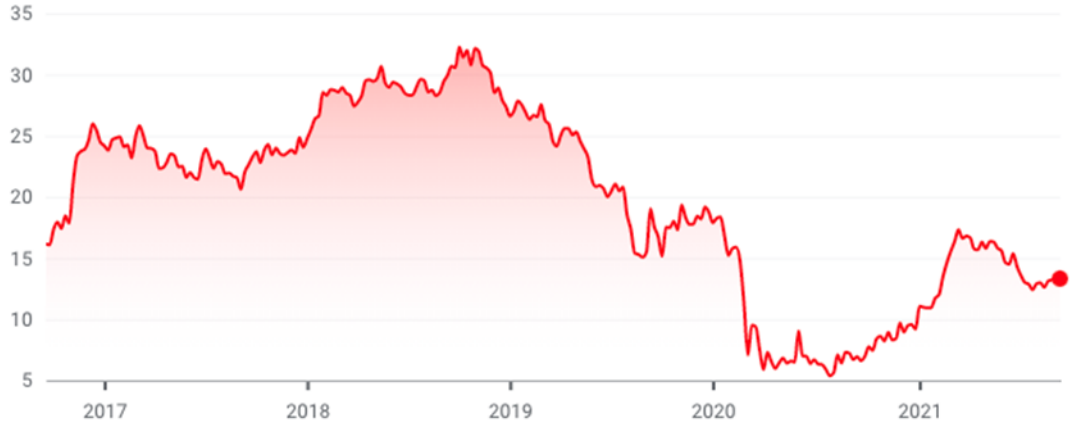

Figure 2: 10-Year Treasury Yield Historical Performance

3 Macroeconomic Variable Selection

3.1 Macroeconomic Indicators for Preliminary Testing

We selected a total of 25 independent variables from the FRED database of which 13 are monthly variables, 10 are daily/weekly variables and 2 are seasonal variables. We then utilized linear interpolation to convert the daily/weekly/seasonal variables into monthly.

- Monthly
  1. CPIAUCSL - Consumer Price Index for All Urban Consumers: It is a measure of the average monthly change in the price for goods and services paid by urban consumers between any two time periods.
  2. UNRATE - Unemployment Rate: It is the number of unemployed as a percentage of the labor force.
  3. EMVOVERALLEMV - Equity Market Volatility Tracker: Overall
  4. APU00007471A - Average Price for Gasoline for all unleaded gasoline and all types
  5. PAYEMS - All Employees, Total Nonfarm: is a measure of the number of U.S. workers in the economy that excludes proprietors, private household employees, unpaid volunteers, farm employees, and the unincorporated self-employed.
  6. MCOILWTICO: Crude Oil Prices: West Texas Intermediate (WTI) - Cushing, Oklahoma
  7. DSPIC96 - Real Disposable Personal Income
  8. PCE - Personal Consumption Expenditures
  9. GS3M - 3-Month Treasury Constant Maturity Rate
  10. GS5 - 5-Year Treasury Constant Maturity Rate
  11. CSUSHPINSA - S&P/Case-Shiller U.S. National Home Price Index: It is the leading measure for U.S. residential real estate prices
  12. PPIACO - Producer Price Index by Commodity: All Commodities: This variable measures the average change over time in the selling prices received by domestic producers for their output.
  13. FEDFUNDS - Effective Federal Funds Rate: It is the interest rate where depository institutions trade funds with one another overnight
- Weekly
  1. WM2NS - M2 for the United States
  2. WALCL - Total Assets (Less Eliminations from Consolidation): Wednesday Level
  3. MORTGAGE30US - 30-Year Fixed Rate Mortgage Average in the United States: Data is provided 'As-Is' by Freddie Mac.
- Daily
  1. BAMLC0A4CBBBEY - ICE BofA BBB US Corporate Index Effective Yield: It is a subset of the ICE BofA US Corporate Master Index which tracks the performance of US dollar-denominated investment grade rated corporate debt publicly issued in the US domestic market.
  2. T10YIE - 10-Year Breakeven Inflation Rate: It is a measure of the expected inflation rate derived from 10-Year Treasury Constant Maturity Securities and 10-Year Treasury Inflation-Indexed Constant Maturity Securities

