Assignment 1
Random Number Generation
MA/FIM 548 Monte Carlo Methods for FM
Spring 2021

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Deadline: February 2, 2022

Instructions:

- Please submit one work per group.
- It counts for 10% of your final grade.
- Submit your work as single pdf file on Moodle before 11:45 am on February 2.
- Late assignments will not be accepted.
- NO interactions between groups.
- Each group member must make a substantial contribution to each part of the assignment.
- It is not acceptable, e.g., to divide the assignments amongst the team members.
- You can use any software (Matlab, Python, C/C++, R etc.)

1. Assume that you can sample from uniform distribution on (0,1). Explain how to use the inverse transform method to generate a random variate with the standard right-triangular distribution, i.e., the distribution with pdf

$$f(x) = 2(1-x), \quad 0 \le x \le 1.$$

   Provide the screenshot of your code.

2. Assume that you can sample from uniform distribution on (0,1). The double exponential distribution has density

$$f(x) = \frac{1}{2}\mathbf{1}_{(-\infty,0)}(x)e^{x} + \frac{1}{2}\mathbf{1}_{[0,\infty)}(x)e^{-x}$$

   Show how to simulate a random variable with density f using the inverse transform method. Provide the screenshot of your code.

3. Suppose X has a distribution with pdf

$$f(x) = \frac{1}{6}x^3 e^{-x}, \qquad x \ge 0.$$

   Apply the acceptance-rejection (AR) method to simulate X using exponential distribution $Exp(\lambda)$ with pdf

$$g(x) = \lambda e^{-\lambda x}, \qquad x \ge 0.$$

   - For fixed $\lambda > 0$, find the constant $c(\lambda)$ for AR method.
   - Find the optimal value of λ^* that makes AR method most efficient.

4. Assume that you can sample from uniform distribution on (0,1). Compare and report the computational time using following four methods by generating $N = 100,000,000$ independent samples from a standard normal distribution: a) Box-Muller; b) Marsaglia's polar method; c) rational approximation (for inverse of cdf); d) acceptance-rejection. Do not use vectorization, simply run a loop from 1 to N for each method. Provide the screenshot of your code.

5. A deck of 100 cards, numbered $1, 2, \dots, 100$, are shuffled and then turned over one at a time. Say that a 'hit' occurs whenever card i is the i-th to be turned over, $i = 1, 2, \dots, 100$. Write a simulation program to estimate the expectation and variance of the total number of hits. Run the program. You can use the build-in function in your software that simulates discrete uniform distribution on $\{1, \dots, 100\}$.