3. T10Y2Y - 10-Year Treasury Constant Maturity Minus 2-Year Treasury Constant Maturity
4. BAMLH0A0HYM2 - ICE BofA US High Yield Index Option-Adjusted Spread: They are the calculated spreads between a computed OAS index of all bonds in a given rating category and a spot Treasury curve.
5. DTWEXBGS - Trade Weighted U.S. Dollar Index: Broad, Goods, and Services
6. VIXCLS - CBOE Volatility Index: It measures the market expectation of near term volatility as evidenced by stock index option prices
7. GOLDAMGBD228NLBM - Gold Fixing Price 10:30 A.M. (London time) in London Bullion Market, based in U.S. Dollars

- Seasonal

1. GDPC1 - Real Gross Domestic Product: It is the inflation-adjusted value of the goods and services produced by labor and property in the United States
2. GDP - Gross Domestic Product: It is the market value of the goods and services produced by labor and property in the United States

3.2 Macroeconomic variable shortlisting

Once we had the wider universe of macroeconomic variables available to us, the next step for us involved choosing 4 macroeconomic variables for our dependent variables based on our analysis.

The variables selected for our stress testing are as follows:

- S&P 500

1. M2 - WM2NS
2. Crude Oil Prices - MCOILWTICO
3. BBB Corporate Yield - BAMLC0A4CBBBEY
4. Effective Federal Fund Rate - FEDFUNDS

- 10-Year Treasury Yield

1. 5 Year Treasury Rate - GS5
2. Total Assets (Less Eliminations from Consolidation) - WALCL
3. Effective Federal Fund Rate - FEDFUNDS
4. Crude Oil Prices - MCOILWTICO

The variables were shortlisted after determining that they have no multicollinearity. For this we utilized the variance inflation factor function with a threshold of 10 as can be seen in Table 2 and Table 3.

M2 Money Supply	BBB Corporate Yield	Effective Federal Fund Rate	Crude Oil Prices
4.48	7.29	2.03	7.32

Table 2: VIF Examination for Macro Indicators for S&P 500

The reasoning for selecting the above-mentioned macroeconomic variables can be explained using both an economic viewpoint and a statistical viewpoint.

5 Year Treasury Rate	Effective Federal Fund Rate	Total Assets	Crude Oil Prices
9.98	5.35	2.75	5.83

Table 3: VIF Examination for Macro Indicators for 10-Year Treasury Yield

3.3 Economic Reasoning

The independent variables have been selected based on macro and microeconomic reasons. The 6 unique variables selected across the S&P 500 and Treasury Index are all highly influential to the markets and how they move.

M2 Money Supply

M2 is a measure of the money supply that includes cash, checking deposits, and is easily convertible near money. It is an indicator closely monitored by the central bank using which they make monetary policy decisions. Our 2 dependent variables are sensitive to the monetary policy decisions made by the central bank and hence we found it as a useful macroeconomic variable to use for critical stress testing.

Crude Oil Prices

It has been seen historically that US stock and treasury markets have been greatly affected by the increase/decrease in crude oil prices. The change is driven by demand or supply shocks in the oil industry. Further, choosing this variable for both the S&P 500 and Treasury Index made sense since the price of crude oil reflects global demand and therefore, is an important metric for critical stress testing.

BBB Corporate Yield

A bond is considered investment grade if its credit rating is BBB- or higher. It is composed of less risky bonds issued by corporations with a high probability of paying interest and returning principal to debt-holders.

Effective Federal Fund Rate

Federal funds rate is the interest rate at which depository institutions (banks and credit unions) lend reserve balances to other depository institutions overnight on an uncollateralized basis.

5 Year Treasury Rate

The 5 Year Treasury Rate is the yield received for investing in a US government-issued treasury security that has a maturity of 5 years. The 5 Year Treasury yield is used as a reference point in valuing other securities, such as corporate bonds.

Total Assets (Less Eliminations from Consolidation)

This metric checks the total assets (Less Eliminations from Consolidation) held with the Federal Reserve System of the United States. Feds can increase or decrease the number of Assets or Liabilities in their balance sheet which has a direct impact on the money supply within the economy. It is an important variable for critical stress testing for Treasury Index as Feds have the power to expand its asset base during an economic downturn.

3.4 Statistical Reasoning

All of the above macroeconomic indicators have a high correlation coefficient with either the S&P 500 or 10-year Treasury Yield.

Variable	Pearson Correlation
M2	0.91
BBB US Corporate Index Effective Yield	-0.73
Federal Funds Effective Rate	0.49
Crude Oil Prices	0.31

Table 4: The Pearson Correlation Coefficient with S&P 500

Variable	Pearson Correlation
Treasury Yield 5 Years	0.99
BBB US Corporate Index Effective Yield	0.90
Total Assets	-0.85
Crude Oil Prices	-0.67

Table 5: The Pearson Correlation Coefficient with 10-year Treasury Yield

4 Macroeconomic Indicators Forecast

4.1 Forecast Method Selection

In order to better predict changes in macroeconomic variables accurately, we perform a comparative prediction between the ARIMA method and Holt-Winter method on an existing sample.

- ARIMA model - Autoregressive Integrated Moving Average Model: fitted to time series data either to better understand the data or to predict future points in the series (forecasting). Given time series data X_t where t is an integer index and the X_t are real numbers, an $ARIMA(p, q)$ model is given by

$$X_t - \alpha_1 X_{t-1} - \cdots - \alpha_p X_{t-p} = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q}$$

- Holt-Winters model - Holt-Winters Exponential Smoothing: Holt-Winters is a model of time series behavior. Forecasting always requires a model, and Holt-Winters is a way to model three aspects of the time series: a typical value (average), a slope (trend) over time, and a cyclical repeating pattern (seasonality)

We take the first 90% of the existing data as the training set to train the model with parameters. The latter 10% of the sample is used for the comparison forecasting and the mean square error of the two models is calculated. The model with the smaller mean square error is taken as the final model for forecasting this macroeconomic variable. To better present the results, we define the standardized mean square error

$$\hat{MSE} = \frac{MSE}{\hat{\sigma}^2},$$

and the comparative results are shown in Table 6.

Variable	**Holt-Winters**	**ARIMA**
M2	0.53	0.60
BBB US Corporate Index Effective Yield	0.24	0.32
Federal Funds Effective Rate	0.10	0.41
Crude Oil Prices	0.46	0.40
5 Year Treasury Rate	0.46	0.53
Total Assets	2.27	2.18

Table 6: Standardized Mean Square Errors of the Two Methods

4.2 Out-of-Sample Forecast

Based on the result above, we use the following methods to forecast the future data of macroeconomic indicators.

- ARIMA
  - Crude Oil Prices
  - Total Assets

- Holt-Winters
  - M2
  - BBB US Corporate Index Effective Yield
  - Federal Funds Effective Rate
  - 5 Year Treasury Rate

The out-of-sample forecast results are shown in Figure 5-8.

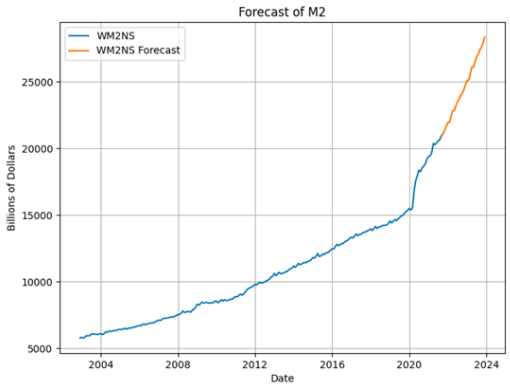

Figure 3: Forecast of M2

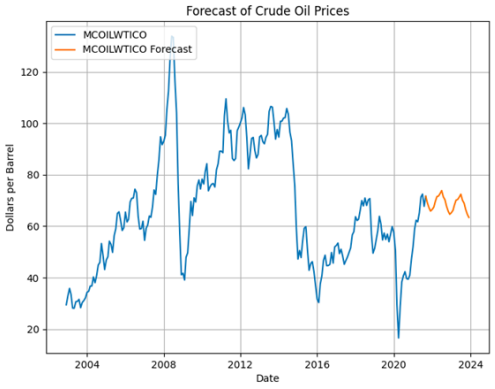

Figure 4: Forecast of Crude Oil Prices

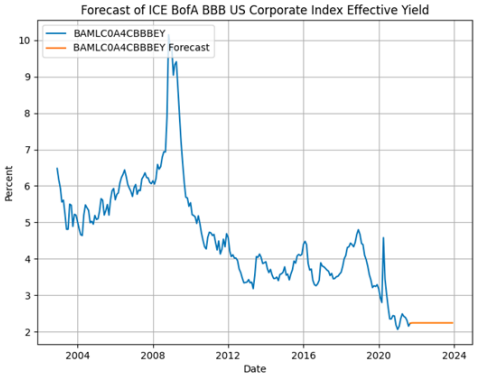

Figure 5: Forecast of BBB US Corporate Yield

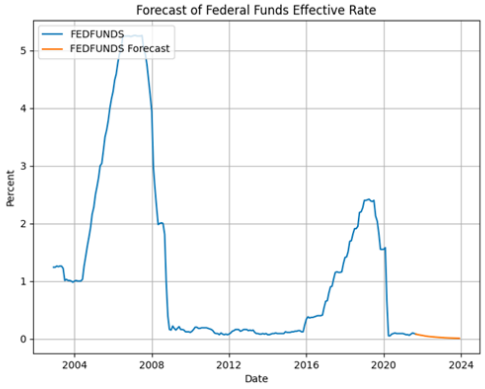

Figure 6: Forecast of Federal Funds Effective Rate

Figure 7: Forecast of 5-Years Treasury Yield

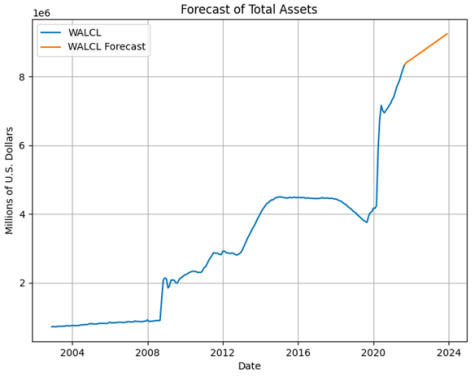

Figure 8: Forecast of Total Assets

5 Linear Regression Stress Testing

Once the macroeconomic variables were shortlisted from the wider universe, macroeconomic variable data was forecasted using time series methods. The next steps involved using the existing and forecasted data to run a linear regression model on our benchmarks i.e., S&P 500 and 10-Year Treasury Yield.

First, the existing data was used to train our regression model for it to be able to understand and memorize how the variables have reacted historically. Training the model with existing data is critical as that acts as a base for when the model is used on test data to forecast future trends and possibilities. The trained model was then used to forecast the benchmarks using the existing data to project S&P 500 and 10-Year Treasury Yield.

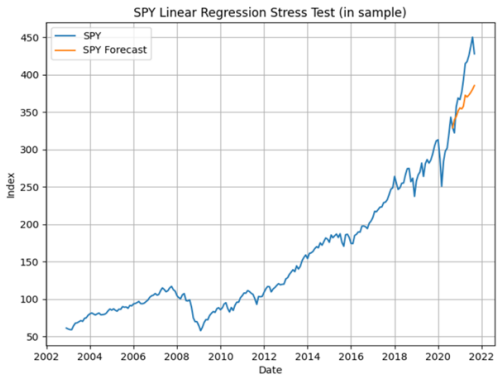

Figure 9: S&P 500 Linear Regression Stress Testing (in sample)

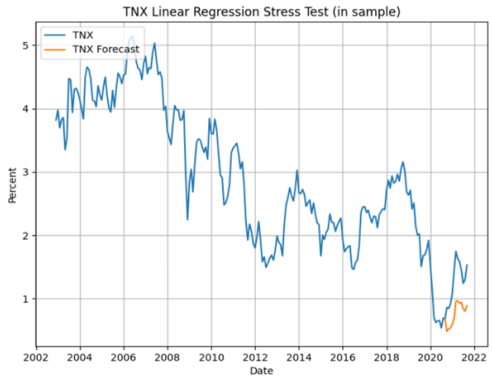

Figure 10: 10-Year Treasury Yield Linear Regression Stress Testing (in sample)

As can be seen from the images, although our predictions do not predict the changes of the benchmarks with complete accuracy, we still capture the trends and fluctuations of the benchmarks.

Afterward, the model was used for prediction using the existing data. The next step involved using the forecasted data from the macroeconomic variables to perform a linear regression for the time series forecast for the benchmarks. The results from the forecasts using out of sample data were as follows.

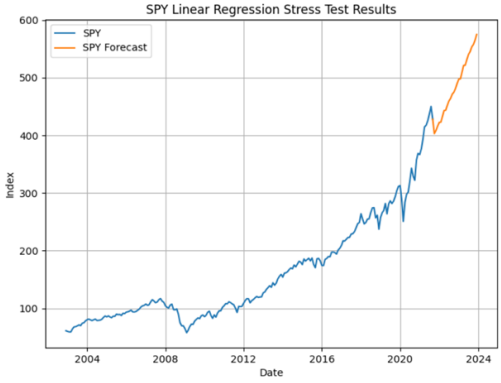

Figure 11: S&P 500 Linear Regression Stress Testing

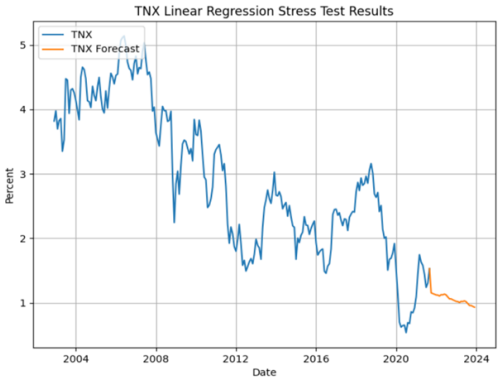

Figure 12: 10-Year Treasury Yield Linear Regression Stress Testing

6 LSTM Neural Network Stress Testing

We constructed the LSTM model and compared the results with the linear regression model for comparative analysis. As we can see from the following plots, the in-sample forecasts for S&P 500 and the 10-Year Treasury Yield have similar predictions with the linear regression model. However, for the 10-Year Treasury Yield index, the linear regression model had a more upward and downward trend compared to the LSTM forecast.

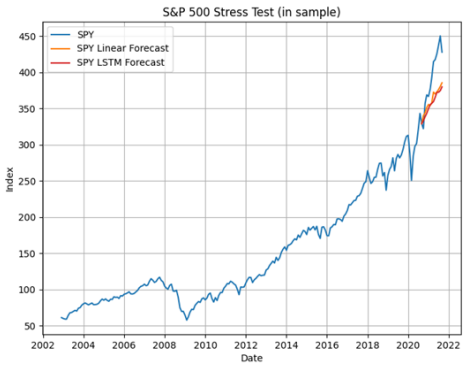

Figure 13: S&P 500 Stress Testing (in sample)

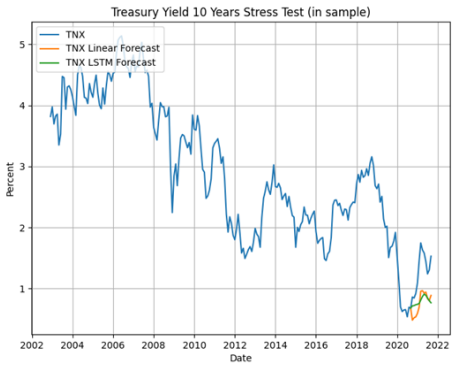

Figure 14: 10-Year Treasury Yield Stress Testing (in sample)

Once we finished training, both indices were predicted based on the forecast variables. We can conclude that the LSTM model had a more downward prediction than the linear regression model.

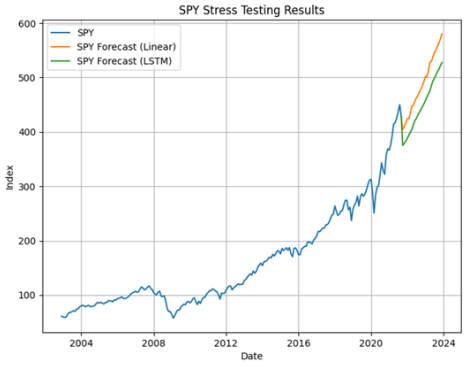

Figure 15: S&P 500 Stress Testing

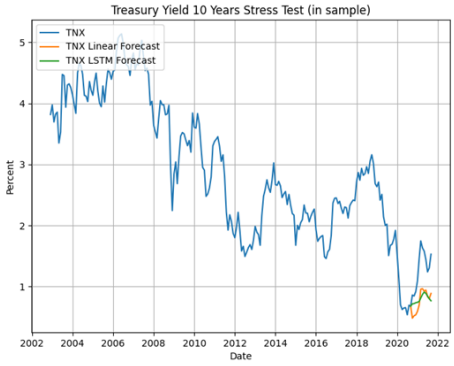

Figure 16: 10-Year Treasury Yield Stress Testing

7 Shocks in a Severely Adverse Scenario

For best comparative and practical purposes, based on the Dodd-Frank Act Stress Test publications, we introduce shocks in a severely adverse scenario to BBB US Corporate Index Effective Yield and 5-year Treasury Yield.

As we can see from the image, a change in the BBB US Corporate Index Effective Yield would cause a more pronounced decline in the S&P 500 in the short term. However, the two curves in the image converge over time, suggesting that the shock to the BBB US Corporate Index Effective Yield would not have a significant negative impact on the S&P 500's long-term growth.

Meanwhile, we note that after introducing a shock acting on the 5-Year Treasury Yield, we see an almost identical curve on the 10-Year Treasury Yield forecast from the linear regression model. Given the high correlation between these two variables, it is not surprising that such a result is obtained. At the same time, the LSTM model gives a much flatter curve.

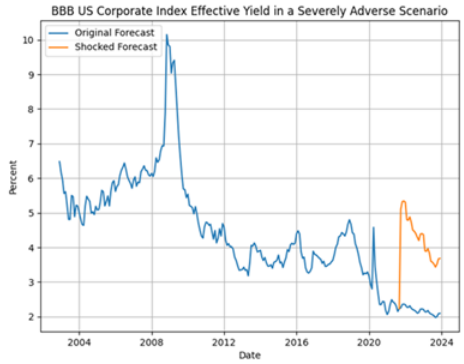

Figure 17: BBB US Corporate Yield in a Severely Adverse Scenario

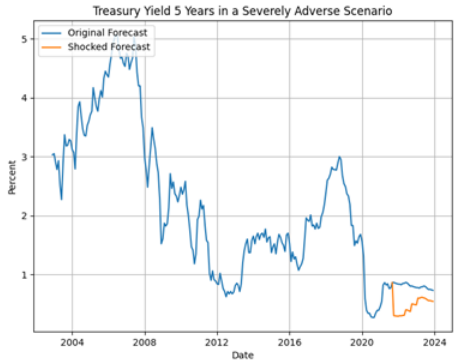

Figure 18: 5-Year Treasury Yield in a Severely Adverse Scenario

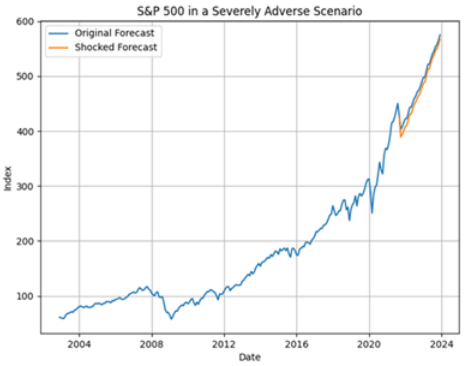

Figure 19: S&P 500 in a Severely Adverse Scenario from the Linear Regression Model

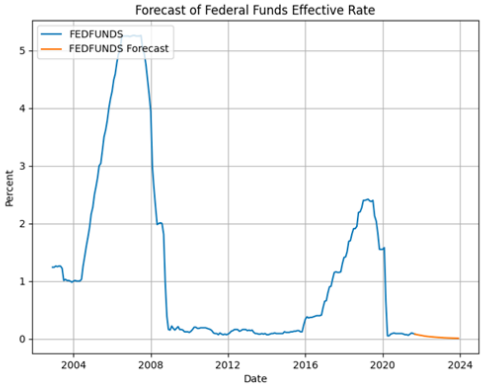

Figure 20: 10-Year Treasury Yield in a Severely Adverse Scenario from the Linear Regression Model

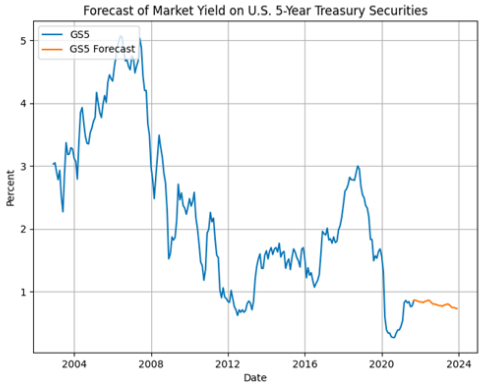

Figure 21: S&P 500 in a Severely Adverse Scenario from the LSTM Model

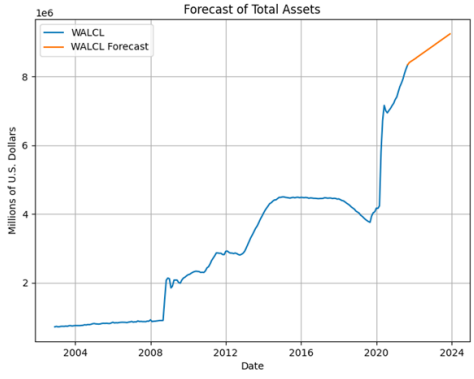

Figure 22: 10-Year Treasury Yield in a Severely Adverse Scenario from the LSTM Model

8 Summary

We learned that raw data is usually more interpretable than interpolated samples. As composite macroeconomic responses, the S&P 500 and the 10-year treasury yields are affected by many of the same economic variables, of which the impact also acts upon each other. The biggest challenges often come from the liquidity risk and interest rate risk which arose with macroeconomic shocks instead of a long-lasting downturn. Although both models had similar conclusions, the LSTM neural network was much more challenging and complex than a model based on the limited available data.

For further development of this project, we will apply the PCA methodology. We would solve the multicollinearity between the macro indicators, maintain the maximum information, and compare the results as an experimental group. Additionally, we would explore the application of other machine learning models and tune the parameters further.